

# Detecting Bidders' Groups in Collusive Auctions: Evidence from Average Bid Auctions

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## Abstract

In this paper we empirically analyze the behavior of firms participating at average bid (AB) public procurement auctions. AB auctions are characterized by the fact that the winner is the bidder submitting the bid closest to (some function of) the average bid. We propose two statistical tests to detect whether firms' bidding and entry decisions are indicative of coordinated behavior. We validate these tests on a subset of auctions where the presence of eight cartels active between 1998 and 2003 has been sanctioned by the judiciary. We then apply the tests to a large set of auctions for road construction works held in the North of Italy between 2005 and 2010 finding strong evidence that multiple groups of not independent bidders are present. We use these results to analyze the effects of the groups' activities on the auctions' revenues and to explain the disappearance of several hundreds of firms after the first price rule gradually replaced the AB rule after 2006.

JEL: DL22, L74, D44, D82, H57.

**PRELIMINARY AND INCOMPLETE - PLEASE DO NOT CIRCULATE**

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*“....At the first meeting they said: "Why should we kill ourselves and make laugh those coming from outside?" Here (i.e., in Turin) firms from the South were coming and getting the jobs, getting the averages, they used to come with 20, 30 or 40 bids, they used to get the jobs and then what was left for us?...”* (Confession of Bruno Bresciani, found guilty of having rigged 94 AB auctions and other related crimes; sentenced to 7 years of jail in April 2008)

## 1 Introduction

In Italy, since 1999 most public works are procured through auctions in which the winner is the bidder submitting the bid closest to (a function of) the average of the bids. In particular, sealed bids are submitted in the form of a discount over an announced reserve price and the discount that is the closest from below to a trimmed average of the bids wins. The winner is paid the price he offered to execute the work. In 2008, contracts worth in total about €6 billion were procured using this format. The origin of this type of auction is uncertain, but the civil engineering literature rediscovered it in the ‘90s and suggested that forms of the Average Bid (AB) auction could outperform the First Price (FP) auction for the procurement of contracts. Indeed, recent studies have shown that an auctioneer might prefer using an AB over an FP auction when there is the risk of a costly default.<sup>1</sup> Nevertheless, these studies show that the AB auction achieves this result at the cost of breaking the link between bids and costs, essentially transforming the auction into a lottery. Furthermore, since the winning price in this auction/lottery depends on the average of the bids, this rule gives strong incentives to bidders to pilot this average by coordinating their bids.<sup>2</sup> This paper seeks to study the behavior of firms bidding in the Italian AB auctions. In particular, it introduces two statistical tests aimed at identifying the presence of groups coordinating their entry and bid decisions to influence the awarding the contract. The application of the tests to a large dataset of auctions for road construction held between 2005 and 2010 reveals the presence of multiple groups and provides a quantification of their effect on the auctions’ revenues and on bidders’ participation.

Although various forms of AB auctions are used in many countries, little is known about how they work.<sup>3</sup> An interesting feature of most AB rules is that they ensure that the highest discount loses whenever there are no ties of the highest discount. Generally, this implies that all these formats share one equilibrium in which all firms submit the lowest possible discount. In this equilibrium the auctioneer pays the highest price for the execution of the contract, exactly

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<sup>1</sup>The original engineering studies, Ioannou and Leu (1993) and Liu and Lai (2000), consider non-strategic bidders. Decarolis (2010) obtains this result in a strategic model with firms that have privately observed costs and asymmetric default types. Burguet, Ganuza and Hauk (2009) and Chillemi and Mezzetti (2009) characterize the optimal procurement mechanism under default risk and show that it has features similar to an AB auction.

<sup>2</sup>This feature of AB auctions has been pointed out first by Albano, Bianchi and Spagnolo (2006).

<sup>3</sup>Various forms of AB auctions have been used to procure public contracts in Chile, China, Italy, Japan, Peru, Taiwan and, in the USA, in Florida and New York, see Decarolis (2009) for a more detailed description.

like when all bidders form one single cartel. Because of this feature, we refer to AB auctions as "collusive auctions". Nevertheless, an equilibrium in which all firms offer the lowest bid is not robust: a subgroup of firms large enough to pilot the awarding threshold can gain by rising their discounts and winning the contract at a slightly higher discount. Although this subgroup of firms rigs the auction, it is not properly a cartel because its behavior benefits the auctioneer.<sup>4</sup> Indeed, the presence of one or more subgroups of bidders is essentially the only form of competition allowed by AB auctions. In this paper we focus on the Italian AB auctions and we try to develop an empirical methodology that could detect the differences in behavior between independent and coordinated firms that attend these auctions.

Our analysis of firms' behavior starts by formulating a model of firms' entry and bidding decisions. We argue that, if firms act independently, the unique equilibrium that should result is not compatible with what we observe in the data. Therefore, we extend the model allowing for the presence of coordinated actions by firms in the same groups. The model indicates that, compared to independent firms, firms that belong to the same group will be more likely to enter together and to bid on the same side of the bid distribution. In fact, clustering bids on the same side of the distribution serves both to shift the average in an unpredictable way and to have the group's bids in the area toward which the average is shifted. Moreover, only a firm within a group would find optimal to bid at the extremes of the support of the bids distribution since such bid will always lose. Finally, joint entry of the group members is essential to ensure that the group has enough bids to influence the average.

Although these behavioral features are not equivalent to a full equilibrium characterization, they represent the essential dimensions along which the salient actions of independent and coordinated firms should differ. Therefore, these features motivate the two statistical tests, one for bids and one for entry, that we develop. These tests seek to identify a group by comparing its behavior to that of comparable sets of randomly grouped firms. In particular, relative to a random group, the test on bids evaluates how much the bids of a suspect group affect the average. The test on participation, instead, compares how likely are firms in a suspect group to enter together. Although the ideas behind the tests are intuitive and have some theoretical foundations, the environment in which the firms operate is quite complex and the capacity of these tests to identify bidders' groups could be a concern.

The way we address this issue consists in evaluating the tests on a "validation" dataset composed by auctions participated by firms with known affiliations to groups. In particular, we use 276 AB auctions for roadworks held by the city of Turin between 1999 and 2002. In 2008, the Turin's Court of Justice ruled that these auctions had been rigged by 8 groups constituted by approximately 95 firms. Each group strategically submitted bids to affect the awarding of the contract. According to the Italian law this activity is a crime, and hence these groups were labelled cartels<sup>5</sup> and their members were fined and, some of them, sentenced to jail. For our

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<sup>4</sup>The purpose of a cartel should be to transfer revenues from the auctioneer to cartel's members.

<sup>5</sup>We refer to these 8 groups as cartels. We use the word "groups" for the sets of not condemned firms.

purposes, this is an ideal sample to validate our tests because we can check whether the tests are able to identify the 8 cartels sanctioned by the Court. The results that we obtain are very much supportive about the capacity of our tests to correctly identify groups. Of the 8 groups, the only one for which we do not find systematic evidence of coordination is the one that the Court sanctioned less because its members rarely coordinated bids.

We then turn to the problem of identifying groups when group membership is unknown. In principle, any subset of firms could be taken and analyzed through the test to check whether its members coordinated entry and bid. Nevertheless, given the large number of firms in the market, this approach is not practical. Therefore, we propose two solutions that differ in the amount of information required. The first method entails constructing candidate groups through a clustering algorithm that links firms on the basis of some observable characteristics. We illustrate this approach using information on subcontracts, legal joint bids and ownership. The second method, instead, requires observing only bids and bidders identities. In this case the candidate groups are chosen on the basis of the joint participation with those firms that most frequently win auctions. The groups formed with the first method can be tested with both tests while the ones formed with the latter can be analyzed only with the bid test.

We apply our methods to a dataset of nearly 900 AB auctions held in the North of Italy between 2005 and 2010. In these data, bids are discounts over the reserve price and the reserve price is set homogeneously across public administrations (PA). In each auction, a large number of bidders (57 on average, with peaks of more than 300) bid to win a contract to perform a simple roadwork (like paving a road). Although we have no prior knowledge of groups in these auctions, many of their observed features resemble those of the validation sample. Indeed, the results of the tests strongly suggest that numerous groups are active in these auctions. For instance, when we apply the tests to the auctions in the Piedmont region, we can classify between 20 and 30% of the firms in the market as members of groups. Accordingly, we can argue that between 20 and 50% of the auctions are participated by groups.<sup>6</sup> We then turn to the problem of evaluating the effects of the activity of these groups. As regards the revenues of the auctioneer, we argue that all groups benefit the auctioneer.<sup>7</sup> However, they harm the independent firms both because they reduce their probability of winning and because they prevent the winning discount to be equal to zero. We present some basic estimates of the damage for firms outside the group and discuss how selection could bias these estimates.

The final set of results that we present concerns the large exit of firms that followed the introduction of the FP auction. In 2006, a reform of the procurement regulation required by the European Union gradually induced the substitution of AB with FP auctions. One of the most visible effects of the switch from AB to FP auctions was the drop in participation. Indeed, for the local administrations that changed the format, the average number of firms per auction decreased from 57 to 7, the 99th percentile decreased from 300 to 20. Our results on bidders'

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<sup>6</sup>The variation in the number of auctions classified as colluded depends on several choices (see section 6).

<sup>7</sup>The result hinges on the counterfactual used being the zero-bid equilibrium (see section 3).

groups help to address what part of this market shakeout is due to the exit of inefficient firms and what part is due to the disappearance of shill bidders. A shill is a replica of another firm which is created only to have one more bid to affect the auction outcome. Since our tests allow to identify which firms belong to groups, we can argue that exiting firms that are not part of a group are most likely inefficient firms. Instead, for exiting firms that belong to a group it is not possible to distinguish between which are shills and which are inefficient members of the group. Applying this idea to the auctions in the North of Italy, we conclude that about 700 firms that abandon the market do so because they are inefficient.

Therefore, the contribution of this paper is twofold. On the one hand, our tests can be used to identify bidders' groups in AB auctions. Therefore, the tests could be useful to a Court investigating a case or to an inspector choosing which firms to monitor. However, on the other hand our analysis suggests that the activity of these groups is not harmful for the auctioneer's revenues. This result would be useful for a Court evaluating the damages imposed by a group. More generally, we believe that the novel results presented by this paper might help the policy discussion about the opportunity of adopting or continuing using AB auctions which are not limited to the procurement of works but, for instance, characterize also part of the procurement process of Medicare.<sup>8</sup>

**Literature:** This paper is related to the literature on collusion in auctions. In general, collusion is regarded as a first order concern in the design of auctions (Klemperer, 2004) and, in particular, its quantitative relevance for the public procurement auctions of roadworks has been recognized in several studies (Porter and Zona, 1993 and Ishii, 2007). The empirical literature on collusion in auctions can be roughly divided into two groups: those that study collusion practices in markets where its existence has been proved (Asker, 2009, Pesendorfer, 2000, Porter and Zona, 1993 and 1999) and those that try to devise methods to distinguish competition from collusion in environments where the presence of collusion is only a possibility (Bajari and Ye, 2003). Both approaches have led to the flourishing of a literature within industrial organization concerned with "screens for collusion" (i.e., statistical tests to detect collusion, see the review by Abrantes-Metz and Bajari, 2010). In this paper, we take an intermediate approach according to which we use information from auctions where collusion was proved, in order to devise an empirical methodology that allows assessing the likelihood of groups in markets where their presence has not been proved yet. The motivation of our approach is based on the idea of Hendricks and Porter (1989) who explain that collusion is tailored to the specific rules of the auction and the environment. Therefore, we use data from auctions with collusion to learn about the behavior of groups and then search for evidence of this behavior in other similar auctions. Finally, our analysis of the effects of groups' behavior also contributes to the literature on the AB auctions (Albano, Bianchi and Spagnolo, 2006, Engel, Guanza, Hauk and Wambach, 2006, Decarolis, 2010).

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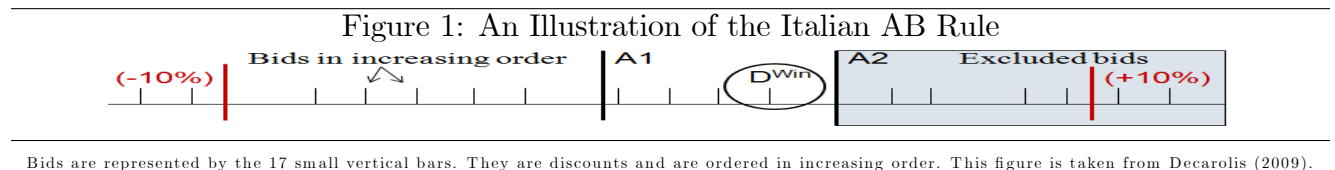
<sup>8</sup>Katzman and McGeary (2008) document the use by Medicare of a multiunit median price auction for the procurement of durable medical equipment. The main difference between this rule and the one that we study consists in its multiunit nature and not in the use of the median instead of the average.

The outline of the (preliminary version of the) paper is as follows: the next section provides a description of the market, section 2 presents a model of firms' entry and bidding, section 3 introduces the tests and provides their validation on the sample of auctions with known groups, section 4 extends the testing to the case of no prior knowledge about groups, section 5 illustrates the results obtained by applying the tests to the auctions in the north of Italy and, finally, section 6 concludes.

## 2 Description of the Market

In this section, we describe the market studied, focusing on the description of the AB awarding rule and of a "rule of thumb" bidding strategy. Since 1999, firms active in the market for public works in Italy have to compete in a type of auction called Average Bid (AB) auction.<sup>9</sup> Until 2006 the use of this auction was compulsory for the procurement of almost every work worth less than (approximately) €5 million. In that period, about 80 percent of all contracts for work were awarded through AB auctions, for a total yearly value of about €10 billion. In 2006 and 2008, two reforms required by the European Union fostered the introduction of First Price (FP) auctions. Nevertheless, in 2008 the AB auctions were still the mechanism used to procure the vast majority of contracts, for a total yearly value of about €6 billion.

Every public administration (PA), periodically publishes a description of a job that it needs (like paving a road) and announces the maximum price (reserve price) that it is willing to pay. Then, every firm qualified to bid for public contracts can submit its sealed bid consisting in a discount over the reserve price. Under the AB auction the winner is calculated as follows: a trim mean (A1) is calculated excluding the 10 percent of the highest and lowest bids; a new threshold (A2) is calculated as the sum of A1 and the average difference between discounts greater than A1 (but lower than the excluded top 10 percent of bids) and A1; the winning discount (Dwin) is the highest discount strictly below A2. Figure 1 offers an example with 17 bids. Ties are broken with a fair lottery. The winner is paid his bid to perform the work.<sup>10</sup>



<sup>9</sup>The historic evolution of the regulation of the public procurement of works is rather complex. Before 1994 several different procurement methods were available. In 1994 a national reform (Legge Merloni) made the FP auction (almost) the only mechanism. Problems related to firms offering high but not credible discounts induced modifications of the FP rule. Various forms of automatic elimination of high discounts were introduced until the format stabilized around 1998. The rule studied in this paper became effective on 1/1/1999.

<sup>10</sup>The rule described applies only if 5 (or 10 after 2006) or more bidders participated. Otherwise the winner is the highest responsive bidder. In practice, AB auctions with less than 5 bidders are rare.

This rule is remarkably different from a standard FP auctions and previous studies have documented that the AB rule has relevant effects on bidders behavior. Decarolis (2010) looks at the effects the policy changes that supplanted the AB with the FP auction and documents sharp declines in entry and increases in the winning discounts. His analysis is based on a sample of simple roadwork jobs procured by various counties and municipalities in five regions of the North of Italy between November 2005 and May 2010. In this paper, we analyze the same dataset but we focus only on the AB auctions. However, for the sake of comparing how different entry and bidding are under the two auction formats, the left panel of Table 1 reports summary statistics for both the sample of AB and FP auctions. Although the number of bidders is several times larger in AB than in FP auctions, the winning discount is much higher in the latter format. Moreover, the within auction dispersion of all bids is very low in AB auctions and the difference between the winning bid and the next lower discount is almost zero.

Table 1: Summary Statistics

Statistics by Auction							Statistics by Firm						
	Mean	SD	Med	Min	Max	Obs		Mean	SD	Med	Min	Max	Obs
AB Auct.							Entry	13.1	22.1	4	1	205	4005
HighBid	17.8	5.4	17.8	1.6	38.6	871	Wins	.31	.87	0	0	18	4005
WinBid	13.7	5.1	13.9	.51	36.8	871	Pr.Win	.03	.12	0	0	1	4005
Win-2Bid	.24	.66	.07	0	9.4	871	Reven	170	1081	0	0	4e <sup>04</sup>	4005
With.SD	2.9	1.4	2.7	.14	9.2	871	Age	22.3	13.8	21	1	106	3611
No.Bids	57.2	46.7	46	5	326	871	Capital	447	2411	52	10	8e <sup>04</sup>	2484
							Subct	.65	2.9	0	0	53	4005
FP Auct.							Miles	159	234	47.8	0	1102	4005
WinBid	31.1	9.9	31.8	1.2	56.3	335	Closed	3.4%					
Win-2Bid	4.5	5.0	3.0	.01	41	335	North5	69.6%					
With.SD	6.9	3.1	6.6	.07	19.1	335	N. & C.	18.4%					
No. Bids	8.5	6.8	7.0	1.0	62	335	S. & I.	12.0%					

Notes: As regards the statistics by auction, HighBid is the highest discount and WindBid is the winning discount. In all the AB auctions High Bid > Win Bid, this is also true for 10 percent of the FP auctions in which the highest bid is reputed not credible. Win-2'Bid is the difference between the winning bid and the bid immediately below it. In standard FP auctions it is referred to as "money left on the table". In the AB auctions, Win-2'Bid is particularly small and it is frequently equal to zero. In general, bid ties occur often among the AB auctions: in total 209 AB auctions, at least two bids are identical, for a total of 720 couples and 38 triplets. With.SD is the within-auction standard deviation of bids. No.Bids is the number of bids. Although not reported among the statistics, it should be noticed that the reserve price has mean €650,000 (SD 225,000) and that all contracts are below €5 million. Decarolis (2009) discusses why for these auctions it can be assumed that all the auctioneers set the reserve price using the same rules. As regards the statistics by firm, we observe the number of auctions attended (Entry), the number of victories (No.Win), the probability of winning in the sample (Pr.Win), the total revenues earned (Reven), the age (Age, measured in years) and the capital (Capital, measured in 2005), the number of subcontracts received (Subct, available only for auctions held in the Piedmont between 2000 and 2007), the miles between the firm and the work (Miles), whether the shuts down between 2005 and 2010 (Closed) and whether it is located in the same five regions in the North where also the auctions were held (North5, these regions are Piedmont, Liguria, Lombardia, Veneto, Emilia-Romagna), in other northern or central regions (N. & C.) or in the southern regions or the islands (S. & I.). Revenues and capital are in thousands of Euro. See Decarolis (2009) for more details on the data.

Bidders' statistics are reported in the right panel of Table 1. There are 4,005 firms that bid at least once and they exhibit substantial differences among many dimensions, which is not surprising given their numerosity. Strong asymmetries appear clearly in firms characteristics (like their capital) and in their performance in the auctions (like their number of victories). Nevertheless, a rather surprising result that was documented in Decarolis (2010) is that firms characteristics play a role in explaining firms bids exclusively in the sample of FP auctions but not in that of AB auctions.<sup>11</sup> In the next section, we develop a model that aims to explain why firms' bids could be disconnected from all proxies of firms' costs. At the heart of the explanation there will be the idea that, in addition to the standard reasons<sup>12</sup> for why firms might form groups, the AB auction generates very strong incentives to coordinate entry and bids.

### 3 Model of Participation and Bidding

This section presents a model of firms' entry and bidding decisions. The model's implications are used to develop our tests for coordinated participation and bidding. We assume that there is a single contract auctioned off and we indicate with  $M$  the set of firms that might bid for it. Firms are either independent or part of groups. We model the decision problem of independent firms in two stages: in the first stage firms observe their cost for preparing the bid and then, in the second stage, those firms that decided to pay the preparation cost learn their cost of completing the job and then bid. At the time of bidding, a firm does not know how many other firms will also bid. However, it is common knowledge in the market that there are  $|M|$  potential entrants and that  $|M^g|$  belong one or more groups. A group is a collection of firms that delegate to a common mediator their entry and bidding decisions in exchange for a share of the group's joint profits. The group's mediator is assumed to observe both the participation cost of each firm of its group and the entry decision of each other firm in the market. This latter assumption intends to capture the superior information possessed by groups relative to independent firms. The mediator observes the production cost of the group's firms that enter.<sup>13</sup>

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<sup>11</sup>In particular, firms' covariates are not significant determinants of bids once auction fixed effects are used (or when there are controls for the identity of the auctioneer and the contract type), see Table 8 in Decarolis (2009).

<sup>12</sup>Porter and Zona (1993) suggest various reasons for why cartels could emerge in the type of market studied in this paper: (1) bids are evaluated only along the price dimension and so product differentiation is absent; (2) firms are relatively homogeneous because of the similar technology and inputs; (3) every year there are many lettings and they take place quite regularly; (4) there are legal forms of joint bidding; (5) the same firms repeatedly interact, (6) ex post the auctioneer discloses the identities and bids of all bidders. These six reasons likely played an important role for both the formation and the stability of the groups that we study.

<sup>13</sup>The bid preparation cost might capture very different costs for independent and groups' firms. In particular, for a firm in a group the pure administrative cost of preparing and submitting a bid might be lower because of scale economies but it might also be higher if it accounts for the probability of being sanctioned by a court.



## Characterizing the behavior in the bidding stage

We indicate with  $N$  the set of firms that place a bid. Moreover, by indicating with  $I$  the independent firms and with  $N^g$ ,  $g = 1, \dots, G$ , the firms belonging to the  $G$  group of coordinating firms we can write  $N$  as:  $N = \{I, N^1, \dots, N^G\}$ . We assume throughout the paper that  $N > 4$ . Each firm  $j$  has cost  $c_j$  of completing the job. Assume that  $c_j \in [c^l, c^h]$  for all  $j=1, \dots, N$  and that each firm that enters draws its cost from a continuous distribution  $F_C(.)$ .<sup>14</sup> Before bidding, firms also observe the maximum price,  $R$ , that the auctioneer is willing to pay (the reserve price). This price is not binding:  $R > c^h$ . A firm submits a sealed bid,  $b \in [0, 100]$ , consisting in a discount over  $R$ . Therefore, the expected profit for an independent firm  $j$  that entered is:  $E_I(\pi) = [(1/100)(100 - b_j)R - c_j] \Pr(b_j \text{ wins})$ . Whether  $b_j$  wins is determined according to the Italian AB rule: the discounts' trim mean, A1, is computed as the average bid disregarding the highest and lowest 10 percent (rounded to the highest integer) of bids; then A2 is calculated as the average of the bids equal or greater than A1 and below the disregarded top 10 percent bids; the discount closest from below to A2 wins. The winner is paid his own price and ties of winning bids are broken with a fair lottery. If all bids are equal, the winner is selected with a fair lottery. Finally, if there is a tie at the highest bid among the bottom 10 percent of bids (or at the lowest bid among the highest 10 percent of bids), the bids to eliminate are chosen with a fair lottery. We begin the analysis by looking at the case in which there is no group.

**Proposition 1:** In the unique Bayesian Nash equilibrium (BNE) all firms bid a discount of zero percent (zero-bids equilibrium).<sup>15</sup>

The idea of the proof is simple: since the highest discount always loses if it is the lone highest bid, then there must be pooling at the top. However, if the highest bid on which bids are pooled is greater than zero, then there exists a unilateral deviation toward a lower discount. The fact that this deviation is profitable is ensured by the trimming of the top 10 percent of bids together with the requirement that the winning discount lies strictly below A2. Therefore, in the unique equilibrium the auctioneer pays the highest price,  $R$ , and the allocation is inefficient since each firm wins with probability  $1/N$ .<sup>16</sup> However, this equilibrium is not robust to the presence of groups as the following proposition illustrates.

**Proposition 2:** Unless all bidders belong to the same group or all groups are smaller than the minimum winning coalition (which is equal to 2 plus 10 percent of  $|N|$  rounded to the next highest integer), then the strategy profile in which all bidders bid zero is not an equilibrium.

The minimum winning coalition,  $N^*$ , that is defined in proposition 2 is the smallest group of

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<sup>14</sup>The symmetry assumption is not essential for the results of the bidding model but it simplifies addressing also entry. Moreover, this assumption is consistent with the finding that firms covariates do not affect bids.

<sup>15</sup>The proof is a simple extension of Proposition 3 in Decarolis (2009). All other proofs are in the Appendix.

<sup>16</sup>Despite these undesirable properties, Decarolis (2009) shows that in a richer environment in which there is cost uncertainty and in which firms can default at a cost that is their private information, a revenue maximizing auctioneer may prefer the AB auction over a standard first price (FP) auction. In this paper, instead, we are not concerned with the auctioneer's behavior but only with that of firms given the AB rule.

bidders that can break the zero-bids equilibrium. If all firms are bidding zero, then a coalition of  $N^*$  firms can submit all bids strictly greater than zero (for instance  $N^* - 1$  bids equal to  $\varepsilon > 0$  and one equal to  $\varepsilon/2$ ) and win for sure. Although the winning firm will receive a lower payment from the auctioneer, there is always an  $\varepsilon$  small enough to make this strategy strictly more profitable for the group than bidding zero. If all groups are smaller than  $N^*$ , then no individual firm or group has an incentive to bid more than zero. On the other hand if all firms belong to a single large group, the winning bid must be equal to zero. Completing the characterization of equilibria beyond these cases is complicated by the need to explicitly computing the probability of winning, which is a rather intractable object. Nevertheless, the following proposition offers an ex post argument to characterize what any group would do regardless of the opponents strategy.

**Proposition 3:** Assume that there is a group  $N^g$  and that  $N^* \leq |N^g| < |N|$ , then all the strategy profiles in which the group's bids do not alter the location of the trim mean are dominated by at least one strategy in which the group's bids shift A1. If the bid distribution is not degenerate, clustering all bids to the right of A1 dominates any strategy that places at least one bid below A1.

Notice that this proposition holds regardless of whether there are other groups present or of whether the other firms are playing according to equilibrium strategy. These latter properties are also true for the following proposition.

**Proposition 4:** Assume that  $F_B(\cdot) \sim [b_l, b_h]$  with  $b_h > b_l \geq 0$  and  $A1(B) = \bar{b}$  is the bid distribution that independent bidders expect to face. Then there cannot be any Bayesian-Nash equilibrium in which an independent firm bids  $b_h$ . There cannot be an equilibrium in which the group submits at least one bid equal to  $b_h$  and does not cluster its other bids above  $\bar{b}$ .

The last two propositions contain the essential features of what coordination of bids implies in the AB auction. First of all, since for any non degenerate distribution of bids the winning bid must lie in  $[A1, A2)$ , then, if the group does not alter A1, it does not take into advantage the possibility of tilting the winning interval toward the area where the group places its bids. The second key aspect is that clustering the bids is the best way to shift A1 because it allows to have the bids exactly where the winning interval is moved. Finally, bidding high discounts is advantageous only for a group and never for an individual firm.

The argument in proposition 4 does not necessarily extend to  $b_l$  as  $b_l$  might be the only individually rational bid for high enough production costs. Nevertheless, if we assume that firms only care about winning and not about the winning bid, then again only groups would bid  $b_l$ . Although this assumption is unusual in the context of auctions, in our application it may be justified by the fact that a large fraction of the contract is subcontracted<sup>17</sup> and that

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<sup>17</sup>In principle there is a legal limit to subcontracts, but in practice firms systematically exceed it with contracts that are identical to subcontracts but that are not legally considered such. Therefore, the winner wins also the option of reselling the job.

the winning bid is likely much lower than the one acceptable by the most efficient firm<sup>18</sup>.

In addition to clustering, another relevant feature of groups' bidding should be mixing. Essentially, this is due to the matching-penny nature of the game. Indeed, the group wants to move the winning interval but it wants to do so in a way that the other firms cannot anticipate it. A group will be particularly likely to mix when other groups are present since, by proposition 4, only another group might try to outguess the group when it is bidding near  $b_h$ . In practice, the observed interval around which the winning bid lies is rather narrow around fixed value that bidders call focal bid (see section 4.1). Therefore, it seems plausible to conjecture that what we observe in the data is the result of groups that mix and cluster on some interval and of independent firms that respond by mixing<sup>19</sup> on a narrower subinterval.<sup>20</sup> Both interval being around the focal bid.

To understand how a focal bid might be sustained through time, we now assume that all independent bidders bid in a very narrow interval around the focal bid and we ask what a group can do. We want to show that both strategies that push the winning bid above or below the focal bid have obstacles. Indeed, even a group larger than  $N^*$  might avoid trying to drag the winning discount toward zero if it expects that many other bids will be clustered on both sides of the focal bid. Efforts to win by moving the winning discount toward zero face three problems: 1) since zero is the lowest possible bid, no overshooting is possible; 2) the only group bids that have a positive probability of winning must be at least equal to  $A1$ ; 3) finally, pushing the winning discount down implies that more firms might find convenient to enter. On the other hand, the group might not be able to profitably push the winning bid above the focal bid. In fact, doing so would reduce its profit conditional on winning. Moreover, the auctioneer sometimes does not admit discounts judged too high to be reasonable.<sup>21</sup> These considerations explain why a focal bid might be persistent. Notice that the behavior that we are conjecturing entails having independent firms submitting independent bids and group's bids that are positively correlated. The "bid-test" that we illustrate in the next section aims at capturing these features. However, since we cannot argue that in all equilibria groups cluster and mix while independent firms mix around the focal bid, it is crucial to assess the empirical relevance of this behavior. Therefore, in the next section we apply the bid-test to the sample for which the presence of groups is known.

### **Characterizing the behavior in the participation stage**

The above discussion makes clear that in the AB auction a group can improve its expected payoff by coordinating bids. However, a group needs the participation of at least the minimum

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<sup>18</sup>This is consistent with the substantial increases of the winning bid observed under the FP rule.

<sup>19</sup>Analogously, they could play a pure strategy that is the purification of their mixing strategy.

<sup>20</sup>Notice also that if the firms' objective is exclusively to win the auction, regardless of the bid, then a large group  $N^g$  achieves almost the certainty of winning if its size is at least  $N^{**} = 1 + \text{integer}^+ \{(.10)N\}$  and if it mixes and clusters all its bids close enough so that there is a zero probability that rivals' bid will fall between any of the group's bids.

<sup>21</sup>For instance, 14 firms that bid 100% discounts in an auction held in the in Pisa in 2005 were all disqualified.

winning coalition if it wants to gain from bids' coordination. From the Court case concerning the cartels in Turin, we know that groups play complex participation strategies sometimes involving bribing independent firms to bid with them for a single auction. However, since our data is not rich enough to measure precisely this phenomenon, we abstain from modeling it and simply assume that, having observed how many firms will bid, a group decides how many of its own firms should bid given the bid preparation cost of each member. For the independent firms, instead, we assume that every firm  $j$  independently draws a participation cost  $q_j \sim F_Q(.)$ . Therefore, if we define the expected profit before independent firm  $j$  observes its cost as  $E_I(\pi_I)$ , then such a firm follows the following cutoff rule: enter if  $q_j \leq E_I(\pi_I)$  and stay out otherwise.

Since independent firms are not aware of how many groups and of how many firms per group will enter, their expected profit from participating is constant. Therefore, the independent firms' entry decision is independent across these firms. On the contrary, a firm in a group is more likely to enter if also other  $N^* - 1$  firms from the same group enter because of the greater expected payoff. Our "participation-test" is based on these ideas. However, since firms might be using more complex entry strategies, the next section evaluates our test using known groups.

## 4 Testing Coordination with Known Groups

The theoretical analysis shows how the behavior of independent and groups' firms should differ. However, given the complexity of the environment, it is crucial to investigate whether these theoretical predictions have empirical support. This is particularly relevant for the case of the bidding behavior given the multiplicity of possible strategies. Therefore, in this section we first present a dataset of auctions where collusion is known. Then we formulate both our statistical tests and apply them to this "validation sample". Therefore, this part of our analysis is similar to Porter and Zona (1993) who construct a test for collusion based on some observed relevant features of a known cartel.

### 4.1 Turin's Rigged Auctions

On April 2008 the Court of Justice of Turin condemned the owners and managers of various construction firms that rigged the AB auctions for roadwork held in the area of Turin between 1999 and 2002. The condemn sentence identifies a network of about 95 firms that operated through 8 groups, referred to as cartels.<sup>22</sup> This case is particularly useful for our study because it

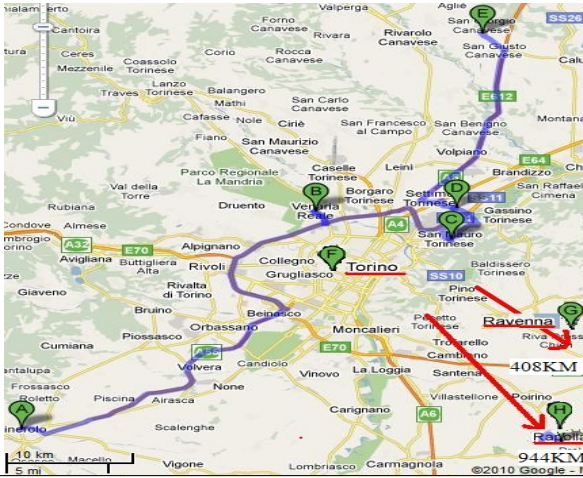
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<sup>22</sup>Turin Court of Justice, 1st criminal section, April 28th, 2008, sentence N. 2549/06 R.G.. Of the 95 suspect firms, the sentence condemns 29. Prescription lead to acquaintance for 2 firms. The judgment of the other firms was scorporated into different court cases. In our study we consider the original full network of 95 firms.

involves the same type of firms, auctions and contracts that we observe in our main dataset.<sup>23</sup> The Turin's groups were very successful in their activity. Despite representing no more than 10 percent of the firms in the market, they won about 80 percent of all the auctions in the Piedmont region between 2000 and 2003. Interestingly, the groups were formed mostly on the basis of firms' geographical proximity, as Figure 2 shows. This is likely due to the lower costs of coordinating actions and of exchanging favors. Moreover, two groups, despite having all members close to each other, are located far from Turin. According to the sentence, these groups did not want to win the auctions to perform the jobs, but just to resell them through subcontracts. Finally, as Table 2 shows, these 8 groups are quite different in their size, entry and victories.

Figure 2: Localization of the 8 Cartels

Table 2: Cartels in Turin's Auctions



Cartel ID	No.Firms	No.Win	Entry
1 - Torinisti B	18	83	247
2 - San Mauro C	12	35	234
3 - Coop G	18	73	240
4 - Pineroli A	12	1	110
5 - Canavesani E	11	7	155
6 - Settimo D	6	10	220
7 - Provvisiero F	7	11	73
8 - Tartara/ Ritonaro H	11	1	62

Capitals letters pinpoint cartels on the map.

Notes: The map gives the exact location of the 6 cartels located around Turin. Cartel G, instead, is centered 408 Km away in the North-East of the country. Cartel H is located in the South, 944 Km away from Turin. However it frequently operates together with some firms located in the same municipality of cartel A. This is the only exception to the fact that firms of the same cartel are all located within few kilometers away from each other.

In addition to the asymmetries across groups, there are also significant asymmetries within groups. The bottom panel of Table 3 reports summary statistics for both the firms inside and outside the groups. Given that this sample was assembled to build a case against the firms accused of collusion, it is normal to see that all variables measuring outcomes of the auctions (entry, victories, subcontracts, etc.) take larger values for the members of the cartels. As regards the auctions themselves, instead, the top panel of Table 3 suggests that these auctions are rather close to those of our sample reported in Table 1 on the basis of entry and of dispersion of the bids. Interestingly, the average winning bid is higher in these colluded auctions than in those reported in Table 1, 17.4 compared to 13.7.

<sup>23</sup>Other Courts (Vicenza, Treviso and Reggio Calabria) have recently started similar trials but none has arrived to a sentence. All cases involve many firms and, often, more than one cartel. The Antitrust Authority (AGCM, 1992) has also expressed concerns about the pro-collusive role of the AB auctions from the very first attempts to introduce these auctions.

Table 3: Summary Statistics - Turin's Cartels Sample													
Statistics by Auction													
	Mean	SD	Med	Min	Max	Obs		Mean	SD	Med	Min	Max	Obs
HighBid	22.8	5.6	22.1	12.5	47.5	276	With.SD	3.6	3.9	1.7	.34	10	276
WinBid	17.4	5.0	17.3	6.7	37.7	276	No.Bids	73.3	37.1	70	6.0	199	276
W-2Bid	.09	.23	.05	0.0	2.9	276	No.Joint	3.0	4.8	1.0	0.0	24	276
Statistics of Independent Firms							Statistics of Cartels Firms						
Entry	17.2	22.3	9.0	1.0	186	717	Entry	82.9	71.1	54	1.0	263	95
Wins	.13	.42	0.0	0.0	3	717	Wins	1.9	3.1	1.0	0.0	19	95
Reven	51.8	19.6	0.0	0.0	2319	717	Reven	822	1466	327	0.0	1e <sup>04</sup>	95
Miles	237	284	101	0.0	1071	504	Miles	101	207	15	0.0	991	86
Age	27.1	14	25	2.0	106	559	Age	29.6	14.1	30	1.0	72	91
Subct	1.8	5.0	0.0	0.0	53	717	Subct	6.8	8.6	4.0	0.0	44	95

Notes: The variables used to describe the auctions are the same of those in Table 1. The only additional variable is No.Joint which measures the number of (legal) bidding consortia present in the auction. Each consortium places one single bid. The type of jobs and the reserve price of contracts is similar to those in Table 1. The set of 717 independent firms contains 24 firms that share part of their owners and managers with the cartels' firms. Their presence makes the summary statistics of the independent firms slightly closer to those of the cartels. The missing values for miles and age are due to the impossibility of identifying with certainty some firms.

As both the confessions of some entrepreneurs and the intercepted mails and phone calls reveal, the strategic environment is complex. The cartels compete against each other (although in some occasions some of them form short term agreements) and against numerous independent firms. Nevertheless, there are at least two features of bidding behavior that emerge clearly from both the confessions and the data. The first feature is that all firms appear to know that in Turin the focal bid is 18 percent, meaning that all firms always expect the winning bid to be somewhere near 18 percent.<sup>24</sup> Indeed, in Table 3, the average winning bid is 17.4. Secondly, joint bids were intended to affect the location of the two means, A1 and A2. For example, some entrepreneurs explain that sometimes some cartel's firm place extremely high (or low) bids not aimed at winning but at helping another member to win. These extreme bids are referred to as "support bids". The definition of what exactly constitutes a support bid is necessarily subjective. However, two conditions seem necessary: (a) that the bid is discontinuously greater than the bid immediately below it and (b) that the bid is in the top (bottom) end of the bid distribution. Any bid greater than a support bid is also a support bid. Figure 3 illustrates an example drawn from one of Turin's auctions which has these characteristics. In the figure, the horizontal axis lists the bidders which are ordered according to their bid (i.e. discount), which is the vertical axis. Different symbols indicate different cartels with the cross representing

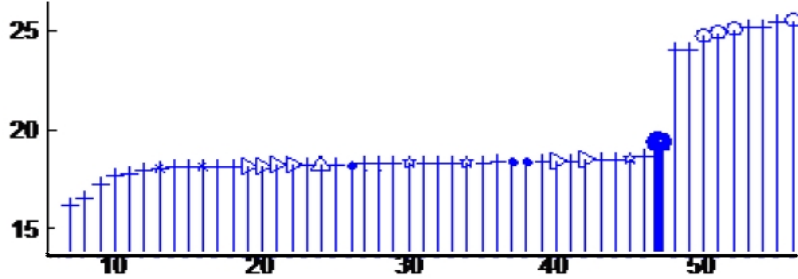
<sup>24</sup>In general, players in this market appear to be aware that there is one focal bid for each administration/contract type pair. Consistently with the relevance of the focal bid idea, Decarolis (2009) shows that in a panel of AB auctions the average of the past winning discounts (for a given types of contract and a given auctioneer) almost perfectly predicts the future winning discounts (for the same type of contract and the same auctioneer).

firms not in cartels. It can be noticed that the vast majority of bids is around 18%, the focal bid. However, it is also evident that several members of the cartel represented by a circle submitted bids that are discontinuously greater than those of all other bidders. Their strategy was successful in making a member of their coalition winning the auction. Numerous similar cases are present in the Turin's data.

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Figure 3: A Cartel Submits "Support Bids" and Wins the Auction

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The Court documents also reveal interesting aspect of the entry behavior of cartels. In particular, they disclose that some members of the cartels are shill bidders for some other firm. This means that a firm, in order to have one additional bid has created a replica of itself, a shill, that exists for the sole purpose of allowing the original firm to place multiple bids. The Turin's case reveals that at least seven firms have one or more shills. The relevant aspect of this behavior is that once the fixed cost of constructing a shill is paid, the marginal cost of submitting one extra bids should be negligible compared to the possible increase in the expected payoff. Therefore, the joint participation pattern of a firm and its shill should clearly violate the assumption of independent entry in favor of strong positive correlation. Analogously, a cartel submitting support bids violates the assumption of independent bids. On the other hand, firms outside the groups should still behave independently if our informational assumptions are satisfied even if cartels are using these strategies. Therefore, we can construct statistical tests to distinguish a group from a random set of firms on the basis of its joint entry and bidding.

## 4.2 The Statistical Tests

**Bid Test:** Our test cannot be based on predictions from a unique equilibrium. Nevertheless, we argued in proposition 4 that bidding far from the focal bid is not rational for an independent firm. Moreover, we conjectured the existence of an equilibrium in which independent firms mix around the focal bid and groups cluster their bids and mix. These indications seem consistent with the evidence from the case in Turin. Therefore, we formulate the following assumption:

Assumption 1: Conditional on the focal bid  $B^*$  and on the firms' observable characteristics  $Z$ , for any firm  $i$  that is not part of a group its bid  $b_i$  is an i.i.d. draw from a distribution  $F_B(\cdot)$ .

To present the test, we start from a situation where Assumption 1 holds without conditioning on any  $Z$ . In this case, our test compares how much the bids of a certain group affect the trim mean relative to the bids of a randomly taken group of the same size. If the group's bids have a significantly greater effect, this is likely because they are clustered on the same side of the bid distribution and we will interpret it as evidence of coordination.

Let us define  $B^g = \{b_1^g, \dots, b_{N^g}^g\}$  as the ordered (from small to large) set of bids of the group and  $B^{-g} = \{b_1^{-g}, \dots, b_{|N|-|N^g|}^{-g}\}$  as the ordered (from small to large) set of remaining bids. Indicating with  $N' = (\text{integer}^+\{(.10)|B^{-g}|\})$ , the trim mean without the group's bids is:

$$A1^g = \frac{1}{|B^{-g}| - 2N'} \sum_{i=N'+1}^{|B^{-g}|-N'-1} b_i^{-g}$$

We can also think of randomly drawing (without replacement and without regard for the order)  $|N^g|$  bids out of  $B^g \cup B^{-g}$ . The number of combinations that we obtain is  $\binom{|N|}{|N^g|} = \frac{|N|!}{|N^g|!(|N|-|N^g|)!}$ . Let us define  $S$  the set of all these combinations of ordered (from small to large) bids and notice that  $B^g \in S$ . Therefore, the trim mean without a combination  $s \in S$  is:

$$A1^s = \frac{1}{|B^{-g}| - 2N'} \sum_{i=N'+1}^{|B^{-g}|-N'-1} b_i^{-s}$$

Our test decides whether a group of firms coordinates bids by checking how extreme is the realization of  $A1^g$  relative to the others  $A1^s$ . If we acknowledge that both upward and downward shifts of  $A1$  are equally informative of coordination, we shall use test a null assumption that  $A1^g$  and  $A1^s$  have the same distribution versus an alternative that their distribution differ. Rejecting independent behavior in favor of coordination at the 5 percent significance level corresponds to the following decision: reject the null if  $A1^g \notin [P_{.025}^T, P_{.975}^T]$  where  $P_x^T$  is the  $x$ th percentile of the empirical distribution of all the  $A1^s$ . The distribution is obtained by making for  $T$  times a draw from  $S$  and calculating for each  $s$  the corresponding  $A1^s$ .

Alternatively, we can restrict our attention to clustering on the upper end of the bid distribution. We saw that only groups might want to do so. In this case, our testing procedure rejects independence when  $A1^g < P_{.05}^T$ .

We can extend this test along two directions. First of all, the above procedure applies to a single auction. Nevertheless, if we have multiple auctions, then we can repeat the test for each of them and then combine them together using the fact that under Assumption 1 the event of passing or not test for an auction is independent from passing or not the test for a different auction. Therefore, the probability of rejecting independence in favor of coordination at least once in  $T$  auctions is  $\sim \text{Binomial}(T, .05)$ . Therefore, to keep the overall significance level of our



test the significance level of each individual test has to be readjusted.<sup>25</sup> The second extension of our test is to account for firms' observable characteristics,  $Z$ . In principle, if  $Z$  had few possible values, we could assign each firm to one of them. Then, when we construct the random groups to match exactly the frequency of  $Z$  in the suspect group.

**Results:** We present the results of the bid test for the 8 cartels in Turin. The tests are not conditional on any firm's covariates because, despite the richness of our data, we could not find any attribute that (alone or jointly with others) was robustly associated with firms' bids.<sup>26</sup> To conduct the test, we fix a cartel. Then for each auction we remove the bids of firms in this cartel<sup>27</sup> and compute the trim mean  $A1^g$ . Then for each auction we repeat 1000 times the calculation of the trim mean but each time we exclude a new set of randomly drawn firms. For each auction we look at these 1000 trim means  $A1_{1000}^s$  and we identify to the percentile of this empirical distribution corresponding to the trim mean without cartel  $A1^g$ . The values taken by these percentiles are reported in the histograms in Figure 4. If the group of firms in the cartel were bidding as a randomly selected group of bidders the histogram should be uniform. Instead, if firms in the cartel coordinate their bids to push up  $A1$ , then the histogram should have all values in the leftmost bin. Instead, if the firms in the cartel push the trim mean down, then the histogram would have all the values in the rightmost bin. If also mixing is present, both extreme bins will be full and the histogram will look like a pair of horns. Indeed, Figure 4 shows that with the only exception of the sixth group, all others are remarkably different from a random group of firms. Interestingly, cartels 1, 2, 3 and 7 seem somewhat more prone to push up the average bid. Although this behavior has positive effects for the auctioneer's revenues, it likely has positive returns for these firms since they have achieved a number of victories (and revenues) relative to the firms in cartels 5 and 8 which bid to push down the trim mean. This implies that focusing on the one tail test, would be enough to identify the most interesting cartels. This matters because the one tail test is the one for which we have more theoretical justifications. Finally, also the result on cartel 6 is not negative for our methodology. In fact, although we do not find evidence of systematic bids' coordination, this is the only cartel whose members were not charged for "criminal association" because their coordination was sporadic.

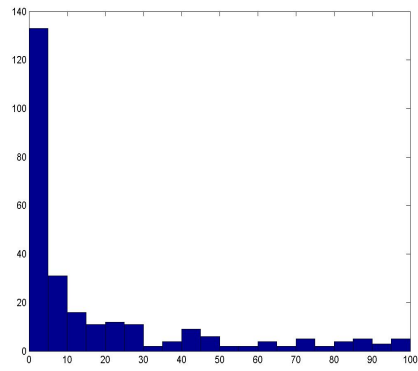
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<sup>25</sup>For a sample of 250 auctions, this implies require a significance of .00205 percent in each test.

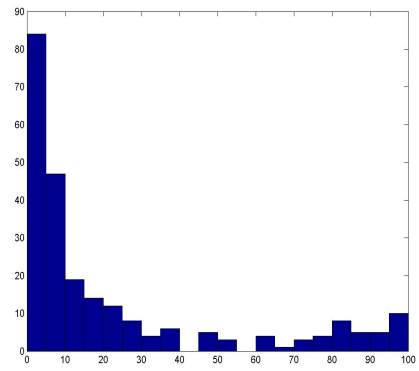
<sup>26</sup>The only exception is the firms' groups affiliation on which, however, we cannot condition. This finding, is consistent with our description of focal point bidding

<sup>27</sup>We consider legal joint bids as cartel's bids if at least a member of the consortium belongs to a cartel.

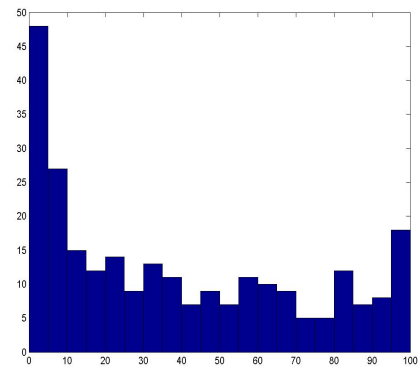
Figure 4: Bid Test for the 8 Known Groups



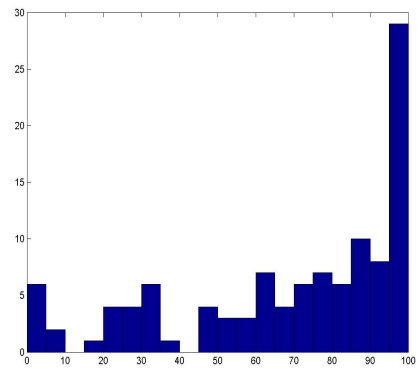
(a) Test Histograms - Cartel 1



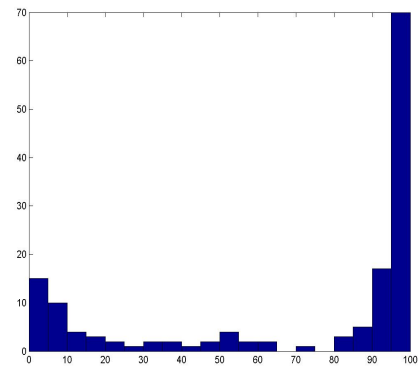
(b) Test Histograms - Cartel 2



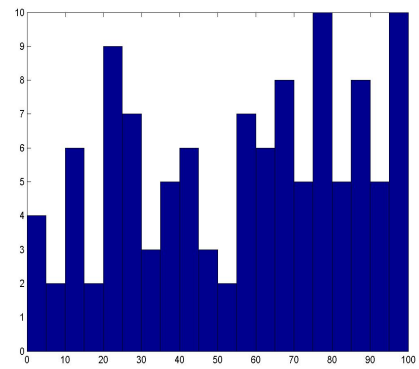
(c) Test Histograms - Cartel 3



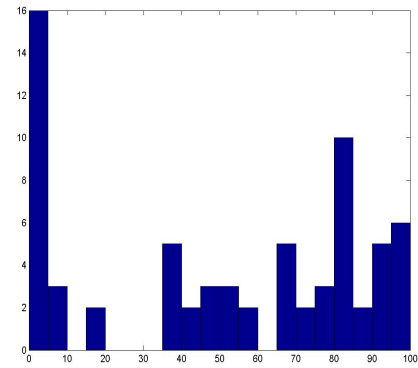
(d) Test Histograms - Cartel 4



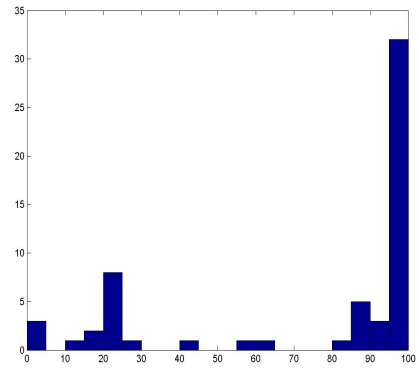
(e) Test Histograms - Cartel 5



(f) Test Histograms - Cartel 6



(g) Test Histograms - Cartel 7



(h) Test Histograms - Cartel 8

**Participation Test:** According to our model of entry, independent firms randomize with the same probability their entry. Members of a group, instead, coordinate their participation. They are more likely to participate when the group's size achieves the size of the minimum winning coalition. The descriptive evidence from Turin suggests that coordinated entry is clear when firms have their skills. The problem of translating these features into a test of coordinated entry is that, on the contrary to what happens for bidding, participation is a function of firms characteristics. A firm can bid in an auction only if it has a certification for both the job's type of work and for at least the contract reserve price. Moreover, among all the firms that qualify for the job, not all of them are equally likely to enter. In particular, the firms further away from the job are less likely to enter than the firms nearby. However, if we observe a matrix  $Z$  containing the relevant firms' characteristic we can formulate the following assumption:

Assumption 2: For any firm  $i$  that is not part of a group, conditional on the firm characteristics  $Z$ , the decision to participate in the auction,  $d_i$ , is an i.i.d. draw from a *Bernoulli*( $p$ ).

As before, we start by introducing the test disregarding  $Z$ . The logic behind our participation test is to compare the frequency with which a group of firms participates together versus the same frequency for a comparable group of randomly selected firms. Therefore, if for instance a group has size 10 we want to compare in how many auctions all the 10 firms show up together versus how many times randomly selected groups of 10 firms show up together.

Assume that the set of potential entrants is  $M$  and that the group that we want to test is  $N^g$ . Drawing  $|N^g|$  firms from  $M$  without replacement, we obtain  $\binom{|M|}{|N^g|} = \frac{|M|!}{|N^g|!(|M|-|N^g|)!}$  combinations. Define  $H$  the set of all these combinations. Defining  $T$  as the total number of auctions and recalling that  $d_{it} = 1$  means that firm  $i$  attended auction  $t$ , we can define the frequency of auctions participated by all members of  $N^g$  as:

$$f^{N^g, |N^g|} = \frac{\sum_{t=1}^T \prod_{i=1}^{|N^g|} \{1 | d_{it} = 1\}}{T} \quad \text{for all } i \in N^g$$

In the same way, we can define the analogous frequency for firms in the set  $h \in H$  :

$$f^{h, |N^g|} = \frac{\sum_{t=1}^T \prod_{i=1}^{|N^g|} \{1 | d_{it} = 1\}}{T} \quad \text{for all } i \in h$$

Our test decides whether a group of firms coordinated entry by checking how much greater is the realization of  $f^{N^g, |N^g|}$  relative to the others  $f^{h, |N^g|}$ . The following our testing procedure consists in comparing a null hypothesis that  $f^{N^g, |N^g|}$  and  $f^{h, |N^g|}$  have the same distribution versus an alternative that states that the value of  $f^{N^g, |N^g|}$  is too large to belong to the distribution of  $f^{h, |N^g|}$ . Rejecting independent entry in favor of coordinated entry at the 5 percent significance level corresponds to the following decision: reject the null if  $f^{N^g} > P_{.95}^T$  where  $P_x^T$  xth percentile of the empirical distribution of all the  $f^h$ . This distribution is obtained by making

for  $T$  times a draw from  $H$  and calculating for each  $h$  the corresponding  $f^h$ .

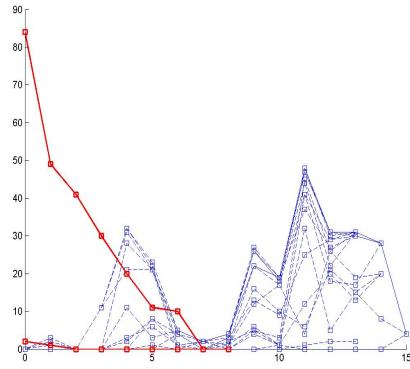
There are two main extensions of our basic tests. First of all, the above test could be repeated for the frequency of auctions attended by  $|N^g| - i$ ,  $i = 1, \dots, |N^g| - 2$ , group members. When the group is large, repeating this test for several large subgroups might strengthen the evidence provided by our basic test. Moreover, this test has special meaning when performed for subgroups of exactly two members. In fact, regardless of  $N$ , the minimum winning coalition must have at least 3 firms. Therefore, the group should be unlikely to have only two of its members showing up at an auction. Therefore, we could test a null hypothesis that  $f^{N^g,2}$  and  $f^{h,2}$  have the same distribution versus an alternative that states that the value of  $f^{N^g,2}$  is too small to belong to the distribution of  $f^{h,2}$ . Rejecting independent entry in favor of coordinated entry at the 5 percent significance level corresponds to the following decision: reject the null if  $f^{N^g} < P_{.05}^T$ .

The second extension, instead, consists in allowing for firms' observable heterogeneity. We already mentioned why this seems crucial for the analysis of participation. Therefore, we consider the case in which the dataset contains a matrix of firms' characteristics,  $Z$ . These characteristics are used to construct the random groups to be compared to the suspect group imposing that each new group has the same distribution of  $Z$  as the original suspect group.

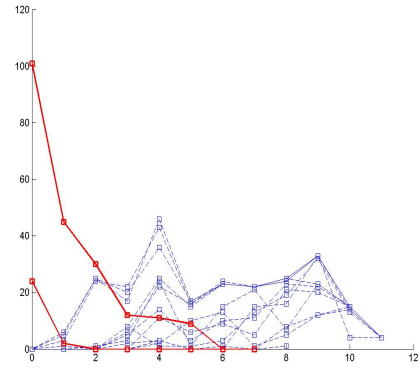
**Results:** Since the test can also be performed firm by firm, we report the results based on this approach. In particular, holding fixed a specific firm  $i$  in the group  $N^g$ , we expect that, conditional on observing  $i$  attending the auction, the number of the other firms in  $N^g$  attending should be significantly higher than that of any other random group. Therefore, first we fix a cartel  $N^g$  and one of its members, say  $i$ . Then we look at all the auctions in which bidder  $i$  bids and we count in how many of these auctions exactly  $0, 1, 2, \dots, N^g - 1$  of the other cartel members bid with firm  $i$ . We then measure how many times the member of a random group participate with  $i$ . We do so for 1000 random groups. Figure 5 reports the results for each firm in the 8 cartels. Each blue line represent the result for a different member of the cartel. In red, instead, we report the 95% and 5% of the distributions obtained by randomly drawing the set of firms. The hypothesis testing is the analogous of the one described above for the whole cartel.

The results are preliminary because we are still in the process of collecting the information regarding the legal qualifications of firms. Therefore, we draw our groups randomly from the universe of firms that submitted at least one bid. Nevertheless, the results of the unconditional test appear encouraging. Based on the participation of the largest possible subgroups, independent entry is rejected for all the 8 cartels. Hence, these results are broadly consistent with the prediction that members of groups are more likely to enter together in an auction. Therefore, for the rest of the analysis we will (temporarily) use this finding to justify our use of the unconditional participation test.

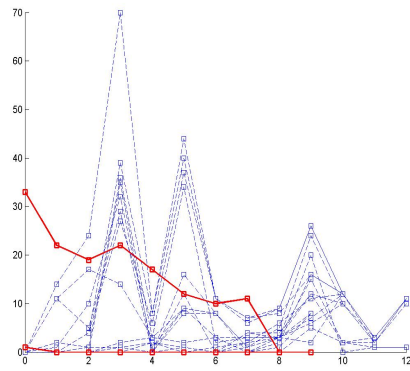
Figure 5: Entry Test for the 8 Known Groups



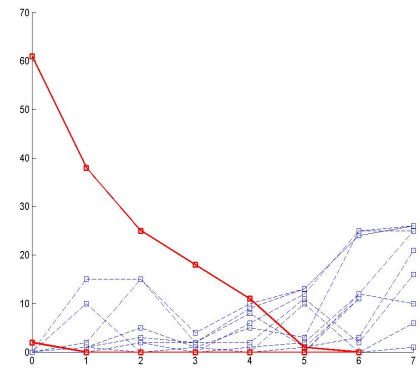
(i) Test Histograms - Cartel 1



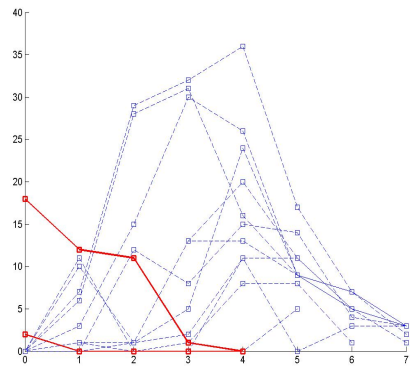
(j) Test Histograms - Cartel 2



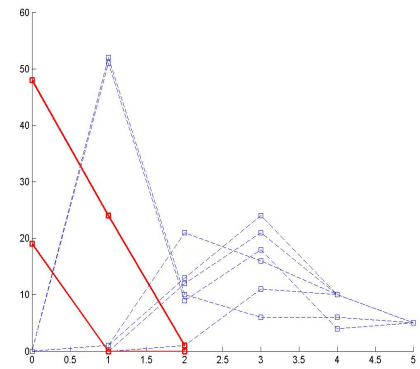
(k) Test Histograms - Cartel 3



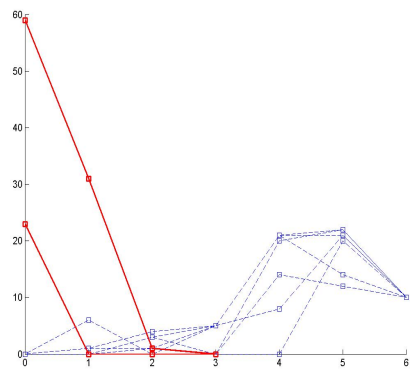
(l) Test Histograms - Cartel 4



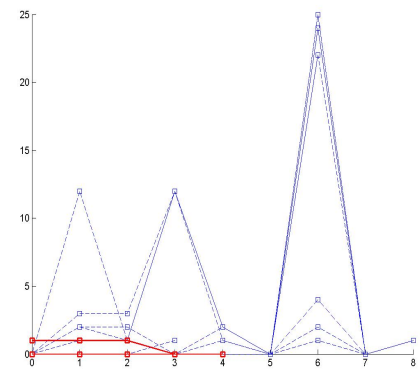
(m) Test Histograms - Cartel 5



(n) Test Histograms - Cartel 6



(o) Test Histograms - Cartel 7



(p) Test Histograms - Cartel 8

## 5 Testing Coordination with Unknown Groups

This section applies the tests for coordination to auctions where there is no prior knowledge of bidders' groups. In principle, this is a simple task because, given a candidate group, we just need to test it with the bidding and entry test to decide whether its members coordinate their actions. However, the problem consists in appropriately choosing the candidate groups so to avoid checking all the enormous number possible firms' combinations. In this section we describe two methods that we propose to solve the problem. The first method can be used when the researcher observes firms' covariates. These covariates are used to measure the probability that couples of firms are linked together. Then a clustering algorithm uses these probabilities as inputs to generate candidate groups. The second method is less demanding in terms of data because it determines potential groups only on the basis of firms' identity and entry. However, groups determined with the latter method can only be tested with the bid test because they would fail the entry test by construction.

Before discussing the details of the two methods, it is worthwhile to stress why it is reasonable to test for group behavior in the sample of AB auctions described in section 2. First, since the environment faced by the firms in the latter dataset is almost identical in every respect to that faced by the firms in Turin, it seems likely that the same incentive to coordinate exist in both cases. Moreover, like in Turin, also in the main dataset both support bids and shill bidders seem to be present.<sup>28</sup> Finally, in 2009 the courts of two cities in the North brought to trial 84 entrepreneurs accused of rigging AB auctions through two cartels.<sup>29</sup> On the basis of these facts, we believe our test could be useful to establish the presence of groups. Notice that this part of our analysis is similar in spirit to Bajari and Ye (2003) which shows how to test for collusion in first price auctions without prior knowledge of groups.

**Method 1: Observable Firms' Characteristics** We use firms' characteristic to construct groups of potentially coordinating firms and then test these groups. We explain the method in steps.

Step 1: In the first step, firms' characteristics are used to construct links between couples of firms. In particular, we identify a link between two firms when: they share some of the owners (managers), or they are geographically close, or they bid together in a consortium, or one did a subcontracting work for the other. First, we quantify the links between the 95 firms in Turin, ending up with a sample of 662 couples of firms connected by at least one link. Since for these firms we know the composition of the cartels, then we are able to tell which of these 662 couples truly belongs to the same group. Hence we can run a probit regression in which the dependent variable is equal to one if the couple is in the same cartel and zero otherwise. For our

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<sup>28</sup>Defining as support bid a bid that belongs to the highest 30% of bids and that is 5 points greater than the preceding one, the data contain 80 auctions with support bids. As regards (possible) shill bidders, in the dataset there various firms registered at the same address, sharing some owners and always participating together.

<sup>29</sup>The courts are those of Treviso and Venice. They have not yet established which firms, if any, are guilty.

favorite specification of the model, Table 8 reports the marginal effect of switching from zero to one the various links. The variable *Personal* is equal to one when the two firms share any owner (top manager). *Subcontract*, instead, equal one if they ever exchanged a subcontract. The estimated marginal effect is the largest for this variable. The joint bidding variables are, instead, equal to one when the firms formed at least once a bidding consortium in the Turin's data (*Joint-Bidding-2*) or when they won at least one auction as a consortium in all the auctions held in Piedmont between 2000 and 2003 (*Joint-Bidding-1*). We exclude firms' geographical proximity, because, although it helps to identify groups, it exposes these groups to the criticism that any observed failure of independence could be due to the spatial correlation of costs.

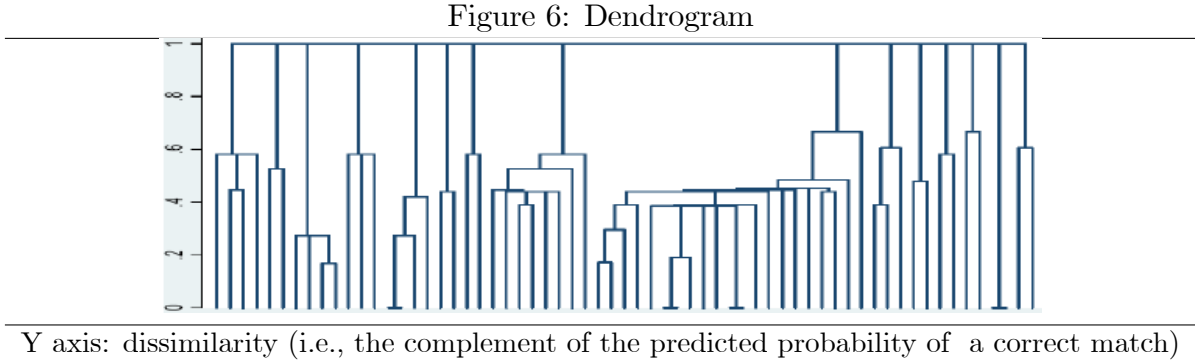
Table 4: Marginal Effects Probit Regression		
	Fixed at:	
	Zero	One
Personal	.00056 (.0016)	.068 (.041)
Subcontract	.00056 (.0016)	.95 (.019)
Joint-Bidding-1	.00056 (.0016)	.049 (.033)
Joint-Bidding-2	.00056 (.0016)	.16 (.037)

For the firms in the dataset where no groups are know, we can use firms' characteristics to create a dataset of linked couples. Then we can use these linkages together with the estimated coefficients from the probit using Turin's data to forecast the probability that two firms are together in a group. Given  $N$  firms, we can then construct an  $N \times N$  "dissimilarity matrix" which is symmetric and has ones on the diagonal. In the off-diagonal entry  $(i, j)$  the matrix has the complement of the predicted probability the firms  $i$  and  $j$  are in the same group.

Step 2: In this step we use a clustering algorithm to create groups. The algorithm is a standard hierarchical algorithm (Gordon, 1999) which uses as its input the dissimilarity matrix constructed in step 1. To give an idea of how the algorithm works, when we applied it to the dataset of Turin's auctions (using it as if we did not know the cartels), the algorithm produced 16 groups.<sup>30</sup> The dendrogram in Figure 6 illustrates the aggregation. Although we did not exactly recover the true 8 cartels, the groups produced are almost all subgroups of the original cartels. Only in one case there is a group containing firms belonging to different cartels. In all other cases the groups are either pure subgroups of the original cartel or they contain at most two firms not belonging to any cartel. Moreover, when we tried these new 16 groups with

<sup>30</sup>We test the validity of these clusters using the Monte Carlo approach described in chapter 7 of Gordon (1999). We rejected that the clusters are identical to random groups of firms.

our tests, the we could identify rather well which of 16 groups were pure subgroups of a cartel because they exhibit high coordination of bids.<sup>31</sup>



Therefore, we can now apply the same algorithm to the dissimilarity matrix constructed for the dataset with unknown groups and obtain candidate groups. For illustrative purposes we focus on the auctions of the Piedmont region. Since firms in groups should be the ones winning more frequently, we focus our attention on the 187 firms that won at least one of the 349 auctions held in Piedmont. For each of these firms, we construct their links to every other bidder in the sample and we obtain the dissimilarity matrix. Depending on the maximum amount of dissimilarity that we admit we end up with clusters of different size. To try to match the size of the cartels observed in Turin we end up having 30 groups with more than two members (we drop two-firms only groups).

Step 3: We apply the entry and bid test to the candidate groups. For the Piedmont case, the results for the groups that appear to exhibit coordination of both entry and bids are reported in Figure A.3 and A.4 in the Appendix. Overall, they convey a consistent picture according to which collusion is pervasive in the Piedmont auctions. Similar results are obtained for the other four regions. However, these results are still preliminary since the usefulness of the unconditional participation test is debatable.

**Method 2: No Observable Firms' Characteristics** Auctions datasets often contain only information on bidders identities and bid. This information is enough to conduct our tests. Nevertheless, when candidate groups need to be found, a possible solution is to preselect firms on the basis of their joint participation. However, this will prevent us from using the participation test, which would reject independence by construction. However, it allows the formation of candidate groups that exhibit a relevant feature of true groups (coordinated entry)

<sup>31</sup>The results are in the Appendix Table A.1 and A.2. An example of a true group detected by the bid test is Group 1 in which 3 out of 4 members of the group are truly members of a cartel. Moreover, the test fails to reject independence for the spurious groups. This is, for instance, the case of Group 8 in Figure A.1. This group has 4 firms that belong to cartel 2 but also 4 more that are not cartel members. The poorer performance of the participation test is likely due to the too small size of most of the groups created.



and that can be tested for coordinated bidding. The idea of this method is first to identify the most frequent winners and then, for each winner, to construct around him a candidate group by looking at those firms that participate with him the most. Since this method requires conditioning on the firms’ legal qualification to bid, we defer discussing both its exact details and its result to when we will have obtained these data.

## 6 Results

### 6.1 Quantifying the Presence of Groups

The methods illustrated above to test coordinated behavior when groups are unknown can be used to quantify the presence of groups in the AB auctions in our dataset. Since we have more than one test (an several variants of each test), we have multiple ways to define which candidate groups should be considered as groups of coordinating firms. Moreover, we could opt for different levels of significance or require a minimum fraction of auctions in which independence has to fail to decide that some firms formed a group. Having made these choices and obtained a classification of firms between independent and not, it is possible to quantify how many auctions are rigged. However, also in this case we shall decide, for instance, whether every auction participated by at least two firms belonging to the same group has to be considered rigged or whether only auctions in which at least one group fails to pass independence according to the bid test have to be considered rigged.

For illustrative purposes, we use the first method described in the previous section with the 164 auctions held in Piedmont. For the candidate groups, we classify them as true groups only if the bid test indicates coordination of bids in at least 50 percent of the auctions that they attend. Having identified the groups, we then count in how many auctions at least two firms from the same group bid. Table 5 reports the results obtained at the different significance level of the two tails bid test.

Table 5: Fraction of Rigged Auctions			
Significance level:	1%	5%	10%
Colluded auctions	21%	48%	52%
Piedmont data. Group is such if rigged 50% or more of the auctions.			

In case we classify as group any set of firms that fails bids’ independence at least once, then the percentage of rigged auctions would be between 80 and 90 percent. More precise results will be provided once we will be able to run the conditional participation test. In any case the basic picture appears clear: a substantially large fraction of the AB auctions is affected by the presence of groups.

## 6.2 The Effect on Revenues

Having established that the presence of groups is pervasive in the AB auctions, we would like to evaluate its effects on revenues. As regards the auctioneer's revenues, the interesting insight is that in the AB auction the presence of groups might benefit the auctioneer. We have already discussed that with multiple, sufficiently large groups their competition will prevent the focal bid to converge to zero. Moreover, since the zero-bid equilibrium is unique under full competition of independent firms, in principle the calculation of the benefits for the auctioneer is trivial. It is simply the difference between the reserve price and the true winning price. Given the large volume of AB auctions and the rather high average winning bid (13%), the savings are substantial. However, the limitation of this naive calculation is that were all the bids to converge to zero, it is unlikely that the auctioneer would not modify the auction format. Indeed, it is known that the legislators introducing the AB auction were not expecting all the bids to go to zero, therefore they might abandon the mechanism if this would happen.<sup>32</sup> The problem with the AB rule is that the incentives to form groups are so strong that a zero-bid situation is very unlikely to emerge.<sup>33</sup>

In the case of the Turin's auctions, the condemned firms are currently facing the risk of having to pay damages to the PA. In the top panel of Table 6, the last two rows report some of the naive damage quantifications proposed. In the first scenario, for instance, the total cost of procuring the 276 auctions in the dataset is recomputed by eliminating all the bids of one cartel per time and assuming that all other bids stay the same. Comparing the first column (the true realized total cost) with the third and the fourth should reveal, according to this very naive method, the total cost that would have emerged without respectively the first and the fifth cartel. However, these counterfactual exercises are hard to make because without all cartels the total cost should have been equal to the sum of all the reserve prices (the value in the second column).

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<sup>32</sup>Decarolis (2010) presents a counterfactual analysis of replacing an AB with an FP rule.

<sup>33</sup>Interestingly, these incentives may be so strong that if, for instance, an auctioneer faces an all inclusive coalition in a second price auction. If he cannot break it by introducing a first price auction because of the fear of defaults, he try to break it by temporarily adopting an AB auction. This was not the case in Italy where, prior to the introduction of the AB, there were no problems related to all inclusive cartels.

Table 6: The Effect of Cartels on Revenues - Turin's Data

<b>EFFECT FOR THE PA</b>	Sum True Cost	Sum Reserve Price	Sum Cost W/O Collusion by C1	Sum Cost W/O Collusion by C5
Scenario 1 (cartel does not bid)	105,937,748	129,345,628	106,579,582	105,701,587
Scenario 2 (cartel members all bid 18%)	105,937,748	129,345,628	105,501,788	105,859,152
Scenario 3 (2/3 of the cartel members bids 18% and 1/3 does not bid)	105,937,748	129,345,628	106,163,857	105,759,604

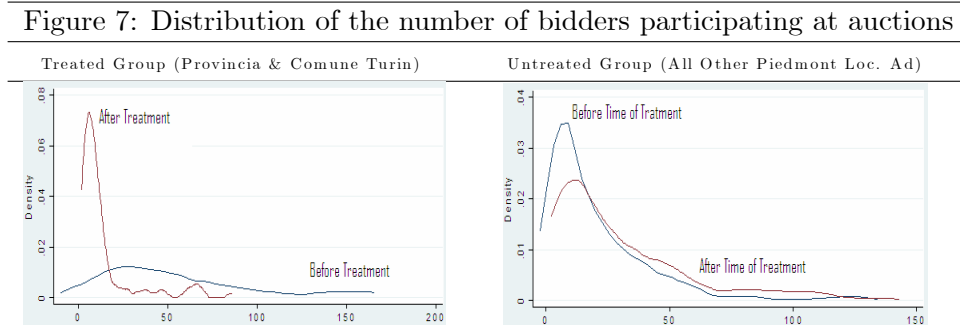
<b>EFFECT FOR THE OTHER BIDDERS (NOT CARTELS)</b>	Expected revenues for a bidder attending every auction under competition	Actual revenues that accrued to the firms not colluded	Difference (i.e. Loss)
Scenario1 (all cartel members are real and there are no shills)	1,873,343	996,051	877,291
Scenario2 (1/3 of the cartel members are shills)	2,619,619	996,051	1,623,567

However, the activity of the groups was likely very harmful for firms outside the groups. The bottom panel of Table 6 illustrates the results of some naive calculations of this damage. The loss for firms outside the groups is due both to their lower probability of winning and to the fact that, when they win, they do so at a discount close to the focal bid and so much greater than zero. The two scenarios in the table compare the total revenues that accrued to the independent firms (second column) with what they would have expected to gain if all auctions were fair lotteries awarded at a discount of zero. The two scenarios distinguish the number of potential entrants on the basis of whether some cartel firms were shills. However these naive calculations fail to account for the fact that at a zero discount entry would have been much larger. Moreover, it is inappropriate to use the sample of Turin's auction to evaluate the damages of independent firms because these auctions were selected by the legal office of the city of Turin as the most representative of the cartels' activities. Therefore, the cartels' probability of winning is likely overestimated in this sample.

However, following the approach of Asker (2009), it may be possible to estimate a structural model to evaluate the losses of independent firms. In our case, following Li (2005) we can estimate a model of entry and bidding in FP auctions with an unknown number of bidders. The dataset would be that of the Turin's FP auctions introduced after 2002. Having recovered the distribution of the entry cost, we could use it together with the observed entry of independent firms in the AB auctions to exactly pin down the expected profit associated with each auction. We can then compare this value to that of a counterfactual zero-bid lottery with the appropriate number of participants. The key idea of this method is that in Turin's data there is no problem of selection for what regards the entry of independent firms. Since a meaningful estimation of the entry cost distribution must be conditional of the firms legal right to enter (in addition other controls like firms' distance) we defer the implementation of this approach to when we will have access to the missing data on firms qualification. The added benefit of this analysis will be to allow to quantify by how much the groups behavior deterred entry of independent firms.

### 6.3 Drop in Participation

A different aspect of firms' entry that can be usefully studied through our test is the drop in participation that followed the introduction of FP auctions. One of the most striking features of the Italian AB auctions is the phenomenally large number of firms bidding. On average, the auctions receive 57 bids and auctions with more than 100 bidders are common. The many studies on FP auctions for road construction contracts in the US report an average bidder turnout that ranges from 3 to 7 firms per auction. In Italy, after the switch to FP auctions the number of bids per auction went down to an average of 8.5. Figure 7 documents this change by reporting in the left panel the distributions of the number of bidders in Turin both under the AB and the FP auctions (i.e. before and after 2003). The right panel, instead, shows that for all the other local administrations comparable to Turin (in terms of geographic location and size of the population served) that remained with the AB format there was an increase in the number of bidders attending the auctions in the period August 2003 - January 2008 as compared to the period December 2000 - December 2002.<sup>34</sup>



The results of the econometric analysis support what indicated by the raw data densities. The results in Table 7 indicate that a switch from AB to FP is associated with a drop of about 40 bidders. Since the variable measuring the number of bidders is highly not normal (the skewness and kurtosis are respectively much greater than zero and three), the model used is a negative binomial regression with robust standard errors. The negative binomial model is preferred to a Poisson regression because the variance of the number of bidders variable is quite larger than its mean and the estimated coefficient on over dispersion in the negative binomial model is statistically different from zero.

<sup>34</sup>Also the dynamic over time of the bidders' turnout indicates an interesting correlation with the auction format. In fact, while over time for Turin's auctions the number of bidders kept declining after the introduction of the FP, for the auctions of the other administrations the turnout drastically increased. This could be explained in part by a greater thirst for work caused by the decline in the number of auctions after 2004 (the total value of all public contracts for works decline from about 25 billions of euro in 2004 to about 20 billions in 2006). However it is also possibly evidence of the fact that over time firms understood that the national law transformed the auctions in lotteries and hence payoff maximization could be more helped by a rise in the probability of winning than by a rise of production efficiency.

TABLE 7: Number of Bidders Regressions				
	Turin Area 2000-2007 <sup>o</sup>		North Regions 2005-2010	
	NEG.BIN <sup>^</sup>	Pred.Change	NEG.BIN <sup>^</sup>	Pred.Change
First Price	-1.84 (.15)***	-38.32	-1.87 (.18)***	-44.03
Observations	2,548		956	
P-Value Chi <sup>2</sup>	.000		.000	

Pred.Change is the predicted discrete change of the number of bidders due to FP switching from 0 to 1.

<sup>o</sup>Control group: all the PA with population > 500,000. Results are very close with the other control groups.

\* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%. Clustered SE by administration and year.

<sup>^</sup>Log(contract value) and dummy variables for type and geographical location of the PA included.

These results are not surprising in light of our previous analysis. However, it is hard to decompose this effect between the disappearance of shills and that of true but inefficient firms. Nevertheless, the large size of the market shakeout produced by the change in the auction format makes disentangling the two effects particularly worthy. In this regard, our tests for collusion allow us to make the following considerations. Suppose that we observe a set of firms bidding in some AB and that we classify them between independent and groups' members. Then if an independent firm disappears from the market after the introduction of the FP auction we can claim that this firm exits because it is inefficient. On the other hand, if a firm that we classified as part of a group exits we cannot tell whether it does so because it is a shill of some other firm or because it is a weak member of a group. If we apply this approach to the auctions held in Piedmont and we use the same classification criteria used for Table 5 we obtain that: 288 firms belong to groups of firms coordinating their actions and 966 do not. Of the latter ones, only 264 keep on bidding after the switch to the FP auctions. Therefore, the exit of the remaining 702 firms is likely due to their inefficiency.

## 7 Conclusions

We constructed two tests that perform well in detecting groups active in AB auctions. Although no statistical test is a final proof, our tests could be useful instruments for the Courts evaluating cases of coordinated bidding. Even if firms were informed about our tests, avoiding detection would require for them renouncing, at least in part, to the benefits of coordination. In this sense our tests have the nice feature of being somewhat "inspector proof". Finally, we believe that the application of our tests to the Italian market have uncovered relevant features of the firms' behavior and of the nature of this market. Our study confirms that firms strategically respond to the incentives generated by the AB rule and that the use of not strategic models by the proponents of the AB rule is incorrect. Therefore, we hope that our results will help to shape the discussion about the various forms of AB rules that are used not only in Italy but in numerous other countries.

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## 9 Appendix: Proofs

*Proof of Proposition 2:* if the coalition is all inclusive, offering a zero discount is the best that can be done. Therefore, if  $|N^g| = |N|$  all equilibria have the winning discount equal to zero and at least  $|N| + 1 - \text{integer}^+\{(.10)|N|\}$  (where  $\text{integer}^+\{x\}$  is  $x$  rounded off to the highest integer) bids equal to zero. Instead, for coalitions that are not all inclusive, the relevant "minimum winning coalition" is defined as  $N^* = 2 + \text{integer}^+\{(.10)(|N|)\}$ . Any group that can submit at least  $N^*$  bids has profitable deviations when all other discounts are equal to zero. One such deviation is to place  $N^g - 1$  identical bids, all equal to  $\varepsilon$ , for small  $\varepsilon > 0$ , and the remaining bids equal to  $\varepsilon/2$ . This strategy gives to the group (approximately) the highest payoff in case of victory and a probability of winning of one (prior to the deviation the probability of winning was  $N^g/N$ ). However, if the group does not reach a size of at least  $N^*$  it cannot profitably deviate from the zero-discount equilibrium because all its bids away from zero would have a zero probability of winning due to the trimming and the requirement that the winner is below A2.

*Proof of Proposition 3:* We first show that a group always gains from altering the location of the trim mean. Consider the set of all bids except those of the group  $N^g$ . Four statistics can be computed with these bids: the trim mean (A1), the trim mean augmented by the positive standard deviation (A2), the lowest bid within the top 10 percent disregarded discounts (Top Bid or TB), and the highest bid within the bottom 10 percent disregarded discounts (Bottom Bid or BB). By definition, the following relationship links the four statistics:  $TB \geq A2 \geq A1 \geq BB$ . Therefore, there are 8 cases:

Case 1	$TB = A2 = A1 = BB$	Case 5	$TB > A2 > A1 > BB$
Case 2	$TB > A2 = A1 = BB$	Case 6	$TB = A2 > A1 > BB$
Case 3	$TB > A2 = A1 > BB$	Case 7	$TB > A2 > A1 = BB$
Case 4	$TB = A2 = A1 > BB$	Case 8	$TB = A2 > A1 = BB$

First of all, notice that cases 7 and 8 are not possible. Let us define "salient bids" all those bids that are not trimmed. If  $A1 = BB$ , then all salient bids must be identical and equal to  $BB$ . However, this is impossible since  $A2 > A1$  implies  $A2 > BB$ . For all the remaining cases, we will show that compared to all group's bids that would leave A1 unaltered, we can find at least one strategy which alters A1 and leads to strictly greater gains for the group. Among the many strategies that leave A1 unaltered, several strategies entail placing some bids that are not salient. However, since non salient bids can never win, we disregard these strategies as they are weakly dominated by strategies placing only salient bids.

Case 1: all bids are identical to some  $b$ . Hence, not affecting A1 requires that the group's bids are also all equal to  $b$ . However, if  $b=0$ , theorem 2 shows that a unilateral profitable deviation exists and that its usage would push A1 up. The same logic gives that, if  $b>0$ , there is a profitable deviation by placing bids between  $b$  and zero. This strategy would push A1 down.



Cases 2, 3 and 4 are almost identical to Case 1. The reason is that  $A2=A1$  implies that all salient bids are identical to the same  $b$ . Therefore, not moving  $A1$  requires bidding  $b$  but this is a dominated strategy. In Case 2, it is possible that  $A1=0$  and we know that in this case a small deviation above zero is profitable. Apart from this situation, in all three cases a deviation towards a bid lower than  $A1$  leads to higher expected profit. Clearly the payoff in case of victory will be strictly higher. Moreover, the probability of winning can be made equal to one in all three cases. Each of these deviations leads to a change in  $A1$ .

Case 5: the winning bid must be in the interval  $(BB, A2)$ . There are two basic situations in which the group's bids leave  $A1$  unaltered: (a) placing all bids equal to  $A1$  and (b) placing at least some bids on both sides of  $A1$ . The former strategy leads to a victory only if there are no bids in  $(A1, A2)$ . Therefore, a strategy that replicates (a) but that places a bid equal to  $A1+\varepsilon$  achieves almost the same payoff in case of victory of strategy (a) but has a strictly higher probability of winning. Now consider strategies (b), they can be replicated for all the bids (if any) that have a positive probability of winning, those bids in  $[A1, A2)$ , and strictly improved by placing the remaining bids strictly within this interval. In particular, these remaining bids are placed symmetrically around  $A2$  so that  $A2$  does not change, this type of strategy leads to a victory every time the strategy (b) was leading to a victory and it strictly increase the probability of those bids that (b) placed below  $A1$ . To ensure that these last bids do not lead to a lower payment in case of victory they can all be placed below the highest bid below  $A2$  that the original (b) strategy was placing. Clearly, this type of strategy pushes  $A1$  up. For Case 6 the the argument is identical to the one of Case 5.

The second part of the proposition says that any strategy that achieves an increase of  $A1$  but leaves some bids below the original  $A1$  is dominated by a strategy that clusters all bids above  $A1$ . The argument is again based on replication: any strategy with just some bids above  $A1$  can be replicated by a strategy in which all bids above  $A1$  are left unchanged and those that were below are moved between the original  $A1$  and the highest group bid below  $A2$ . Through this replication, the group achieves a payoff in case of victory that is at least as large of that of the original strategy and a probability of winning that is strictly larger. The reason why the argument is not symmetric for downward shifts in  $A1$  is that some shifts of  $A1$  might leave  $A2$  unaltered. Therefore, in all these cases, it is not true that all bids between the original  $A1$  and  $A2$  are worthless. Instead, they are the ones most likely to win in these events.

*Proof of Proposition 4:* Proving the first part of the proposition simply requires noticing that a bid equal to  $b_h$  has zero probability of winning unless all bids are equal to  $b_h$ . Therefore, since  $b_h > 0$  there is always a unilateral profitable deviation by bidding in  $(b_l, b_h)$ . Therefore, let us define  $b_h^I < b_h$  the highest bid that an independent bidder would submit. To prove the second part, notice that regardless of the expected distribution of bids, if the group places a bid equal to  $b_h$  this must rise the  $A1$  expected by the group above  $\bar{b}$ . Hence, any bid  $b < \bar{b}$  has now a lower probability of winning. This strategy is therefore dominated by a replication strategy in which all bids are identical with the exception of the lowest one that is now moved to  $b_h^I$ .

Figure A.1: Test for Coordinated Bids, Groups Recovered from Turin's Collusion Data

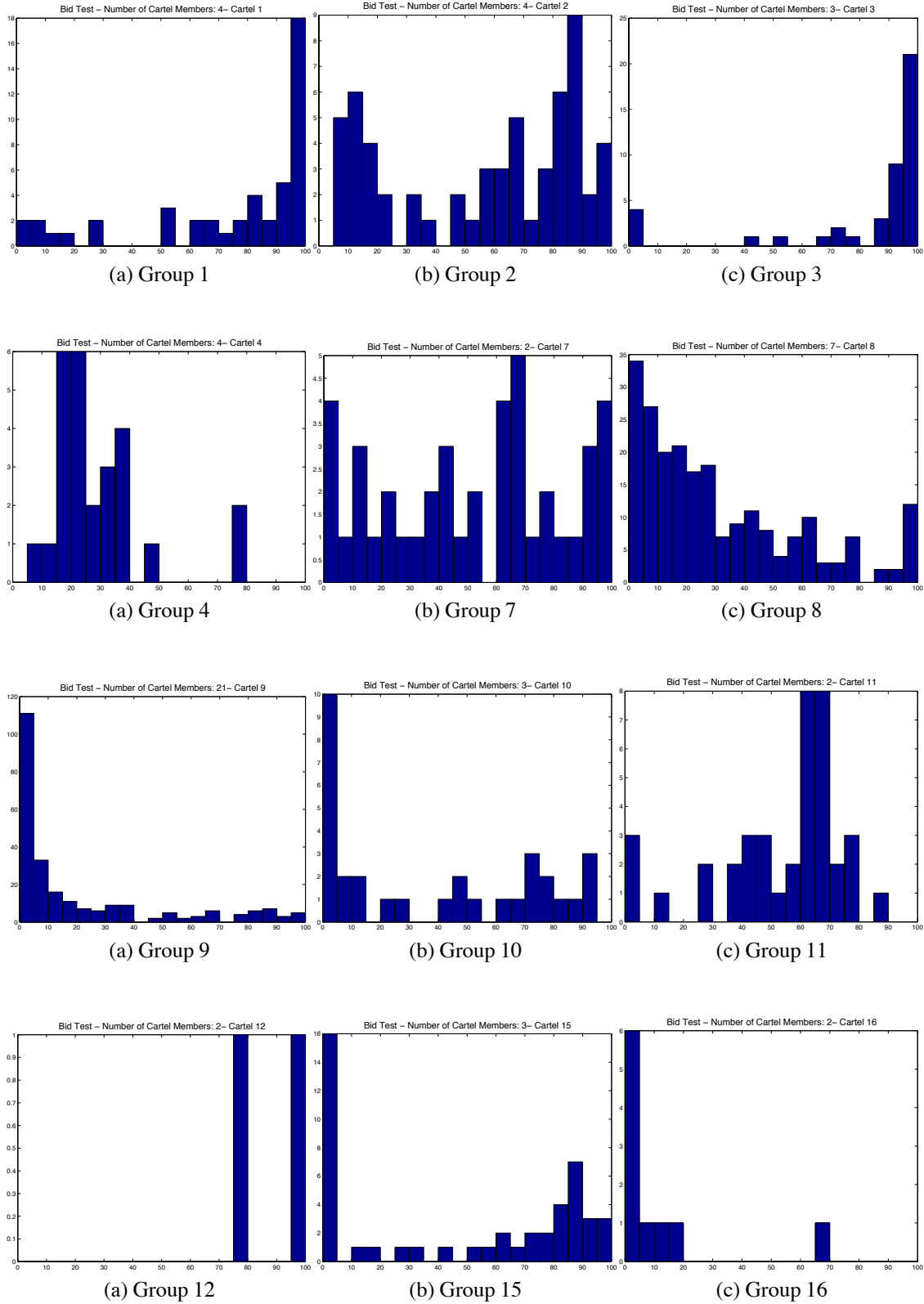


Figure A.2: Test for Coordinated Participation, Groups Recovered from Turin's Collusion Data

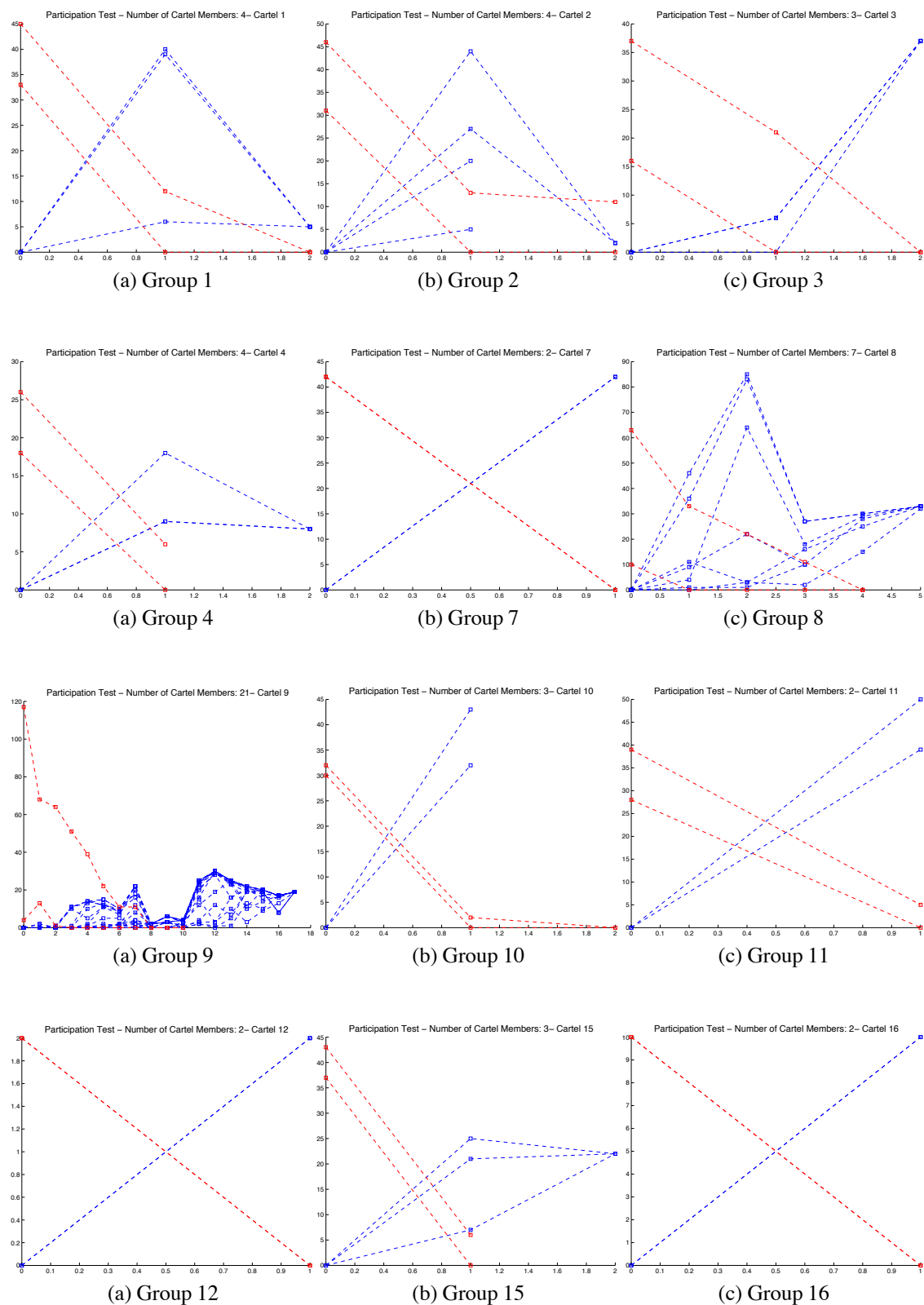


Figure A.3: Test for Coordinated Bids, Groups Found in Piedmont Data

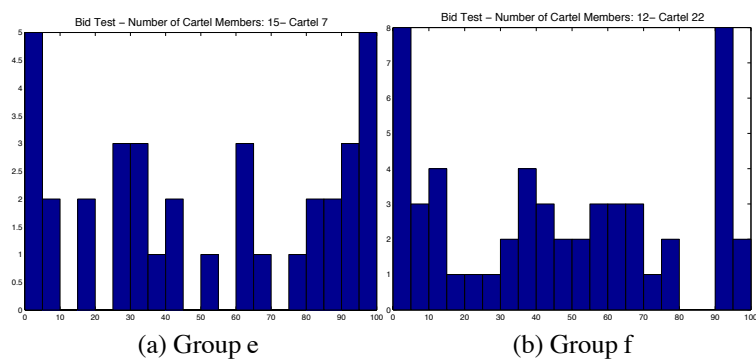
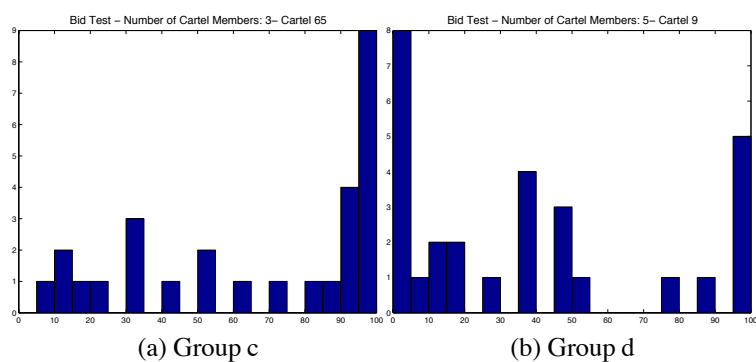
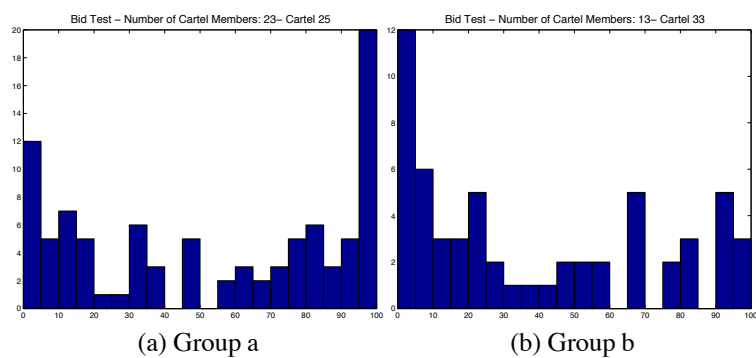


Figure A.4: Test for Coordinated Participation, Groups Found in Piedmont Data

