Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk*

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Abstract

We study the functioning and possible breakdown of the interbank market due to asymmetric information about counterparty risk. We allow for privately observed shocks to the distribution of asset risk across banks after the initial portfolio of liquid and illiquid investments is chosen. Our model generates several interbank market regimes: 1) low interest rate spread and full participation; 2) elevated spread and adverse selection; and 3) liquidity hoarding leading to a market breakdown. The regimes are in line with observed developments in the interbank market before and during the 2007-09 financial crisis. We use the model to examine various policy responses.

JEL CLASSIFICATION: G01, G21, D82

KEYWORDS: Financial crisis; Interbank market; Liquidity; Asymmetric information

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1 Introduction

Interbank markets play a key role in banks' liquidity management and in the transmission of monetary policy. Moreover, the interest rate in the unsecured three-month interbank market acts as a benchmark rate for the pricing of fixed-income securities throughout the economy. In normal times, the short-term market for liquidity works rather smoothly, as credit concerns play hardly any role. Markets tend to be very liquid, with a high turnover and a large number of participants.

Since August 2007, however, the functioning of interbank markets has become severely impaired in several countries, notably in the US and in the euro area. Central banks around the world had to intervene by adjusting their liquidity supply. As the financial crisis unfolded, liquidity in the interbank market has further dried up and banks have kept liquidity on their accounts instead of lending excess funds in the interbank market. The failure of the interbank market to redistribute liquidity has become a key feature of the 2007-09 crisis.

This paper provides a model of the interbank market that can generate such a dry-up of liquidity, and can be used to evaluate the effects of various policy interventions. We use a model in the spirit of Diamond and Dybvig (1983), in which consumers, who are uncertain about the timing of their consumption needs, put their money in the bank in exchange for a deposit contract. Banks face a tradeoff between liquidity and return in their portfolio choice: the long-term asset is more productive than the short-term asset over the long run but its early liquidation entails a loss. Liquidity shocks for each individual bank create a role for an interbank market in which banks with excess liquidity lend to those with a shortage. Since the long-term investment is risky, an interbank market loan may not be repaid, thus giving rise to counterparty risk.

We introduce asymmetric information about counterparty risk and show that it can generate various regimes in the interbank market, akin to those observed in the interbank markets before and during the current financial crisis (see below). In the first regime, there is full participation of borrowers and lenders in the interbank market. In the second regime, the interbank market is characterized by adverse selection. There is still borrowing and lending in the market. However, safe banks in need of liquidity drop out of the market and the interest rate rises to reflect that only riskier banks continue to borrow. In the third regime, the interbank market breaks down. This happens either because lenders prefer to hoard liquidity instead of lending it out to an adverse selection of borrowers, or because even riskier borrowers find the interest rate too high and choose to drop out.

Some interbank market facts. A standard measure of tensions in the (unsecured) interbank market is the spread between three-month bank borrowing costs and the overnight index swap in three months' time. It shows the premium banks have to pay for short-term funds compared with borrowing free of credit risk. Figure 1 plots the spread for the euro area from July 2006 to January 2009 (red line). The blue bars show recourses to the deposit facility of the ECB (daily average per week in billions of euros). Banks may use the deposit facility to make overnight deposits with the Eurosystem at a penalty rate. For completeness, the green bars depict liquidity-absorbing fine tuning operations (daily average per week in billions of euros). In normal times, the Eurosystem carries out such operations (at the policy rate) relatively infrequently to manage the liquidity situation and steer interest rates in the money markets.

From Figure 1, it seems that the interbank market experienced three qualitatively different phases in the time period depicted. The initial period is characterized by a low spread and infinitesimal amounts deposited by banks with the ECB. This is

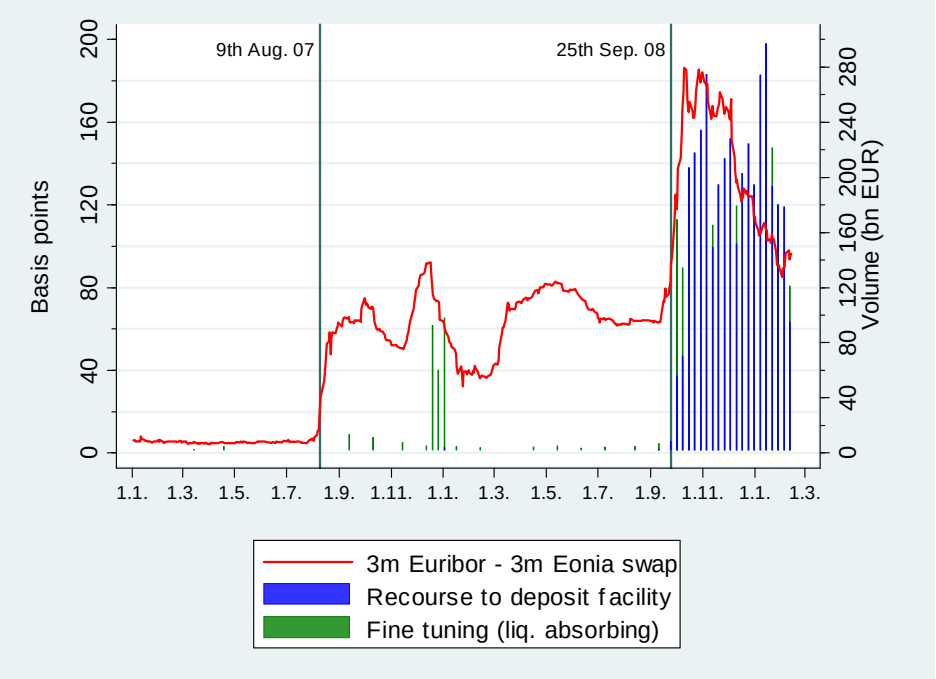

Figure 1: Interbank spread, recourses to the ECB deposit facility, and liquity-absorbing fine tuning operations, 07/2006 - 01/2009

consistent with the fact that in normal times, banks try to avoid using the ECB's overnight deposit facility because the deposit rate is punitive compared with the rates usually available on interbank markets. The second phase is characterized by an increased spread but still very low amounts deposited with the ECB overnight (except the 2007 year-end effect). The spread in the interbank market starts rising following the beginning of the financial crisis on August 9, 2007.

The third phase can be distinguished by a dramatic increase in the usage of the deposit facility by banks, in addition to a continuing rise in the spread. The amounts deposited with the ECB rise from a daily average of 0.09 billion euros in the week starting September 1, 2008 to 169.41 billion in the week of September 29, 2008. Between the week starting September 22 and the next week, the average daily volume in the overnight unsecured interbank market (not shown) in the euro area

almost halved, a drop of 29.28 billion euros, while the amount deposited at the ECB increased by 160.86 billion. Banks seem to prefer depositing funds at a penalty rate rather than lending them out even overnight.[1]

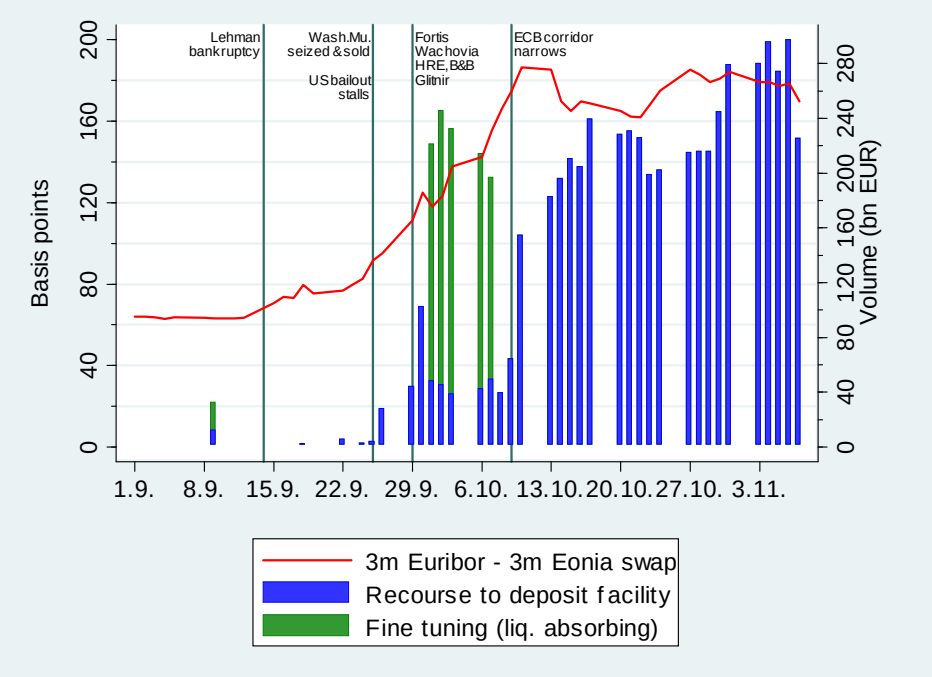

Figure 2: Interbank spread, recourses to the ECB deposit facility, and liquity-absorbing fine tuning operations, 09/2008 - 11/2008

The transition into the third phase and the major developments in the financial crisis at the time are depicted in more detail in Figure 2. The amounts deposited with the ECB start rising after the collapse of Washington Mutual when the crisis spreads outside the investment banking realm. Importantly, this rise precedes the ECB announcement of a change in its tender procedure and in the standing facilities corridor on October 8, 2008.[2]

[1] At the onset of the crisis in August 2007, the overnight interbank market saw an increase in volume. The average daily volume was 40.91 billion euros in the year prior to August 9, 2007. It increased by 27.40%, to 52.12 billion euros, in the second phase (between August 9, 2007 and September 26, 2008). This increase could reflect a substitution towards more short-term financing in the interbank market.

[2] The ECB reduced the corridor of standing facilities from 200 basis points to 100 basis points

A similar pattern of the three-month interbank market spread can be observed in the United States in the aforementioned time period, as documented in Figure 3. It plots the interbank market spread for the US (blue line) as compared to the euro area (red line) from July 2006 to January 2009.

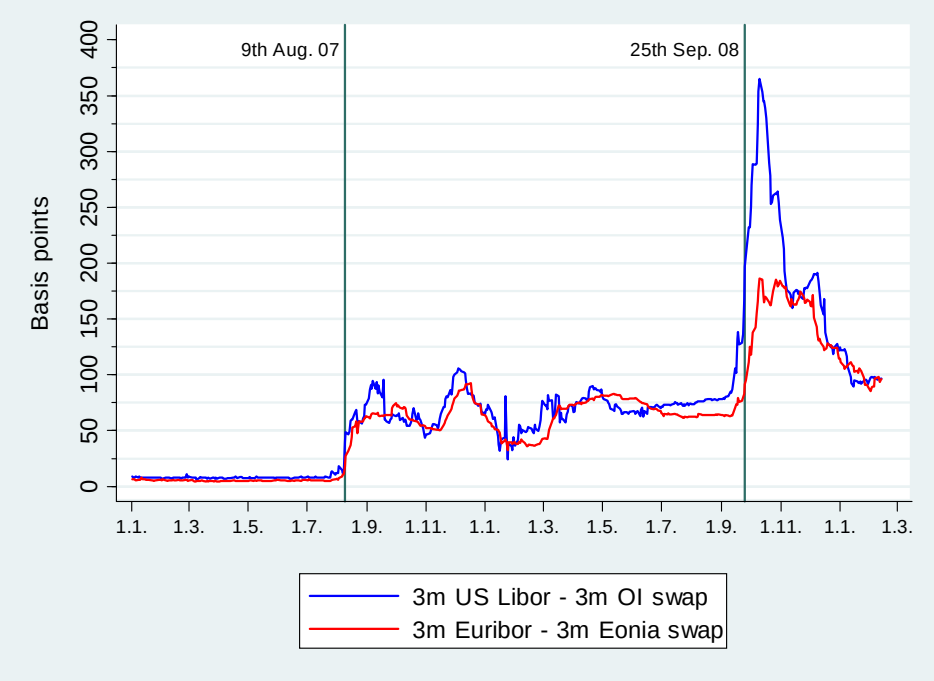

Figure 3: Interbank spreads US and euro area, 07/2006 - 01/2009

Related literature. The role of the interbank market to cope with bank specific liquidity shocks and avoid unnecessary liquidation of long term investments was first acknowledged in Bhattacharya and Gale (1987). Later contributions built upon this role while introducing moral hazard (Rochet and Tirole, 1996), aggregate liquidity

around the interest rate on the main refinancing operation as of October 9, thus making depositing at the deposit facility relatively more attractive. The rate of the marginal lending facility was reduced from 100 to 50 basis points above the interest rate on the main refinancing operation and the rate of the deposit facility was increased from 100 to 50 basis points below the interest rate on the main refinancing operation. Moreover, as from the operation settled on October 15, 2008, the weekly main refinancing operations is carried out through a fixed rate tender procedure with full allotment at the interest rate on the main refinancing operation.

risk (Allen and Gale, 2000), or credit risk (Freixas, Parigi, and Rochet, 2000). Furthermore, Bhattacharya and Fulghieri (1994) analyze the efficiency of an interbank market in a framework where banks face uncertain timing of liquidity returns, and Holmström and Tirole (1998) discuss the role of liquidity provision by the public sector.

The focus of our paper is studying the effects of asymmetric information about credit risk on the functioning of the interbank market. We are particularly concerned about the possibility of a break down due to asymmetric information. From that perspective, our work builds on Stiglitz and Weiss (1981) and is related to Broecker (1990) and to Flannery (1996) who also consider models of asymmetric information and credit risk. Freixas and Holthausen (2004) examine the scope for interbank market integration across countries when there is better information about the solvency of domestic banks than of foreign banks.

Recently, Freixas and Jorge (2007) examine how financial imperfections in the interbank market affect the monetary policy transmission mechanism. Ewerhart and Tapking (2008) analyze the role of collateral in interbank lending. Allen, Carletti, and Gale (2008), Acharya, Gromb, and Yorulmazer (2008), and Freixas, Martin, and Skeie (2008) study different rationales for central bank intervention in the interbank market.

The remainder of the paper is organized as follows. In Section 2, we describe the model setup. In Section 3, we solve for the equilibrium interest rates and banks' portfolio choices in each of the regimes. In Sections 4, we describe transition between regimes. Sections 5 and 6 discuss policy responses, and Section 7 concludes. All proofs are in the Appendix.

2 The model

There are three dates, $t = 0, 1$, and 2, and a single homogeneous good that can be used for consumption and investment. There is no discounting and no aggregate uncertainty.

Consumers and banks. There is a $[0, 1]$ continuum of consumers. Every consumer has an endowment of 1 unit of the good at $t = 0$. Consumers are risk averse with twice differentiable concave utility functions. Ex-ante, consumers are identical. As in Diamond and Dybvig (1983), some of them become "impatient" and only value consumption at $t = 1$ and some become "patient" and only value consumption at $t = 2$.

There is a $[0, 1]$ continuum of risk neutral, profit maximizing banks. We assume that the banking industry is perfectly competitive. Thus, banks make zero profits in equilibrium. At date $t = 0$, consumers deposit their endowment with a bank in exchange for a demand deposit contract which promises them consumption c_1 if they withdraw at $t = 1$ or c_2 if they withdraw at $t = 2$. Deposits are fully insured by deposit insurance and no bank runs occur.

Liquidity shocks. Banks are uncertain about the liquidity demand they will face at $t = 1$. For a fraction π_h of banks, a high fraction of consumers, denoted by λ_h, is impatient and wishes to withdraw at $t = 1$. The remaining fraction $\pi_l = 1 - \pi_h$ of banks faces a low liquidity demand λ_l, with $\lambda_l < \lambda_h$. The aggregate demand for liquidity at $t = 1$, denoted by $\lambda = \pi_h \lambda_h + \pi_l \lambda_l$, is known. Let the subscript $k = l, h$ denote whether a bank faces a low or high need for liquidity.

Assets and banks' portfolio decision. Banks can invest the consumers' endowment at $t = 0$ in two types of real assets, a long-term illiquid asset and a short-term liquid asset. Each unit invested in the liquid asset offers a return equal to 1 after one

period (costless storage). Each unit invested in the illiquid asset yields an uncertain payoff at $t = 2$. The illiquid asset can either succeed and return R or fail and return zero. In the latter case, a bank is insolvent. It is protected by limited liability and is closed down for a value of zero. Let α^I denote the fraction invested in the illiquid asset at $t = 0$. The remaining fraction $1 - \alpha^I$ is invested in the liquid asset.

Importantly, banks are uncertain about the riskiness of their illiquid investment when they make their portfolio allocation at $t = 0$. With probability q, the illiquid investment succeeds with probability p_s and with probability $1 - q$, it succeeds with probability $p_r < p_s$. Let p denote the expected success probability: $p = qp_s + (1-q)p_r$. Each bank becomes privately informed about the risk of its illiquid investment at $t = 1$. While the overall level of counterparty risk, p, is known, banks have private information whether their illiquid investment is safer ($p_s > p$) or riskier ($p_r < p$) than expected. The uncertainty about liquidity demand is assumed to be independent of the uncertainty about the risk of the illiquid asset. Let the subscript $\theta = s, r$ denote whether a bank's illiquid asset is safer or riskier.

The investment in the illiquid asset is ex ante efficient: $pR > 1$. This does not, however, preclude an illiquid investment that turns out to be riskier than expected to be unprofitable ex post: $p_r R < 1$. Any fraction α^L of the illiquid investment can be liquidated early, at $t = 1$, for a constant unit return of less than one (inefficient liquidation). We assume that safer investments are easier to liquidate, $1 > l_s > l_r$.[3]

Bank face a trade-off between liquidity and return when making their portfolio decision. The illiquid asset is ex ante more productive but its early liquidation is

[3]Our results would be qualitatively unchanged if we instead assumed that a riskier asset returns more, $R_s < R < R_r$, and that the per unit liquidation return is the same, $l_s = l_r = l$. What is needed is that the opportunity cost of liquidation, $\frac{R}{l}$, is higher for a riskier bank. Assuming different returns R_θ has the advantage of not requiring the buyer of liquidated assets to be able to assess its risk at $t = 1$. Assuming different liquidation returns l_θ considerably improves the tractability of the analysis.

costly.

Interbank market and liquidity management. Given that banks face differing liquidity demands at $t = 1$, an interbank market can develop. Banks with low withdrawals by impatient consumers can lend any excess liquidity to banks with high early withdrawals. Let L_l and L_h denote the amount lent and borrowed, respectively, and let r denote the interest rate on interbank loans. We assume that the interbank market is competitive, i.e. banks act as price takers.

Due to the risk of the illiquid asset, a borrower as well as a lender in the interbank market may be insolvent at $t = 2$ when the loan is repaid. A solvent borrower must always repay his interbank loan. If his lender is insolvent and has been closed down, the repayment goes to the deposit insurance scheme. In contrast, a solvent lender is only repaid if her borrower is solvent, too.

In sum, a bank can manage its liquidity at $t = 1$ in three ways: 1) by borrowing/lending in the interbank market, 2) by liquidating the illiquid asset early, and 3) by investing in the liquid asset for another period.

The sequence of events is summarized in figure 4 below.

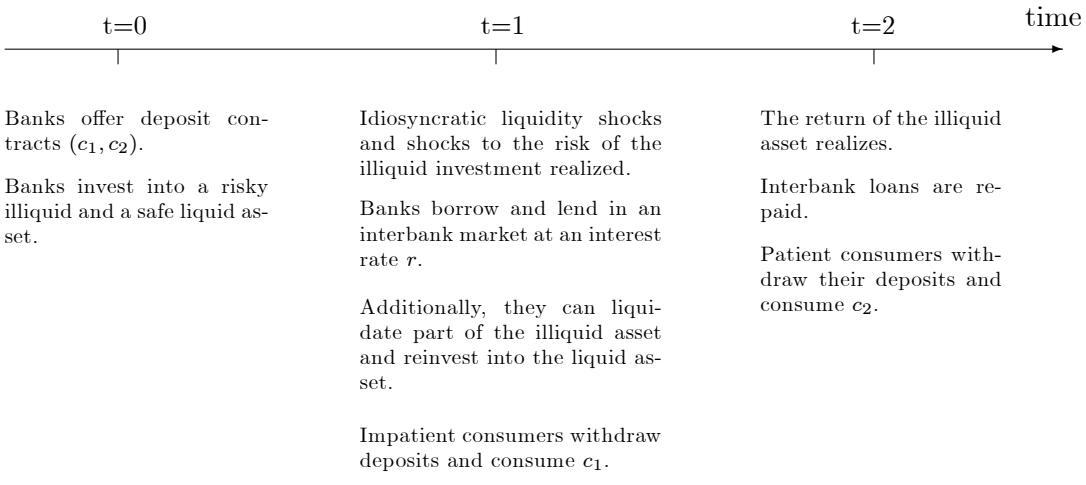

Figure 4: The timing of events

3 Analysis

In this section we solve the model backwards by first examining banks' liquidity management at $t = 1$ and then deriving the price of liquidity from banks' portfolio allocation at $t = 0$. We derive different regimes in the interbank market. First, there can be full participation of borrowers and lenders in the interbank market. Second, there can be adverse selection in the interbank market when borrowers with safer illiquid investments drop out. Third, the interbank market can break down because supply dries up (liquidity hoarding by lenders) or because demand dries up (all borrowers drop out). Which of the regimes occurs depends on the underlying parameters of the model.

3.1 Regime I: Full participation of borrowers and lenders

In order to characterize the regime with full participation in the interbank market, we start by assuming that there is indeed full participation and then verify for which parameters the assumption is met.

Liquidity management. Having received liquidity shocks, $k = l, h$, and being privately informed about the risk of their illiquid investment, $\theta = s, r$, banks need to manage their liquidity at $t = 1$ in order to maximize profits at $t = 2$.

A bank that faces a low level of withdrawals by impatient customers, type-(l, θ), solves the following problem:

$$\max_{\alpha^L_{l,\theta}, \alpha^R_{l,\theta}, L_{l,\theta}} p_\theta[R(1-\alpha^L_{l,\theta})\alpha^I + \alpha^R_{l,\theta}((1-\alpha^I) + \alpha^L_{l,\theta}\alpha^I l_\theta) + p(1+r)L_{l,\theta} - (1-\lambda_l)c_2] \quad (1)$$

subject to

$$\lambda_l c_1 + \alpha^R_{l,\theta}((1-\alpha^I) + \alpha^L_{l,\theta}\alpha^I l_\theta) + L_{l,\theta} \leq (1-\alpha^I) + \alpha^L_{l,\theta}\alpha^I l_\theta.$$

A type-(l,θ) bank has spare liquidity since the level of early withdrawals is low. The bank can thus lend $L_{l,\theta}$ at a rate r in the interbank market. The bank can also reinvest a fraction $\alpha^R_{l,\theta}$ in the liquid asset. Finally, it can liquidate a fraction $\alpha^L_{l,\theta}$ of its illiquid investment.

The budget constraint requires that the outflow of liquidity at $t = 1$ (deposit withdrawals, reinvestment into the liquid asset and interbank lending) be matched by the inflow (return on the liquid asset and liquidation proceeds).

A bank that has received a high liquidity shock, type-(h,θ), will be a borrower in the interbank market, solving:

$$\max_{\alpha^L_{h,\theta},\alpha^R_{h,\theta},L_{h,\theta}} p_\theta[R(1-\alpha^L_{h,\theta})\alpha^I + \alpha^R_{h,\theta}((1-\alpha^I) + \alpha^L_{h,\theta}\alpha^I l_\theta) - (1+r)L_{h,\theta} - (1-\lambda_h)c_2] \quad (2)$$

subject to

$$\lambda_h c_1 + \alpha^R_{h,\theta}((1-\alpha^I) + \alpha^L_{h,\theta}\alpha^I l_\theta) \leq (1-\alpha^I) + \alpha^L_{h,\theta}\alpha^I l_\theta + L_{h,\theta}.$$

A type-(h,θ) bank has a liquidity shortage. It can borrow an amount $L_{h,\theta}$ in the interbank market. It could also liquidate some of its illiquid asset and reinvest into the liquid asset.

There are two key differences between the optimization problems of a lender and a borrower. The first difference is in the objective function. A borrower expects having to repay $p_\theta(1+r)L_{h,\theta}$ while a lenders expects a repayment $p_\theta p(1+r)L_{l,\theta}$. A lender will not be repaid if the illiquid investment of his counterparty fails. With

full participation in the interbank market, a lender expects his counterparty to be solvent and repay the interbank loan with probability $p = qp_s + (1-q)p_r$ since he cannot distinguish safer and riskier borrowers. The second difference is in the budget constraint. The interbank loan is an outflow for a lender and an inflow for a borrower.

We characterize banks' liquidity management at $t = 1$ in a number steps. First, we obtain the marginal value of liquidity from the Lagrange multiplier on the budget constraint, denoted by $\mu^{k,\theta}$.

Lemma 1 (Marginal value of liquidity I) *With full participation in the interbank market, the marginal value of liquidity is* $\mu^{l,\theta} = pp_\theta(1+r)$ *for a lender and* $\mu^{h,\theta} = p_\theta(1+r)$ *for a borrower.*

A lender values liquidity at $t = 1$ since he can lend it out at an expected return $p_\theta p(1+r)$. A borrower values liquidity since it saves the cost of borrowing in the interbank market, $p_\theta(1+r)$. The marginal value of liquidity is lower for a lender because of counterparty risk.

The following result describes banks' decision to reinvest into the liquid asset.

Lemma 2 (Liquid reinvestment I) *With full participation in the interbank market, a borrower does not reinvest in the liquid asset at* $t = 1$*:* $\alpha^R_{h,\theta} = 0$*. A lender does not reinvest in the liquid asset iff* $p(1+r) \geq 1$*.*

It cannot be optimal for a type-(h,θ) bank to borrow in the interbank market at rate $1+r$ and to reinvest the obtained liquidity in the liquid asset since it would yield a negative net return. The same is not true for a lender since his rate of return on the lending in the interbank market is only $p(1+r)$ due to counterparty risk. But if a lender stores his liquidity instead of lending it out, then the interbank market cannot be active.

To have full participation in the interbank market, borrowers must not liquidate their long-term assets early. Otherwise, a borrower could never repay the interbank loan. If he liquidates, he has no inflows at $t = 2$ since he does not reinvest into the liquid asset at $t = 1$ (Lemma 2). Knowing that, no bank would lend in the interbank market.

The next result characterizes banks' liquidation decision.

Lemma 3 (No liquidation I) *With full participation in the interbank market, a borrower does not liquidate his illiquid investment iff* $1 + r \leq \frac{R}{l_\theta}$. *A lender does not liquidate his illiquid investment iff* $p(1 + r) \leq \frac{R}{l_\theta}$.

The decision depends on the benefit of liquidation relative to its opportunity cost. The benefit is given by the expected return on an interbank loan. Is is lower for a lender due to counterparty risk. The opportunity cost of liquidation, $\frac{R}{l_\theta}$, is the rate at which the return on the illiquid asset can be transformed into liquidity at $t = 1$. The opportunity cost is higher for a safer bank since its investment is easier to liquidate. It follows that i) borrowers liquidate earlier, i.e. at lower interest rates, than lenders, and ii) safer banks liquidate earlier than riskier ones.

Banks' liquidity management at $t = 1$ determines an interval of feasible interbank interest rates.

Proposition 1 (Feasible interbank loan rates I) *With full participation in the interbank market, the interbank interest rate satisfies:*

$$\frac{1}{p} \leq 1 + r \leq \frac{R}{l_s}.$$

The lower bound on the interest rate is given by the participation constraint of lenders. Their outside opportunity is to reinvest in the liquid asset. The upper bound

is given by the participation constraint of safer borrowers. Their outside opportunity is to liquidate the illiquid asset. Safer borrowers drop out of the interbank market earlier than riskier ones since their illiquid investment is easier to liquidate. The upper bound, unlike the lower one, depends on banks' risk type.

Pricing liquidity. At $t = 0$ banks decide how much to invest in the illiquid asset, fraction α^I, in order to maximize expected profits. Recall that at $t = 0$ banks are identical since the shocks to liquidity and to the riskiness of the illiquid asset have not yet realized. Under full participation, a bank solves

$$\begin{aligned} \max_{0\leq\alpha^I\leq 1} \quad & \pi_l p[R\alpha^I + p(1+r)L_l - (1-\lambda_l)c_2] \\ & +\pi_h p[R\alpha^I - (1+r)L_h - (1-\lambda_h)c_2] \end{aligned} \tag{3}$$

subject to

$$L_l = (1-\alpha^I) - \lambda_l c_1 \tag{4}$$

$$L_h = \lambda_h c_1 - (1-\alpha^I). \tag{5}$$

where we have used the fact that $\alpha^R_{k,\theta} = \alpha^L_{k,\theta} = 0$ (Lemma 2, Lemma 3 and Proposition 1). Since all banks are assumed to borrow or lend in the interbank market, L_k is given by the budget constraint at $t = 1$. The amounts lent and borrowed are independent of the risk-type of the illiquid investment, θ.

The first-order condition for a bank's optimal portfolio allocation across the liquid

and illiquid asset requires that[4]

$$\pi_h p_\theta (1+r) + \pi_l p_\theta p (1+r) = \pi_h p_\theta R + \pi_l p_\theta R$$

or, equivalently,

$$(\pi_h + \pi_l p)(1+r) = R. \tag{6}$$

The interbank interest rate r, the price of liquidity traded in the interbank market, is given by a no-arbitrage condition. The right hand side is the expected return from investing an additional unit into the illiquid asset, R. The left hand side is the expected return from investing an additional unit into the liquid asset. With probability π_h a bank will have a shortage of liquidity at $t = 1$ and one more unit of the liquid asset saves on borrowing in the interbank market at an expected cost of $p_\theta(1 + r)$. With probability π_l a bank will have excess liquidity and one more unit of the liquid asset can be lent out at an expected return $p_\theta p(1 + r)$. Lenders' expected counterparty risk is the average probability of repayment at $t = 2$ given that all borrowers participate in the interbank market, $p = qp_s + (1-q)p_r$. Note that banks' own probability of being solvent at $t = 2$, p_θ, cancels out in (6) since it affects the expected return on the liquid and the illiquid investment identically.

We rewrite (6) as:

$$\delta(1+r) = R \tag{7}$$

where

$$\frac{1}{\delta} \equiv \frac{1}{\pi_h + \pi_l p} > 1 \tag{8}$$

is the premium of lending in the interbank market due to counterparty risk. Liquidity

[4]It is straightforward to show that a corner solution cannot be optimal. The profitability of the illiquid asset implies a strictly positive investment in it. The presence of liquidity shocks implies a non-zero investment in the liquid asset.

becomes more costly when i) there are fewer suppliers of liquidity ($\pi_l = 1 - \pi_h$ decreases), and ii) counterparty risk, p, increases. Counterparty risk increases when is it less likely that the illiquid investment turns out to be safer than expected (lower q) or when the probability of success decreases (lower p_θ).

The next result summarizes the discussion on the pricing of liquidity at $t = 0$, taking into account the conditions obtained from the management of liquidity at $t = 1$ (Proposition 1).

Proposition 2 (Pricing I) *When there is full participation in the interbank market, then the risk premium must be smaller than the illiquidity premium of the safer illiquid asset:* $\frac{1}{\delta} \leq \frac{1}{l_s}$. *The interbank interest rate is given by* $1 + r = \frac{R}{\delta}$.

Under full participation in the interbank, there is no impairment to market functioning due asymmetric information about counterparty risk. The price of liquidity reflects the opportunity cost of not investing into illiquid asset, R, and the premium due to *average* counterparty risk, $\frac{1}{\delta}$.

Portfolio allocation. The amounts invested in the liquid and illiquid asset are determined by market clearing in the interbank market and competition among banks for deposits.

Proposition 3 (Illiquid investment I) *When there is full participation in the interbank market, then the fraction invested in the illiquid asset is given by*

$$\alpha^I = \frac{1 - \lambda - \pi_h(1-p)(1-\lambda_h)}{1 - (1-\delta)\lambda - \pi_h(1-p)(1-\lambda_h)}. \tag{9}$$

We can rewrite equation (9) as

$$\delta \frac{\alpha^I}{1-\alpha^I} = \frac{1 - \lambda - \Delta}{\lambda}, \tag{10}$$

where $\Delta = \pi_h(1-p)(1-\lambda_h)$. The relative amounts invested in the liquid and the illiquid asset multiplied by the discount due to counterparty risk is equal to the relative outflows at $t=1$ and $t=2$. Counterparty risk reduces the pay-out to depositors at $t=2$ by an amount Δ since some borrowers fail to repay their interbank loan.

It is useful to consider the benchmark case when there is no counterparty risk, $p=1$.

Corollary 1 (First Best) *Without counter-party risk, i) there is always full participation in the interbank market, ii) the interest rate is equal to R, and iii) the fraction invested in the illiquid and the liquid asset is equal to expected amount of late and early withdrawals:* $\alpha^I_{FB} = 1-\lambda$.

Without counterparty risk there is no friction in the economy. All banks participate in the interbank market since lending is riskless and not borrowing, i.e. liquidation, is inefficient. The returns from investing in the liquid and illiquid asset are equal. The amount invested in the liquid asset exactly covers the expected amount of early withdrawals since the interbank market fully smoothes out the problem of uneven demand for liquidity across banks. The fraction invested in the illiquid investment exactly covers the expected amount of late withdrawals.[5]

3.2 Regime II: Adverse selection in the interbank market

The previous section analyzed the regime with full participation in the interbank market. In that regime, borrowers whose illiquid investment is safer than expected subsidize borrowers whose illiquid investment turns out to be riskier. The subsidy becomes too costly when the risk premium is larger than the liquidation premium,

[5]It is easy to see that the pay-out to impatient and patient depositors is $c_1^{FB}=1$, $c_2^{FB}=R$, respectively.

$\frac{1}{\delta} > \frac{1}{l_s}$ (Proposition 2). In that case, the interest rate in the interbank market is so high that safer banks prefer to liquidate their illiquid asset instead of borrowing. Lenders therefore face an adverse selection of risky borrowers.

We follow the same steps as in the previous section. We start by assuming that there is adverse selection in the interbank market and then verify for which parameters there is indeed adverse selection. As before, we first examine banks' liquidity management at $t = 1$ and then consider banks' portfolio choice at $t = 0$.

Let r_r denote the interest rate and α_r^I the fraction invested in the illiquid asset when there is an adverse selection of risky borrowers in the interbank market.[6] Lenders' objective function $t = 1$ is the same as under full participation (equation (1)), except that the expected return on the interbank loan is now $p_r(1 + r_r)$ instead of $p(1 + r)$. Borrowers' expected interest repayment is now $1 + r_r$ instead of $1 + r$ (as in equation (2)). The budget constraint of banks active in the interbank market is unchanged. The analogue of Lemma 1 under adverse selection is:

Lemma 4 (Marginal value of liquidity II) *With adverse selection in the interbank market, the marginal value of liquidity is* $\mu^{l,\theta} = p_r p_\theta(1 + r_r)$ *for a lender and* $\mu^{h,r} = p_r(1 + r_r)$ *for a risky borrower.*

Adverse selection affects the marginal value of liquidity. It increases counterparty risk, $p_r < p$, and it changes the interest rate. We expect, and will confirm below, that the interest rate under adverse selection is higher than with full participation, $r_r > r$. As before, the marginal value of liquidity is higher for borrowers than for lenders.

The changes in the marginal value of liquidity modify banks' decisions to reinvest in the liquid and to liquidate the illiquid asset.

[6] For notational simplicity, we do not index by r the other choice variables.

Lemma 5 (Liquid reinvestment II) *With adverse selection in the interbank market, a risky borrower does not reinvest in the liquid asset at* $t = 1$*:* $\alpha^R_{h,r} = 0$*. A lender does not reinvest in the liquid asset iff* $p_r(1 + r_r) \geq 1$*.*

Lemma 6 (No liquidation II) *With adverse selection in the interbank market, a risky borrower does not liquidate his illiquid investment iff* $(1 + r_r) \leq \frac{R}{l_r}$*. A lender does not liquidate his illiquid investment iff* $p_r(1 + r_r) \leq \frac{R}{l_\theta}$*.*

As in the case with full participation in the interbank market, banks' liquidity management at $t = 1$ determines an interval of feasible interest rates under adverse selection.

Proposition 4 (Feasible interbank loan rates II) *With adverse selection in the interbank market, the interbank interest rate satisfies:*

$$\frac{1}{p_r} \leq 1 + r_r \leq \frac{R}{l_r}.$$

Under adverse selection, the lower bound on the interest rate is higher than with full participation (Proposition 1). Facing only risky borrowers, lenders' outside opportunity of reinvesting in the liquid asset is more attractive. Since only riskier banks borrow, the upper bound is also higher.

The portfolio allocation between the liquid and the illiquid asset at $t = 0$ determines again the interest rate in the interbank market. Anticipating adverse selection in the interbank market, a bank solves

$$\begin{aligned}\max_{0\leq\alpha^I_r\leq 1} \quad & \pi_l p[R\alpha^I_r + p_r(1 + r_r)L_l - (1 - \lambda_l)c_2] \\ & +\pi_h(1 - q)p_r[R\alpha^I_r - (1 + r_r)L_h - (1 - \lambda_h)c_2]\end{aligned}$$

subject to

$$L_l = (1 - \alpha_r^I) - \lambda_l c_1$$

$$L_h = \lambda_h c_1 - (1 - \alpha_r^I)$$

where we used the results in Lemma 5, Lemma 6 and Proposition 4. Compared to full participation (equation (3)), banks' objective function at $t = 0$ under adverse selection differs in two respects. First, the interest rate is given by r_r instead of r. Second, a bank expects to drop out of the interbank market if it receives a high liquidity shock and if its illiquid investment is safer than expected. With probability $\pi_h q$, a bank therefore liquidates its illiquid asset. The total value of its assets, $(1-\alpha_r^I)+\alpha_r^I l_s < 1$, is not enough to pay depositors both at $t = 1$ and $t = 2$ and thus the bank is insolvent.[7] The amounts lent and borrowed per bank are the same as with full participation.

The first-order condition for an optimal portfolio allocation under adverse selection is given by:

$$(\pi_l p p_r + \pi_h(1 - q)p_r)(1 + r_r) = (\pi_l p + \pi_h(1 - q)p_r)R \tag{11}$$

Comparing (11) to the condition with full participation (6) shows that adverse selection has two effects on the price of liquidity in the interbank market. First, lenders get repaid less often, $p_r < p$. Second, composition of banks in the interbank market changes since only risky banks borrow, which is reflected by the term $\pi_h(1 - q)p_r$.

We can rewrite the no-arbitrage condition (11) as

$$\delta_r(1 + r_r) = R \tag{12}$$

[7] The bank has effectively destroyed part of their customers' initial deposit. As in the case of insolvency due to the illiquid investment failing, the bank is protected by limited liability. It is closed down for a value of zero and its depositors are reimbursed by deposit insurance.

where

$$\frac{1}{\delta_r} \equiv \frac{\pi_l + \pi_h \zeta}{\pi_l p_r + \pi_h \zeta} \tag{13}$$

and

$$\zeta \equiv \frac{1}{1 + \frac{q}{1-q}\frac{p_s}{p_r}}. \tag{14}$$

Adverse selection affects the risk premium in the interbank market $\frac{1}{\delta_r}$ first via higher counterparty risk, p_r, and second via the composition effect ζ. Higher counterparty risk (lower p_r) and a worse composition (lower ζ) both increase the risk premium. Adverse selection in the interbank market therefore unambiguously increases the price of liquidity. The next Proposition confirms our initial hypothesis.

Proposition 5 *The interest rate under adverse selection, r_r, is higher than the interest rate with full participation, r, since the risk premium under adverse selection is higher, $\frac{1}{\delta_r} > \frac{1}{\delta}$.*

The next Proposition summarizes the pricing of liquidity under adverse selection in the interbank market.

Proposition 6 (Pricing II) *When there is adverse selection in the interbank market, then i) the risk premium must be smaller than the illiquidity premium of the risky illiquid asset: $\frac{1}{\delta_r} \leq \frac{1}{l_r}$; and ii) the risk discount must be smaller than the expected return of the risky illiquid asset, $\delta_r \leq p_r R$. The interbank interest rate is given by $1 + r_r = \frac{R}{\delta_r}$.*

The effect of adverse selection on banks' investment in the illiquid asset, α_r^I, depends on the parameters of the model. Assuming that a bank is equally likely to face a high or a low liquidity shock at $t = 1$, and reducing the number of free parameters, we can show that adverse selection leads to an overinvestment in the illiquid asset.

Proposition 7 *Assume that* $\pi_h = \pi_l$, $p_r = \frac{1}{2}$ *and* $\lambda_l = \frac{1}{2}$. *The amount invested in the illiquid asset under adverse selection is higher than under full participation,* $\alpha_r^I > \alpha^I$.

3.3 Regime III: Breakdown of the interbank market

When the interest rate under adverse selection is outside the bounds imposed by Proposition 4, then there will be a breakdown of the interbank market. Liquidity will no longer flow from banks with small liquidity shocks to banks with large liquidity shocks. The market can break down either because lenders stop providing liquidity to an adverse selection of borrowers (lack of supply) or because even risky banks find it too expensive to borrow (lack of demand).

Lack of supply. Adverse selection in the interbank market leads to a higher interest rate (Proposition 5). But is the increase in the interest rate high enough to compensate lenders for the larger counterparty risk when facing an adverse selection of borrowers? Lenders prefer to hoard liquidity by reinvesting it in the liquid asset when the lower bound in Proposition 4 is violated:

$$p_r(1 + r_r) < 1. \tag{15}$$

The condition can also be written as in Proposition 6

$$p_r R < \delta_r. \tag{16}$$

Since $\delta_r < 1$, lenders only hoard liquidity if the illiquid investment not only turns out to be riskier than expected, but it is also unprofitable. Note that this is compatible with the assumption about the ex ante efficiency of the illiquid investment, $pR > 1$.

Lack of demand. Even risky borrowers may choose to drop out of the interbank market if adverse selection drives up the interest rate too much. The upper bound on the interest rate in Proposition 4 is violated when

$$\frac{1}{\delta_r} > \frac{1}{l_r},$$

that is, when the risk premium is higher under adverse selection than the illiquidity premium for risky borrowers (see Proposition 6).

4 Interbank market regimes

The previous section shows that depending on parameters, three different regimes in the interbank market can occur as a unique equilibrium: i) full participation and no impairment to the functioning of the interbank market, ii) adverse selection and higher interest rates, and iii) market breakdown due to liquidity hoarding or a complete lack of demand.

Figure 4 shows which regime occurs under different parameters for average counterparty risk, p, and the dispersion of counterparty risk, $\Delta p \equiv p_s - p_r$. The set of feasible $(p, \Delta p)$ pairs is represented by the quadrangle of the three bold lines and the x-axis.

When the average level of counterparty risk is low, there is full participation in the interbank market (Regime I) regardless of the dispersion of counterparty risk. Asymmetric information about the risk of illiquid assets does not impair the functioning of the interbank market as long as the overall level of risk is low.

Once the average counterparty risk rises, driving up the interest rate in the interbank market beyond a certain threshold, safer banks with a liquidity shortage drop

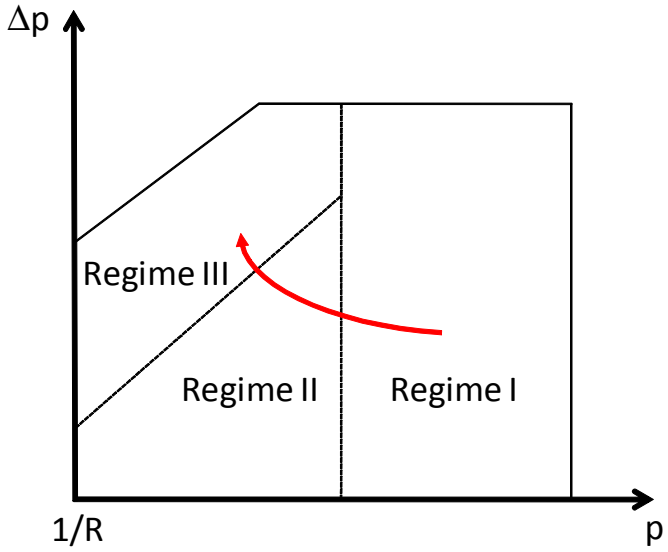

Figure 5: Comparative statics: Transition between regimes

out. Only an adverse selection of riskier banks keeps borrowing, causing the interest rate to increase even further. Once there is adverse selection in the interbank market (Regime II), the dispersion of counterparty risk matters. An increase in the dispersion of risk alone, without an increase in the level of risk, can lead to a breakdown of the interbank market and the hoarding of liquidity.

The arrow in figure 4 depicts a change in the level and the dispersion of counterparty risk and a corresponding transition between regimes that echoes the experience in interbank markets before and during the financial crisis of 2007-09. As explained in Figure 1, three different phases seem to characterize the situation in the interbank market: i) normal times, ii) elevated spreads but no recourse to the ECB's deposit facility, and iii) further increase in spreads with a substantial depositing of funds overnight with the ECB. The phases resemble the different equilibria of our model: i) no impairment, ii) adverse selection, and iii) liquidity hoarding. Moreover, the transition across regimes implies a change in the underlying level and dispersion of

counterparty risk that is consistent with the development of actual events. First, the transition from Regime I to II occurs at the start of the crisis in August 2007. At that time, subprime-mortgage backed securities were discovered in portfolios of banks and bank-sponsored conduits leading to a reassessment of the level of risk. The extent of exposures was unknown and counterparties could not distinguish safe from risky banks. The transition from Regime II to III occurs at the moment of the dramatic events surrounding the last week-end of September 2008.[8] Before the week-end, Washington Mutual, the largest S&L institution in the US was seized by the FDIC and sold to JPMorgan Chase. At the same time, negotiations on the TARP rescue package stall in US Congress. Over the week-end, it was reported that British mortgage lender Bradford & Bingley had to be rescued and that Fortis had to be nationalized. On Monday, Germany announced the rescue of Hypo Real Estate, and Iceland nationalized Glitnir. These events are signs of the financial crisis spreading outside the realm of investment banking and into the global financial system. They can be interpreted as a further increase in the level, and importantly, in the dispersion of counterparty risk.

5 Ex ante policy interventions

Section 3 shows that asymmetric information about the risk of the illiquid asset generates an inefficiency in Regime II. The price of liquidity in the interbank market is too high for banks whose illiquid investment turned out to be safer than expected. These banks prefer to liquidate their illiquid asset. Since liquidation is inefficient,

[8] The transition does not occur at the time of the Lehman failure on September 15th. Nor does it occur at the time the ECB announces the narrowing of its interest rate corridor (October 8, 2008). Note also that our model implies that increasing the deposit facility rate reinforces liquidity hoarding when there is a market breakdown. To see this, consider an increase in the right-hand side of (15) from 1 to $1+\tau$, where τ is the interest earned on deposits at the central bank.

there is scope for policy intervention.

In this section, we consider two ex ante policy interventions, liquidity ratios and market transparency. The interventions are put in place at $t = 0$ and we assume that the parameters are such that absent any policy intervention, the interbank market would be in Regime II. The aim of the intervention is to prevent the inefficient liquidation that would occur otherwise, while maximizing the expected return on the aggregate portfolio of the banking sector.

5.1 Liquidity requirements

To ensure full participation in the interbank market, the cap on the illiquid investment, denoted by α^I_{req}, must be such that the interbank interest rate is sufficiently reduced. Let r_{req} denote the interest rate under liquidity requirements. It must satisfy $\frac{1}{p} \leq 1 + r_{req} \leq \frac{R}{l_s}$ (Proposition 1). In addition, the optimal cap on the illiquid investment satisfies market clearing in the interbank market and maximizes the expected return on the aggregate portfolio of the banking sector:

$$p[R\alpha^I_{req} + \delta\left(1 + r_{req}\right)\left(1 - \alpha^I_{req}\right)].$$

The following result characterizes the optimal liquidity requirements.

Proposition 8 (Liquidity requirements) *Suppose that absent any regulation the interbank market is in Regime II, $l_s > \delta$. Liquidity requirements will avoid inefficient liquidation by safer banks by imposing a cap on the illiquid investment. The optimal regulated level of the illiquid investment is lower than the level under full participation, $\alpha^I_{req} < \alpha^I$. The corresponding interest rate is given by $1 + r_{req} = \frac{R}{l_s}$.*

The level of investment in the liquid asset needed to bring safer banks with a

shortage of liquidity back to the interbank market is higher than under full participation. Moreover, the participation constraint of safe borrowers binds and the interest rate is at the highest feasible level.

The benefit of the liquidity requirements is that safer banks with a liquidity shortage no longer liquidate their illiquid asset prematurely. The cost is that all banks are forced to hold more liquidity and thus forego part of the return on the illiquid investment. To assess the overall effect, we compare the expected return on the aggregate portfolio of the banking sector with and without the policy intervention.

Under the liquidity requirements, the expected return is given by[9]

$$pR\left[\frac{\delta}{l_s}(1-\alpha^I_{req})+\alpha^I_{req}\right]. \tag{17}$$

There are two distortions: the distortion of the price of liquidity, $\frac{\delta}{l_s}<1$, and the non-optimal portfolio allocation, α^I_{req}.

Absent the liquidity requirements, there would be adverse selection in the interbank market and the expected aggregate return would be:

$$\left[\pi_l p+\pi_h\left(1-q\right)p_r\right]R. \tag{18}$$

The cost under adverse selection is that a fraction $\pi_h q p_s$ of banks liquidate and thus forgo the return on the illiquid asset.

Comparing (17) and (18), the aggregate expected return on banks' portfolio is higher with the liquidity requirements if and only if

$$\alpha^I_{req}>\frac{\left[p\pi_l+p_r\pi_h\left(1-q\right)\right]l_s-p\delta}{pl_s-p\delta}.$$

[9] The expression is obtained from using $1+r_{req}=\frac{R}{l_s}$ in $\pi_l p[R\alpha^I+p(1+r_{req})(1-\alpha^I)]+\pi_h p[R\alpha^I-(1+r_{req})(-1)(1-\alpha^I)]$.

Since the right-hand side is smaller than 1, it is possible that the benefit of the liquidity requirements outweighs the cost. Imposing liquidity requirements is more likely to be beneficial if i) there are more safer banks with a liquidity shortage (higher $\pi_h q$), and ii) liquidating the illiquid asset is more inefficient (lower l_s).

5.2 Transparency

Another ex ante policy intervention we consider is market transparency. In the model, we assume that information about the riskiness of the illiquid asset is private to each bank. This asymmetry of information may cause adverse selection and lead to inefficient liquidation. One possible regulatory measure is thus to improve transparency in the banking sector. If, for example, bank supervisors can assess banks' risk and communicate it to the market, then lenders will be able to distinguish safer and riskier borrowers. Two markets then emerge, one for riskier banks with an interest rate, r^r_{tr}, and one for safer banks with an interest rate, r^s_{tr}.

The two interest rates are determined by two no-arbitrage conditions. As before, the first one follows from banks' portfolio allocation at $t = 0$:

$$\begin{aligned}\max_{0\leq\alpha^I_{tr}\leq 1} \quad & \pi_l p[R\alpha^I_{tr} + qp_s(1+r^s_{tr})L_l + (1-q)p_r(1+r^r_{tr})L_l - (1-\lambda_l)c_2] \\ + \quad & \pi_h q p_s[R\alpha^I_{tr} - (1+r^s_{tr})L_h - (1-\lambda_h)c_2] \\ + \quad & \pi_h(1-q)p_r[R\alpha^I_{tr} - (1+r^r_{tr})L_h - (1-\lambda_h)c_2]\end{aligned}$$

subject to

$$\begin{aligned}L_l &= (1-\alpha^I_{tr}) - \lambda_l c_1 \\ L_h &= \lambda_h c_1 - (1-\alpha^I_{tr}).\end{aligned}$$

The first-order condition with respect to α_{tr}^I yields

$$\pi_l p[qp_s(1+r_{tr}^s)+(1-q)p_r(1+r_{tr}^r)]+\pi_h[qp_s(1+r_{tr}^s)+(1-q)p_r(1+r_{tr}^r)]=pR. \quad (19)$$

The second no-arbitrage condition requires that in equilibrium, lenders are indifferent between lending to safer or riskier borrowers:

$$p_s(1+r_{tr}^s)=p_r(1+r_{tr}^r). \quad (20)$$

Combining (19) and (20) results in the following interest rates under market transparency:

$$\begin{aligned}\delta(1+r_{tr}^s) &= \frac{p}{p_s}R,\\ \delta(1+r_{tr}^r) &= \frac{p}{p_r}R.\end{aligned}$$

For market transparency to avoid the inefficient liquidation of safer banks, the interest rate, r_{tr}^s, must be sufficiently low:

$$1+r_{tr}^s \leq \frac{R}{l_s}, \quad (21)$$

or, equivalently $\frac{p}{p_s} \leq \frac{\delta}{l_s}$. This is possible since $\frac{p}{p_s}<1$ and $\frac{\delta}{l_s}<1$ in Regime II.

Market transparency lowers the interest rate for safer banks in need of liquidity since they are no longer pooled with riskier banks. In contrast, riskier banks will be charged a higher interest rate than under full participation. The interest rate, $1+r_{tr}^s$, must therefore not be too high. As long as:

$$1+r_{tr}^r \leq \frac{R}{l_r}, \quad (22)$$

or, equivalently, $\frac{p}{p_r} \leq \frac{\delta}{l_r}$, riskier banks participate in the market. Again, this is possible. Although $\frac{p}{p_r} > 1$, we also have that $\frac{\delta}{l_r} > 1$ since riskier banks would still borrow in Regime II, $l_r < \delta_r < \delta$ (Propositions 5 and 6). Market transparency enlarges the set of parameters for which all types of borrowers participate in the interbank market.

It is easily verified that under market transparency, the expected return on the aggregate portfolio of the banking sector is given by pR, which is always higher than the expected return without the intervention (18). Unlike liquidity requirements, it restores full participation without distorting the price of liquidity. The drawback of market transparency is, however, that it cannot be implemented when the conditions (21) and (22) are not satisfied.

6 Crisis resolution

This section analyzes the situation when an *unanticipated* adverse shock to counterparty risk, p, moves the economy from full participation (Regime I) to adverse selection (Regime II). Since the shock is not anticipated, banks made their portfolio allocation expecting no impairment to the functioning of the interbank market. But instead, safer banks in need of liquidity find the interest rate, which reflects the increased level of counterparty risk, too high.

We examine three policy interventions: the provision of liquidity by a central bank, interbank loan guarantees, and asset purchases by the government. As in the case of ex ante regulation, the aim of these "crisis" measures is to prevent the inefficient liquidation by safer banks. Since we assume that the shock to counterparty risk is unanticipated, the regulatory intervention is also unexpected. Thus, we abstract from moral hazard issues that can be an important consideration when examining policy

responses to crises.

The aim of the policy interventions is to avoid the inefficient liquidation of safer banks occurring in Regime II. The aim also carries over to Regime III as long as the market breaks down due to a lack of demand. When the market breaks down due to the lack of supply, it means that the illiquid investment of riskier banks must be unprofitable, $p_r R < 1$ (see equation (16)). If a supervisor knew the risk of banks' illiquid assets, then the optimal policy response would of course be to close riskier, insolvent banks.

6.1 Liquidity provision by the central bank

A central bank can offer to provide liquidity directly to banks in need. A central bank has no informational advantage over the market and thus it has to offer liquidity to all banks at an interest rate r_{CB}. The interest rate at which safer banks are just indifferent between borrowing from the central bank and liquidation is (see Proposition 1):

$$1 + r_{CB} = \frac{R}{l_s}.$$

This rate is necessarily lower than the interest rate following the unanticipated shock because safer banks in need of liquidity would find it optimal to drop out of the interbank market.

The central bank's net return from lending (an amount $\pi_h L_h$) to all banks is given by:

$$\pi_h L_h \left(p\frac{R}{l_s} - 1 \right),$$

which is positive since $pR > 1 > l_s$. Even though the central bank lends at a subsidized rate, it makes a profit. The reason is that a central bank can raise liquidity

at unit cost. That is, it can "print money". In contrast, the private supply of liquidity is costly since banks have to forgo investing in the illiquid asset if they want to be able to provide liquidity at $t = 1$. Moreover, banks have to bear liquidity and counterparty risk. The no-arbitrage condition (6) shows that the cost of private liquidity is $\frac{R}{\pi_h + \pi_l p} > R > 1$.

If a central bank provides liquidity to banks with a liquidity shortage, then it crowds out the private supply of liquidity. Banks with excess liquidity are no longer able to find a counterparty. In order to have a more balanced intervention, the central bank can offer to take on the excess liquidity and, possibly, offer a return on it. The central bank would effectively become an intermediary. It would be the counterparty for all liquidity transactions and replace the interbank market.[10]

A central bank can always provide liquidity at a lower cost than the interbank market. This is true even without a crisis. While such an intervention may seem desirable ex post (thus disregarding any moral hazard issues), it raises the question of how the central bank would determine the price of liquidity. The importance of markets in information aggregation, price discovery, and peer monitoring is well known (see, for example, the classic account of Hayek (1945), and, more recently, Rochet and Tirole (1996) in the interbank market context).

6.2 Interbank loan guarantees

An alternative policy response to an unanticipated shock to the level of counterparty risk is to guarantee interbank loans. Depending on their scope, guarantees reduce or even eliminate counterparty risk, thus lowering the interbank interest rate. Lower

[10]Indeed, many central banks have become intermediaries for liquidity transactions during the 2007-09 financial crisis. In October 2008, for instance, the ECB started providing unlimited amounts of liquidity in its weekly Main Refinancing Operations and, at the same time, took in significant amounts of deposits from banks (at a penalty rate).

interest rates in turn induce safer banks to borrow again.

Consider first the case of full interbank loan guarantees. Counterparty risk is eliminated, all banks participate in the interbank market and there is no liquidation of the illiquid investment (see Corollary 1). The interest rate in the interbank market drops to $1 + r_{FG} = R$, where r_{FG} denotes the interest rate under full guarantees.

The cost of this intervention for the guarantor is

$$p\left(1 + r_{FG}\right)\pi_h L_h - \left(1 + r_{FG}\right)\pi_h L_h = -R\pi_h L_h\left(1 - p\right). \tag{23}$$

The guarantor has to pay for all the losses due to the risk of the illiquid investment.

Consider next partial guarantees that increase the probability of repayment from p to $\hat{p}$, where $\hat{p}$ is high enough to restore full participation in the interbank market (see Proposition 1):

$$1 + r_{PG} = \frac{R}{l_s},$$

and where r_{PG} is the interest rate under partial loan guarantees.[11] The cost to the guarantor is:

$$p\left(1 + r_G\right)\pi_h L_h - \hat{p}\left(1 + r_G\right)\pi_h L_h = -\frac{R}{l_s}\pi_h L_h\left(\hat{p} - p\right). \tag{24}$$

The following result compares the cost of such partial guarantees to the cost of full guarantees.

Proposition 9 (Cost of partial guarantees) *The cost of partial guarantees that yield an interest rate just ensuring full participation,* $1 + r_{PG} = \frac{R}{l_s}$*, always exceeds the cost of full guarantees.*

[11] To ensure that lenders are still willing to lend, the guarantee must be sufficiently high: $\hat{p}(1 + r_{PG}) > 1$.

The reason is that the interest rate in the interbank market will be higher under partial guarantees to compensate lenders for the remaining counterparty risk. But higher interest rates increase the liability of the guarantor.

Proposition 9 illustrates that interbank loan guarantees should be sufficiently comprehensive to be cost-efficient for the public sector.

6.3 Asset purchases

Another crisis intervention to prevent inefficient liquidation is an asset purchase program. Early liquidation of illiquid assets is costly, $l_\theta < 1$. In a financial crisis, this inefficiency is particularly acute since assets often have to be liquidated at "fire-sale" prices. If the amount of the illiquid asset exceeds the amount of short-term liquidity available to buy it, the market will be characterized by "cash-in-the-market pricing" (see, e.g., Allen and Gale, 2004). In other words, illiquid assets are subject to market liquidity risk.

A central bank or a government authority does not face liquidity risk. They can offer to buy illiquid assets from banks for more than the fire-sale price, $P > l_\theta$, since the liquidity risk does not need to be priced in. The price only needs to reflect the credit risk of the assets. Moreover, by setting the price appropriately, the central bank can attract both safer and riskier borrowers and take advantage of pooling assets whose average quality is known.[12]

In particular, the price P could be set equal to the expected return on the illiquid asset, pR. This ensures that the central bank does not suffer losses on average. Since $pR > 1 > l_s$, safer borrowers prefer selling their illiquid asset to liquidating it. Such pricing effectively lowers the opportunity cost of liquidity to 1. This is beneficial for borrowers, who would otherwise have to pay a premium for obtaining liquidity in the

[12] It is the distribution of risk but not its level that is private information in our model.

interbank market since they have to compensate lenders for counterparty risk.

7 Conclusion

In this paper, we study the functioning and possible breakdown of the interbank market due to asymmetric information about counterparty risk. Banks receive private information about the risk of their illiquid investment after they made their portfolio allocation but before they trade with each in the interbank market.

We show that depending on parameters, reflecting in particular the distribution of counterparty risk among banks, an equilibrium with full participation of borrowers and lenders in the interbank market and no impairment to the reallocation of liquidity may not be reached. The interbank market can break down either because lenders stop providing liquidity to an adverse selection of borrowers or because banks in need of liquidity find it too expensive to borrow. The interbank market regimes obtained in the model echo the developments prior to and during the financial crisis of 2007-09.

Asymmetric information about counterparty risk can lead to the inefficient liquidation of illiquid investments. Thus, a policy intervention which ensures that the interbank market is open to all banks, or which provides alternative financing, can be beneficial. We assess the costs and benefits of different interventions by a regulator or a central bank.

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Appendix

Proof of Lemma 1

Let $\mu_2^{k,\theta}$ be the Lagrange multiplier on the feasibility constraint $L_{k,\theta} \geq 0$. The first-order condition for a type-(l,θ) w.r.t. $L_{l,\theta}$ is

$$p_\theta p(1+r) - \mu^{l,\theta} + \mu_2^{l,\theta} = 0 \tag{25}$$

while the first-order condition for a type-(h,θ) bank w.r.t. to $L_{h,\theta}$ is

$$-p_\theta(1+r) + \mu^{h,\theta} + \mu_2^{h,\theta} = 0 \tag{26}$$

Since we assume that all banks borrow and lend in the interbank market, we have $L_{k,\theta} > 0$ so that $\mu_2^{k,\theta} = 0$. Then (25) and (26) become

$$\begin{aligned} p_\theta p(1+r) &= \mu^{l,\theta} \\ p_\theta(1+r) &= \mu^{h,\theta} \end{aligned}$$

Proof of Lemma 2

Let $\mu_3^{k,\theta}$ and $\mu_4^{k,\theta}$ be the Lagrange multipliers on $0 \leq \alpha_{k,\theta}^R \leq 1$. The first-order condition for a type-(k,θ) bank w.r.t. to $\alpha_{k,\theta}^R$ is:

$$((1-\alpha^I) + \alpha_{k,\theta}^L \alpha^I l_\theta)(p_\theta - \mu^{k,\theta}) + \mu_3^{k,\theta} - \mu_4^{k,\theta} = 0 \tag{27}$$

Substituting $\mu^{h,\theta} = p_\theta(1+r)$ (Lemma 1) into (27) yields

$$((1-\alpha^I) + \alpha_{h,\theta}^L \alpha^I l_\theta)(-r) = -\mu_3^{h,\theta} + \mu_4^{h,\theta} < 0 \tag{28}$$

since left hand side is negative. It cannot be zero since $\alpha^I = 1$ and $\alpha_{h,\theta}^L = 0$ cannot be optimal. A type-(h,θ) bank would have to finance its entire need for liquidity by borrowing in the interbank market at a rate $r > 0$ whereas it could just store some liquidity without cost using the short-term asset. Since $-\mu_3^{h,\theta} + \mu_4^{h,\theta} < 0$ we have $\alpha_{h,\theta}^R = 0$.
Consider now the case of a lender. Substituting $\mu^{l,\theta} = p_\theta p(1+r)$ (Lemma 1) into (27) yields

$$((1-\alpha^I) + \alpha_{l,\theta}^L \alpha^I l_\theta) p_\theta (1 - p(1+r)) = -\mu_3^{l,\theta} + \mu_4^{l,\theta}$$

The first term on the left hand side is negative since $\alpha^I = 1$ and $\alpha_{l,\theta}^L = 0$ cannot be optimal. A type-(l,θ) bank cannot invest everything into the illiquid asset, not liquidate any part of it and still lend in the interbank market. Hence, $\alpha_{l,\theta}^R = 0$ iff $p(1+r) \geq 1$ (we assume that a type-(l,θ) bank does not reinvest into the liquid asset when the condition holds as an equality).

Proof of Lemma 3

Let $\mu_5^{k,\theta}$ and $\mu_6^{k,\theta}$ be the Lagrange multipliers on $0 \leq \alpha_{k,\theta}^L \leq 1$. The first-order condition for a type-(k,θ) bank w.r.t. to $\alpha_{k,\theta}^L$ is:

$$-p_\theta R\alpha^I + \alpha^I l_\theta(\alpha_{k,\theta}^R + \mu^{k,\theta}(1-\alpha_{k,\theta}^R)) + \mu_5^{k,\theta} - \mu_6^{k,\theta} = 0 \tag{29}$$

Substituting $\mu^{h,\theta} = p_\theta(1+r)$ and $\mu^{l,\theta} = p_\theta p(1+r)$ (Lemma 1) and $\alpha_{k,\theta}^R = 0$ (Lemma 2 and the assumption that there is full participation in the interbank market so that a type-(l,θ) bank does not reinvest into the liquid asset) into (29) yields

$$\begin{aligned} -p_\theta\alpha^I(-R+(1+r)l_\theta) &= -\mu_5^{h,\theta} + \mu_6^{h,\theta} \\ -p_\theta\alpha^I(-R+p(1+r)l_\theta) &= -\mu_5^{l,\theta} + \mu_6^{l,\theta} \end{aligned}$$

Since it cannot be optimal to invest nothing into the illiquid asset (if $\alpha^I = 0$ then $\mu_5^{h,\theta} = \mu_6^{h,\theta} = 0$ since they cannot both strictly positive, and thus $\alpha_{k,\theta}^L \in (0,1)$, which contradicts that nothing was invested into the illiquid asset), we have $\alpha_{h,\theta}^L = 0$ iff $(1+r)l_\theta \geq R$ and $\alpha_{l,\theta}^L = 0$ iff $p(1+r)l_\theta \geq R$ (we assume that a bank does not liquidate when the conditions hold as an equality).

Proof of Proposition 1

The lower bound on the feasible interest rate in the interbank market is given by Lemma 2. The upper bound is given by Lemma 3 where the lowest upper bound is given by a safer borrower since $l_s > l_r$.

Proof of Proposition 2

We need to check when the interest rate (equation (6)) is feasible under full participation (Proposition 1). The lower bound requires that $\frac{1}{p} \leq \frac{R}{\delta}$, or equivalently, $\delta \leq pR$, which is always satisfied since $\delta < 1$ and $pR > 1$. The upper bound is $\frac{R}{\delta} \leq \frac{R}{l_s}$, which simplifies to the condition in the Proposition.

Proof of Proposition 3

Using (4) and (5), market clearing in the interbank market, $\pi_l L_l = \pi_h L_h$, yields

$$\lambda c_1 = 1 - \alpha^I \tag{30}$$

Since competition forces banks to pay out everything to depositors at $t = 2$, we have that

$$(1-\lambda_l)c_2 = R\alpha^I + p(1+r)L_l \tag{31}$$

for a lender and

$$(1-\lambda_h)c_2 = R\alpha^I - (1+r)L_h \tag{32}$$

for a borrower. Eliminating c_2 yields

$$\frac{R\alpha^I + p(1+r)L_l}{1-\lambda_l} = \frac{R\alpha^I - (1+r)L_h}{1-\lambda_h} \tag{33}$$

The per depositor outflows for a lender and for a borrower have to be equal. This implies that an increase in counterparty risk, p, reduces the amount invested in the illiquid asset, α^I, ceteris paribus. In order to counter a decrease in p, a lender would increase α^I (left-hand side) but a borrower would decrease α^I (right-hand side). Since borrowers have fewer late withdrawals, $1-\lambda_h < 1-\lambda_l$, the negative borrower effect prevails on a per depositor basis.
Using (4) and (5) to substitute for L_l and L_h in (33), eliminating c_1 using (30), and solving for α^I yields the equilibrium relation between α^I and $1+r$ in Regime I:

$$\frac{R}{1+r}\frac{\alpha^I}{1-\alpha^I} = \frac{1-\lambda-\Delta}{\lambda}. \tag{34}$$

Using (7) to substitute for $1+r$, we can re-write (34) as

$$\delta\frac{\alpha^I}{1-\alpha^I} = \frac{1-\lambda-\Delta}{\lambda},$$

which simplifies to the condition in the Proposition.

Proof of corollary 1

Without counterparty risk, $p=1$, there is no risk premium, $\delta=1$, and no reduction in pay-out at $t=2$, $\Delta=0$. The corollary follows immediately from propositions 2 and 3.

Proof of Lemma 4

See the proof of Lemma 1 and replacing r and p with r_r and p_r in (1) and (2). Type-(h,r)) banks do not participate in the interbank market.

Proof of Lemma 5 and Lemma 6

See the proofs of Lemma 2 and Lemma 3 using $\mu^{k,\theta}$ from Lemma 4.

Proof of Proposition 4

The lower bound on the feasible interest rate in the interbank market is given by Lemma 5. The upper bound is given by Lemma 6.

Proof of Proposition 5

We need to show that $\delta_r < \delta$ Since $p = qp_s + (1-q)p_r$, we can write

$$q = \frac{p - p_r}{\Delta p}$$

where we used $p_s = p_r + \Delta p$ and $\Delta p = p_r - p_r$. Using these expressions to substitute for p_s and q in (13) and using $\pi_h = 1 - \pi_l$ we can write $\delta_r < \delta$ as

$$\frac{p_r(-(1-\pi_l)(p_r + \Delta p) + p((1-\pi_l) - \pi_l \Delta p))}{-(1-\pi_l)p_r(p_r \Delta p) + p(p_r(1-\pi_l) - \pi_l \Delta p)} < \pi_l p + (1-\pi_l)$$

where the right-hand side comes from (8). The condition holds iff

$$(1-p)(1-\pi_l)p_r + \Delta p(1-(1-p)\pi_l) > 0$$

which simplifies to

$$\pi_l < \frac{p_r(1-p) + \Delta p}{(1-p)(p_r + \Delta p)} = \frac{p_s - p_r p}{p_s - p_s p}$$

This always holds since the right-most expression is larger than 1.

Proof of Proposition 6

We need to check when the interest rate (equation (12)) is feasible under adverse selection (Proposition 4). The lower bound requires that $\frac{1}{p_r} \leq \frac{R}{\delta_r}$ and the upper bound is $\frac{R}{\delta_r} \leq \frac{R}{l_r}$, which simplify to the conditions in the Proposition.

Proof of Proposition 7

Available upon request.

Proof of Proposition 8

Choosing liquidity requirements that maximize the expected return on the aggregate portfolio of the banking sector requires solving the following problem:

$$\max_{\alpha^I_{req}} p[R\alpha^I_{req} + (p\pi_l + \pi_h)\left(1 + r_{req}\right)\left(1 - \alpha^I_{req}\right)] \tag{35}$$

subject to

$$\frac{1}{p} \leq 1 + r_{req} \leq \frac{R}{l_s},$$
$$\frac{R}{1+r_{req}} \frac{\alpha^I_{req}}{1-\alpha^I_{req}} = \frac{1-\lambda-\Delta}{\lambda},$$

where the first constraint is given by the condition on the interest rate (Proposition 1) and the second constraint is given by the equilibrium relation between the illiquid investment and the interest rate in Regime I (34).
Using (34) to substitute for $1 + r_{req}$, and taking the first-order condition yields

$$pR\left[1 + (p\pi_l + \pi_h)\frac{\lambda}{1-\lambda-\Delta}\right] + (\phi_1 - \phi_2)\frac{R\lambda}{1-\lambda-\Delta}\frac{1}{\left(1-\alpha^I_{req}\right)^2} = 0,$$

where $\phi_1 \geq 0$ and $\phi_2 \geq 0$ denote the Lagrange multipliers on $\frac{1}{p} \leq 1 + r_{req}$ and $1 + r_{req} \leq \frac{R}{l_s}$, respectively.
It follows that $\phi_2 > 0$ must hold (and thus $\phi_1 = 0$), implying that $1 + r_{req} = \frac{R}{l_s}$. The portfolio allocation that achieves this interest rate is given by

$$l_s \frac{\alpha^I_{req}}{1-\alpha^I_{req}} = \frac{1-\lambda-\Delta}{\lambda}, \tag{36}$$

implying that $\alpha^I_{req} = \frac{1-\lambda-\Delta}{1-\lambda-\Delta+\lambda l_s}$, $0 \leq \alpha^I_{req} \leq 1$.
From (10) and (36) we have

$$l_s \frac{\alpha^I_{req}}{1-\alpha^I_{req}} = \frac{1-\lambda-\Delta}{\lambda} = \delta\frac{\alpha^I}{1-\alpha^I}$$

Since absent any regulation, participation in the interbank market would not be full, we have that $l_s > \delta$. Thus, it must be that $\alpha^I > \alpha^I_{req}$ and the claim in the Proposition follows.

Proof of Proposition 9

Comparing (24) and (23), we see that the cost of partial guarantees exceeds the cost of full guarantees if and only if

$$l_s > \frac{\hat{p} - p}{1 - p}.$$

Since the participation constraint of safe borrowers is binding at the interest rate r_{PG}, we know that $l_s = \hat{p}\pi_l + \pi_h$ (see Proposition 2). Thus, the condition above can

be written as

$$\hat{p}\pi_l + \pi_h > \frac{\hat{p} - p}{1 - p},$$

which simplifies to

$$\hat{p} < 1$$

and hence the claim in the Proposition follows.