INPUT CONTROL AND INFORMATION ASYMMETRY.

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ABSTRACT. In many agricultural production contracts, downstream agents exercise control over one or more of the inputs used by upstream agents. In the U.S., for example, chicken processors typically provide growers with chicks and feed, while vintners specify production parameters that wine-grape growers must comply with. This paper provides a mechanism-design-theoretic rationale for such practices. In a production process with a labor and a capital input, we compare a *basic contract*, in which the agent chooses input levels, with a *restricted contract* in which the principal controls the capital input. We show that the principal's profits are always higher under the restricted contract. In order to compare output and social surplus under the optimal restricted and basic contracts, we develop a construction that preserves the standard mechanism-design framework, but allows us to vary continuously the degree of information asymmetry, and then apply calculus/comparative statics techniques in a neighborhood of the symmetric information benchmark. Output is higher under the restricted contract, because the principal allocates capital to mitigate her information costs. However, this mitigation distorts the capital-labor ratio away from the efficient (neo-classical) ratio and this distortion is socially costly. The net effect of this tradeoff depends on the elasticity of substitution between inputs: the restricted contract results in higher social surplus than the basic contract if labor and capital are sufficiently complementary, and in lower social surplus if labor and capital are sufficiently substitutable.

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1. Introduction

Non-labor inputs often play an important role in principal-agent relationships. A principal may supply an agent with necessary non-labor inputs, or may specify contractually the inputs that the agent must use. For example, construction contracts may specify building materials. Military procurement contracts usually specify component materials. In agriculture, production contracts between farmers and processors often specify allowable fertilizers, seedstock, and other production inputs. There are a number of reasons a principal may seek to control inputs. Input quality may affect output quality, and be cheaper or easier to measure. Agents' input choices may be subject to a moral hazard problem; by specifying the input the principal may entirely avoid associated costs. We focus on another information-driven motivation for input control by the principal: by controlling non-labor inputs, the principal can reduce the information rents he incurs due to adverse selection.

When agents' abilities (their effectiveness in production) differ and are unknown to the principal, he must design an incentive-compatible contract that will induce agents to reveal their true types. When there are two possible agent types, the standard principal-agent solution involves offering a low ability agent a contract that pays him his reservation utility and distorts his production below his full information production level (due to the need to induce truthful revelation by high ability agents), and offering a high ability agent a contract that pays him his costs of production plus the returns she would obtain from choosing the low ability agent's contract (his information rents) and requires him to produce his full information output level. We show that by specifying non-labor inputs the principal can always lower the information rents for a given pair of ability-specific output levels, and that the principal's optimal contract menu will always result in higher profits when he controls inputs relative to when he does not control inputs.

Our conceptualization of input specification by the principal can be viewed as encompassing two cases: first, the prsncipal simply provides the input(s) in question to the agent; second, the principal specifies inputs in the contract with the agent and the (non-labor) inputs actually used by the agent are verifiable by a third party. There are other considerations regarding non-labor inputs in a principal-agent relationship. For example, the principal may be less informed regarding the precise nature of the production function than the agent is. This asymmetric information will impose a cost of input specification on the principal, since he may incorrectly choose the input. Similarly, an agent's choice of inputs may provide information regarding his ability. Here, we maintain that the principal and agent are equally informed about the production function, so that the only information asymmetry is that the principal does not know the agent's type.

Proposition 1, which states that input control always increases profits for the principal, can be viewed as a relatively straightforward application of the LeChatelier Principle. That is, the principal is better off when he can choose the input-labor mix for each contract output level than when the agent chooses. Here, the strict inequality is due to the fact that the agent considers only neoclassical production costs, while the principal considers information costs. Similarly, the principal's control of inputs under the optimal restricted contract is an additional constraint facing the *agent* relative to his maximization problem under the basic contract. Given the differing interests of the principal and the agent, the LeChatelier Principle cannot be applied to rank total social surplus under the two contracts.

While input control always results in larger profits for the principal under her optimal contract menu relative to her optimal contract menu without input control, the consequences for society as a whole are less clear. We develop a construction in which an arbitrarily small amount of asymmetric information is introduced into the principal's maximization problem. We assume that with probability close to one, the high ability agent's type is revealed to the principal, while the low ability agent's type is revealed with probability one.

We begin our analysis of this experiment by establishing, in Propositions 2 and 4, that under both the basic and the optimal contracts, output produced by the low ability agent under the optimal contract declines as the degree of information asymmetry increases We also show (in Proposition 3) that the capital-labor ratio for the low ability agent's restricted contract is higher than the neoclassical, cost-minimizing ratio for any given q. The principal distorts the capital-labor ratio in order to reduce her information rents. Proposition 5 demonstrates that for any given q, information rents for the low ability agent are lower under the restricted contract than the basic contract.

Proposition 6 establishes that optimal contract assigns a higher level of output to the low ability agent when non-labor inputs are specified than when they are not. This gain in output clearly enhances the principal's revenues. However, there is an offsetting distortion in the labor-input ratio which may reduce the total social surplus generated. Proposition 7 identifies conditions under which one or other of these distortions dominates. If the degree of information asymmetry is sufficiently small, and labor and capital are sufficiently complementary, the restricted contract will result in higher social surplus than the basic contract. If are sufficiently substitutable, the reverse relationship holds.

Our analysis is related to the literature initiated by Averch & Johnson (1962) on the effect of cost-plus pricing regulation on firm behavior. In their seminal paper, public utilities regulated under cost-plus pricing have an incentive to overinvest in capital, since it will increase the base for their rate of return. This distorts the capital-labor ratio from its first-best level. In their case, the distortion is not a response to a market failure, such as the asymmetric information case we examine. Since the distortion moves the utility away from the most efficient solution, it always reduces social welfare. In contrast, we find that in the presence of asymmetric information, a distorted capital-labor ratio may be associated with a higher level of social surplus than would a non-distorted ratio. While there are formal differences between our setups (most importantly their assumption of a natural monopoly while we assume decreasing returns to scale and a constant price), our findings suggest that in some cases a divergence of the ratio of marginal revenue products from the ratio of input prices may be associated with efficient, rather than inefficient, regulation.

The effect of the principal's control of non-labor inputs on information rents under adverse selection has largely been ignored in the agency theory literature. Implicitly, the literature has assumed that there is no substitutability between labor and inputs that may be controlled by the principal. Perhaps the closest line of research focuses on the principal's choice between monitoring output and monitoring agent effort when both are feasible but costly. Maskin & Riley (1985) find that the principal prefers to monitor output when the agent is the residual claimant, since high ability agents exert more effort when their marginal incentives are not distorted. Khalil & Lawarree (1995) find that the principal will prefer to monitor labor when he is the residual claimant and output when the agent is the residual claimant, provided that input and output monitoring are feasible and equally costly to the principal.

The paper is organized as follows: Section Two introduces the basic model, and proves that the principal's profits are higher under the optimal restricted contract than under the optimal basic contract. Section Three establishes results regarding quantity changes for the two contracts, and compares them. Section Four compares social surplus under the two optimal contracts. Section Five concludes.

2. The model

We begin with a standard principal-agent model. The agent may be one of two types; each type has access to a distinct production function, and one type's function is more productive than the other. Both principal and agent are perfectly informed about the specification of these functions and the probability distribution over types. The agent's realized type, however, is unknown to the principal. The principal's goal is to maximize her profits from production, which depend on the agent's ability. To induce the agent to reveal his true type, she must provide him with a menu of contracts that provide him with adequate incentives to do. We assume, as is the convention in models of this type, that the principal cannot observe the level of labor supplied by the agent. Further, we assume that the principal can not observe *capital* when it is supplied by the agent, nor can a third party verify the level of capital supplied by the agent. She can, however, observe capital if she chooses to control it by supplying it herself. We assume that capital is homogeneous, so that only the level of capital, and the agent who uses it, are relevant to production. In this section we formally develop the components of our analysis, and examine the principal's problem when she can and cannot specify capital.

<u>The Production Function</u>: Production depends on capital, labor and the agent's ability level, or type. There are two types, "low" and "h". $\theta \in \{\theta^\ell, \theta^h\}$ is the agent's true type, with $\theta^\ell < \theta^h$. $\Pr(\theta) > 0, \theta \in \{\theta^\ell, \theta^h\}$ is the probability that an agent's type is θ. $\theta' \in \{\theta^\ell, \theta^h\}$ is the agent's announced type. We will refer below to agents "ℓ" and "h".

We make a number of assumptions regarding the production function f. For each θ, the marginal products of labor, capital and ability are all positive ($f_e, f_k, f_\theta > 0$), and f is strictly concave in (e,k), i.e., $f_{ee}, f_{kk} < 0$ and $f_{ee} f_{kk} > f_{ek}^2$. An increase in ability positively affects the marginal products of labor and capital ($f_{e\theta} > 0$, and $f_{k\theta} > 0$). The following additional conditions on f are satisfied:

A1: for each θ, f is homogeneous of degree $\alpha < 1$ in e and k, and

A2: θ is "technologically neutral" in the sense that for each θ, θ', $\frac{f_e(e,k,\theta)}{f_k(e,k,\theta)} = \frac{f_e(e,k,\theta')}{f_k(e,k,\theta')}$ for fixed e and k.

In sections 3 and 4, we shall be even more specific than A2 and assume

A3: there exists f^* satisfying A1 such that for $\theta \in \{\theta^\ell, \theta^h\}$, $f(\cdot,\cdot,\theta) = \theta f^*$.

These assumptions ensure that isoquants for different ability levels are parallel. If the isoquants were allowed to cross, the analysis would become much more complex, with little insight added. This restriction is functionally similar to the single-crossing property that is often imposed in single input principal-agent problems.

Agent's Utility Function: The agent will receive a lump-sum transfer payment from the principal and in return will deliver a specified level of output, contributing labor and, in the 'basic' contract, capital. The agent's outside alternative is to provide her labor at the given wage-rate w per unit labor supplied. The wage rate, w, exactly compensates for the agent's constant marginal disutility of labor, so that her reservation utility when she does not supply labor is zero. In order to induce the agent to participate at a labor level e and capital level k, the principal's transfer payment must at least cover the agent's cost, $we + rk$.

Input levels: If an agent of type θ accepts a basic contract (defined below as a contract in which the agent chooses input levels) to produce q she will solve the (neo-classical) optimization problem $\min_k we + rk$ s.t. $f((e,k,\theta) = q$. Let $\left(\tilde{e}(q,\theta),\tilde{k}(q,\theta)\right)$ denote the solution to this problem. We will refer to this input vector as the *neoclassical input mix for q*. Because f is strictly concave, the neoclassical mix is uniquely defined by the first order condition:

$$0 = r f_k\left(\tilde{e}(q,\theta),\tilde{k}(q,\theta),\theta\right) + w f_e\left(\tilde{e}(q,\theta),\tilde{k}(q,\theta),\theta\right) \tag{1}$$

Let $\tilde{C}(q,\theta)$ denote the type θ agent's *production cost* of delivering the output level q with the neoclassical input mix:

$$\tilde{C}(q,\theta) = w\tilde{e}(q,\theta) + r\tilde{k}(q,\theta) \tag{2}$$

Similarly, let $\overline{C}(q,k,\theta)$ denote the type θ agent's production cost of delivering the output level q with capital level k:

$$\overline{C}(q,k,\theta) = w\overline{e}(q,k,\theta) + rk \tag{3}$$

For future reference, note that by definition of $\tilde{k}(q,\theta)$

$$\frac{\partial\overline{C}(q,\tilde{k}(q,\theta),\theta)}{\partial k} = 0, \quad \text{for all } q \text{ and all } \theta. \tag{4}$$

Contracts: A *basic contract* is a mapping from types to output levels and transfers, $\theta \rightarrowtail (\tilde{\mathbf{q}},\tilde{\mathbf{t}}) = (\tilde{q}(\theta),\tilde{t}(\theta))$. We will sometimes write $(\tilde{\mathbf{q}},\tilde{\mathbf{t}})$ as $\left((\tilde{q}^\ell,\tilde{t}^\ell),(\tilde{q}^h,\tilde{t}^h)\right)$. A *restricted contract* is a mapping from types to output levels, capital levels and transfers. We will write $(\overline{\mathbf{q}},\overline{\mathbf{k}},\overline{\mathbf{t}})$ either as $(\overline{q}(\theta),\overline{k}(\theta),\overline{t}(\theta))$ or as $\left((\overline{q}^\ell,\overline{k}^\ell,\overline{t}^\ell),(\overline{q}^h,\overline{k}^h,\overline{t}^h)\right)$.

Our model has the standard property that in any optimal contract, the difference between the transfer offered to agent "ℓ" and the agent's production cost of delivering the designated output level must just equal the agent's reservation utility, which in our case is zero. That is, for a basic contract $(\tilde{\mathbf{q}},\tilde{\mathbf{t}})$,

$$\tilde{t}^\ell = \tilde{C}(\tilde{q}^\ell,\theta^\ell) \tag{5-$\tilde{t}^\ell$}$$

while for a restricted contract:

$$\overline{t}^\ell = \overline{C}(\overline{q}^\ell,\overline{k}^\ell,\theta^\ell). \tag{6-$\overline{t}^\ell$}$$

On the other hand, the transfer offered to agent "h" includes a premium, referred to as her *information rent.* In an optimal basic contract $(\tilde{\mathbf{q}},\tilde{\mathbf{t}})$, this premium $\left(\tilde{t}^h - \tilde{C}(\tilde{q}^h,\theta^h)\right)$ must be just sufficient to offset the utility, $\left(\tilde{t}^\ell - \tilde{C}(\tilde{q}^\ell,\theta^h)\right)$, that agent "h" would derive by adopting agent "ℓ"'s contract. It follows from (5-$\tilde{t}^\ell$) that:

$$\tilde{t}^h = \tilde{C}(\tilde{q}^h,\theta^h) + \left(\tilde{C}(\tilde{q}^\ell,\theta^\ell) - \tilde{C}(\tilde{q}^\ell,\theta^h)\right) \tag{5-$\tilde{t}^h$}$$

while for a restricted contract:

$$\overline{t}^h = \overline{C}(\overline{q}^h,\overline{k}^h,\theta^h) + \left(\overline{C}(\overline{q}^\ell,\overline{k}^\ell,\theta^\ell) - \overline{C}(\overline{q}^\ell,\overline{k}^\ell,\theta^h)\right) \tag{6-$\overline{t}^h$}$$

In what follows, we shall sometimes use the terminology *production costs* and *information costs* to distinguish between costs incurred through production and costs (usually called rents) paid out to ensure truthful revelation. The terms "marginal production" and "marginal information" costs will then have the obvious interpretation.

The principal's problem: basic contracts. For all contracts, we assume that output is sold on a perfectly competitive market at a price of p. Given a basic contract $(\tilde{\mathbf{q}},\tilde{\mathbf{t}})$, the principal's profit from an agent who declares a type of θ' is $p\tilde{q}(\theta') - \tilde{t}(\theta')$. [1] Thus, the principal's problem is to choose the contract $(\tilde{\mathbf{q}},\tilde{\mathbf{t}})$ that maximizes $\sum_{\theta\in\{\theta^\ell,\theta^h\}}\left\{\Pr(\theta)\left(p\tilde{q}(\theta)-\tilde{t}(\theta)\right)\right\}$ subject to incentive and participation constraints. By invoking the necessary conditions (5), we can reduce the principal's program to the problem of finding an

[1] Notice that the principal's profit depends only on agents' *announced* types. The reason is that the contract is written in terms of the agent's deliverable, q. This would not be the case if the principal specified a piecerate and the agent's deliverable were not verifiable.

(unconstrained) maximum over $\mathbf{q}$ of the following expression:

$$\max_{\mathbf{q}} \quad \left\{ \sum_{\theta\in\{\theta^\ell,\theta^h\}} \Pr(\theta)\Big(pq(\theta) - \tilde{C}(q,\theta)\Big) \right\} \quad + \quad \Pr(\theta^h)\tilde{I}(q) \tag{7}$$

where $\tilde{I}(q) \quad = \quad \Big(\tilde{C}(\tilde{q}^\ell,\theta^\ell) - \tilde{C}(\tilde{q}^\ell,\theta^h)\Big)$ denotes the *information cost* of having agent "ℓ" produce q under a basic contract. Together with (5-$\tilde{t}^\ell$) and (5-$\tilde{t}^h$), the necessary conditions for $(\tilde{\mathbf{q}},\tilde{\mathbf{t}}) \quad = \quad \Big((\tilde{q}^\ell,\tilde{t}^\ell),(\tilde{q}^h,\tilde{t}^h)\Big)$ to maximize (7) are:

$$p \quad = \quad \frac{\partial\tilde{C}(\tilde{q}^h,\theta^h)}{\partial q} \tag{5-$\tilde{q}^h$}$$

$$p \quad = \quad \Pr(\theta^\ell)\frac{\partial\tilde{C}(\tilde{q}^\ell,\theta^\ell)}{\partial q} \quad + \quad \Pr(\theta^h)\frac{\partial\tilde{I}(\tilde{q}^\ell,\theta^\ell)}{\partial q} \tag{5-$\tilde{q}^\ell$}$$

The standard results follow immediately: while agent "h" will produce the neoclassical level of output for her type, agent "ℓ" will produce less than the neoclassical level of output for her type, provided that $\frac{\partial\tilde{I}(\cdot,\theta^\ell)}{\partial q}$ is positive. To see that $\frac{\partial\tilde{I}(\cdot,\theta^\ell)}{\partial q}$ is positive, observe that since production technology exhibits decreasing returns to scale, $\frac{\partial\tilde{C}(\cdot,\theta^\ell)}{\partial q}$ is increasing in q. Hence if $\frac{\partial\tilde{I}(\cdot,\theta^\ell)}{\partial q} > 0$, (5-$\tilde{q}^\ell$) can hold only if $\tilde{q}^\ell$ is lower than the q-value at which marginal cost equals price.

The principal's problem: restricted contracts. Now consider a restricted contract $(\overline{\mathbf{q}},\overline{\mathbf{k}},\overline{\mathbf{t}})$. As before, by invoking the necessary conditions (6), we can reduce the principal's program to the problem of finding an (unconstrained) maximum over $(\mathbf{q},\mathbf{k})$ of the following expression:

$$\sum_{\theta\in\{\theta^\ell,\theta^h\}} \Pr(\theta)\Big(pq(\theta) - \overline{C}(q,k,\theta)\Big) \quad + \quad \Pr(\theta^h)\overline{I}(q,k) \tag{8}$$

where $\overline{I}(q,k) \quad = \quad \Big(\overline{C}(q,k,\theta^\ell) - \overline{C}(q,k,\theta^h)\Big)$ denotes the *information cost* of having agent "ℓ" produce q with capital level k under a restricted contract. Together with (6-$\overline{t}^\ell$) and (6-$\overline{t}^h$), the necessary conditions for

$(\overline{\mathbf{q}}, \overline{\mathbf{t}}) \quad = \quad \left((\overline{q}^{\ell}, \overline{k}^{\ell}, \overline{t}^{\ell}), (\overline{q}^{h}, \overline{k}^{h}, \overline{t}^{h})\right)$ to maximize (8) are:

$$p = \frac{\partial \overline{C}(\overline{q}^h, \overline{k}^h, \theta^h)}{\partial q} \tag{6-$\overline{q}^h$}$$

$$0 = \frac{\partial \overline{C}(\overline{q}^h, \overline{k}^h, \theta^h)}{\partial k} \tag{6-$\overline{k}^h$}$$

$$p = \Pr(\theta^\ell) \frac{\partial \overline{C}(\overline{q}^\ell, \overline{k}^\ell, \theta^\ell)}{\partial q} + \Pr(\theta^h) \frac{\partial \overline{I}(\overline{q}^\ell, \overline{k}^\ell, \theta^\ell)}{\partial q} \tag{6-$\overline{q}^\ell$}$$

$$0 = \Pr(\theta^\ell) \frac{\partial \overline{C}(\overline{q}^\ell, \overline{k}^\ell, \theta^\ell)}{\partial k} + \Pr(\theta^h) \frac{\partial \overline{I}(\overline{q}^\ell, \overline{k}^\ell, \theta^\ell)}{\partial k} \tag{6-$\overline{k}^\ell$}$$

As with a basic contract, agent "h" will produce the neoclassical level of output for her type, while agent "ℓ" will produce less than the neoclassical level for her type, provided that $\frac{\partial \tilde{I}(\cdot,\cdot,\theta^\ell)}{\partial q}$ is positive. Analogous to output, agent "h" will use the neoclassical input mix, while the input mix for agent "ℓ" will be affected by the information problem. Since $\frac{\partial \overline{C}(\overline{q}^\ell, k(\overline{q}^\ell,\theta^\ell),\theta^\ell)}{\partial k}$ is zero (see (4)), the neoclassical capital choice $k(\overline{q}^\ell, \theta^\ell)$ will satisfy (6-$\overline{k}^\ell$) only if $\frac{\partial \overline{I}(\overline{q}^\ell, k(\overline{q}^\ell,\theta^\ell),\theta^\ell)}{\partial k}$ is zero also. We will establish below that this will *not* be the case given the assumptions we have imposed, so that agent "ℓ"'s prescribed input mix under an optimal restricted contract will differ from the neoclassical mix.

The following result follows immediately from expressions (5), (6), (7) and (8).

Proposition 1. *The principal's profits under the optimal restricted contract are always strictly higher than her profits under the optimal basic contract.*

The proof, in a sentence, is that the basic contract is a feasible restricted contract (though cheats that would be feasible under the basic contract are proscribed under the restricted one), but the basic contract is not profit maximal within the set of restricted contracts. More formally:

Proof. Let $(\tilde{\mathbf{q}}, \tilde{\mathbf{t}}) = \left((\tilde{q}^\ell, \tilde{t}^\ell), (\tilde{q}^h, \tilde{t}^h)\right)$ denote the optimal basic contract. Construct the restricted contract $(\hat{\mathbf{q}}, \hat{\mathbf{k}}, \hat{\mathbf{t}}) = \left((\hat{q}^\ell, \hat{k}^\ell, \hat{t}^\ell), (\hat{q}^h, \hat{k}^h, \hat{t}^h)\right)$, where $\hat{\mathbf{q}} = \tilde{\mathbf{q}}$ and for $\theta \in \{\theta^\ell, \theta^h\}$, $\hat{k}(\theta) = \tilde{k}(\tilde{q}, \theta)$. That is, under this constructed restricted contract, the outputs that were produced under the original basic contract are once again produced using the (neoclassical) input mix that was endogenously selected under the original basic contract. Thus for each θ, the production cost of $\hat{q}(\theta)$ is identical under both contracts. On the other hand, the *information cost* associated with $(\hat{\mathbf{q}}, \hat{\mathbf{k}}, \hat{\mathbf{t}})$ is lower than the information cost associated with $(\tilde{\mathbf{q}}, \tilde{\mathbf{t}})$. To see this, note that since $\theta^\ell < \theta^h$, $\hat{k}^\ell = \tilde{k}(\hat{q}^\ell, \theta^\ell)$ is distinct from the unique solution $\tilde{k}(\hat{q}^\ell, \theta^h)$ to the first order

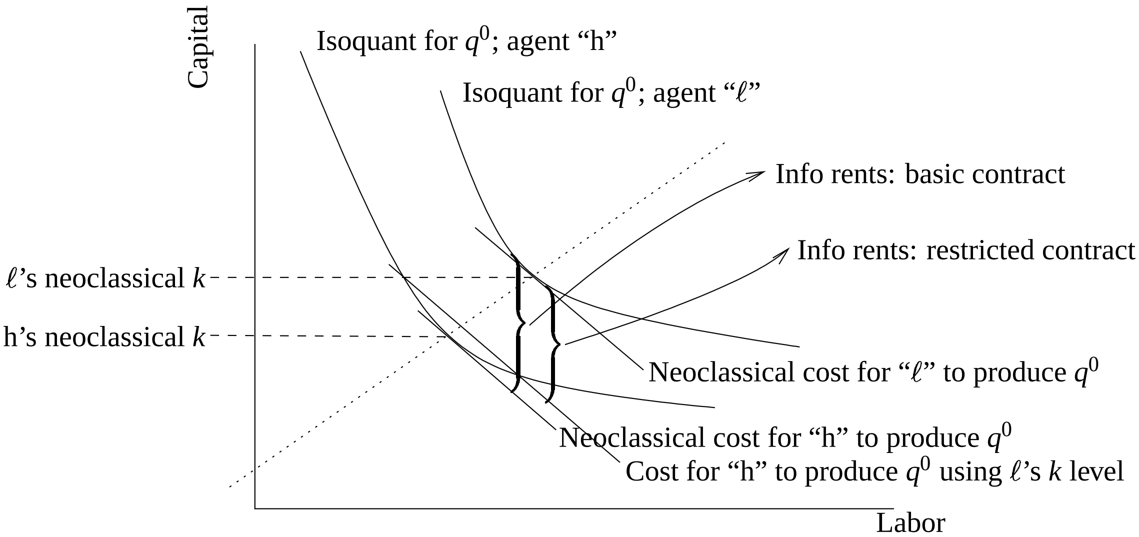

FIGURE 1. Information Cost of Producing q^0

condition (1) for the "h"-type producer. Hence, we have $\overline{C}(\hat{q}^\ell, \hat{k}^\ell, \theta^h) > \overline{C}(\hat{q}^\ell, \tilde{k}(\hat{q}^\ell, \theta^h), \theta^h) = \tilde{C}(\tilde{q}^\ell, \theta^h)$. Hence

$$
\begin{aligned}
\tilde{I}(\hat{q}^\ell) &= \tilde{C}(\tilde{q}^\ell, \theta^\ell) - \tilde{C}(\tilde{q}^\ell, \theta^h) \\
&> \tilde{C}(\tilde{q}^\ell, \theta^\ell) - \overline{C}(\hat{q}^\ell, \hat{k}^\ell, \theta^h) \qquad (9) \\
&= \overline{C}(\hat{q}^\ell, \hat{k}^\ell, \theta^\ell) - \overline{C}(\hat{q}^\ell, \hat{k}^\ell, \theta^h) = \overline{I}(\hat{q}^\ell, \hat{k}^\ell) \qquad (10)
\end{aligned}
$$

The restricted contract we have constructed thus delivers the same output at a strictly lower cost to the principal, and hence delivers higher profits than the original basic contract. Hence the *optimal* restricted contract must yield the principal higher profits still. □

(Note that the constructed basic contract $(\hat{\mathbf{q}}, \hat{\mathbf{k}}, \hat{\mathbf{t}})$ is not optimal within the set of restricted contracts. To see this, observe that $\hat{k}^\ell = \tilde{k}(\hat{q}^\ell, \theta^\ell)$ fails the first order condition (6-$\overline{k}^\ell$), since $\frac{\partial \overline{C}(q, \hat{k}^\ell, \theta^\ell)}{\partial k} = 0$ (display (4)) while $\Pr(\theta^h)$ and $\frac{\partial \overline{I}(\overline{q}^\ell, \overline{k}^\ell, \theta^\ell)}{\partial k}$ are both nonzero.)

Some intuition for Proposition 1 is provided by Fig. 1. The higher isoquant indicates the set of input combinations that agent "ℓ" could use to produce a given output level q^0. The lower isoquant indicates combination that agent "h" would use to produce *the same* output level. The parallel lines represent isocost curves. The brace to the left indicates the cost differential if both kinds of agents were to produce q^0 using their respective cost-minimal (i.e., neoclassical) input combinations. The brace to the right indicates the reduced cost differential when agent "h" is penalized by being forced to use the capital level that is optimal for agent "ℓ", i.e., $\hat{k}^\ell = \tilde{k}(q^0, \theta^\ell)$. The left and right braces also represent *information rents* that the principal would have to pay agent "h", under, respectively, a basic and restricted contract that specified an output level of q^0 and, in the restricted contract, imposed on agent "h" the neoclassical input ratio for agent "ℓ". We thus demonstrate that the principal can construct a restricted contract which exactly mimics any basic contract, except for the restriction on the input mix that "h" must use if she picks the contract designed for "ℓ". Comparing the two contracts, the principal's revenues are the same under both, since outputs are the same. Production costs are also the same, since the input mixes are identical. But information rents are lower under the constructed contract, and so profits are higher. It follows that profits must be strictly higher under the *optimal* restricted contract than under the basic contract.

This result can be viewed as a direct application of the Le Chatelier Principle. The optimization problems facing the principal when designing either a restricted or a basic contract are identical except that in the latter case, the principal faces an additional constraint. Under the restricted contract, the principal is free to specify both capital and output levels for agents of each type, and thus, implictly, labor levels as well. Under the basic contract, by contrast, the principal faces the additional constraint that the agent will always combine capital and labor in the neoclassical, production cost-minimizing ratio.

3. MARGINAL ANALYSIS OF THE BASIC AND RESTRICTED CONTRACTS

In this section we isolate and compare the effect of a small increase in the degree of information asymmetry on the structures of the basic and restricted contracts. When information is symmetric, the restricted and basic contracts yield identical, first-best outcomes for the low ability agent. By introducing a "small" amount of information asymmetry, we can use standard calculus and comparative statics techniques to compare the properties of the two kinds of contracts.

One, particularly simple way to vary the degree of information asymmetry is to vary the probability of realizing each type of agent. Specifically, in a two-type model, consider the probability that the agent is "h". Obviously if this probability is either zero or one then information is perfectly symmetric. The degree of asymmetry increases as the probability that the agent is "h" moves toward one-half, and is maximized at this point. From our perspective, this kind of variation in information is not fully satisfactory, because it necessarily involves changing the principal's *production possibilities* along with her information. In other words, the first-best, symmetric information benchmark changes along with the possibility that the agent is "h". For this reason we propose a test that holds *everything* constant except information asymmetry.

Consider the following thought experiment inspired by the fable of Cinderella. The two types of agent in this story are Cinderella, whose ability level is $\theta^h \in (1/2, 1)$ and shoe-size is petite, and her less able, Ugly Stepsister, whose ability level is $\theta^\ell = 1 - \theta^h$ and shoe-size is extra large. At the time of contracting, the agent's identity is unknown to the principal; to eliminate one piece of notation, we assume that the probability that the agent is Cinderella is equal to her ability level. However, it is commonly known that soon after the contract has been signed, a prince's footman will, with probability $1-\chi$, deliver a petite glass slipper to the house where both Cinderella and her sister live, at which point the identities of both ladies will be revealed to the world. With probability χ, however, the footman will be waylaid en route to Cinderella's house and the slipper will be smashed to pieces.

To minimize information rent payments, the principal should offer a contract menu in which *production instructions* are specified ex ante but *fixed transfers* are specified on a *contingent* basis, depending on whether the footman is able to deliver the slipper and if so, whether the agent is revealed to have lied about her type:

(1) conditional on the slipper revealing the agents' identities, the payment corresponding to each set of production instructions will equal the sum of the reservation utility and production costs of the

agent for whom the instructions are designed; in the event of revelation, if the agent has accepted the contract *designed for her true type*, then she will receive zero information rents.

(2) if the agent is revealed to have lied, she will receive zero compensation;

(3) if the slipper fails to arrive, the payment targeted for Cinderella will include an information rent in the usual way.

Clearly, this kind of contract will induce truth-telling behavior while paying out information rents only with probability $\chi\theta^h$. To reduce notation, we set $\gamma = \chi\theta^h$. Stated formally, the principal's task is to maximize the following objective function:

$$\max_{\tilde{q}} \quad \left\{ \sum_{\theta_i \in \{\theta^\ell, \theta^h\}} \left\{ \theta_i \Big(pq(\theta_i) - C(q, \theta_i) \Big) \right\} \quad + \quad \gamma I(q) \right\} \tag{11}$$

That is, the principal is required to pay all *production* costs with probability one, but the need to pay an information rent arises only with the probability γ that the footman fails to deliver the slipper. By standard arguments, whether the contract is basic or restricted, the production instructions targeted for Cinderella will specify first-best inputs and outputs, regardless of the value of γ. For the remainder of the paper, we shall entirely ignore this trivial aspect of the principal's problem, and focus our attention on the contractual specifications for agent "ℓ". Since θ^h is held constant, a change in γ is proportional to a change in χ. The design of this thought experiment ensures that as we vary γ holding $\{\theta^\ell, \theta^h\}$ constant, the first-best, complete information benchmark remains constant. Thus when $\gamma > 0$, the rate at which principal's profits decline as γ increases is a pure measure of the *marginal* cost to the principal of of information asymmetry. With this construction in place, we can compare this shortfall under our two different kinds of production contract.

3.1. **Basic Contract**

For each contract type, we proceed in two steps. First, for fixed q, we determine the minimal cost of producing at least q under the basic contract, given the level of information asymmetry γ. Slightly abusing notation, we call this the *basic cost function,* $\tilde{C}(q,\gamma)$.[2] Second, we select the profit maximal level of q, given $\tilde{C}(q,\gamma)$, by equating the *basic marginal cost function,* $\widetilde{MC}(q,\gamma) = \frac{d\tilde{C}(q,\gamma)}{dq}$, to the price level, p.

In order to compute our marginal cost function, it is, for reasons that will become apparent, necessary for us to set up a cost minimization problem subject not to the usual equality constraints but to *inequality*

[2] In the preceding section, we defined $\tilde{C}(q,\theta)$. As we shall see, $\tilde{C}(q,\gamma)$ is a linear combination of $\tilde{C}(q,\theta^\ell)$ and $\tilde{C}(q,\theta^h)$.

constraints. That is, we minimize the cost to the principal of having the low-ability agent produce *at least* q, while requiring that if the high agent imitates, he produces *at most* q, The point here is that if the principal were able to, she could reduce information rents by requiring that an imitating agent produce *more than* q. Obviously she cannot impose this requirement, hence the constraint. While this setup is nonstandard, it clearly produces the right result, which is that the principal chooses to have both the low-ability and the imitating high-ability agent produce q.

In the basic contract, under homotheticity, the ratio between capital and labor is fixed, i.e., capital and labor lie on the "neoclassical" ray along which $f^i_e/f^i_k = v/r$, where v denotes the wage rate. Let $\tilde{\beta}$ be the scalar defining the neoclassical ray, i.e., for each i, $k^i = \tilde{\beta}e^i$. Since the k's are not choice variable, the problem under the basic contract of minimizing the cost of producing at least q, when imitators can be required to produce at most q, is

$$\min_{\mathbf{e}} \tilde{C}(\mathbf{e};q,\gamma) \text{ s.t. } f^\ell(e^\ell,\tilde{\beta}e^\ell) \geq q \text{ and } f^h(e^h,\tilde{\beta}e^h) \leq q,$$

where $\mathbf{e}$ is a vector of effort levels and

$$\begin{aligned}\tilde{C}(\mathbf{e};q,\gamma) &= \left(ve^\ell + r\tilde{\beta}e^\ell\right) + \gamma\left(v(e^\ell - e^h) + \tilde{\beta}r(e^\ell - e^h)\right) \\ &= (\theta^\ell + \gamma)(ve^\ell + r\tilde{\beta}e^\ell) - \gamma(ve^h + r\tilde{\beta}e^h)\end{aligned}$$

The Lagrangian for the cost minimization problem under the basic contract is:

$$\begin{aligned}\tilde{L}(\mathbf{e},\boldsymbol{\lambda};q,\gamma) &= \tilde{C}(\mathbf{e};q,\gamma) + \lambda^{\ell B}(q - f^\ell(e^\ell,\tilde{\beta}e^\ell)) + \lambda^{hB}(f^h(e^h,\tilde{\beta}e^h) - q) \\ &= (\theta^\ell + \gamma)(ve^\ell + r\tilde{\beta}e^\ell) - \gamma(ve^h + r\tilde{\beta}e^h) + \lambda^{\ell B}(q - f^\ell(e^\ell,\tilde{\beta}e^\ell)) + \lambda^{hB}(f^h(e^h,\tilde{\beta}e^h) - q) \qquad (12)\end{aligned}$$

<u>Cost minimization under the basic contract</u>:

The first order condition for $\tilde{L}$ has four equations in four unknowns.

$$\nabla\tilde{L} = \begin{bmatrix} \tilde{L}_{e^\ell} \\ \tilde{L}_{e^h} \\ \tilde{L}_{\lambda^{\ell B}} \\ \tilde{L}_{\lambda^{hB}} \end{bmatrix} = \begin{bmatrix} (\theta^\ell + \gamma)(v + \tilde{\beta}r) - \lambda^{\ell B}(f^\ell_e + \tilde{\beta} f^\ell_k) \\ -\gamma(v + \tilde{\beta}r) + \lambda^{hB}(f^h_e + \tilde{\beta} f^h_k) \\ q - f^\ell(e^\ell,\tilde{\beta}e^\ell) \\ f^h(e^h,\tilde{\beta}e^h) - q \end{bmatrix} = 0. \qquad (13)$$

Since at the solution $\left(\mathbf{e}^B(q,\gamma),\boldsymbol{\lambda}^B(q,\gamma)\right)$ to the Lagrangian (12) of the basic problem the constraints are identically zero, the *basic cost function* $\tilde{C}(q,\gamma)$, defined as the minimum attainable value of total cost under the basic contract for each (q,γ) pair, is identically equal to $\tilde{L}(\mathbf{e}^B(q,\gamma),\boldsymbol{\lambda}^B(q,\gamma);q,\gamma)$. For future reference it is useful to decompose $\tilde{C}(q,\gamma)$ into $\tilde{C}^P(q,\gamma)+\tilde{C}^{\mathrm{I}}(q,\gamma)$, where $\tilde{C}^P(q,\gamma) = (ve^{\ell B}(q,\gamma)+r\tilde{\beta}e^{\ell B}(q,\gamma))$ is the *production cost* and $\tilde{C}^{\mathrm{I}}(q,\gamma) = \gamma\Big((ve^{\ell B}(q,\gamma)+r\tilde{\beta}e^{\ell B}(q,\gamma)) - (ve^{hB}(q,\gamma)+r\tilde{\beta}e^{hB}(q,\gamma)\Big)$ is the *information cost* of producing q under the basic contract.

Note from (13) that

$$\lambda^{\ell B} \quad = \quad \frac{(\theta^\ell+\gamma)(v+\tilde{\beta}r)}{f_e^\ell+\tilde{\beta}f_k^\ell} \quad > \quad \lambda^{hB} \quad = \quad \frac{\gamma(v+\tilde{\beta}r)}{f_e^h+\tilde{\beta}f_k^h} \tag{14}$$

The strict inequality holds because the numerator of $\lambda^{\ell B}$ is larger than the numerator of λ^{hB}, while—since "h" is more efficient than "ℓ" and the f_j^i's are evaluated at the same level of output—the denominator of $\lambda^{\ell B}$ is smaller than the denominator of λ^{hB}.

<u>Profit maximization under the basic contract</u>:

The *basic marginal cost function*, denoted by $\widetilde{MC}$, is identically equal to the total derivative of $\tilde{L}(\mathbf{e}^B(q,\gamma),\boldsymbol{\lambda}^B(q,\gamma);q,\gamma)$ w.r.t. q, which in turn, by the envelope theorem (Varian, 3rd edition p. 502), is equal to the *partial* derivative of $\tilde{L}(\mathbf{e}^B(q,\gamma),\boldsymbol{\lambda}^B(q,\gamma);q,\gamma)$ w.r.t. q. This partial derivative in turn equals the difference between the two Lagrangians, $\lambda^{\ell B}$ and λ^{hB}, i.e., $\widetilde{MC}(q,\gamma)\equiv\lambda^{hB}(q,\gamma)-\lambda^{\ell B}(q,\gamma)$.

Proposition 2. *As the degree of information asymmetry increases, output decreases for the optimal basic contract.*

Proof. At the principal's optimum, $\widetilde{MC}(\tilde{q}(\gamma),\gamma)\equiv p$, where $\tilde{q}(\gamma)$ is the profit maximizing level of output produced by the agent of type "ℓ" at price p under the basic contract. Applying the implicit function theorem, we obtain.

$$\begin{aligned}\frac{d\tilde{q}(\gamma)}{d\gamma} &= -\frac{d\widetilde{MC}(q,\gamma)}{d\gamma}\Big/\frac{d\widetilde{MC}(q,\gamma)}{dq}\\ &= -\left(\frac{d\lambda^{\ell B}(q,\gamma)}{d\gamma}-\frac{d\lambda^{hB}(q,\gamma)}{d\gamma}\right)\Big/\left(\frac{d\lambda^{\ell B}(q,\gamma)}{dq}-\frac{d\lambda^{hB}(q,\gamma)}{dq}\right)\end{aligned} \tag{15}$$

In order to compute the derivatives of the λ^{iB}'s w.r.t. q and γ, we need to apply the implicit function theorem to the first order conditions (13) for the basic problem. This involves the Hessian of $\tilde{L}$ (expression (12)), which is:

$$\widetilde{HL} = \begin{bmatrix} -\lambda^{\ell B}(f^{\ell}_{ee}+2\tilde{\beta} f^{\ell}_{ek}+\tilde{\beta}^2 f^{\ell}_{kk}) & 0 & -(f^{\ell}_e+\tilde{\beta} f^{\ell}_k) & 0 \\ 0 & \lambda^{hB}(f^{h}_{ee}+2\tilde{\beta} f^{h}_{ek}+\tilde{\beta}^2 f^{h}_{kk}) & 0 & (f^{h}_e+\tilde{\beta} f^{h}_k) \\ -(f^{\ell}_e+\tilde{\beta} f^{\ell}_k) & 0 & 0 & 0 \\ 0 & (f^{h}_e+\tilde{\beta} f^{h}_k) & 0 & 0 \end{bmatrix}.$$

Taking the inverse of $\widetilde{HL}$, we obtain:

$$\widetilde{HL}^{-1} = \begin{bmatrix} 0 & 0 & -(f^{\ell}_e+\tilde{\beta} f^{\ell}_k)^{-1} & 0 \\ 0 & 0 & 0 & (f^{h}_e+\tilde{\beta} f^{h}_k)^{-1} \\ -(f^{\ell}_e+\tilde{\beta} f^{\ell}_k)^{-1} & 0 & \lambda^{\ell B}\frac{(f^{\ell}_{ee}+2\tilde{\beta} f^{\ell}_{ek}+\tilde{\beta}^2 f^{\ell}_{kk})}{(f^{\ell}_e+\tilde{\beta} f^{\ell}_k)^{-2}} & 0 \\ 0 & (f^{h}_e+\tilde{\beta} f^{h}_k)^{-1} & 0 & -\lambda^{hB}\frac{(f^{h}_{ee}+2\tilde{\beta} f^{h}_{ek}+\tilde{\beta}^2 f^{h}_{kk})}{(f^{h}_e+\tilde{\beta} f^{h}_k)^{-2}} \end{bmatrix}.$$

We now have

$$\begin{bmatrix} \partial e^{\ell B}/\partial\gamma & \partial e^{\ell B}/\partial q \\ \partial e^{hB}/\partial\gamma & \partial e^{hB}/\partial q \\ \partial\lambda^{\ell B}/\partial\gamma & \partial\lambda^{\ell B}/\partial q \\ \partial\lambda^{hB}/\partial\gamma & \partial\lambda^{hB}/\partial q \end{bmatrix} = -\widetilde{HL}^{-1} \begin{bmatrix} \partial\tilde{L}_{e^{\ell}}/\partial\gamma & \partial\tilde{L}_{e^{\ell}}/\partial q \\ \partial\tilde{L}_{e^{h}}/\partial\gamma & \partial\tilde{L}_{e^{h}}/\partial q \\ \partial\tilde{L}_{\lambda^{\ell B}}/\partial\gamma & \partial\tilde{L}_{\lambda^{\ell B}}/\partial q \\ \partial\tilde{L}_{\lambda^{hB}}/\partial\gamma & \partial\tilde{L}_{\lambda^{hB}}/\partial q \end{bmatrix} \tag{16}$$

$$= \begin{bmatrix} 0 & 0 & (f^{\ell}_e+\tilde{\beta} f^{\ell}_k)^{-1} & 0 \\ 0 & 0 & 0 & -(f^{h}_e+\tilde{\beta} f^{h}_k)^{-1} \\ (f^{\ell}_e+\tilde{\beta} f^{\ell}_k)^{-1} & 0 & -\lambda^{\ell B}\frac{(f^{\ell}_{ee}+2\tilde{\beta} f^{\ell}_{ek}+\tilde{\beta}^2 f^{\ell}_{kk})}{(f^{\ell}_e+\tilde{\beta} f^{\ell}_k)^{2}} & 0 \\ 0 & -(f^{h}_e+\tilde{\beta} f^{h}_k)^{-1} & 0 & \lambda^{hB}\frac{(f^{h}_{ee}+2\tilde{\beta} f^{h}_{ek}+\tilde{\beta}^2 f^{h}_{kk})}{(f^{h}_e+\tilde{\beta} f^{h}_k)^{2}} \end{bmatrix} \begin{bmatrix} (v+\tilde{\beta} r) & 0 \\ -(v+\tilde{\beta} r) & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \tag{17}$$

Hence:

$$\begin{aligned} \frac{d\lambda^{\ell B}(q,\gamma)}{d\gamma} - \frac{d\lambda^{hB}(q,\gamma)}{d\gamma} &= (v+\tilde{\beta} r)\left((f^{\ell}_e+\tilde{\beta} f^{\ell}_k)^{-1}-(f^{h}_e+\tilde{\beta} f^{h}_k)^{-1}\right) \\ &= \frac{(v+\tilde{\beta} r)}{(f^{\ell}_e+\tilde{\beta} f^{\ell}_k)}\left(1-\vartheta^{-1/\alpha}\right) \quad > \quad 0. \end{aligned} \tag{18}$$

where $\vartheta = \frac{\theta^h}{\theta^\ell} > 1$. The second line follows from the first because (Assumption A3) $f^\ell(e^\ell,k^\ell) = f^h(e^h,k^h) = \vartheta f^\ell(e^h,k^h)$. Since f^ℓ is homogeneous of degree α, it follows that $(e^h,k^h) = \vartheta^{-1/\alpha}(e^\ell,k^\ell)$. and $(f^h_e + \tilde{\beta} f^h_k) = \vartheta^{1/\alpha}(f^\ell_e + \tilde{\beta} f^\ell_k)$. To verify that this expression is positive, note that $\vartheta > 1$ and $\alpha < 1$ so that $\vartheta^{-1/\alpha} < 1$.

Similarly, since f^h_{ee}, f^h_{ek} and f^h_{kk} are homog. of deg. $\alpha - 2$, $(f^h_{ee} + 2\tilde{\beta} f^h_{ek} + \tilde{\beta}^2 f^h_{kk}) = \vartheta^{2/\alpha}(f^\ell_{ee} + 2\tilde{\beta} f^\ell_{ek} + \tilde{\beta}^2 f^\ell_{kk})$. Therefore

$$\begin{aligned}
\frac{d\lambda^{\ell B}(q,\gamma)}{dq} - \frac{d\lambda^{hB}(q,\gamma)}{dq} &= -\left(\lambda^{\ell B}\frac{(f^\ell_{ee} + 2\tilde{\beta} f^\ell_{ek} + \tilde{\beta}^2 f^\ell_{kk})}{(f^\ell_e + \tilde{\beta} f^\ell_k)^2} - \lambda^{hB}\frac{(f^h_{ee} + 2\tilde{\beta} f^h_{ek} + \tilde{\beta}^2 f^h_{kk})}{(f^h_e + \tilde{\beta} f^h_k)^2}\right) \\
&= -\frac{(f^\ell_{ee} + 2\tilde{\beta} f^\ell_{ek} + \tilde{\beta}^2 f^\ell_{kk})}{(f^\ell_e + \tilde{\beta} f^\ell_k)^2}\left(\lambda^{\ell B} - \lambda^{hB}\right) \qquad (19) \\
&= -\frac{\alpha - 1}{e^\ell(f^\ell_e + \tilde{\beta} f^\ell_k)}\left(\lambda^{\ell B} - \lambda^{hB}\right) \quad > \quad 0 \qquad (20)
\end{aligned}$$

Expression (20) is positive because $\lambda^{\ell B} > \lambda^{hB}$ (see (14)) and $\alpha < 1$. Expression (20) follows from (19) because, since f^ℓ_e and f^ℓ_k are homogeneous of degree $\alpha - 1$,

$$\begin{aligned}
(f^\ell_{ee} + 2\tilde{\beta} f^\ell_{ek} + \tilde{\beta}^2 f^\ell_{kk}) &= \frac{(e^\ell f^\ell_{ee} + k^\ell f^\ell_{ek})}{e^\ell} + \tilde{\beta}^2\frac{(k^\ell f^\ell_{kk} + e^\ell f^\ell_{ek})}{k^\ell} \\
&= (\alpha - 1)\left(\frac{f^\ell_e}{e^\ell} + \tilde{\beta}^2\frac{f^\ell_k}{k^\ell}\right) \\
&= (\alpha - 1)(e^\ell)^{-1}\left(f^\ell_e + \tilde{\beta} k^\ell\right) \qquad (21)
\end{aligned}$$

Dividing (18) by the negative of (20), we can now write (15) explicitly as:

$$\frac{d\tilde{q}(\gamma)}{d\gamma} = \frac{(ve^\ell + rk)(1 - \vartheta^{-1/\alpha})}{(\alpha - 1)(\lambda^{\ell B} - \lambda^{hB})} < 0 \qquad (15')$$

Expression (15′) is negative because $\lambda^{\ell B} > \lambda^{hB}$, $\vartheta^{-1/\alpha} < 1$ and $\alpha < 1$. □

3.2. **Restricted Contract**

We proceed exactly as for the basic contract. For fixed q and γ, we determine the minimal cost of producing q under the restricted contract. Call this the *restricted cost function*, $\bar{C}(q,\gamma)$. We then select the profit maximal level of q, given $\bar{C}(q,\gamma)$, by equating the *restricted marginal cost function*, $\overline{MC}(q,\gamma) = \frac{d\bar{C}(q,\gamma)}{dq}$, to the price level, p.

The cost minimization problem under the restricted contract is:

$$\min \bar{C}(\mathbf{e},k;q,\gamma) \text{ s.t. } f^{\ell}(e^{\ell},k) \geq q \text{ and } f^{h}(e^{\ell},k) \leq q$$

where

$$\begin{aligned}\bar{C}(\mathbf{e},k;q,\gamma) &= \left(ve^{\ell}+rk\right)+\gamma v(e^{\ell}-e^{h})\\ &= v\left((\theta^{\ell}+\gamma)e^{\ell}-\gamma e^{h}\right)+rk\end{aligned}$$

The Lagrangian for this problem is:

$$\begin{aligned}\bar{L}(\mathbf{e},k,\boldsymbol{\lambda}^{R};q,\gamma) =&\bar{C}(\mathbf{e},k;q,\gamma)+\lambda^{\ell}(q-f^{\ell}(e^{\ell},k))+\lambda^{h}(q-f^{h}(e^{h},k))\\ =&v\left((\theta^{\ell}+\gamma)e^{\ell}-\gamma e^{h}\right)+rk+\lambda^{\ell}(q-f^{\ell}(e^{\ell},k))+\lambda^{h}(f^{h}(e^{h},k)-q))\end{aligned} \tag{22}$$

<u>Cost minimization under the restricted contract</u>:

The first order condition for $\bar{L}$ has five equations in five unknowns;

$$\nabla\bar{L} = \begin{bmatrix}\bar{L}_{e^{\ell}}\\ \bar{L}_{e^{h}}\\ \bar{L}_{k}\\ \bar{L}_{\lambda^{\ell R}}\\ \bar{L}_{\lambda^{hR}}\end{bmatrix} = \begin{bmatrix}(\theta^{\ell}+\gamma)v-\lambda^{\ell R}f_{e}^{\ell}\\ -\gamma v+\lambda^{hR}f_{e}^{h}\\ r-\lambda^{\ell R}f_{k}^{\ell}+\lambda^{hR}f_{k}^{h}\\ q-f^{\ell}(e^{\ell},k)\\ f^{h}(e^{h},k)-q\end{bmatrix} = 0. \tag{23}$$

Once again, at the solution $\left(\mathbf{e}^{R}(q,\gamma),k^{R}(q,\gamma),\boldsymbol{\lambda}^{R}(q,\gamma)\right)$ to the Lagrangian (22) of the restricted problem, the constraints are identically zero and the *restricted cost function* $\bar{C}(q,\gamma)$, defined as the minimum attainable value of total cost under the restricted contract for each (q,γ) pair, is identically equal to $\bar{L}(\mathbf{e}^{R}(q,\gamma),k^{R}(q,\gamma),\boldsymbol{\lambda}^{R}(q,\gamma);q,\gamma)$. As before, we decompose $\bar{C}(q,\gamma)$ into $\bar{C}^{P}(q,\gamma)+\bar{C}^{\mathrm{I}}(q,\gamma)$, where $\bar{C}^{P}(q,\gamma)=(ve^{\ell R}(q,\gamma)+rk^{R}(q,\gamma))$ is the *production cost* and $\bar{C}^{\mathrm{I}}(q,\gamma)=\gamma v(e^{\ell R}(q,\gamma)-e^{hR}(q,\gamma))$ is the *information cost* of producing q under the restricted contract.

Note from (23) that

$$\lambda^{\ell R} = \frac{(\theta^{\ell}+\gamma)v}{f_{e}^{\ell}} > \lambda^{hR} = \frac{\gamma v}{f_{e}^{h}} \tag{24}$$

To see that the strict inequality holds, note first the numerator of $\lambda^{\ell R}$ is larger than the numerator of λ^{hR}. Second note that since "*h*" is more efficient than "ℓ" and both "*h*" and "ℓ," are using the same level of capital while the f_e^i's are evaluated at the same level of output, the denominator of $\lambda^{\ell R}$ is smaller than the denominator of λ^{hR}. An immediate implication of (24) is:

Proposition 3. *In a restricted contract for a given q and positive* γ*, the prescribed capital-labor ratio is greater than the neoclassical ratio.*

Proof. Observe first that after substituting for the λ's in (23), $\bar{L}_k$ can be rewritten as

$$1 - \frac{v}{r}\frac{\bar{f}_k^\ell}{\bar{f}_e^\ell} = \frac{\gamma v}{r}\left(\frac{\bar{f}_k^\ell}{\bar{f}_e^\ell} - \frac{\bar{f}_k^h}{\bar{f}_e^h}\right) \tag{25}$$

Since "*h*" is more efficient than "ℓ" and both are using the same level of capital to produce the same level of output, "*h*"'s effort level under the restricted contract must be less than "ℓ"'s. That is, $\frac{k^R}{e^{hR}} > \frac{k^R}{e^{\ell R}}$ which in turn implies $\frac{\bar{f}_k^h}{\bar{f}_e^h} < \frac{\bar{f}_k^\ell}{\bar{f}_e^\ell}$. Hence the right hand side of (25) is positive. Hence $\frac{\bar{f}_k^\ell}{\bar{f}_e^\ell} < \frac{r}{v}$. □

Fig. 2 below provides some intuition for this result. Its top panel reproduces Fig. 1 above. Consider the effect on the principal's problem of increasing γ from zero, for the moment holding the output level constant at an arbitrary output level q^0. By the envelope theorem, a small increase in capital intensity above the neoclassical level has only a second-order impact on the production costs of agent "ℓ" (see the bottom panel of Fig. 2). On the other hand, since the initial capital level is already super-optimal for agent "h", the given increase would result in a *first*-order increase in agent "h"'s production cost if he accepted the contract designed for "ℓ". Thus, a small increase in capital intensity beyond the neoclassical level for "ℓ" results in a first order reduction in information costs, and a second-order increase in production costs. It follows that whenever $\gamma > 0$, the prescribed level of capital for agent "ℓ" will exceed the neoclassical level for her prescribed level of output.

Profit maximization under the restricted contract:

The *restricted marginal cost function*, denoted by $\overline{MC}$, is identically equal to the total derivative of $\bar{L}(\mathbf{e}^R(q,\gamma),\boldsymbol{\lambda}^R(q,\gamma);q,\gamma)$ w.r.t. q. Applying the envelope theorem as on page 14, $\overline{MC}$ is identically equal to the total derivative of $\bar{L}(\mathbf{e}^R(q,\gamma),\boldsymbol{\lambda}^R(q,\gamma);q,\gamma)$ w.r.t. q, which in turn is equal to the partial derivative of $\bar{L}(\mathbf{e}^R(q,\gamma),\boldsymbol{\lambda}^R(q,\gamma);q,\gamma)$ w.r.t. q. This partial derivative in turn equals the difference between the two Lagrangians, $\lambda^{\ell R}$ and λ^{hR}, i.e., $\overline{MC}(q,\gamma) \equiv \lambda^{hR}(q,\gamma) - \lambda^{\ell R}(q,\gamma)$. At the principal's optimum, $\overline{MC}(\bar{q}(\gamma),\gamma) \equiv p$,

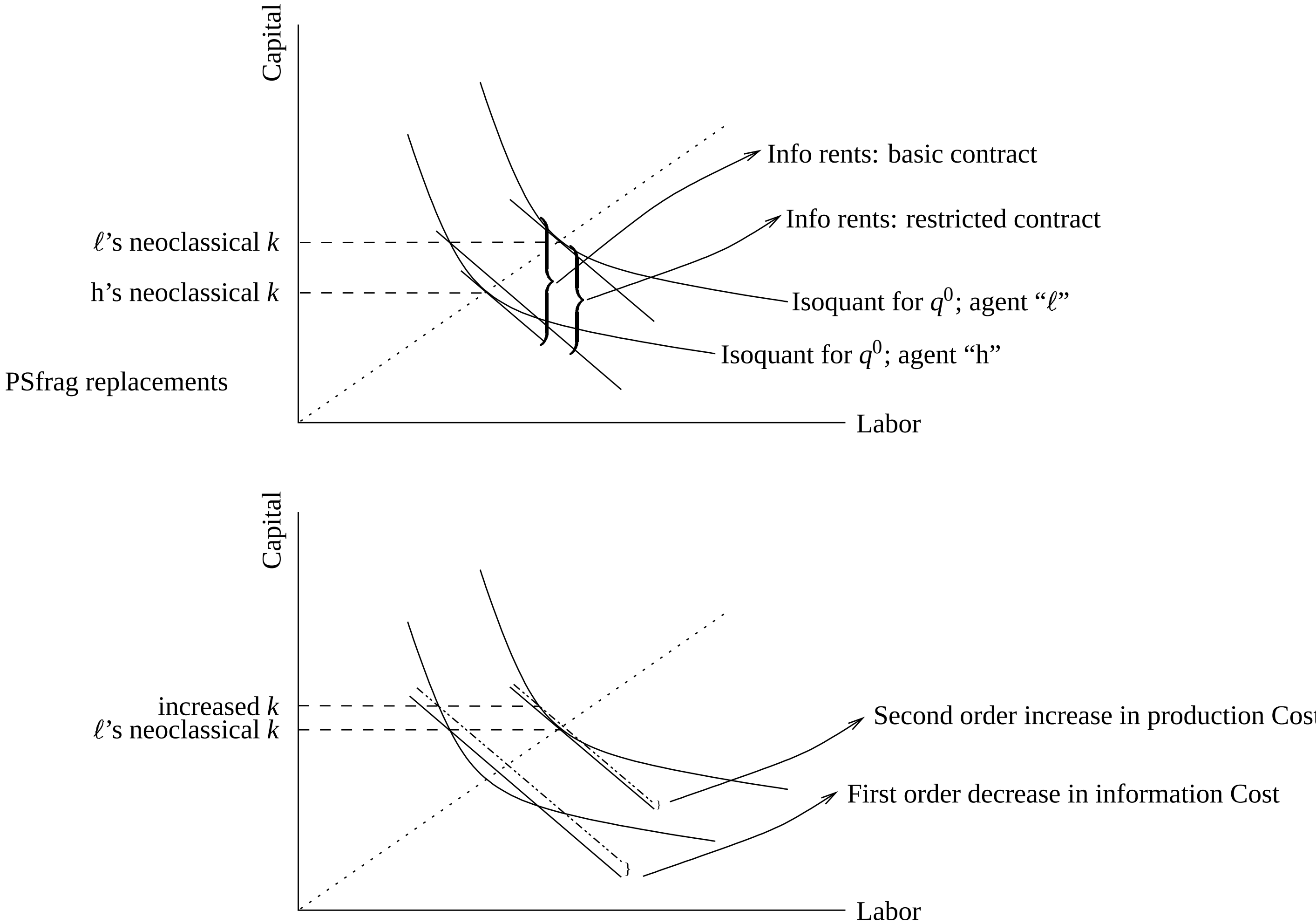

FIGURE 2. Effect on Production and Information Cost of increasing k

where $\overline{q}(\gamma)$ is the profit maximizing level of output produced by the agent of type "ℓ" at price p under the restricted contract. Once again, we apply the implicit function theorem to obtain:

Proposition 4. *As the degree of information asymmetry increases, output decreases for the optimal restricted contract.*

Proof. To obtain an expression for $\frac{d\overline{q}(\gamma)}{d\gamma}$ we proceed exactly as we did to obtain expression (15) on page 14, i.e.,

$$\begin{aligned} \frac{d\overline{q}(\gamma)}{d\gamma} &= -\frac{d\overline{MC}(q,\gamma)}{d\gamma} \Big/ \frac{d\overline{MC}(q,\gamma)}{dq} \\ &= -\left(\frac{d\lambda^{\ell R}(q,\gamma)}{d\gamma} - \frac{d\lambda^{hR}(q,\gamma)}{d\gamma}\right) \Big/ \left(\frac{d\lambda^{\ell R}(q,\gamma)}{dq} - \frac{d\lambda^{hR}(q,\gamma)}{dq}\right) \end{aligned} \tag{26}$$

Once again, we apply the implicit function theorem to obtain the derivatives of the λ^{iR}'s w.r.t. q and γ. The Hessian of $\bar{L}$ (expression (23)) is:

$$\overline{HR} = \begin{bmatrix} -\lambda^{\ell R} f^{\ell}_{ee} & 0 & -\lambda^{\ell R} f^{\ell}_{ek} & -f^{\ell}_{e} & 0 \\ 0 & \lambda^{hR} f^{h}_{ee} & \lambda^{hR} f^{h}_{ek} & 0 & f^{h}_{e} \\ -\lambda^{\ell R} f^{\ell}_{ek} & \lambda^{hR} f^{h}_{ek} & (\lambda^{hR} f^{h}_{kk} - \lambda^{\ell R} f^{\ell}_{kk}) & -f^{\ell}_{k} & f^{h}_{k} \\ -f^{\ell}_{e} & 0 & -f^{\ell}_{k} & 0 & 0 \\ 0 & f^{h}_{e} & f^{h}_{k} & 0 & 0 \end{bmatrix}.$$

Because the inverse of $\overline{HR}$ is complex, we replace terms that we do not need to evaluate in the expression below by $\Box$'s:

$$\begin{bmatrix} \partial e^{\ell R}/\partial\gamma & \partial e^{\ell R}/\partial q \\ \partial e^{hR}/\partial\gamma & \partial e^{hR}/\partial q \\ \partial k^{R}/\partial\gamma & \partial k^{R}/\partial q \\ \partial\lambda^{\ell R}/\partial\gamma & \partial\lambda^{\ell R}/\partial q \\ \partial\lambda^{hR}/\partial\gamma & \partial\lambda^{hR}/\partial q \end{bmatrix} = -\overline{HR}^{-1} \begin{bmatrix} \partial\bar{L}_{e^{\ell}}/\partial\gamma & \partial\bar{L}_{e^{\ell}}/\partial q \\ \partial\bar{L}_{e^{h}}/\partial\gamma & \partial\bar{L}_{e^{h}}/\partial q \\ \partial\bar{L}_{k}/\partial\gamma & \partial\bar{L}_{k}/\partial q \\ \partial\bar{L}_{\lambda^{\ell R}}/\partial\gamma & \partial\bar{L}_{\lambda^{\ell R}}/\partial q \\ \partial\bar{L}_{\lambda^{hR}}/\partial\gamma & \partial\bar{L}_{\lambda^{hR}}/\partial q \end{bmatrix} = \frac{\lambda^{\ell R}\lambda^{hR}}{\Delta^{\overline{HR}}} \times \cdots \qquad (27)$$

$$\begin{bmatrix} \frac{{f^{h}_{e}}^{2} {f^{\ell}_{k}}^{2}}{\lambda^{\ell R}\lambda^{hR}} & \frac{f^{h}_{e} f^{\ell}_{k} f^{h}_{k} f^{\ell}_{e}}{\lambda^{\ell R}\lambda^{hR}} & \Box & \frac{{f^{h}_{e}}^{2}\beta^{\ell}\mu(\ell)}{\lambda^{hR}} - \frac{f^{\ell}_{e}\mathrm{PM}(h)}{\lambda^{\ell R}} & -\frac{f^{\ell}_{e} f^{\ell}_{k}\mu(h)}{\lambda^{\ell R}} \\ \frac{f^{h}_{e} f^{\ell}_{k} f^{h}_{k} f^{\ell}_{e}}{\lambda^{\ell R}\lambda^{hR}} & \frac{{f^{\ell}_{e}}^{2} {f^{h}_{k}}^{2}}{\lambda^{\ell R}\lambda^{hR}} & \Box & -\frac{f^{h}_{e} f^{h}_{k}\mu(\ell)}{\lambda^{hR}} & \frac{{f^{\ell}_{e}}^{2}\beta^{h}\mu(h)}{\lambda^{\ell R}} - \frac{f^{h}_{e}\mathrm{PM}(\ell)}{\lambda^{hR}} \\ \frac{-{f^{h}_{e}}^{2} f^{\ell}_{k} f^{\ell}_{e}}{\lambda^{\ell R}\lambda^{hR}} & \frac{-{f^{\ell}_{e}}^{2} f^{h}_{k} f^{h}_{e}}{\lambda^{\ell R}\lambda^{hR}} & \Box & \frac{{f^{h}_{e}}^{2}\mu(\ell)}{\lambda^{hR}} & \frac{{f^{\ell}_{e}}^{2}\mu(h)}{\lambda^{\ell R}} \\ \frac{(f^{h}_{e})^{2}\beta^{\ell}\mu(\ell)}{\lambda^{hR}} - \frac{f^{\ell}_{e}\mathrm{PM}(h)}{\lambda^{\ell R}} & -\frac{f^{h}_{e} f^{h}_{k}\mu(\ell)}{\lambda^{hR}} & \Box & f^{\ell}_{ee}\mathrm{PM}(h) - \frac{\lambda^{\ell R}(f^{h}_{e})^{2}\Delta(f,\ell)}{\lambda^{hR}} & \mu(\ell)\mu(h) \\ -\frac{f^{\ell}_{e} f^{\ell}_{k}\mu(h)}{\lambda^{\ell R}} & \frac{(f^{\ell}_{e})^{2}\beta^{h}\mu(h)}{\lambda^{\ell R}} - \frac{f^{h}_{e}\mathrm{PM}(\ell)}{\lambda^{hR}} & \Box & \mu(\ell)\mu(h) & f^{h}_{ee}\mathrm{PM}(\ell) - \frac{\lambda^{hR}(f^{\ell}_{e})^{2}\lambda^{\ell R}}{\Delta(f,h)} \end{bmatrix} \begin{bmatrix} \nu & 0 \\ -\nu & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$$

Where for $i = h, \ell$:

$$\mu(i) = f^{i}_{ee} f^{i}_{k} - f^{i}_{ek} f^{i}_{e}$$

$$\beta^{i} = k^{i}/\ell^{i}$$

$$\mathrm{PM}(i) = f^{i}_{ee}(f^{i}_{k})^{2} + f^{i}_{kk}(f^{i}_{e})^{2} - 2 f^{i}_{ek} f^{i}_{k} f^{i}_{e}$$

$$\Delta(f,i) = f^{i}_{kk} f^{i}_{ee} - (f^{i}_{ek})^{2}$$

The determinant of $\overline{HR}$,

$$\Delta^{\overline{HR}} = \lambda^{hR}(f_e^\ell)^2\mathrm{PM}(h) - \lambda^{\ell R}(f_e^h)^2\mathrm{PM}(\ell) \tag{28}$$

can be signed for sufficiently small values of $\gamma = 0$. To see this, first note that the $\mathrm{PM}(i)$'s are the negatives of second principal minors of the bordered hessian of f, and are hence negative. Second, when γ is small, λ^{hR} is small relative to $\lambda^{\ell R}$, so that for sufficiently small values of γ, $\Delta^{\overline{HR}}$ will be positive.

The numerator of $\frac{d\overline{q}(\gamma)}{d\gamma}$ (expression (26) above) can now be written as

$$\frac{v(f_e^\ell f_e^h)^2(e^h - e^\ell)}{k\Delta^{\overline{HR}}}\left(\lambda^{\ell R}\frac{(f_{ee}^\ell f_k^\ell - f_{ek}^\ell f_e^\ell)}{(f_e^\ell)^2} - \lambda^{hR}\frac{(f_{ee}^h f_k^h - f_{ek}^h f_e^h)}{(f_e^h)^2}\right) \tag{29}$$

while the denominator of (26) is

$$\frac{\left(f_e^\ell f_e^h\right)^2}{k\Delta^{\overline{HR}}}(\lambda^{\ell R} - \lambda^{hR})(1-\alpha)\left(\lambda^{\ell R}\frac{(f_{ee}^\ell f_k^\ell - f_{ek}^\ell f_e^\ell)}{(f_e^\ell)^2} - \lambda^{hR}\frac{(f_{ee}^h f_k^h - f_{ek}^h f_e^h)}{(f_e^h)^2}\right) \tag{30}$$

Dividing (29) by the negative of (30), we can now write (26) explicitly as:

$$\frac{d\overline{q}(\gamma)}{d\gamma} = \frac{v(e^\ell - e^h)}{(\alpha-1)(\lambda^{\ell R} - \lambda^{hR})} < 0 \tag{26$'$}$$

Expression (26$'$) is negative because $e^\ell > e^h$, $\lambda^{\ell B} > \lambda^{hB}$ and $\alpha < 1$. □

3.3. **Comparing Restricted and Basic Contracts**

The following result generalizes and extends Proposition 1 above.

Proposition 5. *For a given q and positive γ, both the information cost and the principal's total cost of optimally obtaining q under a restricted contract are less than the information cost and principal's total cost of optimally obtaining this q under a basic contract. That is,*

$$\textit{for all } q \textit{ and all } \gamma > 0, \quad \overline{C}(q,\gamma) < \tilde{C}(q,\gamma) \quad \textit{and} \quad \overline{C}^{\mathrm{I}}(q,\gamma) < \tilde{C}^{\mathrm{I}}(q,\gamma) \tag{31}$$

Proof. Both parts of the proposition are immediate implications of Proposition 3. The first inequality holds because the neoclassical input mix is feasible under the restricted contract, but, by Proposition 3, violates the first order condition (25). Proposition 3 also implies that the *production* cost of producing any given q under the restricted contract, $\overline{C}^P(q,\gamma) = (ve^{\ell R}(q) + rk^R(q))$ strictly exceeds the production cost of producing the same level of q under the basic contract, $\tilde{C}^P(q,\gamma) = (ve^{\ell B}(q) + rk^{\ell B}(q))$. However, as we have just established,

$$\bar{C}(q,\gamma) \quad = \quad \bar{C}^P(q,\gamma) + \bar{C}^{\mathrm{I}}(q,\gamma) \quad < \quad \tilde{C}^P(q,\gamma) + \tilde{C}^{\mathrm{I}}(q,\gamma) \quad = \quad \tilde{C}(q,\gamma)$$

Therefore, $\bar{C}^{\mathrm{I}}(q,\gamma) < \tilde{C}^{\mathrm{I}}(q,\gamma)$. □

Once again, the proposition reflects the fact, illustrated in Fig. 2, that the first order reduction in information costs obtained by moving away from the neoclassical input mix must necessarily offset the resulting, second order increase in production costs.

A less immediate result is:

Proposition 6. *For any level of* $\gamma \in (0,1]$, *output produced by agent* ℓ *is higher under the optimal restricted contract than under the optimal basic contract.*

Proof. We begin by comparing (26′) for the optimal restricted contral to the corresponding expression, (15′), for the optimal basic contract:

$$\frac{d\bar{q}(\gamma)}{d\gamma} \quad - \quad \frac{d\tilde{q}(\gamma)}{d\gamma} \quad = \quad \frac{1}{1-\alpha}\left(\frac{(ve^{\ell B} + rk^{\ell B})(1-\vartheta^{-1/\alpha})}{\lambda^{\ell B} - \lambda^{hB}} \quad - \quad \frac{v(e^{\ell R} - e^{hR})}{\lambda^{\ell R} - \lambda^{hR}}\right) \tag{32}$$

As we observed on pages 14 and 19, $(\lambda^{\ell B} - \lambda^{hB}) = (\lambda^{\ell R} - \lambda^{hR}) = p$ at the optimum basic and restricted contracts. Also, the numerator of the first fraction on the right hand side of (32) can be rewritten as $(ve^{\ell B} + rk^{\ell B}) - (ve^{hB} + rk^{hB})$ (see the explanation following display (18) on page 15). Hence for $\gamma > 0$, (32) reduces to

$$\frac{d\bar{q}(\gamma)}{d\gamma} \quad - \quad \frac{d\tilde{q}(\gamma)}{d\gamma} \quad = \quad \frac{1}{p\gamma(1-\alpha)}\left(\tilde{C}^{\mathrm{I}}(\tilde{q}(\gamma),\gamma) \quad - \quad \bar{C}^{\mathrm{I}}(\bar{q}(\gamma),\gamma)\right) \tag{32′}$$

We use this result to show that $\bar{q}(\gamma) > \tilde{q}(\gamma)$, for all $\gamma \in (0,1]$. First note that inequality (31) plus continuity implies the existence of $\varepsilon > 0$ such that for any $\gamma > 0$, if $|\bar{q}(\gamma) - \tilde{q}(\gamma)| < \varepsilon$, then (32′) will be positive. Summarizing, we have established that

$$\text{there exists } \varepsilon^\star > 0 \text{ such that } |\bar{q}(\gamma) - \tilde{q}(\gamma)| < \varepsilon^\star \text{ implies } \frac{d\bar{q}(\gamma)}{d\gamma} \quad > \quad \frac{d\tilde{q}(\gamma)}{d\gamma} \tag{33}$$

Now suppose that there exists $\gamma > 0$ such that $\tilde{q}(\gamma) \geq \overline{q}(\gamma)$ and let $\gamma^\star$ be the infimum of such γ's. We will establish a contradiction. Since $\tilde{q}(0) = \overline{q}(0)$, statement (33) implies the existence of $\underline{\gamma} > 0$ such that for all $\gamma \in (0, \underline{\gamma}]$, $\tilde{q}(\gamma) = \int_0^\gamma \frac{d\tilde{q}(\vartheta)}{d\gamma} d\vartheta < \int_0^\gamma \frac{d\overline{q}(\vartheta)}{d\gamma} d\vartheta = \overline{q}(\gamma)$. Therefore, $\gamma^\star > \underline{\gamma} > 0$. Now since $\overline{q}(\cdot) < \tilde{q}(\cdot)$ on $[0, \gamma^\star)$ and $\overline{q}(\cdot) - \tilde{q}(\cdot)$ is clearly continuous w.r.t. γ, there exists $\overline{\gamma} < \gamma^\star$ such that $\overline{q}(\cdot) \in (\tilde{q}(\cdot) - \varepsilon^\star, \tilde{q}(\cdot))$ on $[\overline{\gamma}, \gamma^\star)$. Now

$$(\tilde{q}(\gamma^\star) - \overline{q}(\gamma^\star)) - (\tilde{q}(\overline{\gamma}) - \overline{q}(\overline{\gamma})) = \int_{\overline{\gamma}}^{\gamma^\star} \left(\frac{d\overline{q}(\vartheta)}{d\gamma} - \frac{d\tilde{q}(\vartheta)}{d\gamma} \right) d\vartheta > 0$$

which implies that $\left(\frac{d\overline{q}(\cdot)}{d\gamma} - \frac{d\tilde{q}(\cdot)}{d\gamma} \right)$ is positive on some subset of $(\overline{\gamma}, \gamma^\star)$, which contradicts (33). □

An interpetation of the above result is that the marginal cost curve (including both production and information rent costs), is strictly lower under the restricted contract than under the basic contract. Intuitively, this is because with the restricted contract the principal can limit the extent to which agent "h" could substitute between labor and capital if she were to accept the contract designed for "ℓ".

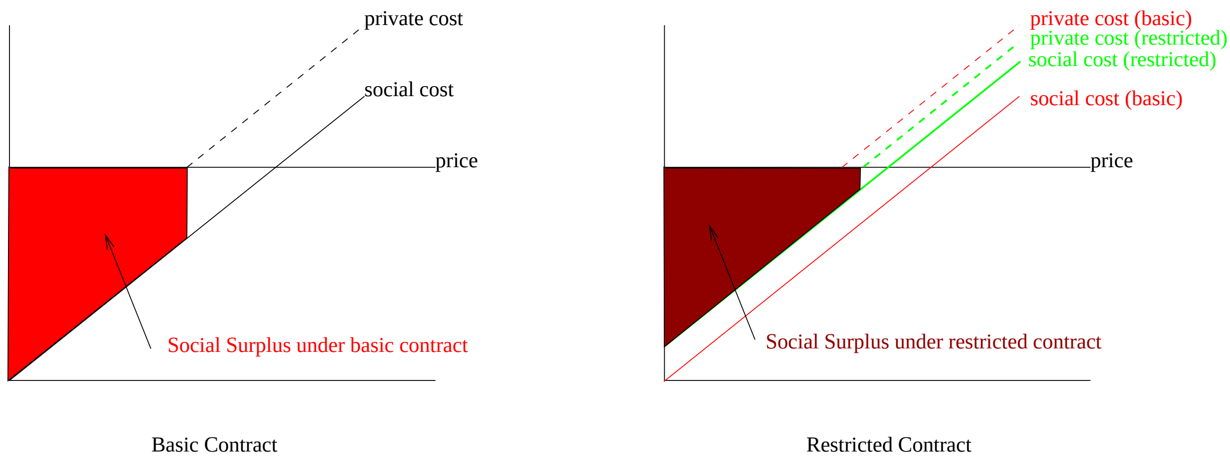

FIGURE 3. Comparison of social surplus under basic and restricted contract

4. THE SOCIAL COST OF INFORMATION ASYMMETRY

As we have seen, the principal's profits are higher under a restricted contract than under a basic contract. This does not necessarily imply, however, that restricted contracts are preferable to basic contracts from a *social* perspective. Briefly, the principal's goal is to minimize the sum of production and information costs, but only production costs matter for social surplus. Information costs are simply a transfer from the principal to agent "h". In terms of the LeChatelier principle, while the principal is made better off, the agent is made worse off, so that the net effect is not pre-determined. In this section we compare the two kinds of contracts from a social perspective when the degree of information asymmetry is "small."

In the present model with perfectly elastic demand, social surplus is the sum of the principal's profit and the information rent received by agent "h". Since the information rent is a pure transfer, social surplus is equal to the principal's total revenue minus *production* cost. Although information rents are lower under the optimal restricted contract, average production costs are higher, because the input mix is sub-optimal from a pure production standpoint. The second factor which affects social surplus is the *level* of production. Proposition 6 establishes that production is always greater under the optimal restricted contract. Whether the production level effect or the input mix effect dominates depends on the degree of substitutability between the two inputs into the production process.[3]

Proposition 7. *There exists* $\underline{\xi} \in (-\infty, 1]$ *and* $\overline{\gamma} > 0$ *such that if the elasticity of substitution between capital and labor is less than* $\underline{\xi}$*, and the degree of information asymmetry is less than* $\overline{\gamma}$*, the optimal restricted*

[3] For the purposes of the following proposition, we define the elasticity of substitution as follows:

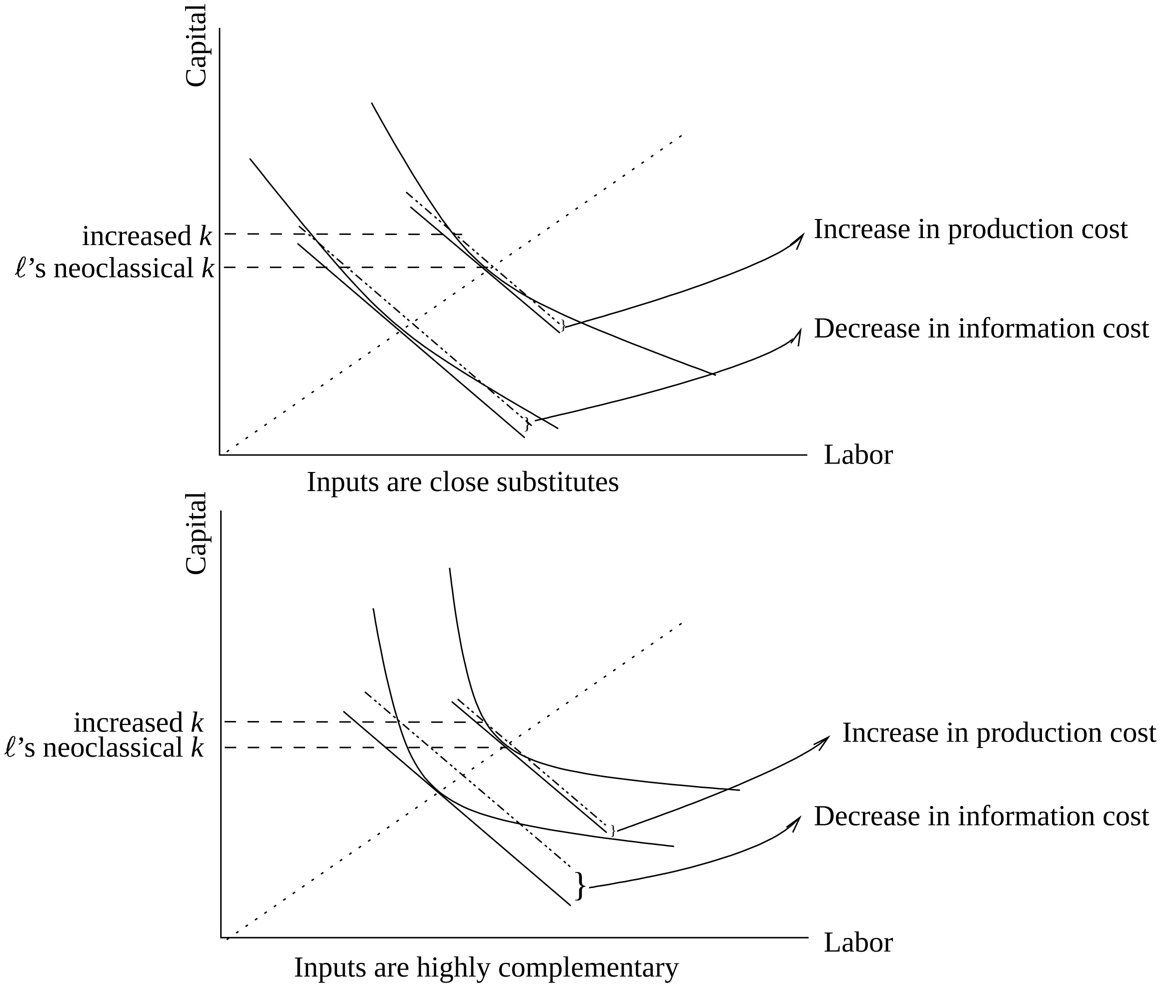

FIGURE 4. Input mix distortions and information rents

contract yields a higher level of social welfare than the basic contract. Similarly, there exists $\overline{\xi} \in (-\infty, 1]$ *and* $\overline{\gamma} > 0$ *such that if the elasticity of substitution between capital and labor is greater than* $\overline{\xi}$*, and the degree of information asymmetry is less than* $\overline{\gamma}$*, the optimal restricted contract yields a lower level of social welfare than the basic contract.*

Formal proof to follow

Intuition for Proposition 7 is provided by Figures 3 and 4. Relative to the basic contract, production costs are higher under the restricted contract, so that the social surplus associated with any given q is lower. However, as we established in Proposition 6, the level of output produced by the low ability agent under the optimal restricted contract is higher than under the optimal basic contract. Which of these two effects dominate? When labor and capital highly complementary, as in the bottom panel of Fig. 4, the principal can obtain a very large "bang for the buck" in terms of information rent by a small distortion in the capital-labor ratio away from the optimal ratio. Indeed, as the elasticity of substitution approaches minus infinity, the quantity produced under the optimal restricted contract approaches the first-best quantity, while the distortion in the capital-labor ratio goes to zero, and the optimal restricted contract is socially superior to the optimal basic contract. On the other hand, when inputs are highly substitutable, input mix distortions have very little leverage in terms of information costs. Indeed, as the elasticity of substitution approaches unity, the quantity produced under the optimal restricted contract approaches that produced under the optimal basic contract, but since the input mix is distorted, the social ordering is reversed.

5. CONCLUSION

We have shown that the principal's profits increase when he controls the non-labor input. Further, output increases, since the principal can allocate capital to help mitigate her information costs. However, this mitigation of information costs distorts the capital-labor ratio away from its production-efficient level. This distortion is socially costly. Provided that labor and capital are sufficiently complementary, the restricted contract will result in higher social surplus than the basic contract. If labor and capital are sufficiently substitutable. the basic contract will result in higher social surplus that the restricted contract.

Our result differs from the classic finding of Averch and Johnson, who found that cost-plus pricing induces overinvestment in capital equipment that is socially costly. Essentially, in the asymmetric information problem we consider there is an off-setting social consideration. Neither of the two factors necessarily dominates the other, so that the social implication of input control by the principal is not a priori determinate. In terms of the LeChatelier principle, this can be explained by observing that input control for the principal is the same as removing a constraint on him, but it simultaneously imposes an additional constraint on the agent. The net effect of these adjustments is not predetermined.

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