Locational Carbon Footprint and Renewable Portfolio Standards

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Abstract

The first part of the paper elaborates on the economic properties of the concept of locational marginal carbon intensity first presented in [1] and formulates a method of decomposing the carbon footprint of the electrical grid between individual generating units, transmission facilities and end users on a real time basis. In the second part of the paper the theory of the marginal carbon footprint is further applied to the derivation of the optimal investment policy underlying Renewable Portfolio Standards (RPS). The argument is made that the existing RPS policies are at best sub-optimal in their goal to reduce emissions of Carbon Dioxide and other greenhouse gases. A proposed optimal investment rule could serve to improve the efficiency of RPS policies.

Introduction

The study of the economics of CO_2 emissions is the study of costs and benefits of CO_2 abatement. Scientists tell us that the emission of greenhouse gases into the atmosphere is a global problem and that when a unit of CO_2 is released into air, the geographic location and, within limitations, the time of this event makes little difference on its consequences. In contrast, as discussed in this article, the time and geographical location of CO_2 abatement actions as well as actions that are not generally considered to be related to CO_2 abatement can make a significant difference in the quantity of emissions as well as the economic efficiency of actions directly or indirectly focused on reduction in CO_2 emissions. This is especially true for the power industry that is characterized by a diverse technological and geographical mix of generation technologies and a constrained transmission network in which avoidance of CO_2 emissions are temporally and spatially dependent in a significant number of ways.

Carbon[1] reduction economic policies are already a reality affecting operations of power systems in many European countries. The U.S. power industry is poised for a national carbon control policy being contemplated by the Federal government and is already subject to certain Regional Greenhouse Gas Initiatives (RGGI), Renewable Portfolio Standards (RPS) and a demand reduction programs at the utility and state

[1] For the purpose of this paper, terms "carbon," "carbon dioxide" or "CO_2" are used interchangeably.

levels, all claiming CO_2 reduction as a policy centerpiece. The investment community and industry project developers have expressed an increasing interest in the development of renewable generation technologies such as wind and solar on a massive scale and in construction of high voltage transmission lines to deliver renewable energy to the markets [2, 3, 4]. These referenced studies provide an example of analyses in which the impact of CO_2 abatement solutions is examined at a highly detailed level of engineering economics involving a combination of security constrained generation dispatch, power flow analysis and emission tracking on a generating unit specific basis. In the process of reviewing [2] and working on [3, 4] and other similar studies, the author had to acknowledge the disconnect between the level of detail required by these analyses and a relatively poor system of power industry specific concepts addressing the economics of CO_2 abatement.

The detailed engineering and mathematical analysis of CO_2 emissions in constrained power networks presented in this paper is initially developed in [1] which introduces the concept of marginal carbon intensity (MCI) of electricity consumption and studies time-dependent and locational properties of MCI within a networked power system that are similar those of locational marginal prices.

Section 1 of the present paper restates basic concepts introduced in [1] and then in Section 2 further expands the MCI theory toward the analysis of the locational carbon footprint of loads, generators and constrained transmission facilities within a power system. In that section of the paper, we provide a formal definition of the carbon footprint of a system element, derive mathematical formulas underlying its calculation and establish the relationship between the total system-wide mass of carbon emissions and carbon footprints of system elements.

Section 3 of the paper provides an application of this theory to the analysis of Renewable Portfolio Standards. Implementation of the RPS policy side-by-side with the nationwide or global carbon regulation offers the power industry a two-prong approach to CO_2 emissions control. Cap-and-trade or carbon tax-based policy directly affects dispatch order of thermal generating plants, electricity prices and thereby sends price signals with respect to new entry and retirement decisions on the part of generators and at the same time affects transmission planning decisions. RPS policies create incentives for renewable generating technologies to enter the market by providing investment subsidies to project developers. The intent here is to attract technological innovations to renewable generation technologies and bring them into the market at an accelerated pace. Generally, this has a potential to provide a relatively soft transitional path for the industry by not forcing thermal generators, primarily coal, into retirement through an introduction of a carbon price shock before the alternative renewable generating technology can take their place in the electricity market.

At the same time, as demonstrated in Section 3 of this paper, presently introduced RPS policies are poorly designed and inefficient. This is primarily because the subsidies provided to participants of RPS programs are not well aligned with the objective of carbon reduction. Under existing RPS programs, developers of renewable resources are paid on a per MWh of renewable generation regardless of the amount of CO_2 they actually displace.

To that end, we introduce a theoretical construct underlying a design of the optimal RPS program. In developing this construct, we provide a systematic comparison of two alternative RPS designs, one in which the subsidy provided to developers of renewable resources is used to maximize the total renewable energy and another one, in which the subsidy is used to minimize CO_2 emissions. We demonstrate that these two designs lead to different investment strategies.

1. Marginal Carbon Impact Indicators

1.1. Marginal Carbon Intensity

Let us consider an electrical grid as a whole and assume that at any moment in time we can measure the total mass of carbon emissions released by all interconnected generators. Thus, $\mathbb{C}(t)$ - total mass of CO_2 emissions produced by the electrical grid measured in tons of CO_2 over time period *t*.

Assume now that a market participant finds it economically beneficial to implement a load reduction measure which reduces electricity demand by a small amount at a given location on the grid. An important question here is how many units of CO_2 emissions will this measure help to avoid? An indicator providing an answer to this question is *Marginal Carbon Intensity (MCI)* which is equal to the decrease in CO_2 emissions in the electrical network in response to an infinitesimal decrease in electricity demand and measured in (T/MWh) [2]. As demonstrated in [1], *MCI* depends on the time and location of the applied demand reduction measure. The following mathematical formula defining the *MCI* reflects that dependency:

$$MCI_k(t) = \frac{\partial \mathbb{C}(t)}{\partial L_k(t)} \tag{1}$$

where $L_k(t)$ denotes demand at time *t* at location *k*. The larger is $MCI_k(t)$ for a given location and time, the greater is the change in the total carbon emission volume in response to the change in electricity demand. A positive value of MCI_k implies that at a given location and time an increase/decrease in electricity demand causes increase/decrease in CO_2 emissions in the system. A negative value of MCI_k implies that at a given location and time changes in electricity demand and CO_2 emissions move in opposite directions. (A statistical analysis of demand reduction measures relying on real-time prices reported in [5] indicates that demand reduction could result in an increase in emissions, in this case NOx and SO_2).

In order to get a better insight into this indicator, consider an unconstrained electrical system dominated by three generating technologies: conventional coal, combined cycle gas-fired (CCg) generation and a simple cycle combustion gas turbine (CTg). Their illustrative characteristics are presented in Table 1.

[2] Here and elsewhere T stands for metric tons of Greenhouse Gases in CO_2 equivalent.

Table 1. Illustrative Characteristics of Generators

Technology	Heat Rate (Btu/kWh)	Fuel Price ($/MMbtu)	VO&M ($/MWh)	CO_2 rate (T/MWh)	CO_2 price ($/T)	Dispatch cost ($/MWh)
Coal	9,500	2.0	1.0	0.9	10	29
CCg	7,000	5.0	3.0	0.4	10	42
CTg	11,000	5.0	5.0	0.6	10	66

Parameters presented in this table are typical for these generating technologies. For the purpose of this example, we assume a $10/T value of CO_2 emissions reflecting a CO_2 control policy in a form of a carbon tax or a price associated with a cap-and-trade program. For generators, this value represents an expenditure which they factor into their dispatch cost along with fuel and variable O&M expenses. The dispatch cost determines a merit order in which generators are deployed while serving system demand. The marginal cost of electricity is set by the cost of the *marginal* generator[3] - the most expensive generator needed to meet the demand in a given time period. All *inframarginal* generators, those below the marginal generator in the merit order, are dispatched at their full capacity, while *supermarginal* generators, those above the marginal generator in the merit order, are not dispatched at all.

A small enough increase or decrease in electricity demand in a given and short enough time period causes an equal (in absence of losses) increase or decrease in output of the marginal generator. Therefore, in each time period the marginal carbon intensity is determined by the emission rate of the generator that is marginal during that period. This is illustrated graphically on Figure 1 which depicts a chronological demand profile for a day, marginal generation technology at each point in time and the resulting dynamics of the *MCI*. As this figure demonstrates, marginal carbon intensity can vary significantly over time by following the change of the marginal generator. In answer to the originally posed question, the amount of carbon avoided by a small load reduction will be changing over time and and will depend on the marginal generator operating at any point in time. Temporal changes in the amount of avoided carbon are significant and could vary by a factor of greater than two when the marginal generating technology switches between convential coal and combined cycle gas-fired generation.

[3] In this and all other examples we assume that generators are always deployed on the basis of their dispatch costs.

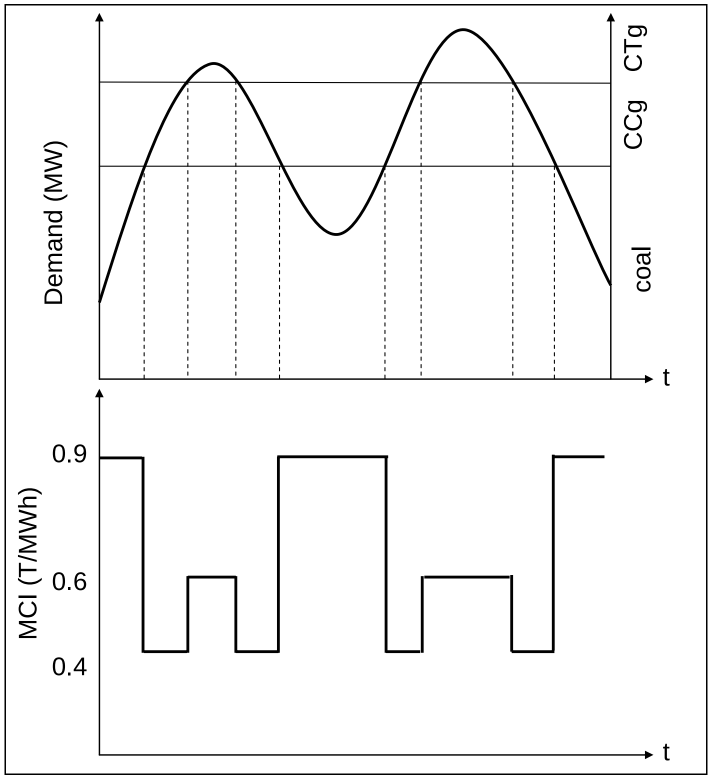

Figure 1. Electricity Demand, Marginal Generators and Marginal Carbon Intensity over Time

The above discussion of the marginal carbon intensity in an unconstrained lossless system could be summarized by the following formula

$$MCI_k(t) = \sigma_*(t) \tag{2}$$

i.e. marginal carbon intensity at all locations is equal to the carbon emission rate of the marginal generator $\sigma_*(t)$.

In a power network, when some transmission constraints bind, there are multiple marginal generating units[4] each with a different emission rate. A demand decrease at a given location requires a redispatch of these marginal units, some of which may have to be ramped down, some ramped up and other should remain unmoved in order to respond to a decremental demand without violating the security of the transmission system. In sum, marginal units have to be moved in tandem, in proportion to each other resulting in the following formula for locational *MCI*.

$$MCI_k = \sum_{j=1}^{m} \alpha_{kj} \sigma_j \tag{3}$$

Where *k* is the location (node) on the electrical grid for which *MCI* is calculated, *m* is the number of marginal units, $\sigma_1,...,\sigma_m$ are CO_2 emission rates of marginal units,

[4] Typically the number of marginal units equals the number of binding transmission constraints plus 1.

α_{kj} are location-specific proportionality coefficients such that $\sum_{j=1}^{m} \alpha_{kj} = 1$. As coefficients α_{kj} vary among locations, so will be the values of *MCI*.

1.2. Marginal Carbon Offset of a Generator

An intorduction of the concept of marginal carbon intensity helps also to answer another question: what is the carbon offset provided by incremental renewable generation, or more geenrally, by any generation deployed at a given location at any point in time?

The answer to this question is given by the difference between the *MCI* at a generator's location and generator's CO_2 emission rate $\sigma_k(t)$ which is equal to

$$\alpha_k^C(t) = MCI_k(t) - \sigma_k(t) \tag{4}$$

$\alpha_k^C(t)$ could be characterized as a *marginal carbon offset* provided by the generator. In absence of transmission losses, increasing generator's output by a small amount displaces output of the marginal generator by the same amount. Marginal carbon offset measures the net impact of this displacement on system-wide carbon emissions.

A positive carbon offset for a generator indicates that if it were possible to increase the generator's capacity (and its optimal output), it would reduce system-wide emissions. A negative carbon offset indicates that increasing this generator's capacity (and its output) increases system-wide emissions.

It is important to note that the carbon offset provided by a generator is dependent on the *MCI* at generator's location which in turn depends on time. Therefore, the fact that the generator has a low or even zero CO_2 emissions does not necessarily guarantee that it will provide a positive carbon offset. As demonstrated in [1], under certain circumstances, marginal carbon offset of renewable generation could be negative. At the same time, depending on system conditions, a non-renewable generator can provide a positive carbon offset as long as its own carbon emission rate is lower than the *MCI* at its location.

1.3. Shadow Carbon Intenstity of a Transmission Constraint

In [1] the impact of transmission congestion on the locational effectiveness of CO_2 reduction is addressed on a systematic level through a study of shadow carbon intensities of transmission constraints. Similarly to the definition of the economic shadow price, a shadow carbon intensity of a transmission constraint *SCI* is defined as a reduction in CO_2 emissions in the entire system in response to an infinitesimal increase in the rating of that transmission constraint and measured in T/MWh. In other words,

$$SCI_r = -\frac{\partial \mathbb{C}}{\partial F_r} \tag{5}$$

where SCI_r is a shadow carbon intensity of transmission constraint *r* and F_r is the rating of that constraint. Transmission constraints which do not bind have zero *SCI* values – increasing line ratings for these constraints would make no impact on overall carbon emissions. Relieving a constraint with a positive *SCI* value reduces carbon emissions. Relieving a constraint with a negative *SCI* value increases carbon emissions.

Another significance of this concept is that locational marginal carbon intensities and shadow carbon intensities of transmission constraints are linked by the same fundamental equation as locational marginal prices (LMPs) and shadow prices of binding transmission constraints:

$$MCI_k = MCI_0 - \sum_{r=1}^{R} \Psi_{kr} \times SCI_r \tag{6}$$

where MCI_0 is the *MCI* at the reference bus, *R*- number of transmission constraints and Ψ_{kr} - are transmission sensitivity coefficients. The derivation of this formula and the computational methodology required to calculate *MCI*s and *SCI*s for power networks is developed in [1].

2. Carbon Footprint Theorem

2.1. Carbon Footprint

A concept of the carbon footprint is widely used in the literature but is loosely defined. A typical definition provided for example in [6] states that carbon footprint is "the total set of greenhouse gas (GHG) emissions caused by an individual, organization, event or product." This definition, as well as other similar definitions, however, implies that this total set of GHG emissions is either known or could be measured. As the above discussion indicates, due to temporal and locational properties of the impact of electricity consumption on carbon emissions, actually measuring carbon footprint of an "individual, organization, event or product" associated with electricity consumption is difficult and ambiguous due to complex network properties of the power system. While the marginal carbon intensity at a given place and time adequately determines the carbon footprint of an incremental (marginal) change in electricity consumption, it is still unclear how to measure the footprint of an entire electricity use at that place and time. One way to establish the carbon footprint of each system element is by distributing total emissions among electricity consumers using an allocation rule. However, a potential set of such rules is infinite and there is no clear guidance on why one rule should be preferred over another. For example, computational methods currently used to perform life cycle assessments associated with electricity consumptions are based on the regional fuel mix of electricity production averaged over some historical period and across all consumers [7]. This simplistic approach is inaccurate, because it does not recognize the temporal and locational impacts of electricity consumption on carbon emissions and therefore provides incorrect signals to electricity market participants. A more accurate approach effectively based on the concept of the marginal carbon intensity is proposed in [8] within the so called “Dispatch data analysis operating margin” methodology. However, this methodology fails to explicitly account for locational properties of the MCI and leads to inaccurate estimates in the presence of transmission constraints and loop flows of power.

A more rigorous and accurate approach to defining the carbon footprint of the element of the power system could be based on the concept of financial responsibility for carbon emissions. Consider, for example, electricity load L at a given location on the grid with locational electricity price P_e with the total cost of serving this load of $P_e L$ expressed, for example, in dollars. Let us further assume that carbon emissions are priced at P_C expressed in dollars per ton of CO_2 and that the cost of carbon emissions are factored into the optimal system dispatch. The carbon footprint associated with this electricity load can be defined as the incremental change in the cost of serving load in response to an infinitesimal increase in carbon price under the assumption that the load is inelastic to price:

$$\mathbf{CF}[L] = \frac{\partial P_e}{\partial P_c} L \tag{7}$$

Note that since the numerator in (7) is expressed in dollars and denominator is expressed in dollars per ton of CO_2 the result is expressed in tons of CO_2. For example, if the cost of serving load equals \$1000 and a \$1/T increase in carbon prices cases the cost of serving load to increase by \$20, the carbon footprint of this load will be equal to 20 tons of CO_2.

Similarly, the carbon footprint could be defined for a generator and for a transmission constraint. The carbon footprint of a generator is a rate at which net revenues of this generator change in response to the change in carbon price. The carbon footprint of a transmission constraint is a rate at which the congestion rent associated with that constraint changes in response to the change in carbon price.

In order to fully formalize the above definition, consider financial flows for key elements of the power system (e.g. cost of serving load, net generators’ revenues and transmission congestion rent) resulting from the optimal environmental dispatch of a power system at carbon price of P_e and corresponding to that dispatch electricity prices. These financial flows could be expressed in the form of a well known identity [9]:

$$Cost(t) = \sum_{n=1}^{N} LMP_n(t) \times L_n(t) - \sum_{n=1}^{N} OM_n(t) \times G_n(t) - \sum_{r=1}^{R} SP_r(t) \times F_r \tag{8}$$

where N is a number of buses in the electrical network, R is a number of monitored transmission constraints, $L_n(t)$ and $LMP_n(t)$ are electricity demand and locational marginal price (LMP), respectively, at a location n, $G_n(t)$ and OM_n - power output and operating margin per unit of output, respectively, for generator at location n, F_r and SP_r -- power flow and shadow price, respectively, of transmission constraint r.

Here system-wide generation costs are decomposed into revenues collected from loads (the first term in (8)), net generators' revenues (the second term in (8)) and transmission congestion rent (the third term in (8)). By definition, an operating margin of each dispatched generator is equal to the difference between generator's LMP and short-run dispatch cost within operational power point

$$OM_n = LMP_n - c_n \tag{9}$$

The short-run dispatch cost c_n for the purpose of this analysis could be represented as the sum of non-carbon component a_n and the product of the emission rate σ_n and carbon price P_C

$$c_n = a_n + \sigma_n P_C \tag{10}$$

Definition.

1. *Carbon footprint of electrical load at a given location at a given moment in time t* $\mathbf{CF}[L_n(t)]$ *is defined as the change in the cost of serving that load in moment t in response to an infinitesimal change in carbon price*

$$\mathbf{CF}[L_n(t)] = \frac{\partial(LMP_n(t))}{\partial P_C} L_n(t) \tag{11}$$

2. *Carbon footprint of a generator n at time t* $\mathbf{CF}[G_n(t)]$ *is defined as the change in operating revenues accrued to that generator in time t in response to an infinitesimal change in carbon price:*

$$\mathbf{CF}[G_n(t)] = \frac{\partial(OM_n(t) \times G_n(t))}{\partial P_C} \tag{12}$$

3. *Carbon footprint of a transmission element r at time t* $\mathbf{CF}[F_r(t)]$ *is defined as the change in congestion rent of that element in response to an infinitesimal change in carbon price:*

$$\mathbf{CF}[F_r(t)] = \frac{\partial(SP_r(t) \times F_r(t))}{\partial P_C} \tag{13}$$

Using this definition, it is possible to establish basic properties of the carbon footprint of each element of the system and of the system as a whole.

***<u>Carbon Footprint Theorem</u>*.**

Carbon footprints of load, generators and transmission satisfy the following formulas:

$$\mathbf{CF}[L_n(t)] = MCI_n(t) \times L_n(t) \tag{14}$$

$$\mathbf{CF}[G_n(t)] = -\alpha_n^C(t) \times G_n(t) \tag{15}$$

$$\mathbf{CF}[F_r(t)] = -SCI_r(t) \times F_r(t) \tag{16}$$

The total carbon footprint of the entire system at a given time moment t could be represented as:

$$\mathbf{CF}(System) = \underbrace{\sum_{n=1}^{N} MCI_n(t) \times L_n(t)}_{\mathbf{CF}(Load)} \underbrace{- \sum_{n=1}^{N} \alpha_n^C(t) G_n(t)}_{\mathbf{CF}(Generation)} \underbrace{- \sum_{r=1}^{R} SCI_r(t) \times F_r}_{\mathbf{CF}(Transmission)} \tag{17}$$

The total carbon footprint is always equal to the total mass of carbon emissions of the power system

$$\mathbf{CF}(System) = \mathbb{C}(t) \tag{18}$$

Here $MCI_n(t)$ *is marginal carbon intensity at location n,* $\alpha_n^C(t)$ - *marginal carbon offset provided by generator at location n as defined in* (4), $SCI_r(t)$ -- *shadow carbon intensity of transmission constraint r.*

Proof.

The proof of equations (14)-(16) is provided in the Appendix at the end of this paper. Equation (17) is simply a definition of the carbon footprint of the system as a sum of the results of equations (14)-(16).

In order to prove equation (18), consider the following identity

$$\mathbb{C}(t) = \sum_{n=1}^{N} \sigma_n(t) G_n(t) \tag{19}$$

Equation (4) implies that

$$\mathbb{C}(t) = \sum_{n=1}^{N} (MCI_n(t) - \alpha_n^C(t)) G_n(t) \tag{20}$$

At the same time, as shown in [1], the following relationship holds

$$\sum_{k=1}^{N} MCI_k(t) \times \left[L_k(t) - G_k(t) \right] = \sum_{r=1}^{R} SCI_r(t) \times F_r(t) \qquad (21)$$

By substituting (21) into (20), we obtain that

$$\mathbb{C}(t) = \sum_{n=1}^{N} MCI_n(t) \times L_n(t) - \sum_{n=1}^{N} \alpha_n^C(t) G_n(t) - \sum_{r=1}^{R} SCI_r(t) \times F_r \qquad (22)$$

By comparing (22) and (17), we get (18).

Q.E.D.

The Carbon Footprint Theorem offers a straightforward method for allocating the total emission of CO_2 among elements of the power system represented by consumer loads, generators and congested transmission facilities in real time based on system operations.

It is important to compare equations (17) and (19). Equation (19) directly traces carbon emissions to their physical source, i.e. power generation. From that perspective, contribution of power generation to carbon emissions is at best non-negative and in most cases positive. In contrast, equation (17) allocates the responsibility for carbon emissions among system elements. The emphasis of this equation is on economic activities which cause carbon emissions.

As follows from the theorem, carbon footprints of power system elements depend on time and location on the grid, could be positive or negative. Elements with a positive carbon footprint could be considered as *virtual* sources of carbon. Elements with negative carbon footprint could be interpreted as *virtual* sinks of carbon. The virtual source does not point to the location on the grid where carbon is being released into the atmosphere, instead it points to the location of economic activity which causes carbon emissions. Similarly, virtual sinks do not physically absorb carbon emissions but they point to locations of economic activities that serve to offset carbon emission caused by virtual sources. According to the Carbon Footprint Theorem, virtual sources and sinks are always in balance with the actual CO_2 emissions released by the entire gird.

Virtual sources are loads with positive *MCI* values, generators with negative values of marginal offset (those whose own emission rate exceed *MCI* at their locations) and congested transmission elements with negative shadow carbon intensities (those for which congestion relief will increase carbon emissions in the system). Virtual sinks are loads with negative values of *MCI*, generators with positive values of marginal offsets (such as renewable generation at locations with a positive *MCI*) and transmission elements with positive *SCI* (those for which congestion relief will reduce carbon emissions).

According to the Carbon Footprint Theorem, in presence of transmission congestion carbon footprint cannot be fully attributed to loads and generators, some portion of the footprint has to be attributed to congested transmission. This attribution is informative: relieving constraints with positive carbon footprint will increase overall carbon emissions in the system while relieving constraints with negative footprint will reduce overall carbon emissions in the system. This assessment will be important for the purpose of transmission planning and assessing its impact on the system-wide carbon emissions.

In order to illustrate the locational properties of the carbon footprint, let us consider several illustrations of the optimal environmental dispatch using a simple case of a three-bus power grid. These illustrations are based on examples presented in [1] and discussed in the next section.

2.2. Illustrative Examples of Locational Carbon Footprint of a Three-bus Network

By following [1] we begin with several examples of a three-bus electrical network under different system parameters. Figure 2 depicts an optimal dispatch of the system in a transmission unconstrained case.

The system includes two generators G1 and G2 located at buses A and B, respectively and three loads, one attached to each bus. Loads at buses A and B are relatively small, 1 MW each. Bus C has the largest load, 50 MW and no generation attached to it. Each generator is characterized by capacity (MW), bid prices ($/MWh) and CO_2 emission rate (T/MWh). For the purpose of this example, we consider two generator bid price parameters. Parameter labeled "Bid price" reflects generator's fuel costs and non-fuel variable O&M expenses. "C-Bid price" includes also the cost of CO_2 emissions computed as a product of the generator's emission rate and the price of carbon, $10/T in this example. The optimal dispatch of this system should be performed on the basis of C-Bids thus internalizing the cost of carbon. Parameters of generators G1 and G2 correspond to parameters of the combined cycle gas fired and conventional coal generators presented in Table 1.

In absence of transmission constraints in this system and assuming no transmission losses, the optimal dispatch is obvious: we should utilize the least expensive resource (generator G2) up to its capacity of 30 MW and meet the remaining 22 MW of demand from generator G1.

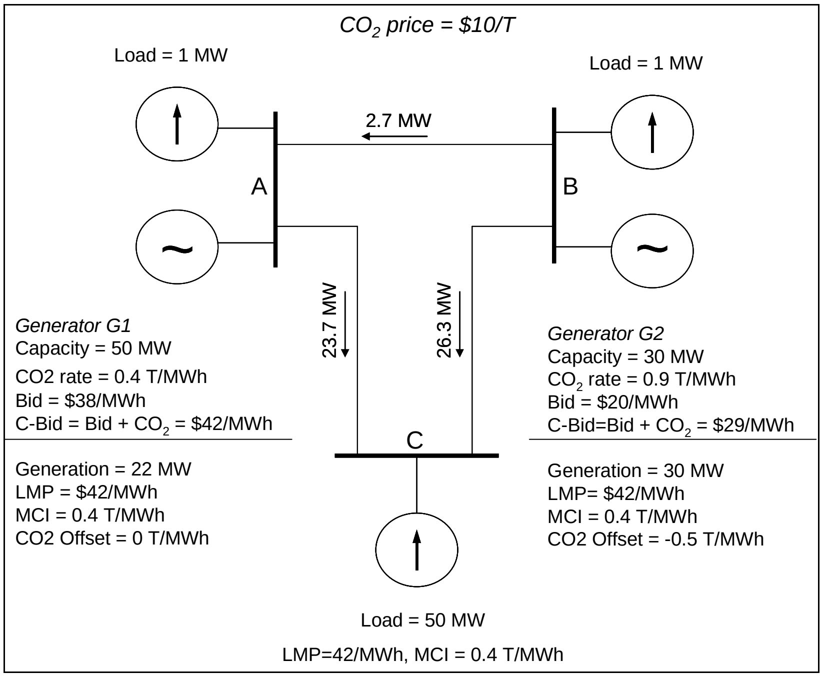

Figure 2. A Three-bus Network, Unconstrained Case

Generator G1 located at bus A is marginal and in absence of transmission constraints it sets the price for the entire system, i.e. LMPs at all three buses are the same and equal the C-Bid of generator G1, i.e. $42/MWh.

Assuming that all three lines A – B, A – C and B – C have the same impedance, 1/3 of power injected at bus A flows to load at bus C along the long path A – B – C and 2/3 flows along the short path A – C. The same rule holds for power injected at B, 1/3 flows over the long path B – A – C and 2/3 over the short path B – C. Resulting flows are shown on Figure 2.

In an unconstrained example presented on Figure 2, a single marginal unit (G1) sets the price and at the same time defines the *MCI* for all locations, 0.4 T/MWh. In this example, reducing demand by 1 MWh at any location would reduce carbon emission by 0.4 T. Generator G1 has CO_2 offset of zero, while generator G2 has a negative CO_2 offset of -$5/MWh. The resulting locational carbon footprint for this system is shown on Figure 3.

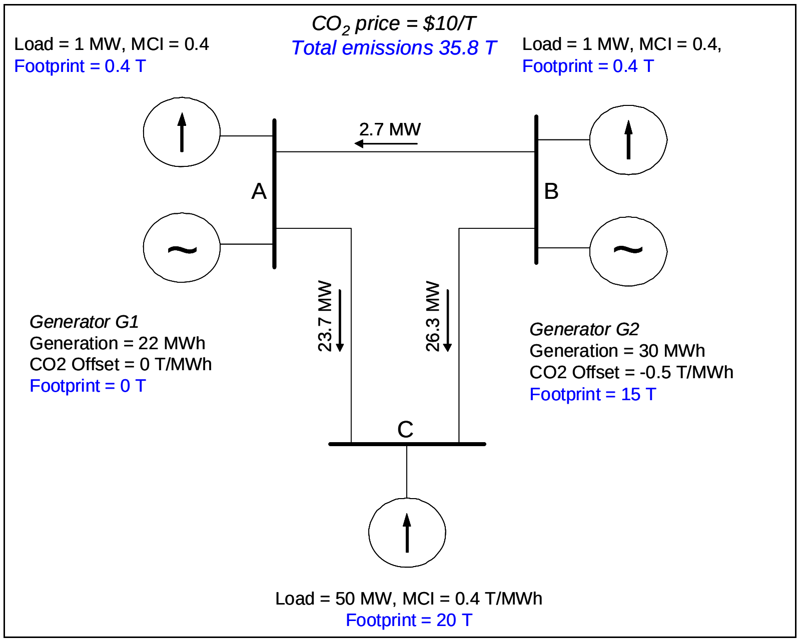

Figure 3. Carbon Footprint for the Unconstrained Case

Loads at buses A, B and C carry carbon footprint of 0.4T, 0.4T and 20T, respectively. Generator G1 has a zero marginal carbon offset and therefore zero contribution to the carbon footprint. Since generator G2 has a negative CO_2 offset of -$5/MWh its contribution to system-wide carbon emissions amounts to 15T. Altogether generators and loads are responsible for 35.8 T of carbon emissions which matches the total emissions produced by two generators: G1 emits 8.8T (22 MWh time 0.4 T/MWh), G2 emits 27T (30 MWh times 0.9 T/MWh).

A more interesting example is presented on Figure 4 depicting the case of a constrained network. In this and the next example we assume that the flow on the line B – C is limited at 20 MW and the flow on the line A – C is limited by 32 MW. A dispatch presented on Figure 2 is not feasible, because it results in a B – C flow of 26.3 MW which is above the limit. A redispatch is necessary in order to accommodate this constraint. Optimal dispatch and corresponding LMPs are shown on Figure 4 with line B – C now operating at its maximum rating of 20 MW while line A – C remains unconstrained. In this case, both generating units G1 and G2 are marginal. As shown in [1], LMPs at their buses are equal to their C-Bids of $42/MWh and $29/MWh, respectively. LMP at bus C is equal to $55/MWh. *MCI* at buses A and B are set by emission rates of generators located at these buses, since they are marginal and equal 0.4 and 0.9 T/MWh, respectively. *MCI* at bus C is negative -0.1 T/MWh[5].

[5] As explained in [1], a 1 MW load reduction at bus C will require an optimal redispatch of both generators in order to maintain the power flow along the line B – C within 20 MW limit. This cost minimizing redispatch is a decrease of generation at bus A by 2 MW and an increase of generation at bus B by 1 MW. The result of this redispatch is a decrease in carbon emissions of generator G1 by 0.8T = 2 MW x 0.4 T/MWh and an increase of carbon emissions at bus B by 0.9 T adding up to an overall 0.1 T increase in carbon emissions.

Since transmission line B – C binds, there is a shadow carbon intensity, associated with this constraint equal to negative -1.5 T/MW[6].

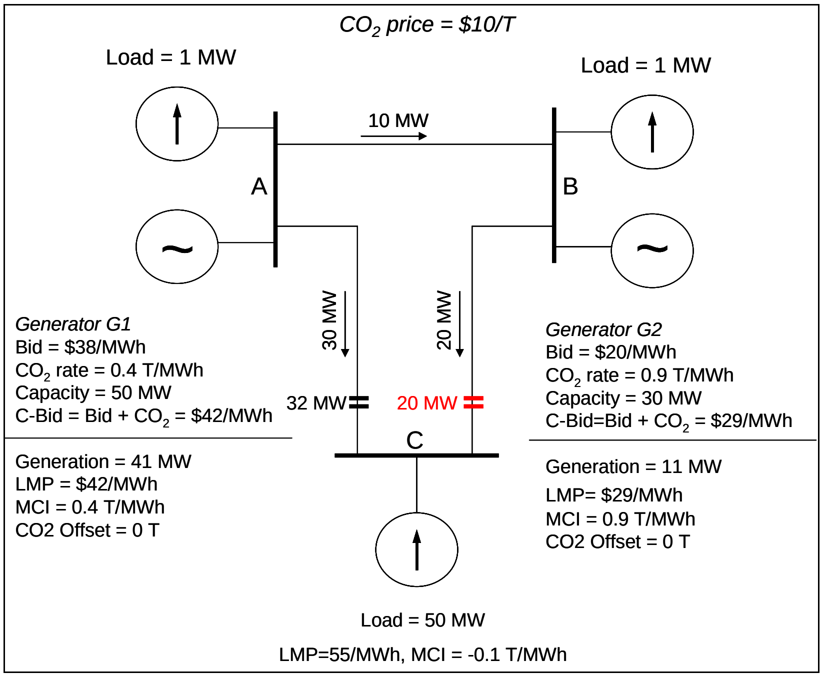

Figure 4. Three-bus Network, Constrained Case, $10/T CO_2

The interesting property of the system revealed by this example is the possibility for *MCI* value to become negative: reduction of electricity demand at location C results in an increase in carbon emission in the system which leads to the negative carbon footprint for this location, as shown on Figure 5. The distribution of the carbon footprint among system elements in this case is very different from that of the unconstrained example. Since both generators are marginal, their marginal offsets are zero and they make no contribution to the carbon footprint. Loads at buses A and B have carbon footprint of 0.4T and 0.9T, respectively, while load at bus C has a negative carbon footprint of –5T. The positive carbon footprint of 30T is concentrated at the binding constraint B – C.

In this example loads at A and B and constrained transmission line B – C serve as virtual sources of carbon while load at bus C serves as a virtual sink. Generators are virtually carbon-neutral. The overall balance matches system-wide carbon emissions of 26.3T. The bulk of the carbon footprint is neither attributable to loads, nor to generators, but is concentrated on the congested transmission line B – C.

[6] As explained in [1], increasing the rating of this constraint by 1 MW would allow unit G2 to increase its dispatch by 3 MW while reducing by 3 MW dispatch of unit G1. Doing so will reduce dispatch costs by –$39 = 3 x $29 – 3 x $42, but will increase carbon emissions by 1.5 T = 3 x 0.9 T – 3 x 0.4 T. In other words, the shadow carbon intensity of constraint B – C is negative 1.5 T.

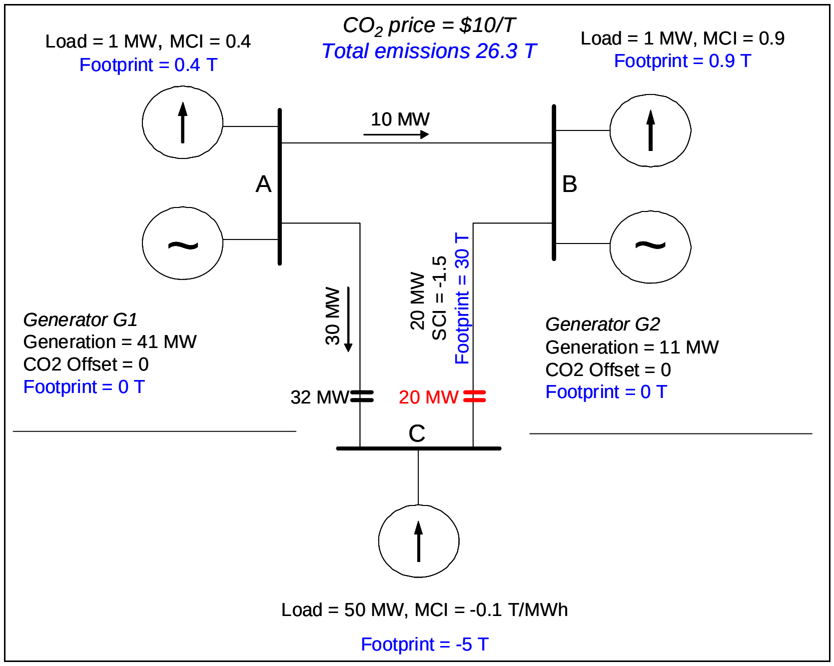

Figure 5. Carbon Footprint, Constrained Case, \$10/T CO_2

It is important to note, however, that the above results are not absolute and depend on the underlying price of CO_2 emissions. To illustrate that, consider yet another example presented on Figure 6. The key difference between this example and the one presented on Figure 4 is the underlying price of carbon. Instead of \$10/T used earlier, we now consider a \$40/T price of CO_2 emissions. This higher carbon emission price changes the merit order of generators G1 and G2. As a result, the dispatch changes such that line B – C is no longer constrained, but congestion moves to line A – C which now operates at its maximum rating of 32 MW. Both generators are again marginal with prices at buses B and C being equal \$54/MWh and \$56/MWh, respectively. LMP at bus C is now equal to \$58/MWh since now a redispatch needed to accommodate a 1 MW demand reduction at bus C requires a 2 MW decrease of generator G2 and 1 MW increase of generator G1. Marginal carbon intensities at buses B and C remain the same as in the previous example, but marginal carbon intensity at bus C now equals +1.4 T/MWh. Indeed, a 1 MW reduction in demand at C will result in a 2 MW decrease of output of G2 and 1 MW increase of G1. Hence, MCI at C equals 2 x 0.9 T/MWh – 1 x 0.4 T/MWh = 1.4 T/MWh.

Carbon footprint of loads, generators and transmission associated with this example is shown on Figure 7. Carbon footprint of generators and loads on buses A and B remain the same is in the previous example. However changes in the carbon footprint of load on bus A and of transmission constraints change very significantly.

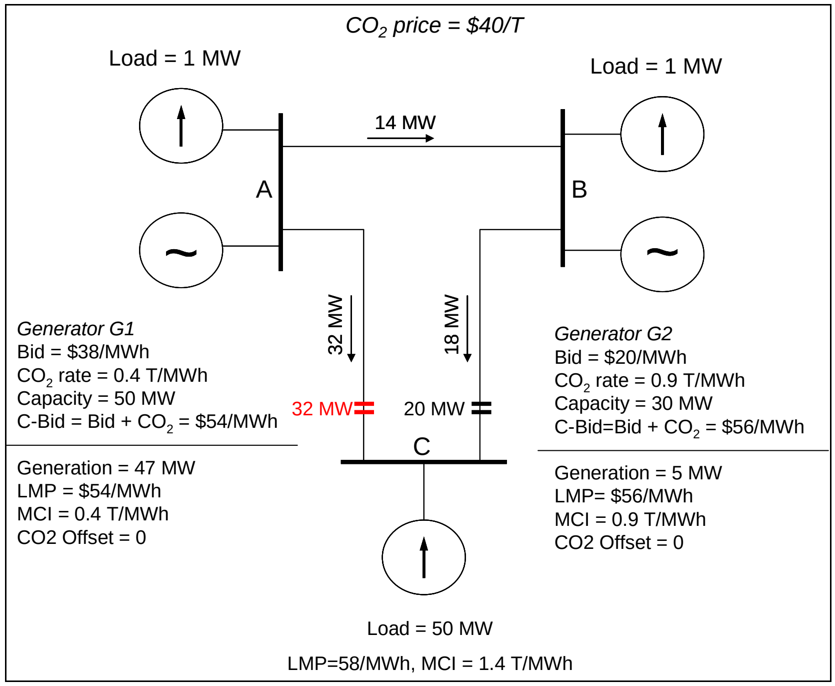

Figure 6. Three-bus Network, Constrained Case, $40/T CO_2

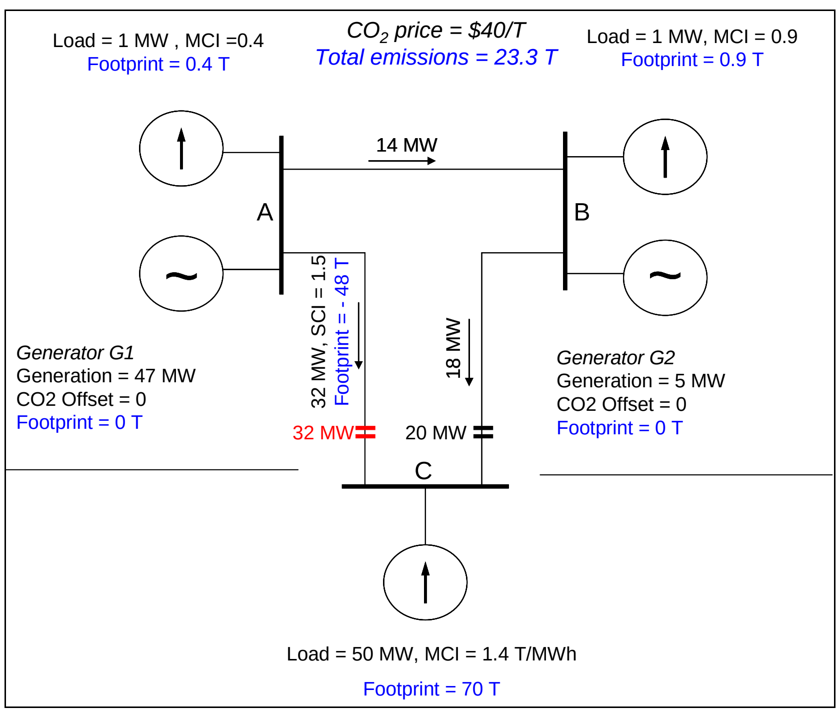

Figure 7. Carbon Footprint, Constrained Case, $40/T CO_2

In the $40 carbon case carbon footprint of load C is 70 T, well above total carbon emissions in the system. The load carbon footprint is offset by the negative footprint of constrained transmission line A – C which serves as a virtual carbon sink with negative footprint of -48 T.

Comparison of the last two examples shows that an increase in the price of carbon could make a significant change in transmission congestion. With shifting of congestion from B – C to A – C, the carbon intensity of bus C changes in both the sign and magnitude. As a result, bus C is transformed from being a virtual carbon sink in the $10 carbon case to a virtual carbon source in the $40 carbon case. In the $10 carbon case constraint B – C is a virtual carbon source: increasing flow through this constraint by relieving it would increase system-wide emissions. In the $40 carbon case constraint A – C is a virtual carbon sink: increasing flow through this constraint by relieving it would reduce system-wide emissions.

3. Theoretical Analysis of Traditional and Carbon Controlling RPS Programs

3.1. Supply Curves of Renewable Resources

In this section of the paper, we apply the results obtained above to the analysis of the Renewable Portfolio Standards (RPS) policy.

For the purpose of this analysis, we consider an RPS policy as an investment optimization problem. Let us assume that there are multiple developers of renewable resources interested in brining their projects online. In doing so, they evaluate the physical potential of the resource and the revenue potential in the electricity market (for energy and installed capacity) against costs to develop and finance their projects and on that basis determine locational capacity supply curves for each resource. A supply curve for a given resource indicates the level of subsidy per unit of incremental installed capacity the developer seeks in order to bring the resource online.

Developers then convey these supply curves to the RPS Agency, a hypothetical decision maker responsible for selecting the RPS portfolio. The objective of the RPS Agency is to meet RPS requirements at lowest costs measured as the total subsidy distributed to project developers on the basis of submitted supply curves.

We consider two alternative formulations of this problem: a traditional formulation in which the RPS requirements are set as the desired level of energy generated by renewable resources and a carbon controlling formulation in which the requirements are set in terms of CO_2 emissions released into the atmosphere by the entire generating system. In both formulations, we assume that the RPS Agency has a perfect knowledge of all future operational details of the power system and therefore is capable of reaching an optimal decision. Decision rules derived from the analysis of these two problems provide valuable theoretical insights that could be used to develop an efficient market mechanism for practical administering of actual RPS programs.

We consider an electrical grid with *n* potential interconnection points for renewable resources. We assume that each renewable resource is characterized by a specific temporal availability profile $g_j(t)$ such that $0 \le g_j(t) \le 1$ and that the maximum available generation from that resource is equal to

$$G_j^{\max}(t) = R_j g_j(t) \tag{23}$$

where R_j is the installed capacity of resource number *j*.

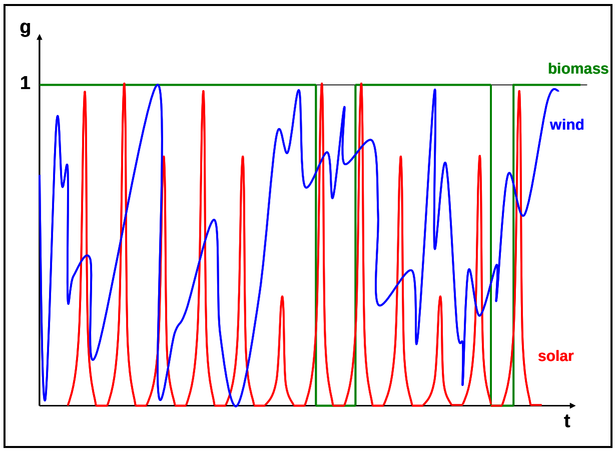

Figure 8. Stylized Availability Profiles for Wind, Solar and Biomass Resources

Figure 8 presents stylized availability profiles for wind, solar and biomass resources. Availability profiles for wind and solar generation are determined primarily by weather conditions, whereas biomass profiles are assumed to be driven by forced and planned outages in absence of which the resource is assumed to be available to generate at full capacity. Without loss of generality we assume that each grid interconnection point is characterized by a unique availability profile and a unique supply curve. In case when multiple resources are interconnected to the same physical point on the grid, we assume multiple replicas of that interconnection point with a unique numerical index assigned to each replica.

We further assume that for each interconnection point the RPS Agency develops a cost curve $Z_j(R_j);\ j = 1, 2, ..., n$ representing the total level of capital subsidy required in order to bring online R_j MW of renewable capacity installed at that location. By definition, the cost curve is constructed by integrating a capacity supply curve over quantity. A locational capacity supply curve is therefore will be equal to the derivative of the cost curve, $Z_j'(R_j)$ and is assumed to be a non-negative monotonically non-descending function of cumulative capacity[7]. By definition, the

[7] Thus we assume that capacity could be added in infinitesimally small increments, which is not an unreasonable assumption for wind and solar technology which can be increased in relatively small blocks. A more restrictive is the assumption that the supply functions are monotonic. A non-monotonic behavior of supply functions in itself does not alter major conclusion of this paper, but create some technical difficulties both from the theoretical and market design perspectives which are not discussed in this paper.

cost curve represents a convex continuous differentiable monotonically increasing function of cumulative capacity[8].

3.2. Marginal Impacts of Renewable Generation

In the above section of this paper, we studied the impact on carbon emissions of incremental renewable generation added at a given location on the grid which is expressed in the concept of the marginal carbon offset of the generator defined by equation (4).

As explained in [1], the same mathematics underlying the computations of marginal carbon intensity could be applied to other characteristics of the power system, for example to tracking the locational marginal intensity of renewable generation. To define this concept, consider the total level of renewable generation dispatched by the power system at any given time period and denote it as $\Re$. Assume now an infinitesimal change in electricity demand $dL_j(t)$ at location *j* on the grid and define a locational marginal renewable intensity as

$$\rho_j(t) = \frac{\partial \Re}{\partial L_j(t)} \tag{24}$$

Similarly to the behavior of marginal carbon intensity, marginal renewable intensity will be equal to the weighted average of per unit renewable generation by marginal generating units with exactly the same weighting coefficients that would be used to compute marginal carbon intensities. For a given generating unit, its per unit renewable generation will be equal to 1 for a renewable generator and 0 for a non-renewable generator. If no renewable generators are on the margin, marginal renewable intensity for all locations will be zero. Although the dispatch costs of renewable generation such as wind and solar are low, significant penetration of these resources can make them marginal in hours of low demand or local transmission congestion. When renewable generators become marginal, the locational marginal renewable intensity could become non-zero and could be positive or negative depending on location.

Imagine now that we interconnect a renewable generator to the grid at point number *j*. Each incremental MWh of power produced by that generator will inject into the grid 1 MWh of renewable power and emit σ_j tons of CO_2[9]. At the same time, the overall system-wide renewable generation will be reduced by $\rho_j(t)$ and the system-wide CO_2

[8] For the purpose of this paper we assume smooth and convex cost curves. The introduction of non-smooth, piece-wise differentiable cost curves and capacity constraints would not change the major conclusions. However, while the resulting mathematical problem would not become intractable, it would require the use of a more sophisticated mathematical technique making it more difficult to follow the important concepts of the analysis.

[9] σ_j represents own CO_2 emission rate of a renewable resource. Typically this number is zero, however, for the sake of generality, we assume that it may deviate from zero (for example if some auxiliary fuel is being used to support the renewable technology).

emissions will reduced by $MCI_j(t)$. The net result of this would be an increase in system-wide renewable generation by

$$\alpha_j^R(t) = 1 - \rho_j(t) \tag{25}$$

and a reduction in system-wide CO_2 emissions by

$$\alpha_j^C(t) = MCI_j(t) - \sigma_j \tag{26}$$

Parameters $\alpha_j^R(t)$ and $\alpha_j^C(t)$ represent the *marginal increase* of renewable generation and a marginal offset of carbon emissions, respectively, provided by 1 unit of energy produced by a given renewable resource.

Consider now a build out scenario of renewable resources characterized by values $R_1, R_2, ..., R_n$ and assume the dispatch profile of each renewable resource determined by economic dispatch of the grid and denote these respective dispatch profiles as $r_1(t), r_2(t), ..., r_n(t)$.

***Proposition 1*.**

1. *If a renewable resource j is marginal in time period t, its marginal impacts on the system are equal to zero:* $\alpha_j^R(t) = 0$ and $\alpha_j^C(t) = 0$.
2. *The sensitivity of system-wide renewable generation and carbon emissions to the addition of renewable capacity satisfy the following equations:*

$$\frac{\partial \Re}{\partial R_j} = \sum_{t \in S_j} \alpha_j^R(t) g_j(t) = \frac{1}{R_j} \sum_{t=1}^{T} \alpha_j^R(t) r_j(t) \tag{27}$$

$$\frac{\partial \mathbb{C}}{\partial R_j} = -\sum_{t \in S_j} \alpha_j^C(t) g_j(t) = -\frac{1}{R_j} \sum_{t=1}^{T} \alpha_j^C(t) r_j(t) \tag{28}$$

where S_j *is the set of all time periods over which the resource* R_j *is inframarginal.*

***Proof*.**

If the renewable resource is marginal, an incremental demand $dL_j(t)$ at its location should cause an additional output of this resource of $dL_j(t)$, hence $\rho_j(t) = 1$ and $\alpha_j^R(t) = 1 - \rho_j(t) = 0$. Similarly, $MCI_j(t) = \sigma_j$ and therefore $\alpha_j^C(t) = MCI_j(t) - \sigma_j = 0$. This proves the first part of the Proposition.

To prove the second part of this Proposition, consider the following identities:

$$\frac{\partial \Re}{\partial R_j} = \sum_{t=1}^{T} \frac{\partial \Re}{\partial r_j(t)} \frac{\partial r_j(t)}{\partial R_j} \tag{29}$$

$$\frac{\partial \mathbb{C}}{\partial R_j} = \sum_{t=1}^{T} \frac{\partial \mathbb{C}}{\partial r_j(t)} \frac{\partial r_j(t)}{\partial R_j} \tag{30}$$

As follows from the above discussion incremental dispatch of the renewable resource will change system-wide renewable generation in proportion to the marginal renewable ,

$$\frac{\partial \Re}{\partial r_j(t)} = \alpha_j^R(t) \tag{31}$$

$$\frac{\partial \mathbb{C}}{\partial r_j(t)} = -\alpha_j^C(t) \tag{32}$$

Based on optimal dispatch rules for the power system,

$$r_j(t) = \begin{cases} R_j g_j(t) \text{ if the resource is inframarginal} \\ \in [0, R_j g_j(t)] \text{ if the resource is marginal} \\ 0 \text{ if the resource is supermarginal} \end{cases} \tag{33}$$

If the resource is inframarginal (i.e. $t \in S_j$), equation (33) implies that $\frac{\partial r_j(t)}{\partial R_j} = g_j(t)$.

If the resource is supermarginal, equation (33) implies that $\frac{\partial r_j(t)}{\partial R_j} = 0$.

If the resource is marginal, a small change in its installed capacity will not change its output[10] and therefore the derivative in this case is also zero. Therefore

$$\frac{\partial r_j(t)}{\partial R_j} = \begin{cases} g_j(t) \text{ if } t \in S_j \\ 0 \text{ otherwise} \end{cases} \tag{34}$$

By combining equations (29), (31) and (34), we obtain the first part of equation (27). Similarly, by combining (30), (32) and (34), we get the first part of equation (28).

Two identities $\sum_{t \in S_j} \alpha_j^R(t) g_j(t) = \frac{1}{R_j} \sum_{t=1}^{T} \alpha_j^R(t) r_j(t)$ and $\sum_{t \in S_j} \alpha_j^C(t) g_j(t) = \frac{1}{R_j} \sum_{t=1}^{T} \alpha_j^C(t) r_j(t)$ follow from the earlier observations that $r_j(t) = R_j g_j(t)$ when the resource is inframarginal, $\alpha_j^R(t) = 0$ and $\alpha_j^C(t) = 0$ in time periods when the resource is marginal, and $r_j(t) = 0$ when the resource is supermarginal.

Q.E.D.

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[10] Except for a degenerate case in which marginal resource is dispatched at full capacity which could be ignored as it makes no material difference for further analysis.

For the purpose of further analysis we introduce average resource impacts over the entire time period [1,T]:

average marginal renewable increase

$$\bar{\alpha}_j^R = \frac{\sum_{t \in S_j} \alpha_j^R(t) g_j(t)}{T} = \frac{\sum_{t=1}^{T} \alpha_j^R(t) r_j(t)}{TR_j} \tag{35}$$

it is easy to see that if the resource is never marginal, $\bar{\alpha}_j^R$ is simply equal to the resource capacity factor;

average marginal carbon offset

$$\bar{\alpha}_j^C = \frac{\sum_{t \in S_j} \alpha_j^C(t) g_j(t)}{T} = \frac{\sum_{t=1}^{T} \alpha_j^C(t) r_j(t)}{TR_j} \tag{36}$$

Proposition 1 provides us with a tool necessary to analyze an RPS problem both in the traditional and carbon controlling formulations.

3.3. Optimal RPS Investment Rules

3.3.1. A Traditional RPS Problem

A traditional RPS problem could be formulated as a cost-minimization mechanism targeting a desired level of generated renewable energy.

$$\min_R \sum_{j=1}^{n} Z_j(R_j) \tag{37}$$

s.t.

$$\Re(R_1,...,R_n) \geq \Re_0 \tag{38}$$

where $\Re_0$ is the target level of renewable generation.

A different formulation of the traditional RPS problem would be to maximize renewable generation subject to the budgetary constraint:

$$\max_R \Re(R_1,...,R_n) \tag{39}$$

s.t.

$$\sum_{j=1}^{n} Z_j(R_j) \leq B_0 \tag{40}$$

where B_0 is the total budget subsidy available.

3.3.2. A Carbon Controlling RPS Problem

A Carbon Controlling RPS problem could be formulated as a cost minimization mechanism targeting a desired level of CO_2 emissions.

$$\min_R \sum_{j=1}^{n} Z_j(R_j) \tag{41}$$

s.t.

$$\mathbb{C}(R_1,...,R_n) \le \mathbb{C}_0 \tag{42}$$

where $\mathbb{C}_0$ is the target level of carbon emissions in the entire power system.

A different formulation of this problem would be to minimize CO_2 emissions subject to the budgetary constraint:

$$\min_R \mathbb{C}(R_1,...,R_n) \tag{43}$$

s.t.

$$\sum_{j=1}^{n} Z_j(R_j) \le B_0 \tag{44}$$

where B_0 is the total budget subsidy available.

3.3.3. Optimal Investment Rules

RPS Theorem.

1a. If the traditional RPS problem (37)-(38) *is feasible, its solution is provided by the following RPS investment rule*

$$Z_j'(R_j) = P_R \bar{\alpha}_j^R T \tag{45}$$

where $\bar{\alpha}_j^R$ *defined by* (35) *is the resource's generation weighted marginal renewable increase and* P_R *is the RPS energy price (in \$/MWh) such that renewable resource additions satisfying equation* (45) *at that price meet the target level of renewable generation:*

$$\Re(R_1,...,R_n) = \Re_0 \text{ if } P_R > 0 \text{ and } P_R = 0 \text{ if } \Re(R_1,...,R_n) > \Re_0. \tag{46}$$

1b. If the budget constrained problem (39)-(40) *is feasible, then the optimal investment rule would satisfy equation* (45) *and the RPS price* P_R *will be selected* *such that the investments additions satisfying equation* (45) *at that price meet the budgetary limitation*

$$\sum_{j=1}^{n} Z_j(R_j) = B_0 \text{ if } P_R > 0 \text{ and } P_R = 0 \text{ if } \sum_{j=1}^{n} Z_j(R_j) < B_0. \tag{47}$$

2a. If the Carbon Controlling RPS problem (41)-(42) *is feasible, its solution is provided by the following investment rule*

$$Z_j'(R_j) = P_C \bar{\alpha}_j^C T \tag{48}$$

where f_j *is the inframarginal capacity factor of resource number j,* $\bar{\alpha}_j^C$ *is this resource's generation weighted marginal carbon offset defined by* (36) *and* P_C *is the RPS price of carbon (in \$/T) chosen in such a way that renewable resource additions satisfying equation* (48) *at that price meet the target level of* CO_2 *emissions:*

$$\mathbb{C}(R_1,...,R_n) = \mathbb{C}_0 \text{ if } P_C > 0 \text{ and } P_C = 0 \text{ if } \mathbb{C}(R_1,...,R_n) < \mathbb{C}_0 \tag{49}$$

2b. If the budget constrained problem (43)-(44) *is feasible, then the optimal investment rule would satisfy equation* (48) *and the RPS price* P_C *will be selected in such a way that the investments additions satisfying equation* (48) *at that price meet the budgetary limitation*

$$\sum_{j=1}^{n} Z_j(R_j) = B_0 \text{ if } P_C > 0 \text{ and } P_C = 0 \text{ if } \sum_{j=1}^{n} Z_j(R_j) < B_0. \tag{50}$$

***Proof*.**

Consider the Lagrangian of problem (37)-(38)

$$L_{\Re} = \sum_{j=1}^{n} Z_j(R_j) - \lambda(\Re - \Re_0) \tag{51}$$

Using the results of Proposition 1, the optimality condition $\frac{\partial L_{\Re}}{\partial R_j} = 0$ could be presented as

$$Z_j'(R_j) - \lambda \sum_{t \in S_j} \alpha_j^R(t) g_j(t) = Z_j'(R_j) - \lambda T \bar{\alpha}_j^R = 0 \tag{52}$$

Since the problem is feasible, the optimal Lagrange multiplier must be non-negative, $\hat{\lambda} \geq 0$ and should be chosen in such a way that renewable additions obtained from equation (52) for each resource meet the renewable generation target:

$\Re(R_1,...,R_n) = \Re_0$ if $P_R > 0$ and $P_R = 0$ if $\Re(R_1,...,R_n) > \Re_0$.

The RPS price measured in \$/MWh is then defined as

$$P_R = \hat{\lambda} \tag{53}$$

This proves the section 1a of the theorem. The proof of section 1b for the budget constrained problem is similar to the above.

The proof of section 2a follows exactly the same logic by defining the Lagrangian of the problem (41)-(42)

$$L_{\mathbb{C}} = \sum_{j=1}^{n} Z_j(R_j) + \lambda(\mathbb{C} - \mathbb{C}_0) \tag{54}$$

Using the results of Proposition 1, the optimality condition $\frac{\partial L_{\mathbb{C}}}{\partial R_j} = 0$ could be presented as

$$Z_j'(R_j) - \lambda \sum_{t \in S_j} \alpha_j^C(t) g_j(t) = Z_j'(R_j) - \lambda T \overline{\alpha}_j^C = 0 \tag{55}$$

Since the problem is feasible, the optimal Lagrange multiplier must be non-negative, $\hat{\lambda} \geq 0$ and should be chosen in such a way that renewable additions obtained from equation (55) for each resource meet the carbon emission target:

$$\mathbb{C}(R_1,...,R_n) = \mathbb{C}_0 \text{ if } \hat{\lambda} > 0 \text{ and } \hat{\lambda} = 0 \text{ if } \mathbb{C}(R_1,...,R_n) < \mathbb{C}_0 .$$

The RPS price measured in dollars per ton of CO_2 emissions is then defined as

$$P_C = \hat{\lambda} \tag{56}$$

The proof of section 2b for the budgetary constrained problems follows similar logic.

Q.E.D.

Optimal investment rules provided by two RPS models look mathematically similar, but lead to different investment policies. The investment rule derived from the traditional RPS problem is based on equalizing supply curves on the basis of the *average marginal renewable increase* of individual resources:

$$\frac{Z_1'(R_1)}{\overline{\alpha}_1^R} = \frac{Z_2'(R_2)}{\overline{\alpha}_2^R} = ... = \frac{Z_n'(R_n)}{\overline{\alpha}_n^R} = TP_R \tag{57}$$

Simply put, this rule means that at optimum an incremental dollar subsidy given to any location should buy the same amount of renewable energy produced by the power system. The subsidy price of that energy paid to a renewable generator on top of other revenues they receive from energy and capacity markets will be proportional to its renewable increases and equal to $\bar{\alpha}_j^R TP_R$. This combined with (35) leads to the following subsidy formula applied on a temporal (e.g. hourly) basis:

$$Subsidy_j^R(t) = r_j(t)\alpha_j^R(t)P_R \tag{58}$$

If we assume that renewable resources are never marginal, then all marginal renewable increases are equal to 1 and in this case, equation (58) implies that all renewable resources should receive the same amount of subsidy on a per MWh basis in each hour regardless of their location on the grid. The value of the subsidy is set by the RPS price. If, due to a high penetration of renewables or inadequate transmission capacity renewable resources may become marginal, the subsidy should be adjusted to account for that effect and the subsidy rule (57) should provide zero payment to a marginal renewable resource.

In contrast, the investment rule derived from the carbon controlling RPS problem is based on equalizing supply curves on the basis of *average marginal carbon offsets* of individual resources:

$$\frac{Z_1'(R_1)}{\bar{\alpha}_1^C} = \frac{Z_2'(R_2)}{\bar{\alpha}_2^C} = ... = \frac{Z_n'(R_n)}{\bar{\alpha}_n^C} = TP_C \tag{59}$$

This rule implies that at optimum an incremental dollar subsidy given to any location should buy the same amount of CO_2 emission reduction in the power system. The subsidy price of that carbon reduction paid to a renewable generator on a per ton of CO_2 basis on top of other revenues they receive from energy and capacity markets will be the same and equal to $\bar{\alpha}_j^C P_C$. This, combined with (36), leads to the following subsidy formula applied on a temporal basis:

$$Subsidy_j^C(t) = r_j(t)\alpha_j^C(t)P_C \tag{60}$$

This subsidy formula indicates that the optimal method for providing subsidy to renewable generation is to compensate it on the basis of its carbon footprint. Indeed, according to the Carbon Footprint Theorem, $\mathbf{CF}[R_j](t) = -r_j(t)\alpha_j^C(t)$ and therefore formula (60) could be restated as

$$Subsidy_j^C(t) = -\mathbf{CF}[R_j](t)P_C \tag{61}$$

Since carbon footprint varies by location and over time, investment decisions resulting from investment rules (57)-(58) and (59)-(60) are structurally and materially different. Between two resources with identical marginal subsidy requirements, the traditional investment rule (57)-(58) would favor locations with the higher capacity factor $\bar{\alpha}_j^R$, since it will be providing more renewable energy.

An investment prescription under the carbon controlling RPS will favor the resource with a higher average marginal carbon offset, since it will offer more contribution to the reduction of CO_2 emissions.

3.4. Comparing Two RPS Programs: an Illustrative Example

In order to illustrate the RPS Theorem, consider a simple example of a two-zone power system separated by a constrained transmission line, as presented on Figure 9. In this system, Zone B is always short in energy which it imports from Zone A. Zone A is assumed to contain predominantly coal-fired generation. Zone B is dominated by natural gas fired generating technology. During OnPeak hours transmission line connecting these two zones gets constrained leading to price separation between them.

During OnPeak hours, Zone B prices are set by gas-fired technology with average OnPeak market heat rate of 9,000 Btu/kWh; Zone A prices are set by coal-fired

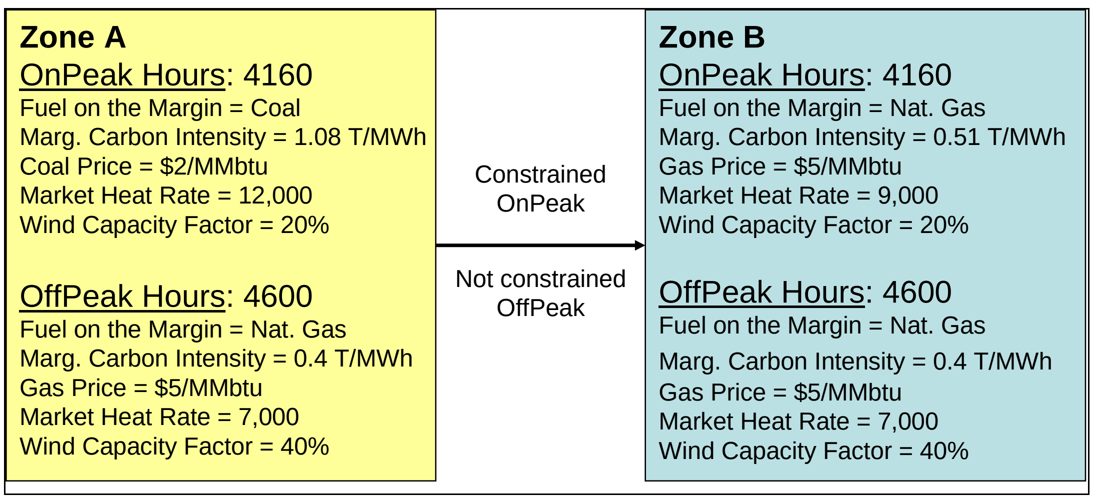

Figure 9. A Two-Zone System Market Assumptions

generation with market coal-based heat rate of 12,000 Btu/kWh. During OffPeak hours, transmission is not constrained and the price for the entire system is set by the gas-fired generation in Zone A at market heat rate of 7,000 Btu/kWh.

We assume that both zones have a significant potential of wind resources. For simplicity, we assume that at both locations technical and cost characteristics of these wind resources are identical.

Finally, we make one more simplifying assumption that the addition of wind resources at any zone makes no impact on market prices and heat rates.

Using the above information we estimate the need for subsidy in order to develop wind resources in each zone computed as the difference between annual carrying costs associated with building and financing a wind farm and revenues the wind farm is expected to receive in the energy market.

The results of these calculations and underlying market assumptions are presented in Table 2. As shown in that table, we assume a CO_2 controlling regulatory policy with an underlying price of $25/T of CO_2, natural gas and coal prices at $5/MMBtu and

$2/MMBtu, respectively and an annual carrying cost of building a wind farm at $200,000 per MW of installed capacity. Based on these assumptions, in order to be financially viable, a wind farm in Zone A requires a $27.98/MWh in subsidy while a similar wind farm in Zone B requires subsidy of $25.85/MWh only. A traditional RPS program focused on the total renewable energy would favor investments into wind in Zone B over zone A.

However, as shown in that table, marginal carbon offsets generated by renewable energy differ significantly between zones. During OnPeak hours, 1 MWh of renewable energy generated in zone A displaces 1.08T of CO_2, compared to 0.51T for 1 MWh of renewable energy generated in Zone B. Due to transmission congestion during OnPeak hours renewable energy in Zone A displaces coal-fired generation with CO_2 emission rate of 1.08 T/MWh. At the same time, in Zone B marginal technology is gas-fired with a much lower emission rate of 0.51 T/MWh. In OffPeak hours, in absence of congestion, renewable energy generated and Zone A or Zone B have identical carbon offsets of 0.4 T/MWh. On average over a year, marginal carbon offset of renewable generation in Zone A is 0.61 T/MWh in Zone A and 0.44 T/MWh in Zone B.

Table 2. Market Assumptions

Input Assumptions	Zone A			Zone B		
	OnPeak	OffPeak	Annual	OnPeak	OffPeak	Annual
Hours	4160	4600	8760	4160	4600	8760
Fuel on the margin	Coal	Gas		Gas	Gas	
Market Heat Rate (Btu/kWh)	12000	7000		9000	7000	
Fuel Price ($/MMBtu)	$2.00	$5.00		$5.00	$5.00	
CO2 Price ($/Ton)	$25.00	$25.00		$25.00	$25.00	
MCI (T/MWh)	1.08	0.40		0.51	0.40	
LMP ($/MWh)	$51.00	$45.00		$57.86	$45.00	
Wind Capacity Factor	20%	40%	31%	20%	40%	31%
Wind Energy Revenue ($K/MW)	$42.43	$82.80	$125.23	$48.14	$82.80	$130.94
Wind Capital Requirements ($K/MW)			$200.00			$200.00
Subsidy needed ($K/MW)			$74.77			$69.06
Marginal renewable increase (MWh/MWh)	1	1	1	1	1	1
Marginal CO2 offsets (T/MWh)	1.08	0.40	0.61	0.51	0.40	0.44
Required subsidy in $/MWh of renewable energy			$27.98			$25.85
Required subsidy in $/T of carbon offset			$45.74			$59.34

Given this difference in marginal carbon offsets, in order to displace 1T of CO_2, a wind farm located in Zone A requires $45.74/T in subsidy while a wind farm located in Zone B requires $59.34/T. As a result, a carbon controlling RPS focused on CO_2 reduction would favor investments into wind in Zone A over zone B, a decision directly opposite to that of the traditional RPS.

A comparison of the results of the traditional and carbon controlling RPS programs is presented in Table 3. In this example, we assume that $10 million dollars in subsidy is available. As explained earlier, under the traditional RPS all subsidy will be given to wind farms in Zone B where it will help to bring online 145 MW of installed wind capacity which will generate 387 GWh of renewable energy annually. Renewable generators will receive additional revenues of $25.85/MWh for energy they inject into the system. In other words, the RPS price for this program is $25.85/MWh. The resulting annual carbon offset under this program will be 169 thousand tons of CO_2. Given the marginal carbon offset of this investment of 0.51 T/MWh, the corresponding marginal cost of CO_2 reduction in the form of the RPS subsidy is $59.34.

In contrast, under the carbon controlling RPS all the subsidy will be diverted to support wind farms in Zone A. Given that more subsidy is needed there per unit of installed capacity, only 134 MW of wind power will be brought online, compared to 145 MW under the traditional RPS. Only 357 GWh of renewable energy will be produced, compared to 387 GWh under the traditional RPS. However, the carbon controlling RPS will displace 219 thousand tons of CO_2 which exceeds the carbon offset of the traditional RPS of 169 thousand tons by 30%. In order to administer the carbon controlling RPS, renewable generators will be paid the RPS price of $45.74/T of CO_2 assessed on the basis of the negative hourly carbon footprint of the windfarm. The marginal cost of subsidy per MWh of renewable energy in this case is $27.98/MWh which is higher than under the traditional RPS.

Table 4. Results for Traditional and Carbon Controlling RPS Programs

	Traditional RPS			Carbon Controlling RPS		
	Zone A	Zone B	Total	Zone A	Zone B	Total
Budget Constraint ($K)	$10,000			$10,000		
Required subsidy in $/MWh of renewable energy	$27.98	$25.85	$25.85	$27.98	$25.85	$27.98
Required subsidy in $/T of carbon offset	$45.74	$59.34	$59.34	$45.74	$59.34	$45.74
Subsidized wind capacity (MW)	-	145	145	134	-	134
Renewable energy generated (GWh)	-	387	387	357	-	357
CO2 Emissions Offset (000 T)	-	169	169	219	-	219
RPS Price ($/MWh)			$25.85			$27.98
RPS Price ($/T of CO2)			$59.34			$45.74

These results are exactly what should be expected: when the program is designed to maximize the renewable energy, it results in the least expensive way of doing just that and yields a higher level of renewable energy produced and lower marginal cost of renewable energy than the alternative. When the program is designed to maximize the amount of carbon displaced (by means of minimizing the total level of carbon emitted), it finds the least expensive way of achieving this goal and yields the higher level of displaced mass of CO_2 emissions and the lower marginal cost of CO_2 abatement than the traditional RPS program.

3.5. RPS Implementation Issues

According to the US Department of Energy [10], presently there are 27 states plus the District of Columbia that have RPS policies in place. Together these states account for more than half of the electricity sales in the United States. Five other states, North Dakota, South Dakota, Utah, Virginia, and Vermont, have nonbinding goals for adoption of renewable energy instead of an RPS. All known RPS programs are designed in a manner which requires electricity providers to obtain a minimum percentage of their power from renewable energy resources by a certain date. In other words all existing in the US RPS programs simply target the total amount of generated renewable energy regardless of the efficiency of CO_2 abatement ultimately achievable by these programs. The American Clean Energy and Security Act already passed by the US House of Representatives and presently under consideration by the US Senate [11] sets the national Renewable Energy Standard also specified in terms of the minimum percentage of total electricity production which has to be provided by renewable resources.

Based on these design goals, as discussed in the previous section, rational developers of renewable generation will prefer renewable projects at locations that are most financially attractive and will seek transmission interconnection for these projects to areas with highest electricity prices regardless of the level of CO_2 emissions these projects will be able to displace. While there is nothing wrong with this strategy on the part of the developers, the current design of RPS programs is highly questionable. If the proclaimed goal of these programs is to promote renewable generation in order to displace CO_2 emissions, the design is not consistent with this goal and will most likely lead to suboptimal levels of CO_2 emission reductions.

Goals of existing RPS programs are set in physical terms (e.g. annual level of renewable generation). Trading of renewable credits set RPS prices. Similarly, a goal of a carbon controlling RPS could be set in physical terms, i.e. in terms of the total carbon footprint of renewable resources and the RPS prices could be determined through trading between buyers and sellers. This alternative approach would provide market participants with a rule leading to an *optimal* investment strategy into renewable resources. Alternatively, the program could be designed by directly setting the RPS price and providing the RPS subsidy to renewable generators based on formula (60).

In order to implement a carbon controlling RPS program, the following need to happen: 1) electricity market operators should calculate and publish locational marginal carbon intensities for all nodes in the system which should be used to compute marginal carbon offsets for renewable generators; 2) the RPS goal should be set either in physical terms or by specifying the RPS price of carbon; 3) the subsidy should be paid to renewable generators according to formula (60); and 4) an RPS cost recovery mechanism should be established in order to equitably allocate RPS costs among electricity consumers. Addressing these issues goes beyond the scope of this paper.

4. Conclusions

A widespread interest in understanding carbon footprint associated with economic activity of individuals and organizations requires means for a precise measurement of that footprint. As an industry, power generation is a single largest source of carbon emissions. For example, based on 2008 data power generation accounts for over 40% of carbon emissions in the United States [12]. This paper presents a rigorous methodology for determining the carbon footprint of electricity consumption based on the economics and engineering rules of the power industry. Understanding and measuring the temporal and locational nature of the carbon footprint associated with electricity consumption is important for the design of all carbon abatement policies affecting the location and timing of the use of electric energy. The range of potential application of this methodology spreads from carbon footprint reporting to very specific policy implications for power industry stakeholders.

There are a number of public policy and individual economic decisions that are directly affected by the discussion of the temporal and locational nature of CO_2. Understanding of this concept provides analytical means for analyzing among other things the efficiency of the design of Renewable Portfolio Standards. However, a proposed RPS analysis is only one among other potential applications of this concept. This determination

The RPS approach presently implemented in the United States and in other countries should not be considered efficient as long as ultimate goal of the RPS policy is the reduction of greenhouse gases. An alternative approach based on the subsidy rule compensating renewable generation for the negative carbon footprint they provide achieves the optimal strategy in targeting carbon emissions through RPS.

This alternative design would require a precise calculation of locational marginal carbon intensity which could be provided by system operators responsible for the operations of regional electrical grids and a design of renewable cost allocation rules by regulatory agencies.

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Appendix

Proof of equations (14)-(16)

Following [1], consider an optimal security constrained dispatch problem of the power system, based on a linearized lossless representation of the power network. According to this representation, power flow on a given transmission element or a flowgate is represented as

$$\mathbf{f} = \mathbf{f}^{o} + \boldsymbol{\Psi}(\mathbf{p} - \mathbf{p}^{o} - (\mathbf{L} - \mathbf{L}^{o})) \tag{62}$$

Where $\mathbf{f}^{o}, \mathbf{p}^{o}, \mathbf{L}^{o}$ represent base case power flow, generator injections and load withdrawal vectors, respectively, $\mathbf{f}, \mathbf{p}, \mathbf{L}$ represent an alternative set of power flows, injections and withdrawals, matrix $\boldsymbol{\Psi}$ is known as a transmission sensitivity matrix and gives the variations in flows due to changes in the nodal injections, with respect to the reference bus assumed to ensure the real power balance.

The optimal dispatch problem solved by the system operator is defined as the following LP problem

$$\min_{\mathbf{p}} \mathbf{c}^{\mathbf{T}}\mathbf{p} \tag{63}$$

s.t.

$$\mathbf{1}^{\mathbf{T}}(\mathbf{p} - \mathbf{L}) = \mathbf{0} \qquad \perp \quad \lambda \tag{64}$$

$$\underline{\mathbf{f}} \leq \boldsymbol{\Psi}(\mathbf{p} - \mathbf{L}) \leq \overline{\mathbf{f}} \qquad \perp \quad \underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}} \tag{65}$$

$$\underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}} \qquad \perp \quad \underline{\boldsymbol{\gamma}}, \overline{\boldsymbol{\gamma}} \tag{66}$$

where $\mathbf{1}$ is a vector of ones, dual variable λ is a price at the reference bus taken at the opposite sign, dual variables $\underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}}$ are shadow prices for transmission constraints. According to a well known LMP decomposition formula,

$$\mathbf{LMP} = -\lambda \mathbf{1} - \boldsymbol{\Psi}^{\mathbf{T}}(\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}}) \tag{67}$$

Assume now that the cost vector $\mathbf{c}$ of all generators is represented as a sum of two components – non-carbon related $\mathbf{a}$ and carbon related $P_C\boldsymbol{\sigma}$:

$$\mathbf{c} = \mathbf{a} + P_C\boldsymbol{\sigma} \tag{68}$$

where $\boldsymbol{\sigma}$-vector of carbon emission rates of generators and P_C is the carbon price. Consider an optimal dispatch for a given price of carbon P_C which results in k binding transmission constraints ($k \geq 0$) and a presence of $k+1$ marginal generators. Denote the dispatch of marginal generator as vector $\mathbf{x}$, their corresponding cost vector as $\tilde{\mathbf{c}} = \tilde{\mathbf{a}} + P_C\tilde{\boldsymbol{\sigma}}$ and a $k \times k$ submatrix of the transmission sensitivity matrix corresponding to binding constraints and marginal generators as $\tilde{\boldsymbol{\Psi}}$.

Let's assume a small perturbation δP_C to carbon price P_C which will lead to a change in optimal generator dispatch. However, as explained in [1], if δP_C is small enough, the set of marginal generators will not change. What may change, however, is their dispatch resulting in a new marginal dispatch vector $\mathbf{x}+\delta\mathbf{x}$. Since the system load remains the same, by following the logic used in [1] it is easy to see that the optimal redispatch could be obtained from the solution of the following LP problem

$$\min_{\delta\mathbf{x}} \tilde{\mathbf{c}}^{\mathbf{T}}\delta\mathbf{x} \tag{69}$$

s.t.

$$\mathbf{1}^{\mathbf{T}}\delta\mathbf{x} = \mathbf{0} \qquad \perp \quad \lambda \tag{70}$$

$$\tilde{\mathbf{\Psi}}\delta\mathbf{x} = \mathbf{0} \qquad \perp \quad \boldsymbol{\mu} \tag{71}$$

The Lagrangian for this problem is equal to

$$\Lambda = \tilde{\mathbf{c}}^{\mathbf{T}}\delta\mathbf{x} + \lambda\mathbf{1}^{\mathbf{T}}\delta\mathbf{x} + \boldsymbol{\mu}^{T}\tilde{\mathbf{\Psi}}\delta\mathbf{x} \tag{72}$$

and the optimality condition $\frac{d\Lambda}{d(\delta\mathbf{x})} = 0$ is represented as the following system of linear equations:

$$\begin{bmatrix} \mathbf{1}^{\mathbf{T}} \\ \tilde{\mathbf{\Psi}}^{\mathbf{T}} \end{bmatrix} \begin{bmatrix} \lambda \\ \boldsymbol{\mu} \end{bmatrix} = -\tilde{\mathbf{a}} - P_C\tilde{\boldsymbol{\sigma}} \tag{73}$$

Its solution is equal to

$$\begin{bmatrix} \lambda \\ \boldsymbol{\mu} \end{bmatrix} = -\begin{bmatrix} \mathbf{1}^{\mathbf{T}} \\ \tilde{\mathbf{\Psi}}^{\mathbf{T}} \end{bmatrix}^{-1} \tilde{\mathbf{a}} - P_C \begin{bmatrix} \mathbf{1}^{\mathbf{T}} \\ \tilde{\mathbf{\Psi}}^{\mathbf{T}} \end{bmatrix}^{-1} \tilde{\boldsymbol{\sigma}} \tag{74}$$

Equation (74) leads to the following:

$$\begin{bmatrix} \frac{\partial\lambda}{\partial P_C} \\ \frac{\partial\boldsymbol{\mu}}{\partial P_C} \end{bmatrix} = -\begin{bmatrix} \mathbf{1}^{\mathbf{T}} \\ \tilde{\mathbf{\Psi}}^{\mathbf{T}} \end{bmatrix}^{-1} \tilde{\boldsymbol{\sigma}} \tag{75}$$

As shown in [1],

$$\begin{bmatrix} MCI_0 \\ -\mathbf{SCI} \end{bmatrix} = \begin{bmatrix} \mathbf{1}^{\mathbf{T}} \\ \tilde{\mathbf{\Psi}}^{\mathbf{T}} \end{bmatrix}^{-1} \tilde{\boldsymbol{\sigma}} \tag{76}$$

where MCI_0 is a marginal carbon intensity at the reference bus and $\mathbf{SCI}$ are shadow carbon intensities of binding transmission constraints.

A comparison of (75) and (76) implies that

$$\frac{\partial \boldsymbol{\mu}}{\partial P_C} = \mathbf{SCI} \tag{77}$$

which proves equation (16) since for non-binding constraints both SCIs and shadow prices are zero. It also implies that

$$\frac{\partial \lambda}{\partial P_C} = -MCI_0 \tag{78}$$

By differentiating the LMP decomposition (67) by price of carbon, omitting zero shadow prices for non-binding constraints and using (77)-(78), one gets that

$$\frac{\partial \mathbf{LMP}}{\partial P_C} = (MCI_0)\mathbf{1} - \tilde{\boldsymbol{\Psi}}^{\mathbf{T}}(\mathbf{SCI}) = \mathbf{MCI} \tag{79}$$

which proves equation (14).

Finally we will prove equation (15). By definition, carbon footprint of generating unit *n* is defined as

$$\mathbf{CF}[G_n(t)] = \frac{\partial(OM_n(t) \times G_n(t))}{\partial P_C}$$

Where

$$OM_n = LMP_n - c_n$$

and

$$c_n = a_n + \sigma_n P_C$$

If the generator is non-marginal, a sufficiently small increase in carbon price will not change its output and therefore for a non-marginal generator

$$\mathbf{CF}[G_n(t)] = \frac{\partial(OM_n(t) \times G_n(t))}{\partial P_C} = G_n(t)\frac{\partial OM_n(t)}{\partial P_C} = G_n(t)\left[\frac{\partial LMP_n(t)}{\partial P_C} - \sigma_n\right]$$
$$= G_n(t)\left[MCI_n(t) - \sigma_n\right] = \alpha_n^C(t) G_n(t)$$

If the generator is marginal its dispatch may change but it its operating margin and its carbon offsets are equal to zero and therefore equation (15) still holds.

Q.E.D.