Grading and Quality Upgrading:

Complements or Substitutes?

by

Abraham Hollander* and Sylvette Monier-Dilhan**

May 2000

*Université de Montréal and Centre de Recherche sur les Transports
**Institut National de Recherche Agronomique, Unité d'Économie de Toulouse

Introduction

When buyers possess little information about product characteristics, markets contract and the quality supplied by sellers is driven down. These effects are mitigated, sometimes eliminated, when grading and certification procedures are implemented. A significant literature explores the channels through which these procedures influence quality, the size of markets and, overall welfare.[1]

The first channel is entry and exit. Grading raises the number of high quality firms at the expense of low quality producers and, in so doing it increases the share of high quality output.[2] The second channel operates through changes in the product specification. The standard finding is that sellers raise quality as buyers become more discerning about product characteristics.

This paper is also concerned with the relationship buyer information and the quality supplied by sellers. It addresses two questions: (1) How do changes in the cost of grading affect the quality choices made by individual firms and, how do these changes influence the average quality produced by the industry as a whole; (2) how do changes in the cost of production affect individual grading decisions, how do they influence the quality composition of output and the share of output that undergoes grading.

However, this paper addresses these questions in a framework that differs from the aforementioned literature. Indeed, by and large, the writings that discuss standards and quality disclosure assume that individual sellers produce a single quality.[3] This assumption fits a manufacturing environment better than an agricultural one. Because our paper is concerned with agricultural markets, it postulates instead that every firm produces output that contains both high and low quality units. When firms differ from one another in regard to quality, it simply means that they produce high and low quality units in different proportions.

Also, the standard assumption in the literature is that absent grading or certification, consumers possess none of the information required to rank firms according to the quality of their output. The roles of grading and certification are precisely to disclose the quality differences among *firms*. Our model by contrast, assumes that *even in the absence of grading*, buyers know the proportion of high quality units produced by each firm.[4] The role of grading is to disclose the quality of *individual units* of output.

The fact that consumers know producers' average quality even when there is no grading, carries the implication that decisions to grade are not influenced by producers' desire to convey signals about the quality of their output. This simplifies the analysis considerably.

In stark contrast to the standard result, our paper finds that better access to grading services - captured as a reduction in the cost of grading- may lower the quality of output. That result

[1] See e.g. Akerlof, Viscusi, De and Nabar.
[2] In equilibrium, that share depends on the disparity of consumer preferences, the accuracy of the tests and, the mandatory or voluntary character of quality disclosure (e.g. Jovanovic, Mason and Sterbenz).
[3] They have examined how market power determines quality (e.g. Spence, Schmalensee); how firms react to mandatory quality standards (e.g. Ronnen, Crampes and Hollander); how the disclosure of quality influences both firms' behavior in general and welfare in particular (Matthews and Postlewaite).
[4] This may be due to the firm's reputation or to the zero (low) cost of conducting tests on a sample of output to obtain a good estimator of the average quality produced by the firm.

applies not only to the industry as a whole, but also to the subset of firms that grades its output.

Sections II and III present respectively the basic set-up and the equilibrium configuration that shows how sellers segregate between graders and non-graders.[5] Section IV explores the effect of changes in the cost of grading and section V examines how the equilibrium is perturbed as a result of changes in the production cost. Results are summarized in a final section.

II. The Model: Assumptions and Notation

Individual units of output come in two qualities: “high” and “low”. Consumers purchase either one unit of output or nothing at all. Their preferences are of the Mussa-Rosen type. Each consumer is indexed by a taste parameter. The consumer indexed θ derives a utility $q\, s_H$ from consumption of a high quality unit and a utility $q\, s_L$ from consumption of a low quality unit, where $s_H > s_L$. The index θ is uniformly distributed on the interval $[0, \bar{q}]$. Prices for high, respectively low quality units are denoted p_H and p_L.

Each firm is assigned an index representing the average quality of its output. Specifically, the firm indexed s produces a proportion λ(s) of high quality units where λ(s)= $(s - s_L)/(s_H - s_L)$. and $s \in [s_L, s_H]$. Hence $l(s) \in [0,1]$. The unit production cost associated with the “average” quality s is given by the function k(s). The latter is increasing in s and convex. The term p(s) denotes the price at which non-graded output of “average” quality s is sold in the market..

The industry produces an experience good; i.e., in the absence of grading consumers do not know the quality of an individual unit of output before they actually consume it. What consumers do recognize even in the absence of grading is the *average* quality of the output produced by each individual firm. Grading discloses to consumers the quality of *individual units* of output. This disclosure occurs without error. Also, consumers are risk-neutral. The expected utility the consumer indexed θ derives from a non-graded unit of output purchased from a firm indexed s is given by $q\,[l(s)s_H + (1 - l(s))s_L]$.

When graded as well as non-graded output is sold, consumers are confronted with the following choices: (a) whether or not to make a purchase; (b) if purchasing, whether to turn to graded or to non-graded output; (c) when buying graded output, whether to go for the high or the low quality; (d) when opting for non-graded output, what quality of non-graded output to select.

Specifically, the choice made by consumer θ is that which maximizes z(θ) where z(θ)=max$\{0, q\, s_L - p_L, q\, s_H - p_H, \max[q\, s - p(s)]\}$ for $s \in S$ with S representing the set of (average) qualities of non-graded output offered for sale.

[5] That set-up is similar to earlier work by Hollander, Monier and Ossard. The difference between this paper and our earlier work is that is the earlier work the average quality produced by a firms was a given; only the decision to grade was determined endogenously. Is this paper firms also decide on the average quality of their production. Hennesy has carried out other work in which the assumption is made that firms produce a combination of high-grade and low-grade output and that the proportion of high-grade output is an endogenous variable.

Figure 1 displays the partition of consumers according to type of purchase, when graded and non-graded output is offered for sale. Consumers with index $\theta \in [0,\theta_0)$ do not make any purchase. Consumers having $\theta \in [\theta_0,\theta_1)$ purchase low quality graded output, while those with $\theta \in [\theta_1,\theta_2)$ buy non-graded output. Consumers having $\theta \in [\theta_2,\bar{q}\,]$ purchase high quality graded output.

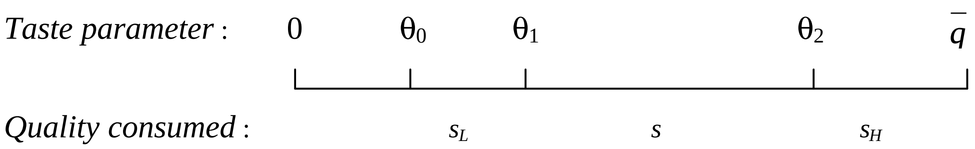

Figure 1: Partition of consumers

The cost of grading is c per unit graded. The problem faced by individual firms is: (a) How to set the average quality of their production; (b) whether or not to engage in grading.[6]

III. Quality choices and grading decisions

Competition insures that all firms earn zero profits. For non-graders the latter entails

(1) $$p(s)-k(s) = 0 \qquad \text{for } s \in S$$

For firms that grade the zero profit condition reads

(2) $$\tilde{l}\, p_H + (1-\tilde{l}\,)\, p_L - k(\tilde{s}) - c = 0$$ where

(3) $$\tilde{l} = \frac{\tilde{s} - s_L}{s_H - s_L}$$

represents the proportion of high quality units produced by these firms and $\tilde{s}$ stand for the average quality of their output. Because firms choose $\tilde{s}$ to maximize profits, the following first order condition must also hold[7]

(4) $$\frac{p_H - p_L}{s_H - s_L} - k'(\tilde{s}) = 0$$

Because the quality chosen by the consumer who purchases non-graded output maximizes $q\,s - p(s)$, it must be true for all $\theta \in [\theta_1,\theta_2]$ that $q = p'(s)$. By virtue of (1) the latter implies

[6] At this stage one cannot yet exclude the possibility that firms grade part of their output. It will be established below that in equilibrium firm grade all their output or nothing at all.

[7]This condition is obtained by differentiation of Π^g using $\lambda(s)=\frac{s - s_L}{s_H - s_L}$ and taking into account that p_H and p_L are a given for the firm.

(5) $$q = k'(s) \quad \text{for} \quad \theta \in [\theta_1, \theta_2]$$

Also, because θ is uniformly distributed, and because each consumer purchases either one unit or nothing at all, the equilibrium must satisfy the condition $(1-\tilde{l})[\bar{q} - q_2] = \tilde{l}[q_1 - q_0]$. Given (5) the latter can be restated as

(6) $$(1-\tilde{l})[\bar{q} - k'(s_2)] = \tilde{l}[k'(s_1) - q_0]$$

In regard to prices, the following must be true: The consumer indexed θ_0 is indifferent between purchasing low quality and not purchasing at all. This follows from the requirement of market clearing. Indeed, if the consumer in question enjoyed positive surplus, then an individual with θ marginally below θ_0 would also get positive surplus from purchasing a low-quality unit at the prevailing price. But, if so, demand for low quality graded output would exceed supply at that price which could therefore not be an equilibrium price. Hence one has,

(7) $$p_L = q_0 s_L$$

Similarly, it must be true that the consumers with the lowest θ purchasing a particular quality must be indifferent between purchasing that quality and purchasing the quality just below it. This entails

(8) $$q_1 s_1 - p(s_1) = q_1 s_L - p_L$$

(9) $$q_2 s_H - p_H = q_2 s_2 - p(s_2)$$

where s_i denotes for $i = \{1,2\}$ the "average" quality on non-graded output purchased by the consumer indexed q_i. By virtue of (1) and (4) the conditions (8) and (9) can be rewritten

(10) $$k'(s_1)s_1 - k(s_1) = k'(s_1)s_L - p_L$$

(11) $$k'(s_2)s_2 - k(s_2) = k'(s_2)s_H - p_H$$

An equilibrium where graded as well as non-graded output is offered for sale is completely characterized by equations (2)-(7) and (10)-(11). Jointly these equations determine $\tilde{l}, \tilde{s}, q_0, p_l, p_h, s_1, s_2$.[8]

Figure 2 illustrates such equilibrium. The utility of the consumers θ_0, θ_1 and θ_2 as a function of the quality consumed are shown as rays through the origin. The unit cost function is displayed as the bold curve k(s).

[8] The latter can be used to calculate θ_1 and θ_2.

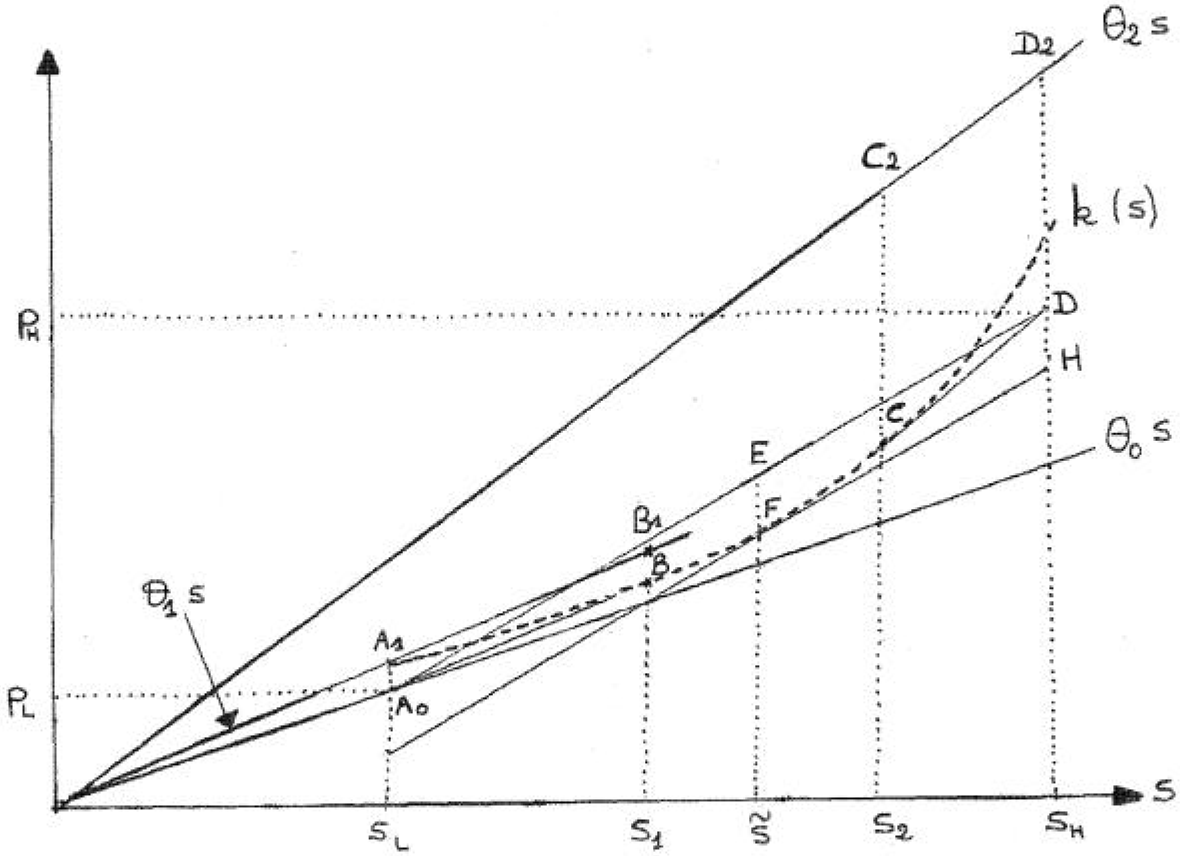

Figure 2

The line segment A_0A_1 represents the surplus that consumer indexed θ_1 derives from the quality s_L. Because the surplus derived by that consumer from non-graded quality s_1 purchased at $k(s_1)$ is the same as the surplus from quality s_L, purchased at p_L, the segment B_1B has the same length as the segment A_0A_1 which represents the surplus that the consumer derives from the non-graded quality s_1.[9] Also, the distance C_2C represents the surplus that consumer θ_2 obtains from quality s_2 purchased at the price $k(s_2)$. That surplus is the same as the surplus the consumer in question would derive from quality s_H purchased at p_H. This surplus is shown as the line segment D_2D.[10]

Because there are a great many firms, each of them views the prices p_L and p_H as parametric. Therefore, the line segment DA_0 represents the locus of the revenue per unit of output faced by a grading firm as a function of $s \in [s_L, s_H]$. The profits made by a firm that grades are highest at $s = \tilde{s}$ i.e. where the cost function k(.) has slope equal to the slope of DA_0. Because each firm earns zero profits, the distance EF represents the unit cost of grading.[11]

[9] At point B the slope of the unit cost function is θ_1.

[10] Point D is located at the horizontal distance s_H from the origin on a line with slope θ_2 going through C. Note that the slope of k(s) at $s = s_2$ is equal to θ_2.

[11] Note that EF=DH=JA_0

Let $\tilde{q}$ denote the preference index of the consumer who is indifferent between purchasing quality s_L and purchasing quality s_H, i.e. $\tilde{q} = (p_H - p_L)/(s_H - s_L)$. By virtue of (5) one knows that the consumer in question purchases non-graded output of quality $\tilde{s}$, i.e. output of the same quality as is produced by the firms that grade. This provides the intuition for the division of consumers into buyers of graded output and buyers of non-graded output.

Indeed, if forced to purchase graded output, consumer $\tilde{q}$ would derive the same surplus regardless of the allocation of his budget among units of different quality. The reason that consumer $\tilde{q}$ purchases quality $\tilde{s}$ in non-graded form is that it sells for a lower price because no grading cost is incurred. Consumer $\tilde{q}$ could reconstitute that basket containing a proportion $\tilde{l}$ of high quality units by purchasing only graded output but when doing so would pay a higher price than for an identical basket of non-graded output. The absence of grading cost explains why consumers whose θ is in the neighborhood of $\tilde{q}$ also purchase non-graded output.

The equilibrium conditions (1)-(11) can be reduced to (12)-(14) below.[12]

(12) $$k(s_1) - k'(s_1)(s_1 - s_L) + k'(\tilde{s})(\tilde{s} - s_L) - k(\tilde{s}) = c$$

(13) $$[k'(s_2)(s_H - s_2) + k(s_2)] - [k(\tilde{s}) + k'(\tilde{s})(s_H - \tilde{s})] = c$$

(14) $$s_L(s_H - \tilde{s})[\bar{q} - k'(s_2)] - (\tilde{s} - s_L)[k'(s_1)s_1 - k(s_1)] = 0$$

The latter are used to perform the comparative statics exercise that is used to examine the effect of changes in grading and production cost.

IV. Shifts in the cost of grading

Note first, that it follows from (12) and (13) that $s_1 = s_2 = \tilde{s}$ when c = 0, i.e., all output is graded when the cost of grading is zero. From (12) and (13) it follows as well that an increase in c increased with $\tilde{s}$ held constant, yields a decrease in s_1 and an increase in s_2. That is for constant (p_H -p_L) it increases the amount of output that is sold in non-graded form. This observation is helpful in understanding how $\tilde{s}$ responds to a change in c.

The total effect of such change-say an increase in c- can be split into two component parts. First, a pure cost increase of graded output. Because that increase raises the cost of low quality proportionately more than the cost of high quality, it should boost $\tilde{s}$. The second adjustment in $\tilde{s}$ is due to the increase in amount of output that is not graded. As θ_2 shifts to the right, the number of high quality units sold to consumers diminishes and, similarly, the number of low quality unit sold to consumers falls as θ_1 shifts to the left. The implication is

[12] See appendix for details of the required substitutions

that $\tilde{s}$ cannot remain constant as c changes unless the number of consumers who make the switch from high quality to non-graded divided by the number of consumers who switch from low quality to non-graded exactly matches the proportion of high to low quality buyers before the change in c. Only by a fluke would the two be equal to each other.

If the ratio of those who switch away from low quality to those that switch away from high quality is larger that the ratio of number of those who initially purchased low quality to those who purchased high quality, then $\tilde{s}$ has to rise to insure that the "accounting" (6) is met. In this case the secondary effect on $\tilde{s}$ reinforces the primary effect. Therefore an increase in c does certainly elicit an increase in $\tilde{s}$. However, when the number of consumers who switch away from low quality is small and the number of those that switch away from high quality is large, then a fall in $\tilde{s}$ is required to satisfy (6). If so, the secondary effect counteracts the first effect. It is possible to specify a cost function k(s) where, locally, this secondary effect dominates the primary effect so lower $\tilde{s}$ results from an increase in c.

Note though, that when c becomes very small, the secondary effect vanishes. The implication is that a reduction in the cost of grading when that cost is already small brings about a decrease in the quality of output that is graded. This and other results are summarized in the proposition below with proof given in the appendix.

Proposition 1

1) $\frac{ds_1}{dc} < 0 \Rightarrow \frac{dq_1}{dc} < 0$

2) $\frac{ds_2}{dc} > 0 \Rightarrow \frac{dq_2}{dc} > 0$

3) $\frac{dp_H}{dc} > 0$ and $\frac{dp_L}{dc} > 0$

4) $\left.\frac{d\Lambda}{dc}\right|_{c=0} = \left.\frac{d\tilde{s}}{dc}\right|_{c=0} > 0$. For c > 0 the signs of both derivatives is ambiguous.

5) The proportion of industry output that is graded is inversely related to c.

The result that θ_1 and θ_2 move is the same direction as s_1 and s_2 follows from (1) and k"(s)>0. Part 3) of the proposition then follows from (10) and (11).

In regard to the effects of a change in c on the average quality Λ produced by the industry note that

$$\Lambda = \left[1 - \frac{q_2 - q_1}{\bar{q} - q_0}\right]\tilde{l} + \left[\frac{q_2 - q_1}{\bar{q} - q_0}\right]\frac{1}{q_2 - q_1}\int_{q_1}^{q_2} l(q)dq$$

The change Λ depends on the change in quality by graders and non-graders as well as on the changes in the proportions produced by each group. [13] In the neighborhood of c=0 one has

[13] The average quality of graded output can be higher or lower than the average quality of non-graded output depending on parameter values.

$\Lambda \approx \tilde{l}$. This and the fact that the share of output graded is still very small imply that the change in the quality of graded output and the change in the quality of total industry output are the same. Using (6) and differentiating yields

$$(15) \qquad \frac{d\Lambda}{dc} = \frac{1}{\bar{q} - q_0}\left[-(1-l_2)\frac{dq_2}{dc} - l_1\frac{dq_1}{dc} + \Lambda\frac{dq_0}{dc}\right] = \frac{1}{\bar{q} - q_0}\left[-k''(\tilde{s})\frac{d\tilde{s}}{dc} + \Lambda\frac{dq_0}{dc}\right]$$

an expression that may be positive or negative.[14] Expression (15) states, rather surprisingly, that a sufficient condition for average industry quality to fall as the cost of grading falls is that the reduction in the cost of grading bring about an increase in the quality of graded output. For c significantly above zero, it is possible to generate examples where Λ falls with c while $\tilde{s}$ increases. For other parameter values Λ increases in c while $\tilde{s}$ falls.[15]

The last result stated by the proportion follows from the fact that θ_0 increases[16] i.e. total output falls as c increases, while the amount of output that does not undergo grading increases.

V. Shifts in production cost

We consider the following perturbations of the cost function: (1) one that shifts the unit cost of all qualities by an equal amount; (2) one that shifts the cost for all qualities in the same proportion.

To do so we set $k(s) \equiv x[F+K(s)]$. The effects of the first type of shift are examined by differentiating with respect to F for x = 1; the effects of the second type are explored by differentiating with respect to x.

Same absolute shift across all qualities

Because the cost of grading does not change, an increase (decrease) in $\tilde{s}$ must be accompanied by an increase (decrease) in both s_1 and s_2.[17] But how does $\tilde{s}$ change? In this regard, one can note that that an increase in F lowers the cost of high quality production relative to low quality production, it should yield an increase in $\tilde{s}$. With $\tilde{s}$ increasing it follows from (4) that p_H must be increasing by more than p_L. The increase in prices results in consumers with the lowest θ exiting he market. These and other results are summarized below

[14] For a cost function of the form $k(s)=0.5+0.75s^2$ we find that the average quality of the industry increases as c increases from c=0 to c=.5. A further increase in c brings about a fall in average quality. For this cost function we also find that all increases in c lead to an increase in the average quality of the graded output and to a *decline* in the average quality of non-graded output. The latter outcome, however, depends on the form of the cost function. Indeed, for $k(s) = 0.5+0.5s^2$ we find that increasing c always leads to an *increase* in the average quality of non-graded output.

[15] It should be noted though that for many cost functions (most of those for which we actually made the calculations) and both the quality of graded output and the quality of total output are positively related to c for all value of c ranging from zero to c^{max} that for the value of c where all grading ceases and all output is sold in non-graded form.

[16] Since p_L increases and $p_L= \theta_0 s_L$

[17] This follows from (12) and (13) and can also be deduced from Figure 2.

Proposition 2[18]

1) $\frac{d\tilde{s}}{dF} > 0$

2) $\frac{ds_1}{dF} > 0$ and $\frac{ds_2}{dF} > 0 \Rightarrow \frac{dq_1}{dF} > 0$ and $\frac{dq_2}{dF} > 0$

3) $\frac{dp_H}{dF} > \frac{dp_L}{dF} > 0$

4) $\left.\frac{d(q_2 - q_1)}{dF}\right|_{c=0} = 0$

5) $Sign \left.\frac{d\left(q_2 - q_1\right)/\left(\bar{q} - q_0\right)}{dF}\right|_{c=0} = Sign\left.\left[\frac{d\left(q_2 - q_1\right)}{dF} + \frac{q_2 - q_1}{\bar{q} - q_0}\frac{dq_0}{dF}\right]\right|_{c=0} = 0$

As indicated by part 4) of the proposition, neither the total amount of non-graded output nor the fraction or output graded vary with F when c is very small. For higher c the total amount of non graded output may increase with F even though the total output falls.[19]

The cost of all qualities changes in the same proportion

To understand how equiproportional changes in the production cost of all qualities affects the equilibrium it is useful to consider first a situation where not only production costs but the cost of grading as well changes by the same proportion. For such changes in cost, the initial equilibrium, would continue to satisfy conditions (12) and (13) because all terms of the left-hand-side of these equalities would be multiplied by the same factor as the right-hand-side term. The reason the initial equilibrium would change is that (14) would no longer be satisfied. It is easy to see that in order to meet (14) $\tilde{s}$ would have to fall. Indeed, suppose that $\tilde{s}$ increased. Because the conditions (12) and (13) are unchanged -i.e. the cost of grading has not changed relative to production cost– s_1 and s_2 would have to increase when $\tilde{s}$ does. The implication however, is that the second term of (14) increase whereas the first term falls. This can not happen if the difference between the two terms is to remain zero. By the same reasoning one shows that $\tilde{s}$ could not remain unchanged. The conclusion must therefore be that an equiproportional increase in all costs –production and grading– results in a decrease in the quality of graded output and of non-graded output.

Now, an equiproportional increase in production cost alone can be thought of as arising from an initial equiproportional increase in all costs, followed by a decrease in the cost of grading. The effects of the latter are known from the previous section. When they results in a fall in

---

[18] For proof see appendix

[19] The latter occurs when the average quality of non-graded output is higher than the quality of graded output. this is more likely to happen the for higher values of F.

quality -as they certainly do when c=0– they reinforce the initial effect. Otherwise they counteract them. This an other results are stated in the proposition below

Proposition 3:[20]

1) $$Sign\left[\frac{d\tilde{s}}{dx}\right]_{c=0} = Sign\left\{-\frac{c}{x}\frac{d\tilde{s}}{dc} - s_L(s_H - s_2)(s_1 - s_L)(s_H - \tilde{s})\bar{q}\right\}_{c=0} < 0$$

2) $$\frac{ds_2}{dx} < 0$$

3) $\frac{ds_1}{dx}$ can be positive or negative

4) $$Sign\left.\frac{d\,(q_2 - q_1)/(\bar{q} - q_0)}{dx}\right|_{c=0} = Sign\left[\frac{d(q_2 - q_1)}{dx} + \frac{q_2 - q_1}{\bar{q} - q_0}\frac{dq_0}{dx}\right]_{c=0} = 0$$

Part 1) shows clearly the aforementioned effects on the quality of graded output. It makes it obvious why the total effect is negative when grading costs are small. Part 2) establishes that s_2 falls for all $c > 0$ regardless of whether $\tilde{s}$ increases on falls. Also, locally, equiproportional changes in production cost do not affect the percentage of output graded.

VI. Final Remarks

The paper has investigated the interrelationships between, on one hand, the cost of grading and the cost of quality, and, on the other hand, the quality choices of firms and the extent to which grading is prevalent. It has found that a decrease in the cost of grading may lower the average quality of the output produced by firms that grade and that it certainly does when the cost of grading is small. Such increase expands the output supplied by firms that do not grade even though total output falls. We found as well that lowering of the cost of grading may increase of decrease the average quality produced by the industry.

The effects of shift in the cost function that changes the cost of all qualities by an equal amount raises the quality of firms that grade as well as the quality of the industry as a whole when the cost of grading is small. When the cost of grading is small such increase in production cost does not affect the percentage of output that is graded.

By contrast a shift in production cost that raises the cost of all qualities by an equal proportion, will, when the cost of grading is sufficiently small, lower the quality produced by the firms that grade and those that do not grade. Also, for a small grading cost such increase in production cost does not change the proportion of industry output that is graded.

[20] see appendix for details

References

Akerlof, G.A. “The Market for Lemons: Qualitative Uncertainty and the Market Mechanism.’’ Quarterly Journal of Economics, 84(1970): 488-500.

Crampes, C and A. Hollander “Duopoly and Quality Standards.” European Economic Review 39 (1995) 71-82.

De S. and P. Nabar. ‘‘Economic Implications of Imperfect Quality Certification.’’ Economics Letters, 37(1991): 333-337.

Hollander A., S.Monier-Dilhan and H. Ossard “Pleasures of Cockaigne : Quality Gaps, Market Structure, and the Amount of Grading”, Am. J. Agr. Econ., 81, 501-511, 1999.

Hennesy D.A., “ Information Asymmetry as a reason for Food Industry Vertical Integration”, Am. J. Agr. Econ., 78, 1034-1043, 1996

Jovanovic, B. “Truthful Disclosure of Information.’’ Bell Journal of Economics 13(1982): 36-44.

Matthews, S. and A. Postlewaite. ‘‘Quality Testing and Disclosure.’’ Rand Journal of Economics 16(1985): 328-340.

Mason, C.F. and F.P. Sterbenz. ‘‘Imperfect Market Testing and Market Size.’’ International Economic Review 35(1994): 61-84.

Mussa M. and S. Rosen. ‘‘Monopoly and Product Quality.’’ Journal of Economic Theory, 18(1978): 301-317.

Ronnen, U. “Minimum Quality Standard, Fixed Cost, and Competition.” Rand Journal of Economics 22(1991) 490-504.

Schmalensee, R.C. “Market Structure, Durability, and Quality: a Selective Survey”, Economic Inquiry, 17, 305-27,1979

Viscusi, K. A. ‘‘Note on Lemons Markets with Quality Certification.’’ The Bell Journal of Economics 9(1978): 277-79.

Appendix

The equilibrium conditions

We have :

(2) $\tilde{l}\, p_H + (1-\tilde{l})\, p_L - k(\tilde{s}) - c = 0$

(3) $\tilde{l} = \dfrac{\tilde{s} - s_L}{s_H - s_L}$

(4) $\dfrac{p_H - p_L}{s_H - s_L} - k'(\tilde{s}) = 0$

(6) $(1-\tilde{l})[\bar{q} - k'(s_2)] = \tilde{l}\,[k'(s_1) - q_0]$

(7) $p_L = q_0 s_L$

(10) $k'(s_1)s_1 - k(s_1) = k'(s_1)s_L - p_L$

(11) $k'(s_2)s_2 - k(s_2) = k'(s_2)s_H - p_H$

From (10) and (11), it follows that :

(I) $p_H - p_L = k'(s_2)(s_H - s_2) + k(s_2) + k'(s_1)(s_1 - s_L) - k(s_1)$

From (4) and (I) one gets :

(II) $k'(s_2)(s_H - s_2) + k(s_2) + k'(s_1)(s_1 - s_L) - k(s_1) = (\mathrm{s_H} - s_L)\,k'(\tilde{s})$

While (2) and (4) entail :

(III) $\tilde{l}\,k'(\tilde{s})(s_H - s_L) + p_L - k(\tilde{s}) - c = 0$

Using (10) the latter yields

(IV) $p_L = k(s_1) - k'(s_1)(s_1 - s_L)$

Using (III) and (IV) : $l\,k'(\tilde{s})(s_H - s_L) + k(s_1) - k'(s_1)(s_1 - s_L) - k(\tilde{s}) - c = 0$
which, using (3), simplifies to

(12) $k'(\tilde{s})(\tilde{s} - s_L) - k(\tilde{s}) + k(s_1) - k'(s_1)(s_1 - s_L) = c$

Using (II) and (12) one gets
$k'(s_2)(s_H - s_2) + k(s_2) + k'(\tilde{s})(\tilde{s} - s_L) - k(\tilde{s}) - c - (\mathrm{s_H} - s_L)\,k'(\tilde{s}) = 0$ or

(13) $[k'(s_2)(s_H - s_2) + k(s_2)] - k(\tilde{s}) - k'(\tilde{s})(s_H - \tilde{s})] = c$

Also, (6) can be rewritten $(1-\tilde{l})[\bar{q} - k'(s_2)] = \dfrac{\tilde{l}}{s_L}[k'(s_1)s_L - q_0 s_L]$ which upon use of (7) and (10) yields $(1-\tilde{l})[\bar{q} - k'(s_2)] = \dfrac{\tilde{l}}{s_L}[k'(s_1)s_L + k'(s_1)(s_1 - s_L) - k(s_1)]$ or $s_L(1-\tilde{l})[\bar{q} - k'(s_2)] = \tilde{l}\,[k'(s_1)s_1 - k(s_1)]$. Using (3) this becomes

(14) $s_L(s_H - \tilde{s})[\bar{q} - k'(s_2)] - (\tilde{s} - s_L)[k'(s_1)s_1 - k(s_1)] = 0$

Shifts in the cost of grading

From (12), (13) and (14) we obtain

$$D\begin{bmatrix} ds_1 \\ ds_2 \\ d\tilde{s} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} dc$$

where $D = \begin{bmatrix} -k''(s_1)(s_1 - s_L) & 0 & k''(\tilde{s})(\tilde{s} - s_L) \\ 0 & k''(s_2)(s_H - s_2) & -k''(\tilde{s})(s_H - \tilde{s}) \\ -s_1 k''(s_1)(\tilde{s} - s_L) & -k''(s_2)s_L(s_H - \tilde{s}) & -\left[s_L(\bar{q} - k'(s_2)) + k'(s_1)s_1 - k(s_1)\right] \end{bmatrix}$

The last term of D can be rewritten : $s_L(\bar{q} - k'(s_2)) + k'(s_1)s_1 - k(s_1) = \frac{1}{1-\tilde{l}}(k'(s_1)s_1 - k(s_1))$

$$|D| = \begin{bmatrix} -k''(s_1)(s_1 - s_L) & 0 & k''(\tilde{s})(\tilde{s} - s_L) \\ 0 & k''(s_2)(s_H - s_2) & -k''(\tilde{s})(s_H - \tilde{s}) \\ -s_1 k''(s_1)(\tilde{s} - s_L) & -k''(s_2)s_L(s_H - \tilde{s}) & -\frac{1}{1-\tilde{l}}\left[k'(s_1)s_1 - k(s_1)\right] \end{bmatrix} = -k''(s_1)k''(s_2)|G|$$

where $G = \begin{bmatrix} s_1 - s_L & 0 & (\tilde{s} - s_L)k''(\tilde{s}) \\ 0 & s_H - s_2 & -(s_H - \tilde{s})k''(\tilde{s}) \\ s_1(\tilde{s} - s_L) & -s_L(s_H - \tilde{s}) & -\frac{1}{1-\tilde{l}}\left[k'(s_1)s_1 - k(s_1)\right] \end{bmatrix}$

$$|G| = (s_1 - s_L)\left[-\frac{1}{1-\tilde{l}}(s_H - s_2)(k'(s_1)s_1 - k(s_1)) - s_L(s_H - \tilde{s})^2 k''(\tilde{s})\right] - s_1(\tilde{s} - s_L)^2(s_H - s_2)k''(\tilde{s})$$

Since $k'(s_1)s_1 - k(s_1) = qs_1 - p_1 > 0$ and $0 < \tilde{l} < 1$, we have, $|G| < 0$, hence $|D| > 0$.

- $ds_1 / dc = |A| / |D| < 0$

$$|A| = -k''(s_2)[(s_H - s_2)(\frac{1}{1-\tilde{l}})\left[k'(s_1)s_1 - k(s_1)\right]$$
$$- s_L(s_H - \tilde{s})^2 k''(\tilde{s})k''(s_2) \quad - s_L(\tilde{s} - s_L)(s_H - \tilde{s})k''(\tilde{s})k''(s_2)] < 0$$

since $A = \begin{bmatrix} 1 & 0 & k''(\tilde{s})(\tilde{s} - s_L) \\ 1 & k''(s_2)(s_H - s_2) & -k''(\tilde{s})(s_H - \tilde{s}) \\ 0 & -k''(s_2)s_L(s_H - \tilde{s}) & -\frac{1}{1-\tilde{l}}\left[k'(s_1)s_1 - k(s_1)\right] \end{bmatrix}$

- From equation (5), it follows that $dq_1 / dc < 0$
- From equation (10), it follows that $dp_L / dc > 0$

- $ds_2 / dc = |B| / |D| > 0$

$$|B| = k''(s_1)k''(\tilde{s})(s_H - \tilde{s})s_1(\tilde{s} - s_L) + k''(s_1)(s_1 - s_L)\frac{1}{1-\tilde{l}}\left[k'(s_1)s_1 - k(s_1)\right] + k''(\tilde{s})k''(s_1)s_1(\tilde{s} - s_L)^2 > 0$$

$$\text{since } B = \begin{bmatrix} -k''(s_1)(s_1 - s_L) & 1 & k''(\tilde{s})(\tilde{s} - s_L) \\ 0 & 1 & -k''(\tilde{s})(s_H - \tilde{s}) \\ -s_1 k''(s_1)(\tilde{s} - s_L) & 0 & -\frac{1}{1-\tilde{l}}\left[k'(s_1)s_1 - k(s_1)\right] \end{bmatrix}$$

- From equation (5), it follows that $dq_2 / dc > 0$
- From equation (11), it follows that $dp_H / dc > 0$

- $d\tilde{s} / dc = |C| / |D|$

$$\begin{aligned} |C| &= [-k''(s_1)k''(s_2)][-s_1(\tilde{s} - s_L)(s_H - s_2) + s_L(s_H - \tilde{s})(s_1 - s_L)] \\ &= [-k''(s_1)k''(s_2)][-s_1(\tilde{s} - s_L)(s_H - \tilde{s}) - s_1(\tilde{s} - s_L)(\tilde{s} - s_2) + s_L(s_H - \tilde{s})(\tilde{s} - s_L) - s_L(s_H - \tilde{s})(\tilde{s} - s_1)] \\ &= [-k''(s_1)k''(s_2)][-(s_1 - s_L)(\tilde{s} - s_L)(s_H - \tilde{s}) - s_1(\tilde{s} - s_L)(\tilde{s} - s_2) - s_L(s_H - \tilde{s})(\tilde{s} - s_1)] \\ &= k''(s_1)k''(s_2)\,[s_1(\tilde{s} - s_L)(s_H - s_2) - s_L(s_H - \tilde{s})\,(s_1 - s_L)] \end{aligned}$$

$$\text{since } C = \begin{bmatrix} -k''(s_1)(s_1 - s_L) & 0 & 1 \\ 0 & k''(s_2)(s_H - s_2) & 1 \\ -k''(s_1)\,s_1(\tilde{s} - s_L) & -k''(s_2)s_L(s_H - \tilde{s}) & 0 \end{bmatrix}$$

Same absolute shift across all qualities

Using $k(s) \equiv F + aK(s)$, from (12), (13) and (14) we obtain :

$$\begin{bmatrix} -k''(s_1)(s_1 - s_L) & 0 & k''(\tilde{s})(\tilde{s} - s_L) \\ 0 & k''(s_2)(s_H - s_2) & -k''(\tilde{s})(s_H - \tilde{s}) \\ -s_1 k''(s_1)(\tilde{s} - s_L) & -k''(s_2)s_L(s_H - \tilde{s}) & -\frac{1}{1-\tilde{l}}\left[k'(s_1)s_1 - k(s_1)\right] \end{bmatrix} \begin{bmatrix} ds_1 \\ ds_2 \\ d\tilde{s} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -(\tilde{s} - s_L) \end{bmatrix} dF$$

- $ds_1 / dF = |E| / |D| > 0$

where $|E| = (\tilde{s} - s_L)k''(\tilde{s})k''(s_2)(s_H - s_2)(\tilde{s} - s_L) > 0$

$$\text{since}: E = \begin{bmatrix} 0 & 0 & k''(\tilde{s})(\tilde{s} - s_L) \\ 0 & k''(s_2)(s_H - s_2) & -k''(\tilde{s})(s_H - \tilde{s}) \\ -(\tilde{s} - s_L) & -k''(s_2)s_L(s_H - \tilde{s}) & -\frac{1}{1-\tilde{l}}\left[k'(s_1)s_1 - k(s_1)\right] \end{bmatrix}$$

- From equation (5) it follows $dq_1 / dF > 0$
- From equation (10), it follows that $dp_L / dF > 0$

- $ds_2 / dF = |H| / |D| > 0$

where $|H| = (\tilde{s} - s_L)\, k''(s_1)(s_1 - s_L)k''(\tilde{s})(s_H - \tilde{s}) > 0$

$$\text{since}: H = \begin{bmatrix} -k''(s_1)(s_1 - s_L) & 0 & k''(\tilde{s})(\tilde{s} - s_L) \\ 0 & 0 & -k''(\tilde{s})(s_H - \tilde{s}) \\ -s_1 k''(s_1)(\tilde{s} - s_L) & -(\tilde{s} - s_L) & -\frac{1}{1-\tilde{l}}\left[k'(s_1)s_1 - k(s_1)\right] \end{bmatrix}$$

- From equation (5) it follows $dq_2 / dF > 0$
- From equation (11), it follows that $dp_H / dF > 0$

- $d\tilde{s} / dF = |I| / |D| > 0$

where $|I| = (\tilde{s} - s_L)k''(s_1)(s_1 - s_L)k''(s_2)(s_H - s_2) > 0$

$$\text{since}: \; I = \begin{bmatrix} -k''(s_1)(s_1 - s_L) & 0 & 0 \\ 0 & k''(s_2)(s_H - s_2) & 0 \\ -s_1 k''(s_1)(\tilde{s} - s_L) & -k''(s_2)s_L(s_H - \tilde{s}) & -(\tilde{s} - s_L) \end{bmatrix}$$

- From (6) one obtains

$$\left(\bar{q} - q_2 + q_1 - q_0\right)\frac{d\tilde{s}}{dF} + \left(s_H - \tilde{s}\right)\frac{k''(s_2)ds_2}{dF} + \left(\tilde{s} - s_L\right)\frac{k''(s_1)ds_1}{dF} = \left(\tilde{s} - s_L\right)\frac{dq_0}{dF}$$

which implies $\dfrac{dq_0}{dF} > 0$.

The cost of all qualities changes in the same proportion

Assume $k(s) \equiv x\left[F + aK(s)\right]$, from (12), (13) and (14) we obtain :

$$\begin{bmatrix} -k''(s_1)(s_1-s_L) & 0 & k''(\tilde{s})(\tilde{s}-s_L) \\ 0 & k''(s_2)(s_H-s_2) & -k''(\tilde{s})(s_H-\tilde{s}) \\ -s_1k''(s_1)(\tilde{s}-s_L) & -k''(s_2)s_L(s_H-\tilde{s}) & -\frac{1}{1-\tilde{l}}[k'(s_1)s_1-k(s_1)] \end{bmatrix} \begin{bmatrix} ds_1 \\ ds_2 \\ d\tilde{s} \end{bmatrix} = \begin{bmatrix} -c/x \\ -c/x \\ s_L(s_H-\tilde{s})\bar{q} \end{bmatrix} dx/x$$

- $|D|\ ds_1/dx =$

$$\left\langle k''(\tilde{s})k''(s_2)s_L\left\{\frac{c}{x}(s_H-\tilde{s})(s_H-s_L)-(s_H-\tilde{s})(s_H-s_2)(\tilde{s}-s_L)\bar{q}\right\}+\frac{c}{x}(s_H-s_2)\frac{k''(s_2)}{1-\tilde{l}}[k'(s_1)s_1-k(s_1)]\right\rangle$$

- $|D|\ ds_2/dx\ =$

$$-k''(\tilde{s})k''(s_1)\left\{\frac{c}{x}s_1(\tilde{s}-s_L)(s_H-s_L)+s_L(s_1-s_L)(s_H-\tilde{s})^2\bar{q}\right\}-\frac{c}{x}(s_1-s_L)\frac{k''(s_1)}{1-\tilde{l}}[k'(s_1)s_1-k(s_1)]$$

- $|D|\ d\tilde{s}/dx =$

$$k''(s_1)k''(s_2)\left\{\frac{c}{x}[-s_1(\tilde{s}-s_L)(s_H-s_2)+s_L(s_1-s_L)(s_H-\tilde{s})]-s_L(s_H-\tilde{s})(s_1-s_L)(s_H-s_2)\bar{q}\right\}$$

$$=-|D|\frac{d\tilde{s}}{dc}-|D|k''(s_1)k''(s_2)s_L(s_H-s_2)(s_1-s_L)(s_H-\tilde{s})\bar{q}$$

- Finally, note that $\frac{d\frac{q_2-q_1}{\bar{q}-q_0}}{dx}=\frac{1}{\bar{q}-q_0}\left\{\frac{d(q_2-q_1)}{dx}+\frac{q_2-q_1}{\bar{q}-q_0}\frac{dq_0}{dx}\right\}$. Because for c = 0 we have $q_2-q_1=0$ and $\frac{d(q_2-q_1)}{dx}=0$ and thus $\left.\frac{d\frac{q_2-q_1}{\bar{q}-q_0}}{dx}\right|_{c=0}=0$.