# *CoVaR* *

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**Abstract**

We propose a measure for systemic risk: *CoVaR*, the value at risk (*VaR*) of the financial system conditional on institutions being under distress. We define an institution's (marginal) contribution to systemic risk as the difference between *CoVaR* and the financial system *VaR*. From our estimates of *CoVaR* for characteristic sorted portfolios of publicly traded financial institutions, we quantify the extent to which characteristics such as leverage, size, and maturity mismatch predict systemic risk contribution. We argue for macroprudential regulation based on the degree to which such characteristics forecast systemic risk contribution.

*Keywords:* Value at Risk, Systemic Risk, Adverse Feedback Loop, Endogenous Risk, Risk Spillovers, Financial Architecture

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# 1 Introduction

During times of financial crisis, losses tend to spread across financial institutions, threatening the financial system as a whole.[1] While comovement of financial institutions' assets and liabilities is primarily driven by fundamentals in normal times, comovement tends to increase during times of crisis. Such increases of comovement give rise to systemic risk—the risk that institutional distress spreads widely and distorts the supply of credit and capital to the real economy. Measures of systemic risk that capture the increase in tail comovement during financial crisis should become supervisory tools and form the basis of any macroprudential regulation.

The most common measure of risk used by financial institutions—the value at risk (*VaR*)—focuses on the risk of an individual institution in isolation. The $q\%$-*VaR* is the maximum dollar loss within the $q\%$-confidence interval; see, e.g., Jorion (2006). However, a single institution's risk measure does not necessarily reflect systemic risk – the risk that the stability of the financial system as a whole is threatened. Following the classification in Brunnermeier, Crocket, Goodhart, Perssaud, and Shin (2009), a systemic risk measure should identify the risk on the system by "individually systemic" institutions, which are so interconnected and large that they can cause negative risk spillover effects on others, as well as by institutions which are "systemic as part of a herd." A group of 100 institutions that act like identical clones can be as precarious/dangerous to the system as the large merged identity.

The objective of this paper is twofold: First, we propose a measure for systemic risk. Second, we outline a method that allows for a countercyclical implementation

[1] Examples include the 1987 equity market crash which started by portfolio hedging of pension funds and led to substantial losses of investment banks; the 1998 crisis started with losses of hedge funds and spilled over to the trading floors of commercial and investment banks; and the 2007/08 crisis spread from SIVs to commercial banks and on to investment banks and hedge funds, see Brady (1988), Rubin, Greenspan, Levitt, and Born (1999), and Brunnermeier (2009).

of macroprudential regulation by predicting future systemic risk using past variables such as size, leverage, and maturity mismatch. To emphasize the systemic nature of our risk measure, we add to existing risk measures the prefix "*Co*", which stands for *co*nditional, *co*movement, *co*ntagion, or *co*ntributing. We focus primarily on $CoVaR$, where institution $i$'s $CoVaR$ relative to the system is defined as the $VaR$ of the whole financial sector conditional on institution $i$ being in distress.[2] The difference between the $CoVaR$ and the unconditional financial system $VaR$, $\Delta CoVaR$, captures the marginal contribution of a particular institution (in a non-causal sense) to the overall systemic risk.

There are several advantages to our $\Delta CoVaR$ measure. First, while $\Delta CoVaR$ focuses on the contribution of each institution to overall system risk, current prudential regulation focuses on the risk of individual institutions. This leads, in the aggregate, to excessive risk-taking along systemic risk. To see this more explicitly, consider two institutions, $A$ and $B$, which report the same $VaR$, but for institution $A$ the $\Delta CoVaR= 0$, while for institution $B$ the $\Delta CoVaR$ is large (in absolute value). Based on their $VaR$s, both institutions appear to be equally risky. However, the high $\Delta CoVaR$ of institution $B$ indicates that it contributes more to system risk. Since system risk might carry a higher risk premium, institution $B$ might outshine institution $A$ in terms of generating returns, so that competitive pressure might force institution $A$ to follow suit. Imposing stricter regulatory requirements on institution $B$ would break this tendency to generate systemic risk.

One might argue that regulating institutions' $VaR$ might be sufficient as long as

[2] Just as $VaR$ sounds like variance, $CoVaR$ sounds like covariance. This analogy is no coincidence. In fact, under many distributional assumptions (such as the assumption that shocks as conditionally Gaussian), the $VaR$ of an institution is indeed proportional to the variance of the institution, and the $CoVaR$ of an institution is proportional to the covariance of the financial system and the individual institution.

each institution's $\Delta CoVaR$ goes hand in hand with its $VaR$. However, this is not the case, as (i) it is not desirable that institution $A$ should increase its contribution to systemic risk by following a strategy similar to institution B and (ii) there is no one-to-one connection between an institution's $\Delta CoVaR$ (y-axis) and $VaR$ (x-axis) as Figure 1 shows. Overall, Figure 1 questions the usefulness of current bank regulation, such as Basel II, which relies primarily on $VaR$.

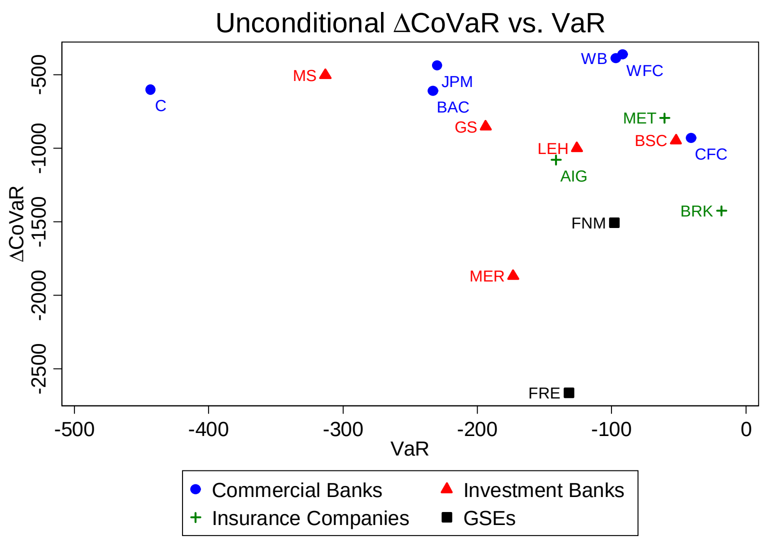


Figure 1: The scatter plot shows the weak link between institutions' risk in isolation, measured by $VaR^i$ (x-axis), and institutions' contribution to system risk, measured by $\Delta CoVaR^i$ (y-axis). The $VaR^i$ and $\Delta CoVaR^i$ are measured in 2006Q4 and are reported in billions of dollars. A list with the names of the institutions corresponding to the tickers in this plot is given in Appendix C.

Another advantage of our co-risk measure is that it is general enough to study the risk spillover effects across the whole financial network. For example, $\Delta CoVaR^{j|i}$ captures the increase in risk of individual institution $j$ when institution $i$ falls into distress. To the extent that it is causal, it captures the risk spillover effects that institution $i$ causes on institution $j$. Of course, it can be that institution $i$'s distress causes a large risk increase in institution $j$, while institution $j$ causes almost no risk spillovers onto institution $i$. That is, there is no reason why $\Delta CoVaR^{j|i}$ should equal $\Delta CoVaR^{i|j}$. Figure 2 shows the directional effects for five U.S. banks with large broker-dealers subsidiaries.

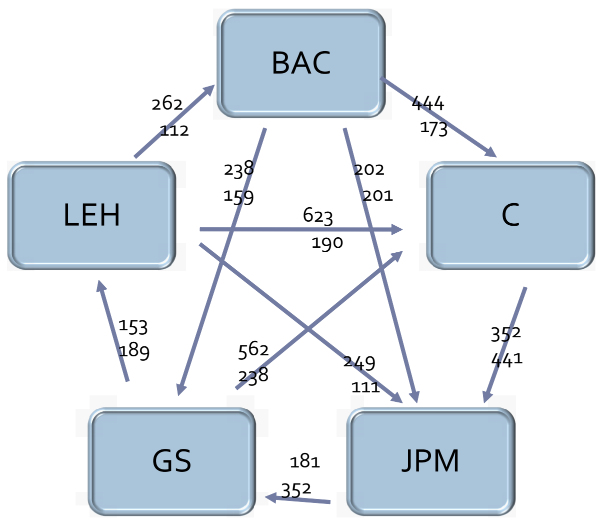


Figure 2: *CoVaR* network structure. The top number represents the *CoVaR* of the pointed institution conditional to the event that the institute at the origin of the arrow is in distress. The bottom number represents the *CoVaR* in the opposite direction.

Another advantage of *CoVaR* is that it is readily extendable from a value at risk measure to other tail risk measures. Several authors have pointed out short-comings of *VaR* and argued in favor of alternative risk measures. One of these measures is the expected short-fall (*ES*), which captures the expected loss conditional on being in the $q$% quantile. It is straightforward to extend our approach to other risk measures, e.g. the Co-Expected Shortfall (*Co-ES*). Just as *ES* is a sum of *VaR*s, *Co-ES* is the sum of *CoVaR*s. The advantage of *Co-ES* relative to *CoVaR* is that it provides less incentive to load on to tail risk below the percentile that defines the $VaR$ or $CoVaR$. In summary, the economic arguments of this paper are readily translatable to expected shortfall.

So far, we have deliberately not specified how to estimate our $CoVaR$ measure, since there are many possible ways. In this paper, we primarily use quantile regressions which are appealing for their simplicity and efficient use of data. Since we want to capture all forms of risk, including not only the risk of adverse asset price movements, but–equally important–also funding liquidity risk, our estimates of $\Delta CoVaR$ are based on (weekly) changes in the market valued assets of public financial institutions. Since the asset and liability composition of any particular financial institution may change over time (e.g., due to mergers, demergers, or ventures into new businesses), we estimate our risk measures over quintile portfolios of financial intermediaries sorted based on leverage, maturity mismatch, size, market-to-book, and volatility.

Our paper also addresses the problem that (empirical) risk measures suffer from the fact that "tail observations" are—by definition—rare. After a string of good news, risk seems tamed, but, when a new tail event occurs, the estimated risk measure may sharply increase. This problem is most pronounced if the data samples are short. Hence, regulatory requirements that are based on estimated risk measures would be stringent during a crisis and lax during a boom. This introduces procyclicality – exactly the opposite of the goal of effective regulation.

In order to construct a countercyclical risk measure, we derive unconditional and conditional measures of $\Delta CoVaR$ using the full length of available data (we use weekly data from the beginning of 1986 to the end of 2008 for all publicly traded commercial banks, broker-dealers, insurance companies, and real estate companies). While the unconditional $\Delta CoVaR$ estimates are constant over time, the conditional ones model time variation of $\Delta CoVaR$ as a function of state variables that model the evolution of tail risk dependence over time. These state variables include the slope of the yield curve, aggregate credit spread, and implied equity market volatility from VIX. To estimate which characteristics of financial institutions contribute to systemic risk, we first estimate $CoVaR$ conditional on the state variables. Using panel regressions, we then relate these time-varying $\Delta CoVaR$ —in a predictive, Granger causal sense—to measures of each portfolio's average maturity mismatch, leverage, market-to-book, and size. These predictive regressions allow preemptive macroprudential policy and ex-ante regulation of systemic risk contribution. The regression coefficients indicate how one should weigh the different funding liquidity measures in determining the capital charge or Pigouvian tax imposed on various financial institutions.

In practice, we argue for a change of the supervisory and regulatory framework that aims at internalizing externalities that an institution's risk taking imposes on the financial system rather than focusing on a bank's risk in isolation. More specifically, the degree to which an institution increases systemic risk—as measured by $\Delta CoVaR$—should determine the macroprudential regulation and capital surcharges of that institution.

**Related Literature.** Our *co-risk measure* is motivated by theoretical research on externalities across financial institutions that give rise to liquidity spirals. $CoVaR$ measures such externalities, together with fundamental comovement. $CoVaR$ also relates to econometric work on contagion and spillover effects.

A "fire-sale externality" gives rise to excessive risk taking and leverage. The externality arises since each individual institution takes potential fire-sale prices as given, while, as a group, they cause the fire sale prices. In an incomplete market setting this precuniary externality leads to an outcome that is not even constrained Pareto efficient. This result was derived in a banking context in Bhattacharya and Gale (1987), applied to international finance in Caballero and Krishnamurthy (2004) and most recently shown in Lorenzoni (2008). Stiglitz (1982) and Geanakoplos and Polemarchakis (1986) show it generically in a general equilibrium incomplete market setting. Runs on financial institutions are dynamic co-opetition games and lead to externalities, as does banks' hoarding. While hoarding might be microprudent from a single bank's perspective it need not be macroprudent (fallacy of the commons). Network effects can also lead to externalities, as hiding one's own contractual commitments increases the risk of one's counterparties and the counterparties of one's counterparties etc, Brunnermeier (2009). In Acharya (2009) banks do not fully take into account that they contribute to systematic risk.

Procyclicality occurs because risk measures tend to be low in booms and high in crises. The margin/haircut spiral outlined in Brunnermeier and Pedersen (2009) then forces financial institutions to delever at fire-sale prices. Adrian and Shin (2009) provide empirical evidence for the margin/haircut spiral for the investment banking sector. Borio (2004) is an early contribution that discusses a policy framework to address margin/haircut spirals and procyclicality.

**Outline.** The remainder of the paper is organized in four sections. In Section 2, we outline the methodology and define $\Delta CoVaR$ and its properties. In Section 3, we outline the estimation method via quantile regressions. We also introduce time-varying $\Delta CoVaR$ conditional on state variables and present estimates of these conditional $\Delta CoVaR$. Section 4 shows how to use $\Delta CoVaR$ to implement preemptive macroprudential supervision and regulation by demonstrating that institutional characteristics such as size, leverage, maturity mismatch, and market-to-book predict future systemic risk contribution. We conclude in Section 5.

# 2 *CoVaR* Methodology

In this section, we introduce and define our systemic co-risk measure, *CoVaR*. Subsequently, in Section 3, we introduce time-varying *CoVaR*s by linking our *CoVaR* estimates to state variables. In Section 4, we outline how countercyclical financial regulation can be achieved.

## 2.1 Definition of *CoVaR*

Recall that $VaR_q^i$ is implicitly defined as the $q$ quantile, i.e.

$$\Pr\left(X^i \leq VaR_q^i\right) = q,$$

where $X^i$ is the variable of institution (or portfolio) $i$ for which the $VaR_q^i$ is defined. Note that $VaR_q^i$ is typically a negative number. In practice, the sign is often switched, a sign convention we will not follow.

**Definition 1** *We denote by* $\mathrm{CoVaR}_q^{j|i}$ *the* VaR *of institution* $j$ *(or the financial system) conditional on* $X^i = \mathrm{VaR}_q^i$ *of institution* $i$*. That is,* $\mathrm{CoVaR}_q^{j|i}$ *is implicitly defined by the* $q$*-quantile of the conditional probability distribution:*

$$\Pr\left(X^{\mathrm{j}} \leq \mathrm{CoVaR}_q^{j|i} | X^i = \mathrm{VaR}_q^i\right) = q.$$

*We denote institution* $i$*'s contribution to* $j$ *by:*

$$\Delta\mathrm{CoVaR}_q^{j|i} = \mathrm{CoVaR}_q^{j|i} - \mathrm{VaR}_q^j.$$

Most of the paper focuses on the case $j = system$, i.e. when the portfolio of all financial institutions is at its VaR level. In this case, we drop the superscript $j$. Hence, $\Delta CoVaR^i$ denotes the difference between the $VaR$ of the financial system conditional on the distress of a particular financial institution $i$, $CoVaR^i$, and the unconditional $VaR$ of the financial system, $VaR^{system}$. It measures how much an institution adds to overall systemic risk. The measure captures externalities that arise because an institution is "too big to fail", or "too interconnected to fail", or takes on positions or relies on funding that can lead to crowded trades. Of course, ideally, one would like to have a co-risk measure that satisfies a set of axioms as, for example, the Shapley value does (recall that the Shapley value measures the marginal contribution of a player to a grand coalition).

The more general definition of $CoVaR^{j|i}$, i.e. the $VaR$ of institution (portfolio) $j$ conditional on institution (or portfolio) $i$ being at its $VaR$ level, allows us to study spillover effects across a whole financial network as illustrated in Figure 2. Moreover, we can also derive $CoVaR^{j|system}$ which answers the question which institutions are most at risk should a financial crisis occur. $\Delta CoVaR^{j|system}$ reports institution $j$'s increase in value-at-risk in the case of a financial crisis. One could argue that focusing on the states of the world in which the financial system is in distress can provide the basis for more focused and efficient regulation.

## 2.2 Properties of *CoVaR*

**Cloning Property.** Our *CoVaR* definition satisfies the desired property that, after splitting one large "individually systemic" institution into $n$ identical clones, the *CoVaR* of the large institution is exactly the same as the *CoVaR*s of the $n$ clones. Put differently, conditioning on the distress of a large systemic institution is the same as conditioning on one of the $n$ identical clones.

**Causality.** Note that the $\Delta CoVaR$ measure does not distinguish whether the contribution is causal or simply driven by a common factor. We view this as a virtue rather than as a disadvantage. To see this, suppose a large number of small hedge funds hold similar positions and are funded in a similar way. That is, they are exposed to the same factors. Now, if only one of the small hedge funds falls into distress, this will not necessarily *cause* any systemic crisis. However, if this is due to a common factor, then all of the hedge funds, all of which are "systemic as part of a herd", will be in distress. Hence, each individual hedge fund's co-risk measure should capture this even though there is no direct causal link, and the $\Delta CoVaR$ measure does so. Moreover, when we estimate $\Delta CoVaR$, we control for lagged state variables that capture variation in tail risk not directly related to the financial system risk exposure.

**Tail Distribution.** $CoVaR$ focuses on the tail distribution and is more extreme than the unconditional $VaR$ as $CoVaR$ conditions on a "bad event", a conditioning which typically shifts the mean downwards and increases the variance in an environment with heteroscedasticity. The $CoVaR$, unlike the covariance, reflects both shifts.

**Conditioning.** Note that $CoVaR$ conditions on the event that institution $i$ is at its $VaR$ level, which occurs with probability $q$. That is, the likelihood of the conditioning event is independent of the riskiness of $i$'s strategy. If we were to condition on an absolute return level of institution $i$, then more conservative institutions could have a higher $CoVaR$ since the conditioning event would be a more extreme event for less risky institutions.

**Directionality.** $CoVaR$ is directional. That is, the $CoVaR$ of the system conditional on institution $i$ does not equal the $CoVaR$ of institution $i$ conditional on the system.

**Endogeneity of Systemic Risk.** Note that each institution's $CoVaR$ is endogenous and depends on other institutions' risk taking. Hence, imposing a regulatory framework that internalizes externalities alters the $CoVaR$ measures. We view the fact that $CoVaR$ is an equilibrium measure as a strength, since it adapts to changing environments and provides an incentive for each institution to reduce its exposure to risk if other institutions load excessively on it.

***CoES.*** Another attractive feature of $CoVaR$ is that it can be easily adopted for other "*co*risk-measures". One of them is the co-expected-shortfall, *Co-ES*. Expected shortfall has a number of advantages relative to $VaR$ and can be calculated as a sum of $VaR$s. We denote the $CoES_q^i$, the *Expected Shortfall* of the financial system conditional on $X^i \leq VaR_q^i$ of institution $i$. That is, $CoES_q^i$ is defined by the expectation over the $q$-tail of the conditional probability distribution:

$$E\left[X^{system}|X^{system} \leq CoVaR_q^i\right]$$

Institution $i$'s contribution to $CoES_q^i$ is simply denoted by:

$$\Delta CoES_q^i = E\left[X^{system}|X^{system} \leq CoVaR_q^i\right] - E\left[X^{system}|X^{system} \leq VaR_q^{system}\right].$$

Acharya, Pedersen, Philippon, and Richardson (2009) modify our approach by proposing the marginal expected shortfall as a measure of systemic risk.

## 2.3 Market Valued Total Financial Assets

Our analysis focuses on the $VaR_q^i$ and $\Delta CoVaR_q^i$ of growth rates of market valued total financial assets. More formally, denote by $ME_t^i$ the market value of an intermediary

$i$'s total equity, and by $LEV_t^i$ the ratio of total assets to book equity. We define the normalized change in market value of total financial assets, $X_t^i$, by:

$$X_t^i = \frac{ME_t^i \cdot LEV_t^i - ME_{t-1}^i \cdot LEV_{t-1}^i}{ME_{t-1}^i \cdot LEV_{t-1}^i} = \frac{A_t^i - A_{t-1}^i}{A_{t-1}^i}, \tag{1}$$

where $A_t^i = ME_t^i \cdot LEV_t^i$. Note that $A_t^i = ME_t^i \cdot LEV_t^i = BA_t^i \cdot (ME_t^i/BE_t^i)$, where $BA_t^i$ are book valued total assets of institution $i$. We thus apply the market-to-book equity ratio to transform book-valued total assets into market valued total assets. Note that the total market value weighted sum of the $X_t^i$ across all institutions gives back the growth rate of market valued total assets for the financial system as a whole:

$$\sum_i \frac{A_{t-1}^i}{\sum_j A_{t-1}^j} X_t^i = \frac{A_t^{system} - A_{t-1}^{system}}{A_{t-1}^{system}} = X_t^{system} \tag{2}$$

Our analysis is constrained by using publicly available data. In principle, a systemic risk supervisor could compute the $VaR_q^i$ and $\Delta CoVaR_q^i$ from a broader definition of total assets which includes off balance sheet items as well as derivative contracts. A more complete description of the assets and exposures of institutions would potentially improve the measurement of systemic risk and systemic risk contribution. Conceptually, it is straightforward to extend the analysis to such a broader definition of total assets.

We focus on the $VaR_q^i$ and $\Delta CoVaR_q^i$ of total assets as they are most closely related to the supply of credit to the real economy. Ultimately, policy makers are concerned about systemic risk as it has the potential to inefficiently lower the supply of credit to the non-financial sector. However, supervisors and regulators might also be interested in $VaR_q^i$ and $\Delta CoVaR_q^i$ measures for equities or liabilities as well. For example, the

$\Delta CoVaR_q^i$ for liabilities captures the extent to which financial institutions rely on debt funding—such as repos or commercial paper—that can collapse during systemic risk events. Equity is the residual between assets and liabilities, so the $\Delta CoVaR_q^i$ measure can give additional information about the systemic risk embedded in the asset liability mismatch. The estimation of $\Delta CoVaR_q^i$ for other items of intermediary balance sheets is left to future research.

## 2.4 Financial Institution Data

We focus on publicly traded financial institutions, consisting of four financial sectors: commercial banks, investment banks and other security broker-dealers, insurance companies, and real estate companies. We start our sample in the beginning of 1986 and end in 2008. We obtain the daily market equity data from CRSP and quarterly balance sheet data from COMPUSTAT. We limit the portfolios to institutions that belong to four industries, as identified by their two digit SIC codes (SIC code 60-61: commercial banks; SIC code 63-64: insurance companies; SIC code 65-66: real estate companies; SIC code 62: broker-dealers; we exclude industry code 67). We have a total of 1340 institutions in our sample.

**Portfolio Sorts.** While we are interested in estimating the evolution of the risk measures $VaR$ and $CoVaR$ for individual financial institutions, the nature of any particular institution might have changed drastically over the 1986-2008 sample period. In addition, many banks either merged with other organizations, or went out of business. One way to control for the changing nature of each individual institution is to form portfolios on balance sheet characteristics that are identified by theories of the margin spiral as being determinants of systemic risk. In particular, for each of the four financial sector industries, we form the following sets of quintile portfolios: maturity mismatch, leverage, market-to-book, size, and equity return volatility. We thus obtain 100 industry / characteristic sorted portfolios. Maturity mismatch is measured as short-term debt relative to total assets. Leverage is the ratio of total book assets to book equity. Equity volatility is calculated each quarter from daily return data. The portfolios are sorted at the beginning of each quarter, based on the characteristics of the previous quarter. The quintile cut-offs are value-weighted so that, within industries, each portfolio has (approximately) the same size.

# 3 *CoVaR* Estimation

In this section we outline one simple and efficient way to estimate *CoVaR* using quantile regressions. In Section 3.1, we describe the basic time invariant regressions that are used to generate Figures (1) and (2). In Section 3.2, we describe estimation of the time-varying, conditional *CoVaR*. Details on the return generating model and the econometrics of quantile regression are given in Appendix A and B. Section 3.3 provides estimates of *CoVaR* and discusses properties of the estimates.

## 3.1 Estimation Method: Quantile Regression

The *CoVaR* measure can be computed in various ways. Using quantile regressions is a particularly efficient way to estimate *CoVaR* but, by no means, the only one. Alternatively, *CoVaR* can be computed from models with time-varying second moments, from measures of extreme events, or by bootstrapping past returns.

To see the attractiveness of quantile regressions, consider the predicted value of a quantile regression of the financial sector $\hat{X}_q^{system,i}$ on a particular institution or portfolio $i$ for the $q-th$ quantile:

$$\hat{X}_q^{system,i} = \hat{\alpha}_q^i + \hat{\beta}_q^i X^i, \tag{3}$$

where $\hat{X}_q^{system,i}$ denotes the predicted value for a particular quantile conditional on institution $i$.[3] In principle, this regression could be extended to allow for nonlinearities by introducing higher order dependence of the system return as a function of returns to institution $i$. From the definition of value at risk, it follows directly that:

$$VaR_q^{system}|X^i = \hat{X}_q^{system,i}. \tag{4}$$

That is, the predicted value from the quantile regression of the system on portfolio $i$ gives the value at risk of the financial system conditional on $i$ since the $VaR_q$ given $X^i$ is just the conditional quantile. Using a particular predicted value of $X^i = VaR^i$ yields our $CoVaR_q^i$ measure. More formally, within the quantile regression framework our $CoVaR$ measure is simply given by:

$$CoVaR_q^i := VaR_q^{system}|\,VaR_q^i = \hat{\alpha}_q^i + \hat{\beta}_q^i VaR_q^i. \tag{5}$$

The unconditional $VaR_q^i$ and $\Delta CoVaR_q^i$ estimates for Figure 1 are based on equations (4) and (5). In the remainder of the paper, we use conditional $VaR$ and $\Delta CoVaR$ estimates which explicitly model the time variation of the joint distribution of asset

[3]Note that a median regression is the special case of a quantile regression where $q = 50\%$.We provide a short synopsis of quantile regressions in the context of linear factor models in Appendix B. Koenker (2005) provides a more detailed overview of many econometric issues.

While quantile regressions are regularly used in many applied fields of economics, their applications to financial economics are limited. Notable exceptions are econometric papers like Bassett and Chen (2001), Chernozhukov and Umantsev (2001), and Engle and Manganelli (2004) as well as the working papers by Barnes and Hughes (2002) and Ma and Pohlman (2005).

returns as a function of lagged systematic state variables. The methodology is explained in the next section and the econometric background is given in Appendix A.

## 3.2 Time-Variation Associated With Systematic State Variables

The previous section presents a methodology for estimating $CoVaR$ that is constant over time. To capture time variation in the joint distribution of $X^i$ and $X^{system}$, we estimate the conditional distribution as a function of state variables. A derivation of our estimated equations from a risk factor model of the underlying asset returns, as well as the analytics of quantile regressions for the purpos of this paper, are given in Appendix A and B.

We indicate time-varying $CoVaR_t$ and $VaR_t$ with a subscript $t$. We estimate the time variation of $CoVaR_t$ and $VaR_t$ conditional on a vector of state variables $M_t$. The one week lag of the state variables is denoted by $M_{t-1}$. We run the following quantile regressions in the weekly data (where $i$ is a portfolio or the whole financial system):

$$X_t^i = \alpha^i + \beta^i M_{t-1} + \varepsilon_t^i, \tag{6a}$$

$$X_t^{system} = \alpha^{system} + \beta^{system} M_{t-1} + \varepsilon_t^{system}, \tag{6b}$$

$$X_t^{system} = \alpha^{system|i} + \beta^{system|i} M_{t-1} + \gamma^{system|i} X_t^i + \varepsilon_t^{system|i}. \tag{6c}$$

We then generate the predicted values from these regressions to obtain:

$$VaR_t^i = \alpha^i + \beta^i M_{t-1}, \tag{7a}$$

$$VaR_t^{system} = \alpha^{system} + \beta^{system} M_{t-1}, \tag{7b}$$

$$CoVaR_t^i = \alpha^{system|i} + \beta^{system|i} M_{t-1} + \gamma^{system|i} VaR_t^i. \tag{7c}$$

Finally, we compute $\Delta CoVaR_t^i$ for each institutions:

$$\Delta CoVaR_t^i = CoVaR_t^i - VaR_t^{system} \tag{8}$$

From these regressions, we obtain a panel of weekly $\Delta CoVaR_t^i$. For the forecasting regressions in section 4, we generate a quarterly time series by summing the risk measures within each quarter.

The systematic state variables are lagged. They should not be interpreted as systematic risk factors, but rather as conditioning variables that are shifting the conditional mean and the conditional volatility of the risk measures. Note that different portfolios can load on these risk factors in different directions, so that any correlation of risk measures across portfolios — or correlations of the different risk measures for the same portfolio are not imposed by construction.

**State variables:** To estimate the time-varying $CoVaR_t$ and $VaR_t$, we include a set of state variables $M_t$ that are (i) well known to capture time variation in conditional means and volatilities of asset returns, and (ii) that are also liquid and easily tradable. We restrict ourselves to a small set of risk factors to avoid overfitting the data. Our factors are:

(i) *VIX*, which captures the implied volatility in the stock market. This implied volatility index is available on the Chicago Board Options Exchange's website.

(ii) A short term *"liquidity spread"*, defined as the difference between the 3-month repo rate and the 3-month bill rate measures the short-term counterparty liquidity risk. We use the 3-month general collateral repo rate that is available on Bloomberg, and obtain the 3-month Treasury rate from the Federal Reserve Bank of New York.

(iii) The change in the 3-month term Treasury bill rate.

In addition, we consider the following two fixed-income factors that are known to be indicators in forecasting the business cycle and also predict excess stock returns:

(iv) The change in the *slope of the yield curve*, measured by the yield-spread between the 10-year Treasury rate and the 3-month bill rate from the Federal Reserve Board's H.15 release.

(v) The change in the *credit spread* between BAA rated bonds and the Treasury rate (with same maturity of 10 years) from the Federal Reserve Board's H.15 release.

We also control for the following equity market returns:

(vi) The weekly equity market return from CRSP.

(vii) The one year cumulative real estate sector return.

## 3.3 *CoVaR* Summary Statistics

Table 1 provides the estimates of our weekly conditional 1%-*CoVaR* measures that we obtain from using quantile regressions. Each of the summary statistics comprises the 100 portfolios generated by forming quintiles along five dimensions: leverage, maturity mismatch, market-to-book, size, and equity volatility for each of the four financial industry portfolios (commercial banks, broker-dealers, insurance companies, and real estate).

Table 1: **Summary Statistics**. The table reports summary statistics for the asset returns and 1% risk measures of the $I = 100$ characteristic sorted portfolios for weekly data from 1986-2008 (T=52·23=1196). The portfolios are formed quarterly based on five characteristics of the previous quarter (leverage, maturity mismatch, size, market-to-book, and equity volatility) for each of four industries (commercial banking, insurance, security broker dealers, real estate companies). $X^i$ denotes the weekly market valued asset returns for the 100 portfolios. The portfolio risk measure $VaR^i$ and the system risk measure $VaR^{system}$ are obtained by running 1-% quantile regressions of returns on the one week lag of the state variables and by computing the predicted value of the regression. $\Delta CoVaR^i$ is the difference between $CoVaR^i$ and $VaR^{system}$, where $CoVaR^i$ is the predicted value from a 1-% quantile regression of the financial system asset returns on the portfolio asset returns and on the lagged state variables. The 1%-Stress $\Delta CoVaR^i$ is the $\Delta CoVaR^i$ computed with the worst 1% of state variable realization and the worst 1% financial system return replaced in the quantile regression.

| | Variable | | Mean | Std. Dev. | Observations |
|---|---|---|---|---|---|
| (1) | $X^i$ | overall | 0.210 | 3.806 | N = 116976 |
| (2) | | between | | 0.129 | I = 100 |
| (3) | | within | | 3.804 | T = 1170 |
| (4) | $VaR^i$ | overall | -7.953 | 4.524 | N = 116976 |
| (5) | | between | | 2.283 | I = 100 |
| (6) | | within | | 3.908 | T = 1170 |
| (7) | $\Delta CoVaR^i$ | overall | -1.615 | 1.351 | N = 116976 |
| (8) | | between | | 0.777 | I = 100 |
| (9) | | within | | 1.110 | T = 1170 |
| (10) | 1% Stress-$\Delta CoVaR^i$ | overall | -2.672 | 3.403 | T = 1196 |
| (11) | $VaR^{system}$ | overall | -6.178 | 3.161 | T = 1195 |

The first three lines of Table 1 give the summary statistics for the market valued total asset growth rates; lines four to six give the summary statistics for the time-series/cross-section of $VaR_t^i$ for each of the portfolios; lines seven to nine give the summary statistics for $\Delta CoVaR_t^i$; line 10 is the 1% stress level of $\Delta CoVaR_t^i$; and line 11 gives the summary statistics for the financial system value at risk, $VaR_t^{system}$. The stress $\Delta CoVaR_t^i$ is estimated by substituting the worst 1% of state variable realizations into the $\Delta CoVaR_t^i$ estimates. Recall that $\Delta CoVaR_t^i$ measures the marginal contribution of portfolio $i$ to overall systemic risk and reflects the difference between two value at risks of the portfolio of the "financial universe".

We report the mean, standard deviation, and number of observations for each of the items in Table 1. All of the numbers are weekly percent returns. The data is a panel, so we report the summary statistics for the overall data set, as well as the between and within summary statistics. The between standard deviations are obtained by first taking the time series average of each series and then computing the cross sectional standard deviation. Per construction, the number of cross sectional observations, $I$, is always 100. The within standard deviations are obtained by first taking the cross sectional average, and then computing the time series standard deviation. There are a total of 1196 weeks in the sample: 23 years $\cdot$ 52 weeks. The portfolio sorts use lagged data, so the effective number of time periods is somewhat smaller.

We obtain time-variation of the risk measures by running quantile regressions of asset returns on the lagged state variables. We report average $t-$stats of these regressions in Table 2. A higher VIX, higher repo spread, and lower market return tend to be associated with more negative risk measures. In addition, increases in the 3-month yield, the term spread, and the credit spread tends to be associated with larger risk. The housing variable is not significant on average, though it is significant for some portfolios (not reported).

## 3.4 *CoVaR* versus *VaR*

Figure 1 in the introduction shows that, *across institutions,* there is only a very loose link between an institution's $VaR^i$ and its contribution to systemic risk as measured by $\Delta CoVaR^i$. Hence, imposing financial regulation that is solely based on the individual

Table 2: **Average $t$-Statistics of State Variable Exposures**. The table reports average $t$-statistics from 1%-quantile regressions. For the portfolio risk measures $VaR^i$ and the system risk measure $VaR^{system}$, 1-% quantile regressions are run on the state variables. For $CoVaR^i$, 1-% quantile regressions of the financial system returns are run on the state variables and the portfolio asset returns. There are $I = 100$ characteristic sorted portfolios for weekly data from 1986-2008 (52·23=1196 weeks). The portfolios are formed quarterly based on five characteristics of the previous quarter (leverage, maturity mismatch, size, market-to-book, and equity volatility) for each of four industries (commercial banking, insurance, security broker dealers, real estate companies).

| Variable | $VaR^{system}$ | $VaR^i$ | $CoVaR^i$ |
|---|---|---|---|
| VIX (lag) | (-11.11) | (-0.98) | (-5.83) |
| Repo spread (lag) | (-9.43) | (-1.91) | (-3.91) |
| 3 Month yield change (lag) | (-2.46) | (-0.25) | (-1.94) |
| Term spread change (lag) | (-1.84) | (-0.73) | (-0.80) |
| Credit spread change (lag) | (-1.64) | (-0.94) | (-1.12) |
| Market return (lag) | (8.86) | (8.17) | (4.49) |
| Housing (lag) | (0.99) | (0.20) | (1.53) |
| Portfolio asset return $X^i$ | | | (4.24) |

risk of an institution in isolation might not be very useful. Figure 3 repeats the scatter plot of $\Delta CoVaR^i$ against $VaR^i$ for the 100 portfolios, grouped by 25 portfolios for each of the four financial industries. While $\Delta CoVaR^i$ and $VaR^i$ have only a weak relationship in the cross section, they have a strong relationship in the time series. This can be seen in Figure 4, which plots the cross sectional average of $\Delta CoVaR^i$ against the cross sectional average of $VaR^i$. Figures 5 and 6, respectively, plot the $\Delta CoVaR^i$ and $VaR^{system}$ for the portfolios of large investment banks and high maturity mismatched commercial banks over time.

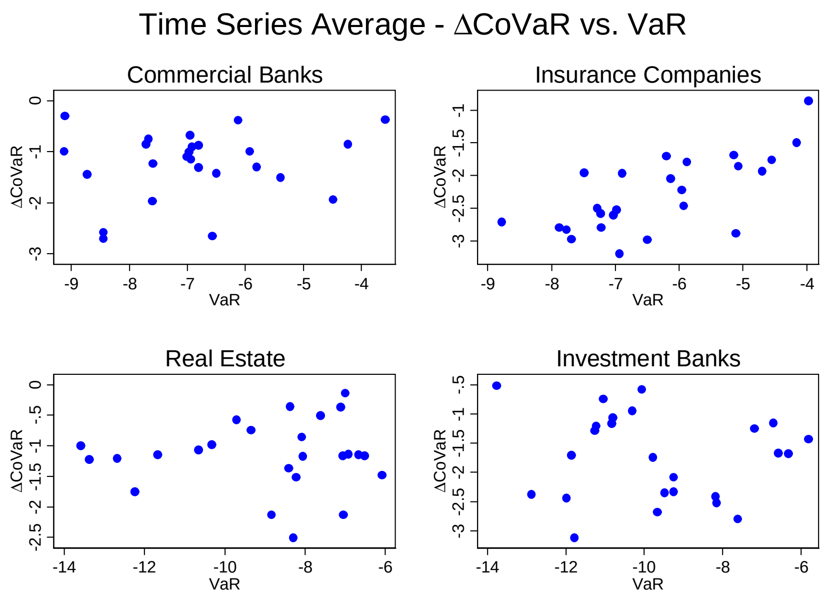


Figure 3: The scatter plot shows the weak cross sectional link between the time series average of a portfolio's risk in isolation, measured by $VaR^i$ (x-axis), and the time series average of a portfolio's contribution to system risk, measured by $\Delta CoVaR^i$ (y-axis). The $VaR^i$ and $\Delta CoVaR^i$ are in units of weekly returns to total market valued financial assets.

# 4 $\Delta CoVaR$ Forecasts and Regulation

From a regulatory or macroprudential policy perspective, the potential for systemic risk builds up before an actual financial crisis occurs. For example, the securities that were at the core of the 2007-2009 crisis were bought by financial institutions between 2005 and 2007, years before the actual crisis occurred. While the estimates of section 3 provide contemporaneous measures of systemic risk contribution, policy should take the potential for future systemic risk into account. Consequently, we provide systemic risk contribution forecasts in this section.

We sum the weekly risk measure estimates of the previous section by quarter and

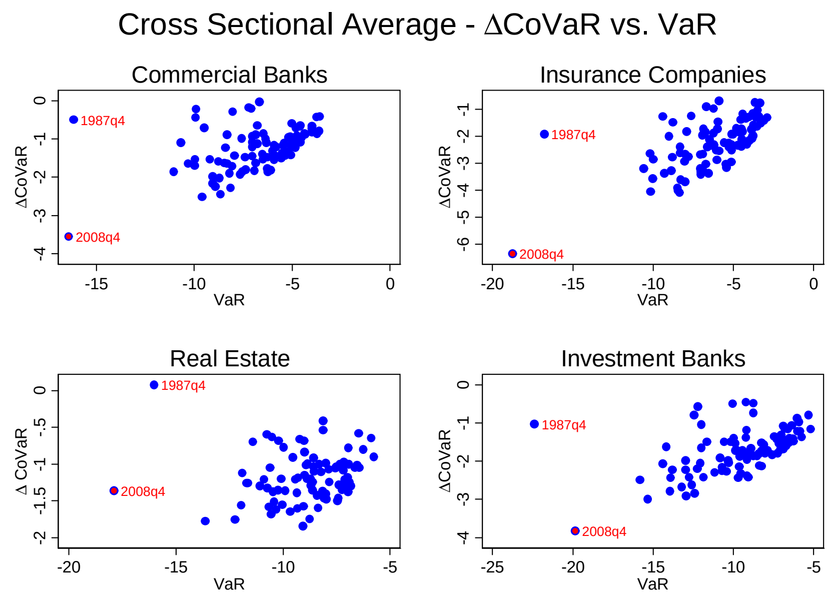


Figure 4: The scatter plot shows the strong time series link between the cross sectional average of $VaR^i$ (x-axis) and the cross sectional average of contribution to system risk, measured by $\Delta CoVaR^i$ (y-axis). The weekly risk measures are time-aggregated by averaging within each quarter, so that $VaR^i$ and $\Delta CoVaR^i$ are in units of weekly returns to total market valued financial assets.

use firm characteristics to predict future systemic risk contribution. We show that more leverage, more maturity mismatch, and larger size all forecast larger systemic risk contribution. We propose to base macroprudential policy on such estimates of systemic risk contribution. In particular, we are able to calculate a weighting scheme for the characteristics which allows for the ex-ante taxation of characteristics which are likely to cause systemic risk problems in the future.

The forecast of systemic risk contribution several quarters into the future addresses the procyclical nature of current regulation. Currently, capital requirements as well as margin and haircut setting are based on contemporaneous risk calculations. When volatility is low, capital requirements are low, which allows the build up of aggregate

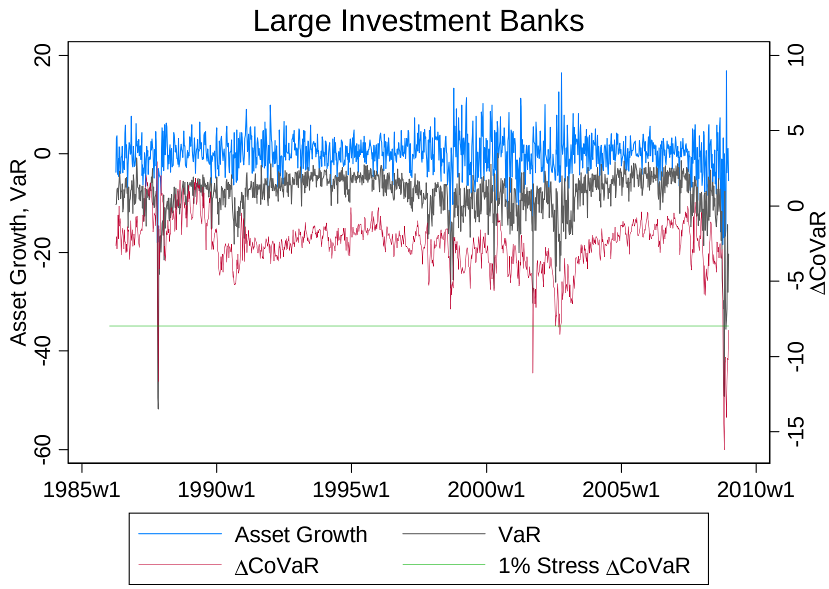


Figure 5: This figure shows the market valued asset returns (blue), the 1%-$VaR$ (gray), and the 1%-$\Delta CoVaR$ (red) for the portfolio of the 20% of largest investment banks. The 1%-stress $\Delta CoVaR$ is also plotted. All risk measures are in weekly returns to total market valued assets.

risk. By basing regulatory requirements on the characteristics that predict future systemic risk contribution, institutions have to hold higher capital ratios in anticipation of future risk contribution, even if the contemporaneous level of measured risk is low. Such a capital regulation scheme is thus forward looking.

## 4.1 Forecasting $\Delta CoVaR$ from Institutional Characteristics

Countercyclical regulation should tighten in booms, in advance of increases of risk. In Table 3, we ask whether systemic risk contributions can be forecasted, portfolio by portfolio, by the lagged characteristics at different time horizons.

Table 3 shows that portfolios with higher leverage, more maturity mismatch, larger

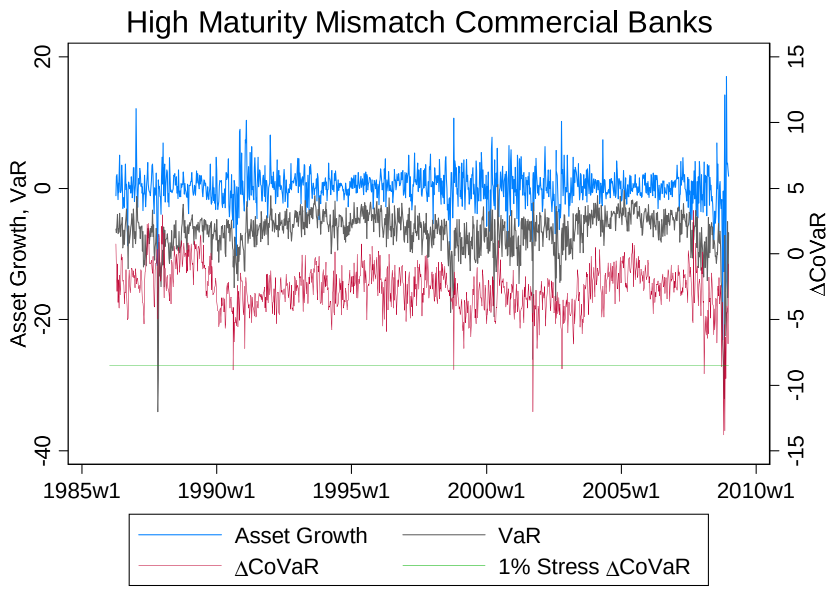


Figure 6: This figure shows the market valued asset returns (blue), the 1%-$VaR$ (gray), and the 1%-$\Delta CoVaR$ (red) for the portfolio of the 20% commercial banks with the largest maturity mismatch. The 1%-stress $\Delta CoVaR$ is also plotted. All risk measures are in weekly returns to total market valued assets.

size, and higher market-to-book tend to be associated with larger systemic risk contributions two years later. These $\Delta CoVaR$ regressions are run with risk measures that are time-aggregated by summing the weekly measures within each quarter. As a result, the coefficients in Table 3 are sensitivities of $\Delta CoVaR$ with respect to the characteristics expressed in units of quarterly returns. For example, the coefficient of -0.083 for the leverage forecast at the one year horizon implies that an increase in leverage (say from 15 to 16) of an institution is associated with an increase in systemic risk of 8.3 basis points of quarterly asset returns. For an institution that has $1 trillion of total market valued assets, that translates into $83 billion of systemic risk contribution.

Table 3 can be understood as a "term structure" of systemic risk contribution by

Table 3: $\Delta CoVaR^i$ **Forecasts by Characteristics at the Quarterly, One Year, and Two Year Horizons for the** 1% **Quantile**. This table reports the coefficients from forecasting regressions of the 1% $\Delta CoVaR^i$ on the quarterly, one year, and two year lag of portfolio characteristics. Each regression has a cross section of 100 portfolios. The methodology for computing the risk measures $VaR^i$ and $\Delta CoVaR^i$ is given in the captions of Tables 1 and 2. Risk measures are summed to a quarterly frequency. All regressions include industry fixed effects and time effects. The foreign dummy is the fraction of foreign firms in each portfolio. *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level. Significance is computed from robust standard errors.

| Variable | 2 Years | 1 Year | 1 Quarter |
|---|---|---|---|
| $\Delta CoVaR^i$ (lagged) | 0.623*** | 0.706*** | 0.876*** |
| $VaR^i$ (lagged) | -0.044*** | -0.033*** | -0.016*** |
| Leverage (lagged) | -0.093*** | -0.083*** | -0.049*** |
| Maturity mismatch (lagged) | -2.799*** | -1.948*** | -1.146*** |
| Relative size (lagged) | -0.731 | -1.002*** | -0.520** |
| Market-to-book (lagged) | -0.002* | -0.001** | -0.001 |
| Foreign dummy | 0.121 | 0.035 | 0.632 |
| Commercial Bank FE | 3.051*** | 2.322*** | 1.290*** |
| Investment Bank FE | -1.103*** | -0.732** | -0.109 |
| Insurance Company FE | -2.562*** | -2.411*** | -0.961*** |
| Constant | -10.168*** | -7.568*** | -3.325*** |
| | | | |
| Observations | 8102 | 8497 | 8798 |
| $R^2$ | 0.597 | 0.650 | 0.800 |

reading from right to left. It should be noted that we include lagged variables of the $\Delta CoVaR^i$ and $VaR^i$ in the regression so as to control for the persistence of systemic risk contribution. Table 4 reports the "tailness" of systemic risk. The forecasting horizon is fixed at the one year level and columns (1), (2), and (3) correspond to the forecasts of $\Delta CoVaR^i$ at the 1%, 5%, and 10% level. Column (2) of Table 3 and Column (1) of Table 4 are identical, per construction. Table 4 indicates that the systemic risk contribution of higher leverage, more maturity mismatch, larger size, and larger

Table 4: $\Delta$ ***CoVaR***$^{i}$ **Forecasts by Characteristics at the One Year Horizon for the** 1%**,** 5%**, and** 10% **Quantiles**. This table reports the coefficients from quarterly forecasting regressions of the $q$%-$\Delta$ $CoVaR^{i}$ on the one year lag of portfolio characteristics for $q = 1\%$, 5%, and 10%. Each regression has a cross section of 100 portfolios. The methodology for computing the risk measures $VaR^{i}$ and $\Delta CoVaR^{i}$ is given in the caption of Tables 1 and 2. Risk measures are summed to a quarterly frequency. All regressions include industry fixed effects and time effects. The foreign dummy is the fraction of foreign firms in each portfolio. *** denotes significance at the 1% level, ** denotes significance at the 5% level, and * denotes significance at the 10% level. Significance is computed from robust standard errors.

| Variable | 1% | 5% | 10% |
|---|---|---|---|
| $\Delta CoVaR^{i}$ (lagged) | 0.706*** | 0.665*** | 0.629*** |
| $VaR^{i}$ (lagged) | -0.033*** | -0.031*** | -0.046*** |
| Leverage (lagged) | -0.083*** | -0.005*** | -0.003*** |
| Maturity mismatch (lagged) | -1.948*** | -0.022 | -0.049*** |
| Relative size (lagged) | -1.002*** | -0.043*** | -0.019** |
| Market-to-book (lagged) | -0.001** | -0.000*** | -0.000 |
| Foreign | 0.035 | -0.020 | -0.028 |
| Commercial Bank FE | 2.322*** | 0.031** | 0.026** |
| Investment Bank FE | -0.732** | -0.109*** | -0.102*** |
| Insurance Company FE | -2.411*** | -0.113*** | -0.091*** |
| Constant | -7.568*** | -0.309*** | -0.402*** |
| | | | |
| Observations | 8497 | 8490 | 8490 |
| $R^2$ | 0.650 | 0.573 | 0.642 |

market-to-book tends to increase as we inspect quantiles further out in the left tail of the return distribution.

## 4.2 Macroprudential Policy

Instead of tying financial regulation directly to $\Delta CoVaR$, we propose to link it to the financial institution characteristics that predict $\Delta CoVaR$. This ensures that financial regulation is implemented in a forward looking way that counteracts the procyclicality of current regulation. Like any tail risk measure, $\Delta CoVaR$ estimates rely on relatively few data points. Hence, adverse movements, especially following periods of stability, can lead to sizable increases in tail risk measures. Any regulation that relies on contemporaneous $VaR$ and $\Delta CoVaR$ estimates would be unnecessarily tight after adverse events and unnecessarily loose in periods of stability. Thus, capital regulation based on current risk measures would amplify the adverse impacts after bad shocks, while also amplifying balance sheet expansions in good times (see Estrella (2004) and Gordy and Howells (2006)).

To overcome this procyclicality of capital regulation, we have shown that the $\Delta CoVaR$ measure is forecasted by institutions' characteristics such as maturity mismatch, leverage, market-to-book, relative size, and industry. Data limitations restrict our analysis, but a systemic risk supervisor can make use of a wider set of institution specific characteristics. We especially emphasize the predictive relationship between $\Delta CoVaR$ and these characteristics since they allow supervisors to act before excessive financial sector leverage and maturity mismatch builds up. The coefficients for each of these characteristic variables determine how systemic risk capital charges should be imposed.

Macroprudential policy tools to mitigate the likelihood of systemic financial crisis include capital regulation, taxation, reverse convertible debt, and insurance schemes. For each of these policies, the forecasting regressions can be used to determine magnitudes. For example, capital requirements can be based on the estimates of relative systemic risk contribution of maturity mismatch, size, and industry dummies. (Capital ratios cannot directly be based on the leverage estimates, as leverage itself is pinned down by the capital requirement.) In a Pigouvian taxation scheme, tax rates would be pinned down by weights from the forecasting regressions. In reverse convertible debt schemes, the requirement for how much reverse convertibles need to be held would be determined by the forecasting coefficients. In insurance schemes, the insurance premium would be based on the relative magnitude of the coefficients.

# 5 Conclusion

During financial crises or periods of financial intermediary distress, tail events tend to spill over across financial institutions. Such risk spillovers are important to understand for supervisors of financial institutions.

The financial market crisis of 2007-2009 has underscored fundamental problems in the current regulatory set-up. When regulatory capital and margins are set relative to $VaR$s, forced unwinding of one institution tends to increase market volatility, thus making it more likely that other institutions are forced to unwind and delever as well. In equilibrium, such unwinding gives rise to a margin/haircut spiral, triggering an adverse feedback loop. An economic theory of such an amplification mechanism is provided by Brunnermeier and Pedersen (2009). These "adverse feedback loops" were discussed by the Federal Open Market Committee in March 2008 and motivated Federal Reserve Chairman Ben Bernanke to call for regulatory reform.[4] Our $\Delta CoVaR$ measure provides a potential remedy for the margin spiral, as the measure takes the risk spillovers which give rise to adverse feedback loops explicitly into account.

We propose to require institutions to hold capital not only against their $VaR$, but also against the characteristics that forecast future $\Delta CoVaR$. By not relying on systemic risk surcharges that are based on contemporaneous risk measurement, but rather on the characteristics that are shown to forecast future systemic risk contribution, the

[4] See http://www.federalreserve.gov/monetarypolicy/fomcminutes20080318.htm. and http://www.federalreserve.gov/newsevents/speech/bernanke20080822a.htm.

proposal addresses the procyclicality of current capital regulation. Capital charges are forward looking, per construction.

For risk monitoring purposes, $\Delta CoVaR$ is a parsimonious measure for the potential of systemic financial risk. Supervisors that monitor systemic risk have traditionally followed the evolution of $VaR$s of individual financial institutions. $\Delta CoVaR$ allows supervisors to complement the individual institution risk estimates with systemic risk contribution estimates. This shifts the focus of supervision to overall financial sector risk and to the potential externalities that actions of individual institutions might impose on the financial system as a whole.

# A Asset Return Generating Model

The purpose of this appendix is to show that the asset return specifications of equations (6$a$)-(6$c$) that we use to estimate $CoVaR$ can be derived from a standard factor model for asset returns. Consider the following model for the returns $R_t^j$ of assets $j$:

$$R_t^j = \lambda^j M_{t-1} + \psi^j X_t^{system} + \kappa^j v_t + e_t^j \tag{9}$$

where

$$\begin{aligned} v_t &= \text{Systematic risk} \\ \kappa^j &= \text{Systematic risk sensitivity} \\ e_t^j &= \text{Idiosyncratic risk} \\ X_t^{system} &= \text{Financial system return} \\ \psi^j &= \text{Exposure to system return} \\ \lambda^j M_{t-1} &= \text{Time-varying expected returns} \end{aligned}$$

Balance sheet returns are the returns to some portfolio of assets $\theta^i$ such that:

$$X_t^i = \theta^i \mathrm{R}_t = \theta^i \left( \lambda M_{t-1} + \psi X_t^{system} + \kappa v_t + \mathrm{e}_t \right) \tag{10}$$

where $\kappa$ are the stacked $\kappa^j$ and $\mathrm{e}_t$ and the stacked $e_t^i$ shocks. We can sum across all institutions and solve (10) to obtain the returns of the system:

$$X_t^{system} = \underbrace{\tilde{\theta}^{system} \lambda}_{\beta^{system}} M_{t-1} + \underbrace{\tilde{\theta}^{system} \kappa v_t + \tilde{\theta}^{system} \mathrm{e}_t}_{\varepsilon_t^{system}} \tag{11}$$

where $\tilde{\theta}^{system} = \left(\left(1-\theta^{system}\psi\right)'\left(1-\theta^{system}\psi\right)\right)^{-1}\left(1-\theta^{system}\psi\right)'\theta^{system}$.

Equation (10) can also be solved for the system as a function of individual asset returns:

$$\begin{aligned} X_t^{system} &= \underbrace{\left(\left(\theta^i\psi\right)'\left(\theta^i\psi\right)\right)^{-1}\left(\theta^i\psi\right)'}_{\gamma^i}X_t^i + \underbrace{-\left(\left(\theta^i\psi\right)'\left(\theta^i\psi\right)\right)^{-1}\left(\theta^i\psi\right)'\theta^i\lambda}_{\beta^i}M_{t-1} \quad (12)\\ &+ \underbrace{-\left(\left(\theta^i\psi\right)'\left(\theta^i\psi\right)\right)^{-1}\left(\theta^i\psi\right)'\theta^i\left(\kappa v_t + \mathrm{e}_t\right)}_{\varepsilon_t^i}\\ &= \gamma^{system|i}X_t^i + \beta^{system|i}M_{t-1} + \varepsilon_t^{system|i} \end{aligned}$$

Note that $X_t^{system} = \beta^{system}M_{t-1} + \varepsilon_t^{system}$ from (11). Replacing into (12) gives:

$$X_t^i = \underbrace{\left(\theta^i\lambda + \theta^i\psi\beta^{system}\right)}_{\beta^i}M_{t-1} + \underbrace{\theta^i v_t + \varepsilon_t^{system} + \mathrm{e}_t}_{\varepsilon_t^i} \quad (13)$$

In summary, we obtain equations $(6a)-(6c)$:

$$X_t^i = \beta^i M_{t-1} + \varepsilon_t^i \quad \text{(14a)}$$

$$X_t^{system} = \beta^{system} M_{t-1} + \varepsilon_t^{system} \quad \text{(14b)}$$

$$X^{system} = \gamma^{system|i}X_t^i + \beta^{system|i}M_{t-1} + \varepsilon_t^{system|i} \quad \text{(14c)}$$

# B *CoVaR* Estimation via Quantile Regressions

This appendix explains how to use quantile regressions to estimate *VaR* and *CoVaR*. Suppose that returns $X_t^i$ have the following linear factor structure:

$$X_t^j = \phi_0 + M_{t-1}\phi_1 + X_t^i\phi_2 + \left(\phi_3 + M_{t-1}\phi_4 + X_t^i\phi_5\right)\varepsilon_t^j \tag{15}$$

where $M_{t-1}$ is a vector of state variables. The error term $\varepsilon_t$ is assumed to be i.i.d. with zero mean and unit variance and is independent of $M_{t-1}$ so that $E\left[\varepsilon_t^j|M_{t-1}, X_t^i\right] = 0$. Returns are generated by a process of the "location-scale" family, so that both the conditional expected return $E\left[X_t^j|M_{t-1}, X_t^i\right] = \phi_0 + M_{t-1}\phi_1 + X_t^i\phi_2$ and the conditional volatility $Vol_{t-1}\left[X_t^j|M_{t-1}, X_t^i\right] = (\phi_3 + M_{t-1}\phi_4 + X_t^i\phi_5)$ depend on the set of state variables $M_{t-1}$ and on $X_t^i$. The coefficients $\phi_0$, $\phi_1$, and $\phi_3$ could be estimated consistently via OLS of $X_t^i$ on $M_{t-1}$ and $X_t^i$. The predicted value of such an OLS regression would be the mean of $X_t^j$ conditional on $M_{t-1}$ and $X_t^i$. In order to compute the *VaR* and *CoVaR* from OLS regressions, one would have to also estimate $\phi_3$, $\phi_4$, and $\phi_5$, and then make distributional assumptions about $\varepsilon_t^j$.[5] The quantile regressions incorporate estimates of the conditional mean and the conditional volatility to produce conditional quantiles, without the distributional assumptions that would be needed for estimation via OLS.

Instead of using OLS regressions, we use quantile regressions to estimate model (15) for different percentiles. We denote the cumulative distribution function (cdf) of $\varepsilon^j$ by $F_{\varepsilon^j}\left(\varepsilon^j\right)$, and its inverse cdf by $F_{\varepsilon^j}^{-1}(q)$ for percentile $q$. It follows immediately that the

[5]The model (15) could otherwise be estimated via maximum likelihood using a stochastic volatility or GARCH model if distributional assumptions about $\varepsilon$ are made. The quantile regression approach does not require specific distributional assumptions for $\varepsilon$.

inverse cdf of $X_t^j$ is

$$F_{X_t^j}^{-1}\left(q|M_{t-1}, X_t^i\right) = \alpha_q + M_{t-1}\beta_q + X_t^i\gamma_q, \tag{16}$$

where $\alpha_q = \phi_0 + \phi_3 F_{\varepsilon^j}^{-1}(q)$, $\beta_q = \phi_1 + \phi_4 F_{\varepsilon^j}^{-1}(q)$, and $\gamma_q = \phi_2 + \phi_5 F_{\varepsilon^j}^{-1}(q)$ for quantiles $q \in (0,1)$. We call $F_{X_t^j}^{-1}(q|M_{t-1}, X_t^i)$ the conditional quantile function. From the definition of $VaR$:

$$VaR_q^j = \inf_{VaR_q}\left\{\Pr\left(X_t \leq VaR_q|M_{t-1}, X_t^i\right) \geq q\right\} = F_{X_t^j}^{-1}\left(q|M_{t-1}, X_t^i\right).$$

The the conditional quantile function $F_{X_t^j}^{-1}(q|M_{t-1}, X_t^i)$ is the $VaR_q^j$ conditional on $M_{t-1}$ and $X_t^i$ is . By conditioning on $X_t^i = VaR_q^i$, we obtain the $CoVaR_q^{j|i}$ from the quantile function:

$$CoVaR_q^{j|i} = \inf_{VaR_q}\left\{\Pr\left(X_t \leq VaR_q|M_{t-1}, X_t^i = VaR_q^i\right) \geq q\right\} = F_{X_t^j}^{-1}\left(q|M_{t-1}, VaR_q^i\right). \tag{17}$$

We estimate the quantile function as the predicted value of the $q-$quantile regression of $X_t^i$ on $M_{t-1}$ and $X_t^j$ by solving:

$$\min_{\alpha_q,\beta_q,\gamma_q} \sum_t \begin{cases} q\left|X_t^j - \alpha_q - M_{t-1}\beta_q - X_t^i\gamma_q\right| & \text{if } \left(X_t^j - \alpha_q - M_{t-1}\beta_q - X_t^i\gamma_q\right) \geq 0 \\ (1-q)\left|X_t^j - \alpha_q - M_{t-1}\beta_q - X_t^i\gamma_q\right| & \text{if } \left(X_t^j - \alpha_q - M_{t-1}\beta_q - X_t^i\gamma_q\right) < 0 \end{cases}.$$

See Bassett and Koenker (1978), and Koenker and Bassett (1978) for finite sample and asymptotic properties of quantile regressions. Chernozhukov and Umantsev (2001) discuss $VaR$ applications of quantile regressions.[6]

[6] Note that our sample size is chosen such that we do not need extreme value adjustments to our estimators. See Chernozhukov and Du (2008) for an overview of extremal quantile regressions, with for $VaR$ applications.

# C List of Financial Institutions for Figure 1

| **PANEL A: BANK HOLDING COMPANIES** | **PERMCO** | **TIC** |
|---|---|---|
| BANK OF AMERICA CORP | 3151 | BAC |
| CITIGROUP INC | 20483 | C |
| COUNTRYWIDE FINANCIAL CORP | 796 | CFC |
| JPMORGAN CHASE & CO | 20436 | JPM |
| WACHOVIA CORP | 1869 | WB |
| WELLS FARGO & CO | 21305 | WFC |
| **PANEL B: INVESTMENT BANKS** | **PERMCO** | **TIC** |
| BEAR STEARNS COMPANIES INC | 20282 | BSC |
| GOLDMAN SACHS GROUP INC | 35048 | GS |
| LEHMAN BROTHERS HOLDINGS INC | 21606 | LEH |
| MERRILL LYNCH & CO INC | 21190 | MER |
| MORGAN STANLEY | 21224 | MS |
| **PANEL C: GSEs** | **PERMCO** | **TIC** |
| FANNIE MAE | 20695 | FNM |
| FREDDIE MAC | 22096 | FRE |
| **PANEL D: INSURANCE COMPANIES** | **PERMCO** | **TIC** |
| AMERICAN INTERNATIONAL GROUP | 137 | AIG |
| BERKSHIRE HATHAWAY | 540 | BRK |
| METLIFE | 37138 | MET |

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