Investment decisions in Liberalized Electricity Markets: A framework of Peak Load Pricing with strategic firms*

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Abstract

In this article we analyze firms investment incentives in liberalized electricity markets. Since electricity is economically non storable, it is optimal for firms to invest in a differentiated portfolio of technologies in order to serve strongly fluctuating demand. Prior to the Liberalization of electricity markets, for regulated monopolists, optimal investment and pricing strategies haven been analyzed in the peak load pricing literature (compare Crew and Kleindorfer (1986)). In restructured electricity markets regulated monopolistic generators have often been replaced by competing and potentially strategic firms.

This article aims to respond to the changed reality and model investment decisions of strategic firms in those markets. We derive equilibrium investment for strategic firms and compare to the benchmark cases of perfect competition and monopoly outcomes. We find that strategic firms have an incentive to overinvest in base–load technologies but choose total capacities too low from a welfare point of view. By fitting the framework to a specific electricity market (Germany) we are able to empirically analyze Investment choices of strategic firms, and quantify the potential for market power and its impact on generation portfolios in restructured electricity markets in the long run.

Keywords: Investment Decisions, Technology Choice, Restructured Electricity Markets, Peak Load Pricing, Strategic Firms.

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1 Introduction

In this article we analyze firms investment incentives in liberalized electricity markets. In those markets firms choose to invest in different types of power plants which allow production of electricity at different levels of marginal cost. Since electricity is not storable at reasonable cost, it is optimal for firms to invest in a differentiated portfolio of technologies in order to serve strongly fluctuating demand.[1] Prior to the liberalization of electricity markets, regulated monopolists decided on optimal investment and pricing strategies. In the course of liberalizing those markets in Europe and the US, which started in the 1990's, regulated monopolistic generators have been transformed into competing, but potentially strategically acting firms. The present chapter aims to respond to the changed reality in restructured electricity markets and model investment decisions of strategic firms in those markets.

For a single regulated firm, optimal investment and pricing decisions have been thoroughly analyzed in the so called peak load pricing literature. All main findings are summarized in Crew and Kleindorfer (1986), the first contributions date back to the seminal work of Boiteux (1949) and Steiner (1956). That literature and all its extensions[2] analyze optimal investment and pricing decisions of a single firm whose product is non-storable and demand fluctuates over time. The classical framework allows to determine welfare maximizing investment in a single technology. This was subsequently extended to the case of optimal investment in several technologies under the objective of either welfare, or profit maximization.[3] The peak load pricing literature was thus perfectly suited (and widely used) to model investment decisions in electricity markets prior to liberalization, where electricity indeed was supplied by regulated monopolies.

Liberalization of electricity markets, which started in the 1990's throughout Europe, has changed this picture dramatically. In many countries electricity generation has been opened to competition and regulated monopolistic generators have been transformed into competing firms. Most interestingly, the results obtained in the peak load pricing literature

[1] Typical industry investment in electricity markets contains for example nuclear, lignite, coal, gas and oil plants. Nuclear and lignite plants are expensive to build but produce at low cost and thus run most of the time. Coal and especially gas and oil plants are less expensive to build, but produce more expensively. They will produce only part of the time in order to serve higher demand and peaks. Compare figure 1 for an illustration of typical industry marginal cost in Germany, 2006.

[2] For the case of a profit maximizing monopolist see for example Oren et al. (1985).

[3] In a so called second best approach this was further extended and allows to determine optimal investment choice, maximizing any weighted sum of profit and welfare.

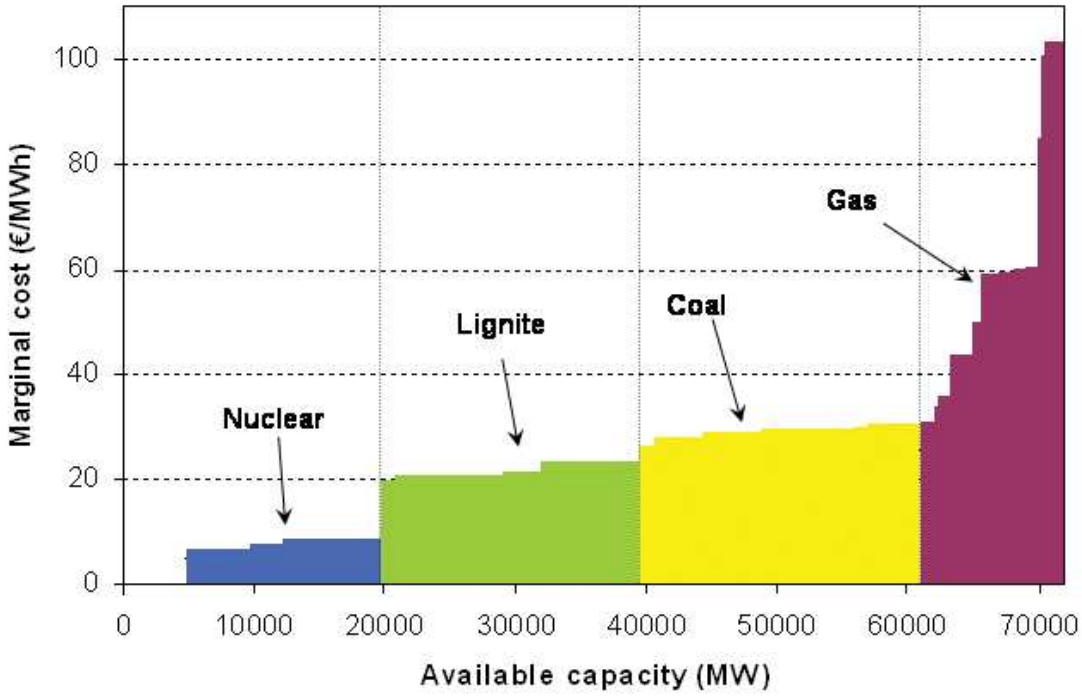

Figure 1: Typical Industry Marginal cost of producing electricity in Germany, 2006.

for a single firm under the objective of welfare maximization can easily be extended to the case of perfectly competitive firms in liberalized markets. In a recent contribution Joskow and Tirole (2007) thoroughly discuss all those results in the light of perfectly competitive restructured electricity markets. All the results obtained in the peak load pricing literature, however, are not applicable in case firms do not behave perfectly competitive, but interact strategically when making their investment decisions.

Especially in Europe policy makers are seriously concerned by the exercise of market power in the electricity sector, which has been extensively analyzed and documented for the wholesale markets.[4] Very little is known, if and how market power is exercised in those markets in the long run, when firms make their investment decisions. The results obtained for the spot markets, however, give little reason to expect perfectly competitive behavior (of the same firms) in the long run. Strategic interaction of several firms has thus to be taken into account, if meaningful predictions regarding firms investment incentives in liberalized

[4]A recent study presented by the European Commission (2007) entitled "Structure and Performance of Six European Wholesale Electricity Markets in 2003, 2004 and 2005" detects considerable market power at several spot markets, e.g. for Germany.

electricity markets are to be made.

In the present article we thus extend the framework of investment in several technologies analyzed in the peak load pricing literature for a single firm to the case of strategically interacting firms.[5] In a two stage market game firms first make their investment decisions prior to the spot market which is subject to uncertain or fluctuating demand, then firms compete at the spot market. Firms can decide to invest in many different available technologies, which all differ in their cost of investment and corresponding cost of production. Firms investment decisions thus determines the precise composition of industry investment in all technologies. That is, we obtain the precise shape of industry marginal cost function.[6]

Our main results can be summarized as follows: Most importantly we derive equilibrium investment of strategic firms, establishing existence and uniqueness.[7] We then compare equilibrium investment choice to the benchmark cases of perfect competition (welfare maximization), monopoly (profit maximization) and the so called second best solution[8] derived in the peak load pricing literature. Interestingly, under imperfect competition firms have a strong incentive to invest into low marginal cost technologies in order to negatively influence their competitors' spot market outputs. We are able to establish properties under which this strategic effect is so intense that equilibrium investment in low–marginal–cost technologies in oligopoly is even above the welfare optimal level.

Based on the theoretical framework developed we then empirically analyze equilibrium investment for the German electricity market. As a main result we find that investment of strategic firms[9] in base–load technologies (producing at marginal cost below 25 €/MWh, such as nuclear and lignite plants) exceeds first best investment levels. Strategic under–investment takes place exclusively in middle– and peak–load technologies (such as gas, or

[5]Notice that strategic investment in a single technology, i.e. capacity choice of strategic firms, already has received attention in the literature. See for example Murphy and Smeers (2005) or Grimm Zoettl (2007) for the case of capacity choice prior to Cournot competition. For case of capacity choice prior to price setting at the spot market , as analyzed by Fehr (1997), Reynolds and Wilson (2000) and recently Fabra and Fructos (2007), it has been shown that symmetric pure strategy equilibria typically cannot exist.

[6]How those look like is illustrated in figure 1 for the case of Germany.

[7]Fehr (1997) analyzes the case of strategic investment in several technologies prior to a spot market of inelastic demand with price setting duopolists. He shows that typically symmetric investment-equilibria in pure strategies cannot exist in such a setting.

[8]This second best approach maximized a weighted sum of welfare and profits.

[9]The German market consists essentially of four large players. Two of them (RWE and E.on) have a market share of 26 % each, while the two smaller ones (ENBW and Vattenfall) together cover 30 % of the market each. Compare, e.g., Monopolkommission (2007).

oil-fired plants). We are furthermore able to determine the impact of strategic behavior on the entire distribution of wholesale prices in the long run.[10] This allows to quantify the potential for the exercise of market power in the German Electricity market, in the long run, when firms investment decisions are taken into account.

The article is structured as follows: In section 2 the framework is introduced. In sections 3 and 4 we derive the benchmark cases of perfect competition and monopoly. In section 5 we analyze the case of imperfect competition, and compare them to the benchmark scenarios (section 6). In section 7 empirically analyze investment decisions in the German electricity market for all different market structures. Section 8 concludes.

2 The Model

We analyze a two stage market game where firms choose cost functions under demand uncertainty and make output choices after market conditions unraveled.

Industry demand is subject to random variations. Denote by $\theta \geq 0$ the range of possible demand scenarios and by $F(\theta)$ the probability distribution over those demand scenarios, with the corresponding density $f(\theta) = F_\theta(\theta)$.[11] Market demand in scenario θ is given by[12] $P(Q,\theta) = \theta - B(Q)$, without loss of generality we assume $B(0) = 0$. Whenever $P(Q,\theta) > 0$ the following assumptions are satisfied:

Assumption 1 (Demand) .

(i) $P(Q,\theta)$ *is twice continuously differentiable in* Q *with* $P_q(Q,\theta) < 0$

(ii) $P_q(Q) + P_{qq}(Q)Q < 0$.

Prior to unraveling of uncertainty, firms decide on the technologies they want to install. Each unit of a technology c allows for production of one unit at marginal cost $c \in \mathbb{R}^+$ and comes at marginal cost of investment denoted by $k(c)$. The Technologies available for investment satisfy the following properties:

[10] Remember for the case of capacity choice in part I of the thesis, assessment of the distribution of electricity prices was possible only for the upper tail (10%) of the price distribution. In part II we are now able to derive the entire price distribution.

[11] Throughout the article we denote the derivative of a function $g(x,y)$ with respect to an argument z, $z = x, y$, by $g_z(x,y)$, the second derivative with respect to that argument by $g_{zz}(x,y)$, and the cross derivative by $g_{xy}(x,y)$.

[12] For the case of linear demand we obtain $B(Q) = b * Q$, with b being a positive constant.

Assumption 2 (Technology) *Each technology is characterized by its constant marginal cost of production denoted by c. Per unit cost of investment in technology c is denoted by $k(c)$, which satisfies:*

(i) No technology comes for free, i.e. $k(c) > 0\ \forall c$. [13] *A technology which produces at zero marginal cost is not available, i.e. $k(0) = \infty$.*

(ii) less efficient technologies (that is technologies with higher c) are less expensive, i.e. $k_c(c) < 0\ \forall c$.[14]

(iii) $k(c)$ is sufficiently convex, i.e.

$$k_{cc}(c) > f(c) \qquad \textit{and} \qquad k_{cc}(c) > f(F^{-1}(k_c(c)+1))\ \forall c. \tag{1}$$

The situation we want to analyze is captured by the following two stage situation. At stage one firms determine their technology mix by choosing their investment function $x_i(c)$, we denote by $x(c) = x_1(c), \ldots, x_n(c)$ the vector of all investment choices and by $X(c) = \sum_{j=1}^{n} x_j(c)$ the industry investment. As illustrated in figure 2, the investment choice $x_i(c)$ of firm i determines which output can be produced at Marginal cost c.[15]

At the second stage firms choose their output at the spot market after having observed both the investment choices of all firms and the realization of demand. We denote by $q(\theta) = (q_1(x,\theta), \ldots, q_n(x,\theta))$ the vector of outputs of the n firms in scenario θ, and let $Q(x,\theta) = \sum_{i=1}^{n} q_i(\theta)$ be total quantity produced in the market.

In the following we now determine profits of firm i for fixed cumulative investment $x(c)$, and given Spot market outputs $q(x,\theta)$. In the subsequent sections we will be more specific on the precise characterization of Spot market competition, which will be given by perfect competition, monopoly and Cournot-Competition. For the framework chosen, Spot market outcomes are always nondecreasing in θ. We can thus characterize the demand realization θ_i^c which will give rise to production cost c:

$$\theta_i^c(x,q) = \quad \{(\theta, c)\ :\ q_i(x,\theta) = x_i(c)\}$$

[13] This assumption is not crucial, whenever $k(c) = 0$ for some $c < \bar{\bar{c}}$, then we would just obtain a corner solution, where capacity could be infinite at that technology. For ease of exposition we exclude this corner solution

[14] This is a natural observation which can already be found in the pioneering contributions on peak load pricing, compare Boiteux (1948).

[15] In a sense it is just the inverse of the marginal cost function, however since firms choose their investment in different technologies it is much more convenient to choose this formulation.

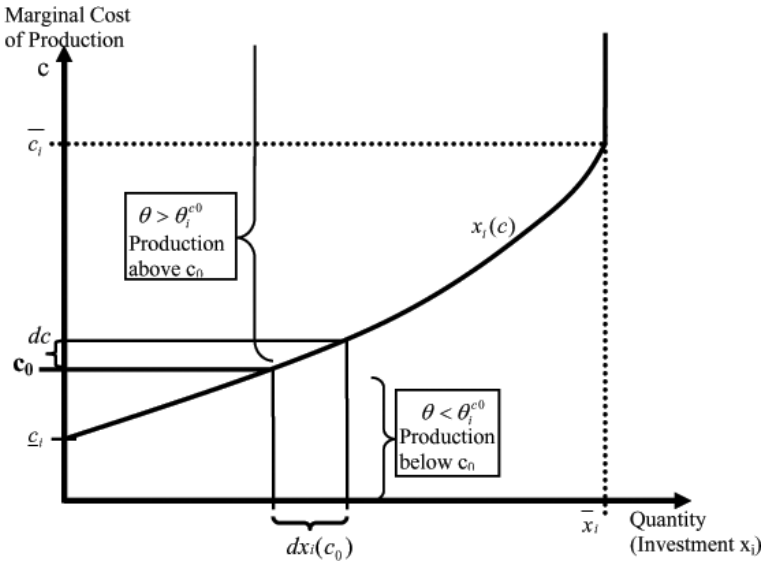

Figure 2: Investment decision $x_i(c)$ of firm i.

i.e. for given $(q_i(x,\theta), x_i(c))$, if $\theta_i^{c_0}$ occurs, then firm i will produce at marginal cost c_0. Furthermore for all $\theta < \theta_i^{c_0}$ firm i will produce at marginal cost below c_0 and for all $\theta > \theta_i^{c_0}$ firm i will produce at marginal cost above c_0. This is illustrated in figure 2.

In order to determine total profits of firm i associated to some investment choice $x_i(c)$, we first determine profits associated to some partial investment $dx_i(c_0)$ as illustrated in figure 2. That is we determine profits generated by technology c_0, the amount invested in this technology is given by $dx_i(c_0)$. Observe that such investment will only yield positive revenues for demand realizations $\theta > \theta_i^{c_0}$. Expected revenues generated by $dx(c_0)$ are given by the expected markup in all those demand realizations where indeed production is at or above marginal cost c_0 times the amount $dx(c_0)$ invested:

$$\int_{\theta_i^{c_0}}^{\overline{\theta}} \left[P\left(Q(x,\theta),\theta\right) - c_0\right] dF(\theta) \ * \ dx_i(c_0)$$

On the other hand, the cost of investment in technologies $dx_i(c_0)$ is given by the (constant) marginal cost $k(c_0)$ of investment times the amount times the amount $dx(c_0)$ invested:

$$k(c_0) \ * \ dx_i(c_0)$$

In the context of electricity generation, the above analysis corresponds to determining expected profits generated by a small power plant of size $dx_i(c_0)$ which produces at marginal cost c_0 and comes at a cost of investment given by $k(c_0)$.

In order to obtain total profits associated to the entire investment-choice $x_i(c)$, we need to sum up for all technologies, where investment took place. Suppose firm i did choose to invest into technologies $c \in [\underline{c}_i, \overline{c}_i]$, with $0 \leq \underline{c}_i < \overline{c}_i$, then integration over all those technologies yields:

$$\pi_i(x,q) = \int_{\underline{c}_i}^{\overline{c}_i} \left(\left[\int_{\theta_i^c}^{\overline{\theta}} [P(Q,\theta) - c]\, dF(\theta) - k(c) \right] \frac{dx_i(c)}{dc} \right) dc$$

This expression can be transformed by applying the rule of integration by parts:

$$\begin{aligned} \pi_i(x,q) \;=\; & \int_{\underline{c}_i}^{\overline{c}_i} \left(-\frac{d}{dc} \left[\int_{\theta_i^c}^{\overline{\theta}} [P(Q,\theta) - c]\, dF(\theta) - k(c) \right] x_i(c) \right) dc + \\ & \int_{\theta_i^{\overline{c}_i}}^{\overline{\theta}} \left([P(Q,\theta) - \overline{c}_i]\, \overline{x}_i \right) dF(\theta) - k(\overline{c}_i)\overline{x}_i \end{aligned} \tag{2}$$

The first summand of (2) yields profits of interior investment $x_i < x_i(\overline{c}_i)$ and the second summand yields profits of investment at the capacity bound $\overline{x}_i = x_i(\overline{c}_i)$.

We now analyze the impact of a variation of the investment function $x_i(c)$ on firm i's total profits. We have to take into account two different types of such variations as illustrated in figure 3.

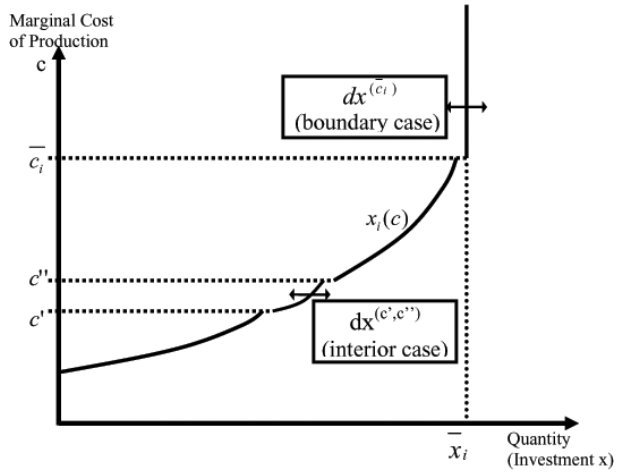

Figure 3: Changes of Investment $x_i(c)$, interior case and boundary case.

First we consider interior cases, where investment is changed by some amount $dx^{(c',c'')}$ for all technologies $c \in [c', c'']$, with $c' < c'' \leq \overline{c}_i$. Such variation is denoted by $\frac{d\pi_i(\cdot)}{dx^{(c',c'')}}$ and is given by expression (3) of lemma 1. Second we consider variations of the overall capacity $\overline{x}_i$

by the amount $dx^{(\overline{c}_i)}$, changing investment for all technologies above the highest technology $c \geq \overline{c}_i$. Such variation is denoted by $\frac{d\pi_i(\cdot)}{dx^{(\overline{c}_i)}}$ and is given by expression (4) of lemma 1.

When computing those first derivatives it is important to notice that the spot market equilibrium $Q(x, \theta)$ depends on firms investment choices. Thus both the critical demand realizations and the realized market prices will change as investment $x_i(c)$ of firm i is modified.

Lemma 1 (First Derivative – General Case) .

(i) ***Interior Case:*** *Consider a variation of investment $x_i(c)$, affecting all technologies $c \in [c', c'']$, where $c' < c'' \leq \overline{c}_i$. This leads to the following change in firm i's profits:*

$$\frac{d\pi_i(x,q)}{dx^{(c',c'')}} = \int_{c'}^{c''} \left(1 - F(\theta_i^c) + k_c(c) + f(\theta_i^c)\frac{d\theta_i^c}{dc}\left[P(Q,\theta) - c + \frac{dP(Q,\theta)}{dx^{(c',c'')}}x_i(c)\right]_{\theta=\theta_i^c}\right) dc \tag{3}$$

(ii) ***Boundary case:*** *Consider a variation of investment $x_i(c)$, affecting all technologies $c \geq \overline{c}_i$. This leads to the following change in firm i's profits:*

$$\frac{d\pi_i(x,q)}{dx^{(\overline{c}_i)}} = \int_{\theta_i^{\overline{c}_i}}^{\overline{\theta}} \left(P(Q,\theta) - \overline{c}_i + \frac{dP(Q,\theta)}{dx^{(\overline{c}_i)}}\overline{x}_i\right) dF(\theta) - k(\overline{c}_i) \tag{4}$$

Proof see Appendix 8. □

Lemma 1 gives the impact of a variation of firm i's investment on it's profits both for the interior and for the boundary case[16]. The analysis up to now does not yet specify the type of (strategic) behavior of firms both at the spot markets and the investment stage. In order to solve for the cases of Perfect competition, Monopoly and of strategic interaction in the subsequent section, these first derivatives of lemma 1 provide a valuable starting point.

3 Perfect Competition – Welfare Maximization

As a Benchmark we determine the case of perfectly competitive behavior both at the spot markets and the investment stage. [17] In our framework, for the case of perfect

[16] A change in total investment by dx which affects all $c \geq c'$ can be determined by setting $c'' = \overline{c}_i$ and just summing then over (3) and (4).

[17] This has already been analyzed in the peak load pricing literature, the first best solution of our framework, however, will serve as a valuable benchmark in order to compare to the case of strategic interaction of firms as analyzed in section 5. Furthermore in the framework chosen, we obtain a smooth solution, which makes its characterization as given in theorem 1 rather short, in contrast to previous contributions on that topic.

competition, only industry investment $X(c) = \sum_{j=1}^{n} x_j(c)$ and industry output $Q(\theta) = \sum_{j=1}^{n} q_j(\theta)$ matter. In order to compare later on to the case of strategic behavior, however, we will explicitly consider n firms which equally share investment and output among each other.

The perfectly competitive spot market outcome for fixed industry investment $X(c)$ and fixed realization of uncertainty $\theta \geq 0$ is given by the well known condition of "price = marginal cost", i.e. $P(X(c), \theta) = c \ \forall \theta$. As in the previous section for the general case, we now define the critical demand realization θ^c_{FB} which makes firms produce at marginal cost c. We obtain for θ^c_{FB} and the corresponding spot-market output $Q^{FB}(\theta)$:

$$\begin{aligned} \theta^c_{FB} &= c + B(X(c)) \ \forall c \\ Q^{FB}(\theta) &= \{(Q, \theta) : \ (Q = X(c), \theta = \theta^c_{FB} \ \forall c)\} \end{aligned} \tag{5}$$

Having specified the outcomes at the spot market under perfect competition for given investment choice, we can now turn towards solving for equilibrium investment choice at stage one. We already have derived general first order conditions in lemma 1. It now remains to adapt those conditions for the case of perfect competition and to verify their sufficiency, as summarized in lemma 2.

Expressions (6) and (7) of lemma 2 can be directly derived from expressions (3) and (4), since the last summand of it's integrands equals zero. This is due to the following two observations: first under perfect competition firms take the market price as given i.e. $\frac{dP(\cdot)}{dx} = 0$ and second, by definition of θ^c_{FB} as established above in (5), we have $P(Q(\theta^c_{FB}), \theta^c_{FB}) = c$. We obtain the following equilibrium conditions under perfect competition:

Lemma 2 (Optimality Conditions, Perfect competition) .

(i) ***First order conditions:***

For an interior change affecting $c \in [c', c'']$ where $(c' < c'' \leq \overline{c}_i)$ we obtain:

$$\frac{d\pi_i(x, q^{FB})}{dx^{(c', c'')}} = \int_{c'}^{c''} (1 - F(\theta^c_{FB}) + k_c(c)) \ dc = 0. \tag{6}$$

For a change of total capacity $\overline{x}$, which affects all $c \geq \overline{c}_i$ we obtain:

$$\frac{d\pi_i(x, q^{FB})}{dx^{(\overline{c}_i)}} = \int_{\theta^c_{FB}}^{\overline{\theta}} P\left(\overline{X}, \theta\right) - c \ dF(\theta) - k(c) = 0. \tag{7}$$

(ii) ***Second order conditions:***

The cross derivatives with respect to different technologies equal zero, i.e. $\frac{d^2\pi_i(x,q^{FB})}{dx^{(c^1,c^2)}dx^{(c',c'')}} = 0$ *(for* $c^1 < c^2 < c' < c''$*) and* $\frac{d^2\pi_i(x,q^{FB})}{dx^{(c',c'')}dx^{(\bar{c}_i)}}$*=0.*

The second derivatives with respect to the same technologies are always negative, i.e. $\frac{d^2\pi_i(x,q^{FB})}{d\left(x^{(c',c'')}\right)^2} < 0$ *and* $\frac{d^2\pi_i(x,q^{FB})}{d\left(x^{(\bar{c}_i)}\right)^2} < 0$.

Computation of the second derivatives is relegated to appendix 8. A crucial property of the way the problem of investment in the optimal technology mix is presented throughout this article lies in the simplicity of the second order conditions, namely the cross derivatives with respect to different technologies always equal to zero. This is due to the fact, that the profitability of changing investment $x_i(c)$ for some technologies $c \in [c', c'']$ is the same for all different investment functions which coincide for the technologies $c \in [c', c'']$ under consideration but exhibit completely different values at other technologies. Optimality of investment $x_i(c_0)$ at some technology c_0 can thus be determined independently from the shape of the remaining function.

Let us now briefly provide some economic intuition for the first order conditions as given in lemma 2. First notice that variation of investment $x_i(c)$ for $c \in [c', c'']$ as illustrated in figure 3 can be interpreted as substituting technology c'' by technology c' (in the case of $dx^{(c',c'')} > 0$, the reverse for $dx^{(c',c'')} > 0$).

The impact of such variation will reduce expected production cost on the one hand, it will increase however the investment cost of firm i. More specifically this implies that firms when choosing their optimal technology–mix face the following tradeoff: The extra revenues from such substitution in the case of perfect competition are given by the savings in production cost $(c'' - c')$, which occur with probability $(1 - F(\theta^c_{FB}))$ and are given by $\int_{c'}^{c''}(1 - F(\theta^c_{FB}))dc$. The extra expenditures of such variation on the other hand are given by the extra investment cost $\int_{c'}^{c''} k_c(c)dc = k(c'') - k(c') < 0$.

Thus whenever $1 - F(\theta^c_{FB}) + k_c(c) > 0$ firms want to increase investment in the more efficient technology c' by substitution with the less efficient technology c''. For the converse, $1-F(\theta^c_{FB})+k_c(c) < 0$ firms want to invest less in the more efficient technology c' by reducing in c''.

We now denote by $\underline{c}^*$ that technology where firms just start to invest and denote by $\underline{\theta}^* = \underline{c}^*$ that demand realization where firms start to produce for given investment. [18] Investment below $\underline{c}^*$ cannot be profitable, this lower bound is characterized by $1 - F(\underline{c}^*) + k_c(\underline{c}^*) = 0$, see lemma 3 (i).

[18] i.e. $X(c)$=0, for $c \leq \underline{c}^*$ and firms do not produce for $\theta \leq \underline{\theta}*$ for given investment.

For $c > \underline{c}^*$, the optimal technology mix then satisfies $1 - F(\theta^c_{FB}) + k_c(c) = 0$ for all technologies $c \in [\underline{c}^*, \overline{c}^*]$. Notice however that this interior first order condition is only a relative statement, establishing the optimal mix of technology choice. It still leaves the question of absolute capacity choice, untouched. This is tackled in expression (7) of lemma 2, the first order condition for a capacity change. It pins down the trade off a firm faces at the capacity bound, where the right hand side is just marginal welfare at the spot market from additional investment in technology $\overline{c}$ while the left hand side is marginal cost of investment.The overall capacity choice $\overline{X}$ and the corresponding boundary technology $\overline{c}^*$ consequently have to satisfy both the interior first order condition (6) and the boundary condition (7). In lemma 3 we now characterize the set of active technologies $[\underline{c}^*, \overline{c}^*]$:

LEMMA 3 (ACTIVE TECHNOLOGIES) *If investment in some technologies is profitable, i.e.* $\int_c^{\overline{\theta}} (\theta - c)\, dF(\theta) > k(c)$ *for some* $c \geq 0$*, then firms will invest in the technologies* $c \in [\underline{c}^*, \overline{c}^*]$.

(i) ***Lower Bound*** $(\underline{c}^*)$***:*** *The lowest technology for which investment is profitable* $\underline{c}^*$ *is uniquely characterized by:* $1 - F(\underline{c}^*) + k_c(\underline{c}^*) = 0$*. The demand realization for which firms start to produce is given by* $\underline{\theta}^* = \underline{c}^*$.

(ii) ***Upper Bound*** $(\overline{c}^*)$***:*** *The highest technology for which investment is profitable* $\overline{c}^*$*, and the demand realization when firms are capacity constrained* $\overline{\theta}^*$ *are uniquely characterized by:*

$$\begin{aligned} (a) \qquad & 1 - F(\overline{\theta}^*) + k_c(\overline{c}^*) = 0 \\ (b) \qquad & \int_{\overline{\theta}^*}^{\overline{\theta}} \theta - \overline{\theta}^* dF(\theta) = k(\overline{c}^*) \end{aligned} \tag{8}$$

PROOF See appendix 8 □

In theorem 1 we put all these results together characterizing Industry investment $X^*_{FB}(c)$ which obtains under perfect competition:

THEOREM 1 (COMPETITIVE SOLUTION) *Under perfect competition the industry investment function* $X^*_{FB}(c)$ *is unique and characterized as follows:*

$$X^*_{FB}(c) = \begin{cases} 0 & c < \underline{c}^* \\ (X, c): \ \{P(X, \theta^c_{FB}) = c\} & \underline{c}^* \leq c \leq \overline{c}^* \\ \overline{X}: \ \left\{P\left(\overline{X}, \overline{\theta}^*\right) = \overline{c}^*\right\} & \overline{c}^* < c. \end{cases} \tag{9}$$

The critical demand realization θ^c_{FB} *for the interior solution solves* $1 - F(\theta^c_{FB}) + k_c(c) = 0$*. Under Symmetry each firm chooses* $x^*_{FB}(c) = \frac{1}{n} X^*_{FB}(c)$.

For an illustration of industry investment obtained under perfect competition compare figure 4.

The entire analysis was conducted under the hypothesis, that all technologies $c \in [\underline{c}^*, \overline{c}^*]$ are part of a solution, i.e. that the solution $X^*_{FB}(c)$ is strictly monotonic in c. We obtain $\frac{dX^*_{FB}(c)}{dc} = \frac{k_{cc}(c) - f(\theta^*_{FB}(c))}{f(\theta^*_{FB}(c))(-P_q)} > 0$ due to assumption 2(iii), there $k(c)$ is assumed to be convex enough, which ensures that at the solution all technologies $c \in [\underline{c}^*, \overline{c}^*]$ indeed are active.

At this point it is important to point out that the analysis of the first best case has already been dealt with extensively in the peak load pricing literature. However, that literature focuses on discrete instead of continuous technology choices. Here, we (re–)solve the first best case within our continuous model since it later serves as a benchmark for the case of imperfect competition, which is the main focus of our analysis. It is still worthwhile to mention, however, that the framework of continuous technology sets not only seems to make it easier to gather some intuition, but also crucially simplifies the mathematical exposition of the results.[19]

4 Profit Maximization – Monopoly Outcome

Next we consider the case that where firms' total profits are maximized, i.e. the case of monopoly (or collusion in case several firms are active in the market). The solution is obtained analogously to the solution of the first best case. Under firms joint Profit maximization again only industry investment $X(c) = \sum_{j=1}^{n} x_j(c)$ and industry output $Q(\theta)$ matter. In order to compare to the case of strategic behavior of firms in section 5, we explicitly n firms which equally share investment and output.

The profit maximizing Spot–market solution for given investment $X_0(c)$ at each demand realization θ solves the standard condition of "Marginal Revenue equals marginal cost", i.e. $P(X(c), \theta) + P_q(X(c))X(c) = c \ \forall \theta$. Again we define the critical demand realization θ^c_M, which makes firm produce at marginal cost given they have invested $X_0(c)$ and maximize profits at the spot market. The critical demand realization and the corresponding spot-

[19] Just to give an example: In their more than 50 page survey on the theory on peak load pricing in Crew and Kleindorfer (1986) exact characterizations of quantities invested and exact production of each technology is omitted: "While the lemma indicates which plants are used it does not indicate the amounts of capacity q_l and amounts produced by each plant in each period q_{li}. To derive this is complicated and calls for a lengthy theorem which we do not state here (see Crew and Kleindorfer 1979a, pp-42–50, 63-65).", see Crew and Kleindorfer (1986), p 45.

market output $Q^M(\theta)$ are given by:

$$\theta_M^c = B(X(c)) - P_q(X(c))X(c) + c \ \forall c, \tag{10}$$

$$Q^M(\theta) = \{(Q,\theta) : \{Q = X(c), \theta = \theta_M^c \ \forall c\}\}. \tag{11}$$

Having specified the outcomes at the spot markets under profit maximization for given investment choice, we can now turn towards solving for optimal investment choice at stage one. Again relying on the general first order conditions derived in lemma 1, we can directly deduce the first order conditions under profit maximization as summarized in lemma 4.

Expression (12) is directly obtained from (3) since it's last integrand equals zero for the case of profit maximization. This is due to the following two observations: first $\frac{dP(Q^M,\theta)}{dx^{(c',c'')}} x_i(c) = P_q(\cdot)\frac{dQ^M(\theta)}{dx^{c',c''}} x_i(c) = P_q X(c)$, and second by the definition of θ_M^c as established in (10), we have $P(Q(\theta_M^c), \theta_M^c) - c + P_q(Q(\theta_M^c))X(x) = 0$. We obtain the following optimality conditions under profit maximization:

Lemma 4 (Optimality Conditions, Profit Maximization) .

(i) ***First order conditions:***

For an interior change affecting $c \in [c', c'']$ where $(c' < c'' \leq \overline{c}_i)$ we obtain:

$$\frac{d\pi_i(x, q^M)}{dx^{(c',c'')}} = \int_{c'}^{c''} (1 - F(\theta_M^c) + k_c(c)) \ dc = 0. \tag{12}$$

For a change of total capacity $\overline{x}$, which affects all $c \geq \overline{c}_i$ we obtain:

$$\frac{d\pi_i(x, q^M)}{dx^{(\overline{c}_i)}} = \int_{\theta_M^{\overline{c}_i}}^{\overline{\theta}} P\left(\overline{X}, \theta\right) + P_q\left(\overline{X}\right)\overline{X} - c \ dF(\theta) - k(c) = 0. \tag{13}$$

(ii) ***Second order conditions:***

The cross derivatives with respect to different technologies equal zero, i.e. $\frac{d^2\pi_i(x,q^M)}{dx^{(c^1,c^2)}dx^{(c',c'')}} = 0$ for $(c^1 < c^2 < c' < c'')$ and $\frac{d^2\pi_i(x,q^M)}{dx^{(c',c'')}dx^{(\overline{c}_i)}} = 0$.

The second derivatives with respect to the same technologies are always negative, i.e. $\frac{d^2\pi_i(x,q^M)}{d\left(x^{(c',c'')}\right)^2} < 0$ and $\frac{d^2\pi_i(x,q^M)}{d\left(x^{(\overline{c}_i)}\right)^2} < 0$.

Computation of the second order conditions is relegated to appendix 8.

Those first order conditions look very similar to the case of perfect competition as obtained in the last section. Most notably the locus of the critical demand realizations,

making firms produce at some level of marginal cost c, θ_M^{c*} is identical to the case of perfect competition since it is also given as the solution to $1-F(\theta_m^{C*})+k_c(c)=0$, in the following we will thus keep the notation θ_{FB}^{c*}. Thus under profit maximization firms will choose to invest into the same active set of technologies $c \in [\underline{c}^*, \overline{c}^*]$ which has already been characterized in lemma 3 for the case of perfect competition. When backing out Industry investment X_M^* which gives rise to those critical demand realizations, we observe different investment behavior, firms will invest less than under perfect competition.

For the boundary case on the other hand, the first order condition pins down the trade off firms faces at the capacity bound. The left hand side is marginal revenues at the spot market from additional investment in technology c_0 while the right hand side is marginal cost of investment.

The following theorem 2 provides a full characterization of firms' investment decision $X_M^*(c)$ under joint profit maximization.

THEOREM 2 (MONOPOLY SOLUTION) *Under profit maximization the industry investment function $X_M^*(c)$ is unique and characterized as follows:*

$$X_M^*(c) = \begin{cases} 0 & c < \underline{c}^* \\ (X,c): \quad \{P(X, \theta_{FB}^c) + P_q(X) X = c\} & \underline{c}^* \leq c \leq \overline{c}^* \\ \overline{X}: \quad \left\{P\left(\overline{X}, \overline{\theta}^*\right) + P_q\left(\overline{X}\right)\overline{X} = \overline{c}^*\right\} & \overline{c}^* < c. \end{cases} \tag{14}$$

The critical demand realization θ_{FB}^c for the interior solution ($c \in [\underline{c}^*, \overline{c}^*]$) is identically to the First best case characterized by $1 - F(\theta_{FB}^c) + k_c(c) = 0$. Under symmetry each firm chooses $x_M^*(c) = \frac{1}{n} X_M^*(c)$.

For an illustration compare figure 4.

REMARK 1 *In the peak load pricing literature (see for example Crew and Kleindorfer (1986), pp. 77–79), a so called "second best solution" is proposed, where welfare is maximized under a profit constraint. The tightness of the profit constraint can be expressed through a shadow price denoted by $\Theta \geq 0$ and normalization yields $\lambda = \frac{\Theta}{1+\Theta} \in [0,1]$. For $\lambda \in [0,1]$ all solutions lying within the two extreme points of welfare maximum and profit maximum can be obtained (see Crew and Kleindorfer (1986) expression (4.58) on p. 79). As it turns out, within our approach of a continuous technology set all those solutions share the same critical demand realizations θ_{FB}^c, which solve $1 - F(\theta_{FB}^c) + k_c(c) = 0$, $\forall c \in [\underline{c}^*, \overline{c}^*]$. Firms investment $x_\lambda(c) = \frac{1}{n} X_\lambda(c)$ differs, however, depending on the tightness of the profit*

constraint, as expressed through the parameter λ. We obtain

$$X^*_\lambda(c) = \begin{cases} 0 & c < \underline{c}^* \\ (X,c): \ \{P(X,\theta^c_{FB}) + P_q(X)\lambda X = c\} & \underline{c}^* \leq c \leq \overline{c}^* \\ \overline{X}: \ \left\{P\left(\overline{X},\overline{\theta}^*\right) + P_q\left(\overline{X}\right)\lambda\overline{X} = \overline{c}^*\right\} & \overline{c}^* < c. \end{cases} \tag{15}$$

Notice that for $\lambda = 0$ (the case of welfare maximization) this statement replicates theorem1 and for $\lambda = 1$ (profit maximization) it replicates theorem2. For an illustration compare figure 4.

The concept of second best solutions allows thus to cover both the case of perfect competition and the case of profit maximization. It is important to emphasize however that even for intermediate values $0 < \lambda < 1$, the concept of second best solutions is not capable of capturing strategic interaction of firms which is the topic of the subsequent section.

5 Strategic Firms – Imperfect Competition

Having reviewed the benchmark cases of welfare and profit maximization in sections 3 and 4, we now turn towards the case of imperfect competition. We derive the equilibrium of the two stage market game where firms first decide on their investment $x_i(c)$ and then after having observed investment decisions and realization of demand decide on production $q_i(x,\theta)$. Analogously to the previous analysis we first solve for the spot market equilibria for given Investment decisions $x^0_i(c)$ for each demand realization θ.

Since marginal cost are nondecreasing (the capacity choice problem exhibits this property by construction) and due to concavity of profits ensured by assumption 1 (ii), there always exists a unique Cournot equilibrium at the spot market. Since firms investment choices can be asymmetric, in that Cournot equilibrium for given demand realization θ_0 and given investment $x_0(c)$ firms will produce at different marginal cost. The Cournot equilibrium at the spot market $x_i(c^{EQ}_i)$, for all $i = 1,\ldots,n$ can be determined by $\overrightarrow{c^{EQ}}(\theta) = c^{EQ}_1(\theta),\ldots,c^{EQ}_n(\theta)$. Those are characterized by of the following well–known equilibrium conditions of the Cournot game:

$$\overrightarrow{c^{EQ}}(\theta) = \left\{ \forall i \ \left[\left(c^{EQ}_i\right) : P\left(\sum_{j=1}^n x_j(c^{EQ}_j), \theta\right) + P_q\left(\sum_{j=1}^n x_j(c^{EQ}_j)\right) x_i(c^{EQ}_i) = c^{EQ}_i \right] \right\}$$

We can now proceed analogously to the previous sections and characterize the critical

demand realizations[20] $\theta^c_{EQ,i}$ and the corresponding spot-market outputs $q_i^{EQ}(\theta)$:

$$\begin{aligned} \theta^c_{EQ,i} &= \left\{(\theta, c) : \ c = c_i^{EQ}(\theta) \ \forall\theta\right\}, \\ q_i^{EQ}(\theta) &= x_i(c_i^{EQ}(\theta)) \ \forall\theta, i \end{aligned} \tag{16}$$

It is worth-wile to notice that the critical demand realization $\theta^c_{EQ,i}$ of the Cournot spot market equilibrium has the following property:

$$P(Q_{EQ}, \theta^c_{EQ,i}) + P_q(Q_{EQ})q_i^{EQ} = c \quad \Leftrightarrow \quad \theta^c_{EQ,i} = B(Q_{EQ}) - P_q(Q_{EQ})q_i^{EQ} + c \tag{17}$$

Where the right expression just makes use of the initial separability assumption of demand which was assumed to be given by $P(Q, \theta) = \theta - B(Q)$.

Having solved for the outcomes at the spot market for fixed investment choice, we can now proceed and solve for the overall equilibrium with respect to firms investment choices. Again we make use of the general first order conditions derived in lemma 1, and derive the optimal capacity of firm i for fixed investment $X_{-i}(c)$ of all other firms. The first order conditions of firm i for the case of strategic capacity choice are summarized in lemma 5.

Expression (18) is directly obtained from (3) This is due to the following two observations: first $\frac{dP(Q^{EQ},\theta)}{dx^{(c',c'')}}x_i(c) = P_q\left(1 + \frac{dQ^{EQ}_{-i}(\theta)}{dx^{c',c''}}\right)x_i(c)$, and second by the definition of θ^c_{EQ} as established in (16), we have $P(Q^{EQ}(\theta^c_{EQ}), \theta^c_{EQ}) - c + P_q(Q^{EQ}(\theta^c_{EQ}))x_i(c) = 0$. Likewise we obtain expression (19) from (4). We obtain the following optimality conditions for the case of strategic interaction among firms:

Lemma 5 (Optimality Conditions, Strategic Firms) .

(i) ***First order conditions:***
For an interior change affecting $c \in [c', c'']$ where $(c' < c'' \leq \overline{c}_i)$ we obtain:

$$\frac{d\pi_i(x, q^{EQ})}{dx^{(c',c'')}} = \int_{c'}^{c''}\left(1 - F(\theta^c_{EQ,i}) + k_c(c) + f(\theta^c_{EQ,i})\left(\frac{d\theta^c_{EQ,i}}{dc}\frac{dQ^{EQ}_{-i}}{dx^{(c',c'')}}\right)P_q\, x_i(c)\right)dc = 0 \tag{18}$$

$$\textit{and} \quad \left(\frac{d\theta^c_{EQ,i}}{dc}\frac{dQ^{EQ}_{-i}}{dx^{(c',c'')}}\right) = \sum_{j\neq i}\frac{\left(P_q + P_{qq}x_j(c_j^{EQ})\right)x_j'(c_j^{EQ})}{1 - P_q x_j'(c_j^{EQ})}$$

[20]Remember: the critical demand realization is that demand realization $\theta^c_{EQ,i}$ that will give rise to production cost c for firm i in the Spot market Cournot equilibrium. In the present context this is just the inverse of $c_i^{EQ}(\theta)$.

For a change of total capacity $\overline{x}$, which affects all $c \geq \overline{c}_i$ we obtain:

$$\frac{d\pi_i(x, q^{EQ})}{dx^{(\overline{c}_i)}} = \int_{\theta^{\overline{c}_i}_{EQ,i}}^{\overline{\theta}} \left(P\left(Q^{EQ}, \theta\right) + P_q\left(Q^{EQ}\right) \overline{x} \left(1 + \frac{dQ^{EQ}_{-i}}{dx^{(\overline{c}_i)}} \right) - \overline{c}_i \right) dF\left(\theta\right) - k(\overline{c}_i) = 0 \tag{19}$$

(ii) ***Second order conditions:***

The cross derivatives with respect to different technologies equal zero, i.e. $\frac{d^2\pi_i(x,q^M)}{dx^{(c^1,c^2)}dx^{(c',c'')}}$*=0 for* $(c^1 < c^2 < c' < c'')$ *and* $\frac{d^2\pi_i(x,q^M)}{dx^{(c',c'')}dx^{(\overline{c}_i)}}$*=0.*

The second derivatives with respect to the same technologies can be shown to be negative, i.e. $\frac{d^2\pi_i(x,q^M)}{d\left(x^{(c',c'')}\right)^2} < 0$ *and* $\frac{d^2\pi_i(x,q^M)}{d\left(x^{(\overline{c}_i)}\right)^2} < 0$*, if the following conditions are satisfied:*
(a) Demand is linear, i.e. $P_{qq} = 0$*,*
(b) $f''(\theta) \leq 0$ *whenever* $f'(\theta) > 0$*,*
(c) $x_j''(c) \leq 0\ \forall c, j \neq i$*.*

PROOF see appendix 8. □

Also for the case of strategic interaction we observe first of all that again the second order conditions have a very special and simple form: all cross derivatives equal to zero. Again (for given investment decisions $X_{-i}(c)$) the profitability of substituting investment in technology c' by investment in technology c'' is solely determined by the investment level $x_i(c)$ $for c \in [c', c'']$ but not by the investment decision in other technologies $c < c'$ or $c > c''$.

Verification of second order condition thus reduces to checking for negative second derivatives with respect to the same technologies, i.e. $\frac{d^2\pi_i(x,q^M)}{d\left(x^{(c',c'')}\right)^2} < 0$ and $\frac{d^2\pi_i(x,q^M)}{d\left(x^{(\overline{c}_i)}\right)^2} < 0$. The computations involved are relatively burdensome, and we restrict to the case of linear demand in order to maintain tractability of the problem (It seems however that there are no major obstacles when extending the present analysis of second order conditions to the nonlinear case). Furthermore in order to ensure concavity of the problem two further assumptions are required: first the density of uncertainty should not increase too steeply (condition (b)) and second the investment functions chosen by all rivals should become flatter and flatter as the capacity bound $\overline{x}_j$ is approached (condition (c)).

The first order conditions can be interpreted similar to the case of welfare or profit maximization as analyzed in sections 3 and 4. For the interior solution, firms face the trade off of substituting investment in technology c' by investment in technology c''. Again this decision is driven by the mass above the critical demand realization versus the difference in investment cost, i.e. $1 - F(\theta^c_{EQ,i}) + k_c(c)$. Under Strategic interaction, however, firms also take into account the impact of their investment decision on the rivals spot market

outputs Q_{-i}, since more aggressive cost functions will give a more advantageous position for the spot market competition.In principle the same argument holds true for the boundary case (again we observe the term $\frac{dQ_{-i}}{dx^{(\overline{c}_i)}}$)), however since there exist only symmetric equilibria of the overall game as we will show later on, the reaction of firms output choice will be irrelevant since firms anyhow are capacity constrained for those demand realizations.

Most importantly it remains to notice that the first order conditions of firm i do only involve levels of investment choice $x_i(c)$, but not on it's slope $x_i'(c)$. When checking for potential deviations from a given equilibrium candidate with fixed investment $X_{-i}(c)$, firm i faces a standard maximization problem can be solved by the point wise first order conditions (pointwise, since all cross derivatives are zero). Derivation of entire equilibrium candidates (e.g. symmetric candidate) on the other hand will involve the solution of a differential equation. We thus restrict attention to differentiable[21] equilibrium candidates $x^*(c)$, but allow deviations to non–differentiable investment functions $x_i^*(c)$. We now provide a full characterization of each firm's investment choice $x_{EQ}^*(c)$ under strategic interaction of firms:

THEOREM 3 (STRATEGIC BEHAVIOR) *There exists a unique equilibrium of the overall game, if the second order conditions established in lemma 5 are satisfied*[22]*. Each firm chooses to invest $x_{EQ}^*(c)$, which is uniquely characterized by:*

$$x_{EQ}^*(c) = \begin{cases} 0 & & c < \underline{c}^* \\ (x,c): & \left\{ \begin{array}{ll} x'(c) = & \frac{F\left(\theta_{EQ}^c\right)-1-k_c(c)}{f\left(\theta_{EQ}^c\right)(n-1)b^2x-b\left(F\left(\theta_{EQ}^c\right)-1-k_c(c)\right)} \\ with & \theta_{EQ}^c = c+b(n+1)x \end{array} \right\} & \underline{c}^* \leq c \leq \overline{c}^* \\ \overline{X}: & \left\{\overline{\theta}^* - (n+1)b\overline{x} = \overline{c}^*\right\} & \overline{c}^* < c. \end{cases} \tag{20}$$

PROOF Proof see appendix 8. □

[21] This ensures that $\frac{dQ_{-i}}{dx^{(\overline{c}_i)}}$) is well defined. Similar restriction to differentiable functions are found in many contributions of the literature, compare for example the article on supply function competition by Klemperer and Meyer (1989).

[22] As stated in lemma 5, the second order conditions are always satisfied if conditions (a), (b) and (c) are satisfied. Especially the assumption of linear demand was made mainly in order to limit the computational burden when determining second order conditions. The symmetric candidate solution for the nonlinear case however would only change slightly and is given by the following differential equation:

$$\begin{array}{lll} x'(c) & = & \dfrac{F\left(\theta_{EQ}^c\right)-1-k_c(c)}{f\left(\theta_{EQ}^c\right)(n-1)\left(P_q^2x+P_qP_{qq}x^2\right)+P_q\left(F\left(\theta_{EQ}^c\right)-1-k_c(c)\right)} \\ \text{with} & & \theta_{EQ}^c = c+B(nx)+P_qx \end{array}$$

For an illustration of industry investment obtained under strategic behavior, compare figure 4. Most notably we obtain the same set of active technologies $c \in [\underline{c}^*, \overline{c}^*]$ (characterized in lemma 3) as for the case under perfect competition and profit maximization. However as explained above, strategic firms take into account their opponents reactions at the spot markets when making their investment decisions. In the following section 6 we provide a detailed discussion, comparing the solutions under imperfect competition to the benchmark cases of welfare and profit maximization.

6 Comparison of the Theoretical Results

In this section we discuss and compare the solutions obtained in sections 3, 4, and 5. That is, we compare the two benchmark cases of welfare– and profit–maximization with the case of imperfect competition. Theorems 1, 2, and 3 characterize industry investment $X^*(c)$ for for the different market structures, they all rely on specifying the locus of critical demand realizations θ^c. Remember, for given industry investment $X_0(c)$, the demand realization $\theta_0^c(X_0(c_0))$ was defined such as to give rise to production at marginal cost $c_0 \in [\underline{c}^*, \overline{c}^*]$ at the spot market. In equilibrium, industry investment $X^*(c)$ relates to those critical demand realizations θ^c by the well known optimality conditions for the different types of spot market competition.[23] Both industry investment and corresponding critical demand realizations for all scenarios analyzed are illustrated in figure 4.

First notice that all solutions discussed in the peak load pricing literature (i.e. welfare– and profit–maximization, and all intermediate X_λ–solutions discussed in remark 1) share the same locus of critical demand realization[24] $\theta^*_{FB}(c)$ for all c. In other words, for any given technology c_0, this technology will start to be operating at the very same demand realization $\theta_0 = \theta^{c_0}_{FB}$, no matter if welfare or total profits or a weighted sum of both is maximized. Industry investment $X^*(c)$ relate to those critical demand realizations by the well known optimality conditions for the different types of spot market competition mentioned above. Since all benchmark cases share the same critical demand realization (θ^c_{FB}), when comparing them with each other, the usual well known arithmetic applies: The monopoly outcome lies

[23] For the case of Perfect competition those are given by "price equals marginal cost", i.e. $P(X^*_{FB}(c), \theta^c_{FB}) = c \ \forall c$, for the case of Monopoly by "marginal revenue equals marginal cost", i.e. $P(X^*_{FB}(c), \theta^c_{FB}) + P_q(X^*_{FB}(c))X^*_{FB}(c) = c \, \forall c$ and for the so called second best solutions by $P(X^*_\lambda(c), \theta^c_{EQ}) + P_q(X^*_{EQ}(c))\lambda \, X^*_\lambda(c) = c \ \forall c$.

[24] Remember this is defined by $F(\theta^*_{FB}(c)) - 1 - k_c(c) = 0 \ \forall c \in [\underline{c}^*, \overline{c}^*]$Compare theorem 1, theorem 2 and Remark 1.

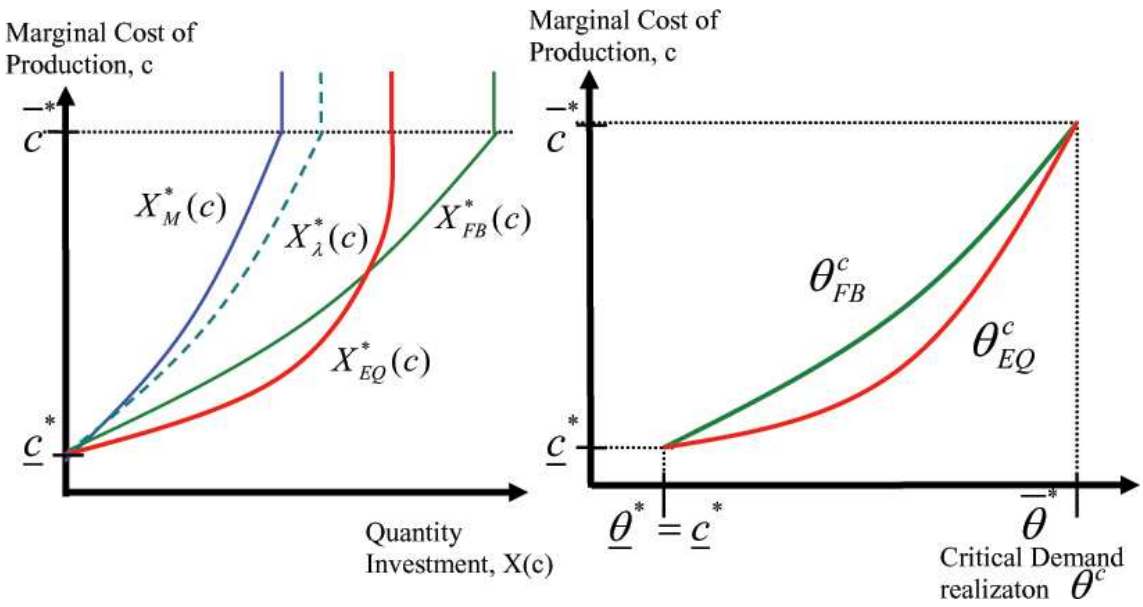

Figure 4: Illustration of the Solutions: Industry investment $X^*(c)$ and critical demand realizations θ^c for the 4 scenarios analyzed.

below the perfectly competitive outcome All second best solutions lie between these two solutions (proportionally according to λ). This is illustrated in figure 4.

The solution under imperfect competition turns out to be qualitatively different. The reason is that strategic firms take into account that their rivals will reduce their spot market production at the second stage in case they invest more heavily in low marginal cost technologies at the first stage. As a result, the locus of the critical demand realizations θ^c_{EQ} always lies above the corresponding locus of the critical demand realizations in the first best solution as illustrated in figure 4. In other words, for any given technology c_0, this technology will start to be operating at a lower demand realization in the First Best and the monopoly case (and all intermediate "second best" solutions) than in the case of imperfect competition (formally: $\theta^c_{FB} < \theta^c_{EQ}$ for all $c \in [\underline{c}^*, \overline{c}^*]$). This strategic effect is irrelevant, however, when either (i) production is zero (for $c \leq \underline{c}^*$), or (ii) firms are capacity constrained and do not react to modified cost functions of the rivals (i.e. for $c \geq \overline{c}^*$). In both cases the critical demand realizations coincide (i.e. $\theta^c_{FB} = \theta^c_{EQ}$ for $c = \{\underline{c}^*, \overline{c}^*\}$), for an illustration again see figure 4. Also for the case of strategic firms, Industry investment $X^*_{EQ}(c)$ relate to the critical demand realizations θ^c_{EQ} by the standard equilibrium conditions of the Cournot spot market competition[25].

[25]It is well known, those are given by: $P(X^*_{EQ}(c), \theta^c_{EQ}) + P_q(X^*_{EQ}(c))\frac{1}{n}X^*_{EQ}(c) = c \; \forall c$.

We can thus easily compare the case of imperfect competition to the monopoly outcome: both, the usual spot market arithmetics and the locus of the critical demand realization imply that the marginal cost function under imperfect competition will be chosen such that strictly more output is produced at each given marginal cost c than in the case of monopoly. (i.e. $x^*_{EQ}(c) > x^*_M(c)\ \forall c \in [\underline{c}^*, \overline{c}^*]$). This is illustrated in figure 4. Finally notice that we could replicate firms' total capacity choice $\overline{x}^*$ under imperfect competition by a "second best" scenario (as analyzed in the peak load pricing literature) by the appropriate choice of λ. For the case of n firms, $\lambda = \frac{1}{n}$ would yield the same total capacity choice as the n–firm equilibrium. However, for technologies below the capacity bound. In equilibrium firms invest strictly more into low marginal cost technologies than predicted by the $\lambda = \frac{1}{n}$ approximation. Thus, even though the approximation would yield the same total capacity, under imperfect competition firms invest relatively more in low marginal cost technologies than predicted. This is nicely illustrated in figure 7 when comparing equilibrium investment for the case of $n = 2$ and $n = 4$ firms (i.e. $X^*_{EQ}(n = 2)$ and $X^*_{EQ}(n = 4)$) with the corresponding λ–Solutions (i.e. $X^*_{\lambda=\frac{1}{2}}$ and $X^*_{\lambda=\frac{1}{4}}$).

When it comes to compare the case of imperfect competition to the first best outcome, clear cut solutions are obtained only for the total capacity choice. Recall that the critical demand realizations for the First Best case and the case of imperfect competition coincide at the upper bound $\overline{c}^*$ (i.e. $\theta_{EQ}(\overline{c}^*) = \theta_{FB}(\overline{c}^*)$. We can thus conclude that total capacity invested under imperfect competition is strictly below total capacity in the First Best solution. For interior technologies below the capacity bound, however, this may turn around, since the locus of the critical demand realization is above the benchmark case at each technology c. In the following lemma we provide a condition under which firms invest more in low marginal cost technologies under imperfect competition than in the First Best solution.

Lemma 6 (Over–investment in low marginal cost technologies) *In the case of imperfect competition firms invest more in efficient technologies (close to $\underline{c}^*$) than in the first best case if and only if $k_{cc}(\underline{c}^*) > \frac{2(n-1)}{n-2} f(\underline{c}^*)$.*

Proof see appendix 8.

As we can see, for $n = 2$ firms we will never observe overinvestment in low marginal cost technologies under imperfect competition. For $n \geq 3$ firms, however, if the set of technologies available on the market is sufficiently convex, overinvestment in low marginal cost technologies occurs.[26]

[26]In the subsequent section 7 we apply our theoretical framework to the case of investment choice in

We conclude this section by a discussion of the assumptions necessary in order to obtain existence of the symmetric equilibrium for the case of imperfect competition. While the assumption of linear demand does not seem to be essential in order to obtain our results, other assumptions made on the nature of uncertainty (the distribution of θ) and the set of available technologies $k(c)$ are crucial. In particular we had to restrict the analysis to those densities which do not exhibit upwards jumps, i.e. $f''(\theta) < 0$ whenever $f'(\theta) > 0$. The reason is that if the distribution had high peaks, firms could want to deviate by "jumping on that peak" creating a situation where they are cheap and the others are relatively expensive just for those values of θ that have a high mass. We furthermore had to choose the framework such that equilibrium investment is a concave function (i.e. $x^{*''}_{EQ}(c) < 0$ for all $c \in (\underline{c}^*, \overline{c}^*)$). The reason is that the reaction of the opponents to more aggressive cost functions is driven by the steepness of their marginal cost functions. The steeper the marginal cost function of the opponents, the smaller their reaction. Notice that the requirement of concave equilibrium investment is closely related to continuity of the technology set $k(c)$. That is, if only a discrete set of technologies is available, the resulting investment choice will be a step function which necessarily violates the above assumption. This implies however that an analysis of equilibrium cost functions under imperfect competition in the framework presented necessarily has to involve a continuous distribution of uncertainty and continuous technology sets. This seems to parallel the findings on supply function equilibria. In a seminal article Klemperer and Meyer (1989) show existence and uniqueness of differentiable supply functions for a continuous distribution of demand uncertainty. Bolle (1992) and Green and Newberry (1992) apply those findings in order to model firms behavior at electricity spot markets. In a subsequent contribution Fehr and Harbord (1994) show, however, that those results do extend to a more realistic discrete setting. Nevertheless, frameworks with smooth supply functions enjoy unchanged popularity when modeling firms behavior at electricity spot markets.

electricity generation. As illustrated in figure 7, for the case of duopoly (X^2_{EQ}) we do not observe over–investment, whereas for the case of 4 strategic firms (X^2_{EQ}) the model predicts over–investment in efficient technologies.

7 Empirical Analysis for the German Electricity Market

In this section we demonstrate how our theoretical insights can be used to assess firms investment decisions in electricity generation facilities in liberalized electricity markets. Here, for the reason of data availability, we use data of the German electricity market. Our aim is to fit the theoretical model as closely as possible to the data of the German Electricity market for the year 2006 and to compute resulting industry investment choice for the different hypothesis of perfect competition, monopoly and strategic behavior of firms.

In order to use our theoretical model for the analysis we chose to make the following specifications. We assume linear, fluctuating demand $P(Q) = \theta - bQ$. and derive the set of available technologies, given by the pairs of annuities of investment cost on the one hand and production cost on the other. For a given demand distribution, and for given investment and production cost structure $k(c)$, firms investment choices can be calculated as given in theorems 1 2 and 3. The resulting investment choices allow us to derive the price distribution for all 8760 hours of the year and to compare to the observed price distribution.

The major purpose of such empirical analysis is to provide a practical illustration how the theoretical results can be used in order to derive firms investment decisions and resulting wholesale electricity prices for different market structures. The Model parameters are determined as follows:

Market demand: To construct fluctuating market demand, we depart from hourly market prices (from the European Energy Exchange (EEX)[27]) and hourly quantities consumed (from the Union for the Co-ordination of Transmission of Electricity (UCTE)[28]) for the year 2006. We chose the value of b in line with other studies on energy markets. Most studies that estimate demand for electricity[29] find short run elasticities between 0.1 and 0.5 and long run elasticities between 0.3 and 0.7.[30] The relevant range of prices is around $P = 100$ €/MWh and corresponding consumption is approximately $Q = 50$ GW. In our empirical analysis we thus use the slope $b = 0.0055$ which corresponds to an elasticity of around 0.4.

[27] See www.EEX.com

[28] See www.UCTE.org

[29] See, for example, Lijsen (2006) for an overview of recent contributions on that issue.

[30] E.g. Beenstock et al. (1999), Bjorner and Jensen (2002), Filippini Pachuari (2002), Booinekamp (2007), and many others.

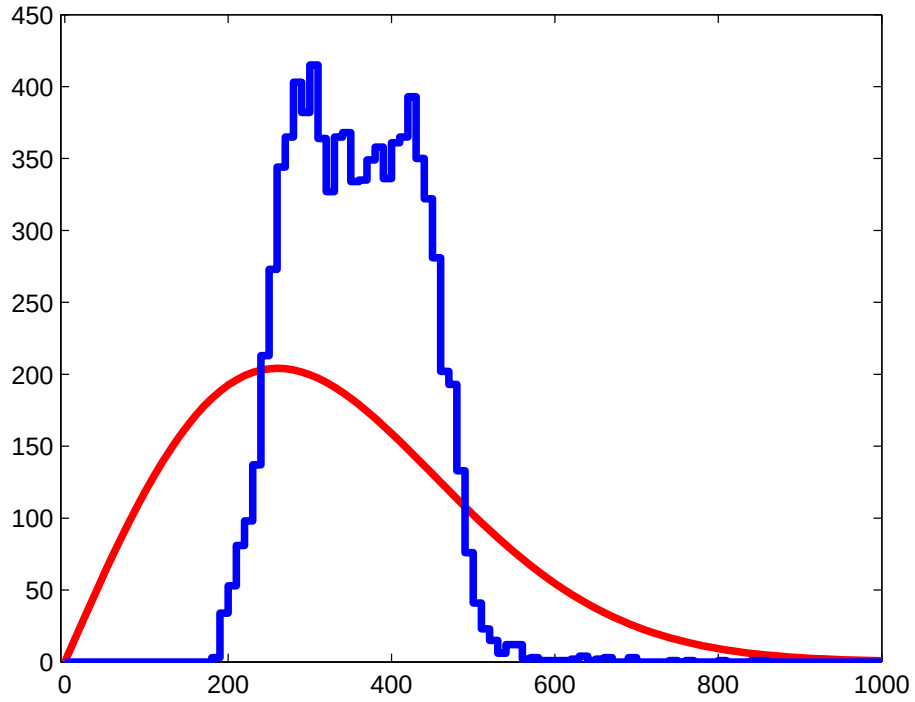

Figure 5: Fitting a Weibull distribution ($\alpha = 2$, MLE for the parameter: $\widehat{\beta} = 368.31$) to the frequency distribution of intercepts $f(\theta)$.

The computed intercepts θ for each of the 8760 of the year are sorted, their frequency-distribution is reported in figure 5. In order to satisfy the smoothness required to match the theoretical framework we fit a Weibull distribution[31] with parameter $\alpha = 2$.

The fitted distribution exhibits fatter tails than the distribution of observed intercepts. Those fatter tails could be motivated by the uncertainty about levels of demand at the time of investment additionally to the fluctuation of demand.

Production cost: The major component of variable production cost are fuel prices at the plant and cost of CO_2 emission allowances, which are determined by the price of allowances (assumed to be 10 €/t CO2) and the emissions coefficient of the different technologies. Production cost can then be computed based on the efficiencies of each technology[32]. No

[31] The Weibull distribution is given by $F(\theta) = 1 - e^{-\left(\frac{\theta}{\beta}\right)^{\alpha}}$ and it's density by $f(\theta) = \alpha\beta^{-\alpha}\theta^{\alpha-1}e^{-\left(\frac{\theta}{\beta}\right)^{\alpha}}$. For $\alpha = 2$ condition (b) of lemma 5 is satisfied.

[32] See 2006 GTW Handbook or EWI and Prognos (2005).

	Nuclear	Lignite	Coal	CCGT	GT
Efficiency	-	43.5%	47.0%	58.0%	37.5%
Fuel-prices at plant €/MWh	-	3.0	7.6	19.8	19.8
Cost of CO2-Cert. €/MWh	-	9.10	7.7	3.5	5.4
c Production Cost in €/MWh	5.0	18.3	25.7	38.1	58.7
Overnight Investment in €/kW	2011.3	1196.4	1073.7	550.0	250.0
Annual fixed cost in €/kWa	40.0	31.6	30.2	27.4	19.01
Free CO2 allocation in €/kWa	0	56.25	56.25	27.38	3.65
k Investment Cost in €/kWa	293.9	126.86	110.46	70.50	47.9

Table 1: Cost of Production c and Cost of Investment k.

information has been found for variable production cost of nuclear power plants,. it is assumed to be given by $5/MWh$. A proxy for final production cost of electricity for all different technologies in 2006 is reported in table 1.

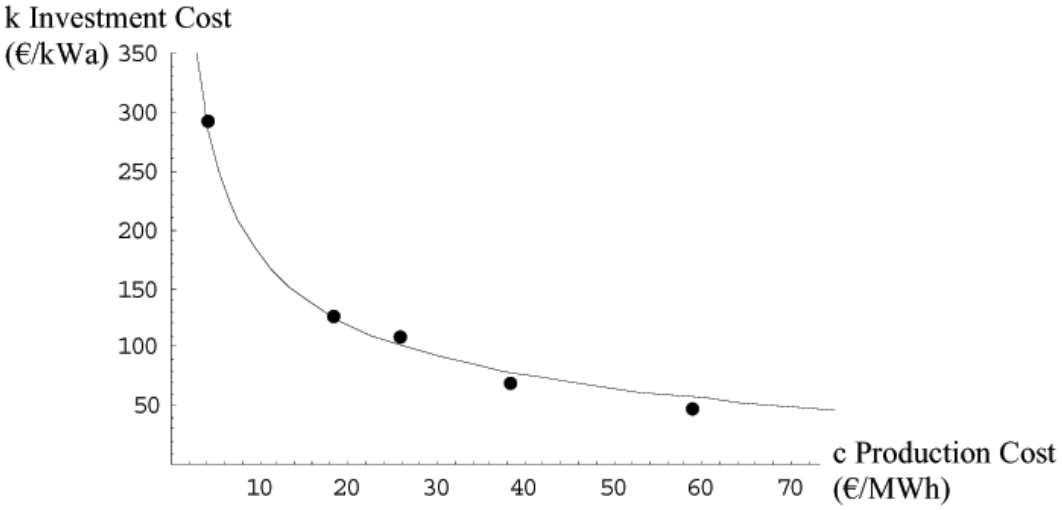

Figure 6: Fitting the pairs of production cost and investment cost to the following hyperbolic function: $k(c) = \frac{635.2}{c^{0.47}} - 34.5$.

Investment Cost: Since we analyze investment incentives based solely on one year, we break down investment cost to annuities.[33] In order to take construction times into account

[33]The results will thus only yield a benchmark for current profitability of investment. Provided, however, that yearly demand is increasing over time (and that strategic timing of investment is not an issue) our procedure should yield accurate predictions, even though once installed capacities cannot be removed the

we consider investment for the years 1995/2000. We furthermore assume perfect foresight, i.e. all cost components have been predicted accurately by the firms at the time of their investment decision. The relevant annuities are determined based on investment cost and annual fixed cost of running the plant. These values are corrected by availability of each technology, we take an average availability of 94%.[34] Based on a financial horizon of 20 years and an interest rate of 10 % we can compute annuities of investment cost. Finally, the free allotment of CO_2 allowances granted to new power plants results in a de facto reduction of the annuity by the net value of the allocated allowances. The resulting annual cost of investment for each technology are reported in table 1. In order to illustrate our theoretical findings we need to specify a continuous technology set which associates investment cost k to any level of production cost c. We do this by simply fitting a continuous function to the pairs c and k in table 1. We choose a simple hyperbolic functional form: $k(c) = \frac{p_2}{c^{p_1}} + p_3$ (least square fit yields: $k(c) = \frac{635.2}{c^{0.47}} - 34.5$).

After solving for firms investment choices (compare theorems 1 2 and 3.) we obtain Industry investments for the scenarios analyzed. This is illustrated in figure 7. Only the case of strategic interaction is sensitive to the numbers of firms, the graph illustrates the case of 2 and 4 firms. The graph illustrates that the presence of market power also has a strong effect on firms investment choices. Most interestingly we observe a strong incentive for overinvestment in efficient technologies, in the case of strategic interaction of 4 firms.[35] Up to a level of production cost of 25€/MWh firms invest more than in the first best scenario.[36] As a main result we thus conclude that predicted investment for the German market with four strategic firms in base–load technologies (producing at marginal cost below 25 €/MWh, such as nuclear and lignite plants) exceeds first best investment levels. Strategic under–investment takes place exclusively in middle and peak load technologies (such as gas, or oil-fired plants). Finally from the predicted capacity levels we now compute the price distribution over all 8760 hours of the year as illustrated in figure 8. Figure 8 provides the observed price distribution (P_{Real}), as well as the predicted price distributions for the benchmark cases of perfect competition (P_{FB}) and Monopoly (P_M) and also for the case of strategic interaction ($P_{EQ,4}$, 4 firm oligopoly and $P_{EQ,2}$ duopoly). In order to make

subsequent year.

[34]Compare VGB Powertech (2006).

[35]The German market consists essentially of four large players. Two of them (RWE and E.on) have a market share of 26 % each, while the two smaller ones (ENBW and Vattenfall) together cover 30 % of the market each. Compare, e.g., Monopolkommission (2007).

[36]Remarkably, this is not the case for a hypothetic duopoly of firms, nicely illustrating our theoretical result of lemma 6.

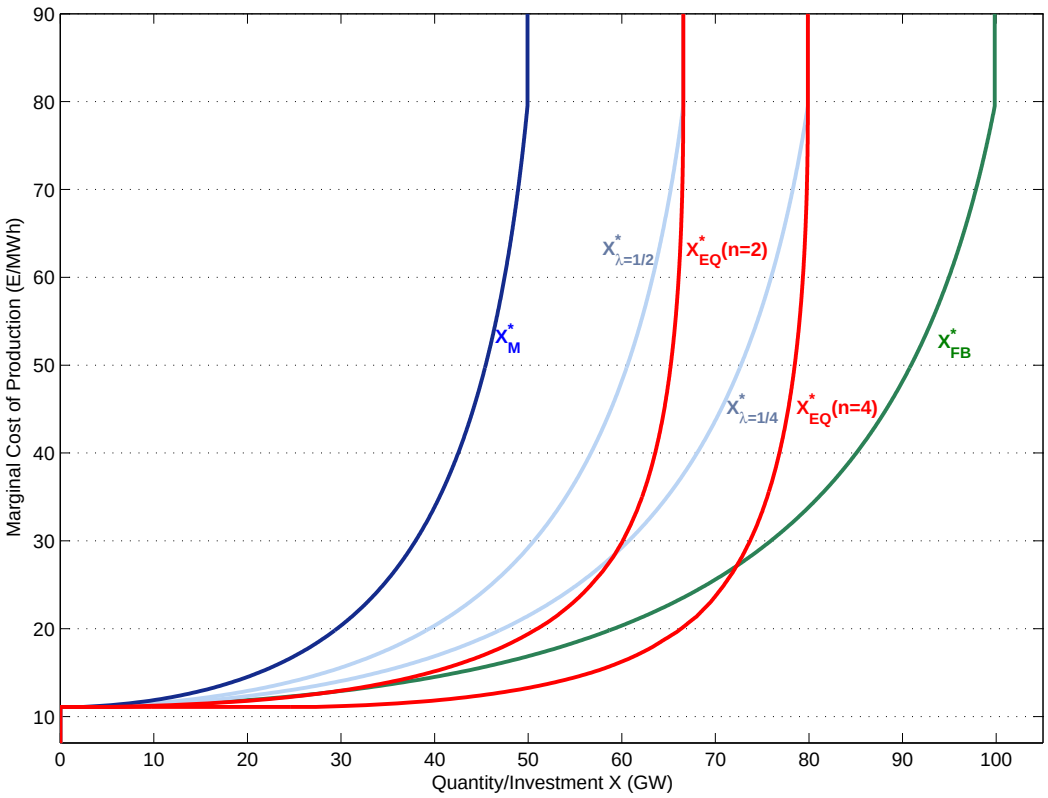

Figure 7: Industry Investment for the different scenarios $X^*_M(c)$, $X^*_\lambda(c)$, $X^*_{EQ}(c)$, and $X^*_{FB}(c)$ for Germany 2006.

the differences more visible, in the figure we focus on prices in the interval $[0, 350]$. We find that for the parameter configuration we chose, observed prices are somewhere in between the first best scenario and strategic interaction of firms.

Notice that the relatively low level of observed prices (as compared to the strategic scenario) may well be due to the fact that currently firms have more capacity installed than they would have chosen in a liberalized regime.[37] Our theoretical analysis implies that the current prices do not yield sufficient investment incentives to sustain the current investment level. Strategic investment would affect the price distribution, as comparison of the curves for the cases FB and EQ illustrates. We can conclude that there seems to be considerable potential for the exercise of market power in the long run when taking firms investment decisions into account.

[37] In the pre-liberalization period, generators where subject to a rate of return regulation that imposed excessive investment incentives.

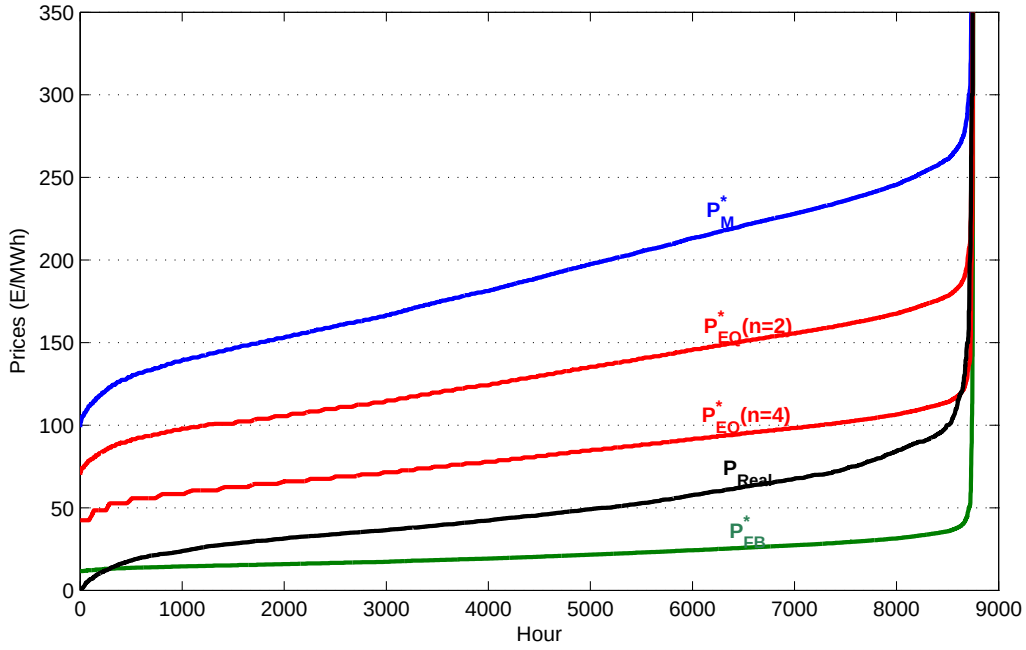

Figure 8: Distribution of market prices for perfect competition (P_{FB}), monopoly (P_M), and the oligopoly cases of $n = 2$ and $n = 4$ firms (P^2_{EQ}, P^4_{EQ}).

8 Conclusion

In this article we analyze firms investment incentives in liberalized electricity markets. Since electricity is economically non storable, it is optimal for firms to invest in a differentiated portfolio of technologies in order to serve strongly fluctuating demand. In the absence of strategic interaction, for a single firm, optimal investment and pricing decisions have been thoroughly analyzed in the so called peak load pricing literature. Those findings were widely used to model investment decisions in electricity markets prior to liberalization, when electricity was supplied by regulated monopolies.

Liberalization of electricity markets which started in the 1990's throughout Europe has changed this picture dramatically. In many countries electricity generation has been opened to competition and regulated monopolistic generators have been replaced by competing firms. All the results obtained in the peak load pricing literature, however, are not applicable in case firms do not behave perfectly competitively, but interact strategically when making their investment decisions. Since electricity markets especially in Europe are thought to be subject to the exercise of market power, however, the formerly used framework of the peak load pricing literature now has only limited use when predicting firms

investment decisions in those markets.

It has been the aim of the present article to derive equilibrium investment choice in liberalized electricity markets when firms behave strategically. We have derived equilibrium investment and compared it to the benchmark cases of perfect competition (welfare maximization), monopoly (profit maximization) and the so called second best solution derived in the peak load pricing literature. Interestingly, under imperfect competition firms have a strong incentive to invest into low marginal cost technologies in order to influence their competitors' spot market outputs. We have been able to establish properties under which this strategic effect is so intense that equilibrium investment in low marginal cost technologies in oligopoly is even above the welfare optimal level.

We finally have calibrated the theoretical framework to the problem of investment choice in the German electricity market. As a main result we find that investment of strategic firms in base–load technologies (producing at marginal cost below 25 €/MWh, such as nuclear and lignite plants) exceeds first best investment levels. Strategic under–investment takes place exclusively in middle– and peak–load technologies (such as gas, or oil-fired plants). Our empirical results confirm that the framework established in the present article provides a new and powerful tool in order to analyze investment behavior of strategic firms in electricity markets. It allows to assess the potential for the exercise of market power in liberalized electricity markets in the long run, by taking firms investment decisions into account.

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Appendix

Proof of lemma 1

Part (i): Since only technologies $c \in [c', c'']$ are affected by that change, we just have to apply the product rule of differentiation to the integrand of the second summand of (2) evaluated within the limits c' and c''. This yields:

$$\frac{d\pi_i(x,q)}{dx^{(c',c'')}} = \int_{c'}^{c''} \left(-\frac{d}{dc}\left[\int_{\theta_i^c}^{\overline{\theta}} [P(Q(\theta),\theta) - c]\, dF(\theta) - k(c)\right] - \frac{d}{dc}\frac{d}{dx^{(c',c'')}}\left[\int_{\theta_i^c}^{\overline{\theta}} [P(Q(\theta),\theta) - c]\, dF(\theta) - k(c)\right] x_i(c)\right) dc$$

This can be further simplified[38] and yields:

$$\int_{c'}^{c''} -\frac{d}{dc}\left[\int_{\theta_i^c}^{\overline{\theta}} [P(Q(\theta),\theta) - c]\, dF(\theta) - k(c)\right] - \frac{d}{dc}\left[\int_{\theta_i^c}^{\overline{\theta}} \left[\frac{dP(Q(\theta),\theta)}{dx^{(c',c'')}}\right] dF(\theta)\right] x_i(c) dc$$

Notice that the lower limits of integration θ_i^c of both summands clearly do depend on c, however only the integrand of the first summand is function of c through the expression "$-c$". We obtain expression (3) by Leibnitz rule after some rearranging:

$$\int_{c'}^{c''} \left[1 - F(\theta_i^c) + k_c(c) + \frac{d\theta_i^c}{dc} f(\theta_i^c)\left(P(Q(\theta),\theta) - c + \frac{dP(Q(\theta),\theta)}{dx^{(c',c'')}} x_i(c)\right)_{\theta=\theta_i^c}\right] dc \quad .$$

Part (ii): Notice that changing total capacity at $\overline{c}_i$ leaves the profits from technologies $c < \overline{c}_i$ unaffected. The First derivative with respect to changing total capacity is thus just given by differentiation of the first summand of (2) with respect to total capacity $\overline{x}_i$, yielding expression (4):

$$\frac{d\pi_i(x,q)}{dx^{(\overline{c}_i)}} = \int_{\theta_i^{\overline{c}_i}}^{\overline{\theta}} \left[P(Q(\theta),\theta) - \overline{c}_i + \frac{dP(Q(\theta),\theta)}{dx^{(\overline{c}_i)}} \overline{x}_i\right] dF(\theta) - k(\overline{c}_i)$$

Proof of lemma 2

The second order conditions for the case of perfect competition can easily be verified: First we observe that the cross partial derivative with respect to different technologies

[38] Maybe mention the continuity of profits issue, so differentiation of the limits θ_i^c wrt dx does not play a role for the first derivatives.

$\frac{d}{dx^{c^1,c^2}}\left[\frac{d\pi_i(x,q^{FB})}{dx^{(c',c''}}\right]=0$, for all $[c^1,c^2]$ different from $[c',c'']$. Formally this is due to the fact that the critical demand realization θ^c_{FB} for $c\in[c',c'']$ as defined by (5) does only depend on $X(c)$ for $c\in[c',c'']$, but not on $X(c)$ for $c\in[c^1,c^2]$. Thus expression (6) is not a function of $X(c)$ for $c\in[c^1,c^2]$ and the cross derivative equals zero. The argument is analogous for $\frac{d}{dx^{(\overline{c}_i)}}\left[\frac{d\pi_i(x,q^{FB})}{dx^{(c',c''}}\right]=0$

The second derivatives with respect to the same technologies are given as follows:

$$\frac{d^2\pi_i(x,q^{FB})}{d\left(x^{(c',c'')}\right)^2}=\int_{c'}^{c''}\left(-f(\theta^c_{FB})\,\frac{\theta^c_{FB}}{dx^{(c',c'')}}\right)\;dc<0. \tag{21}$$

This expression is negative since $\frac{\theta^c_{FB}}{dx^{(c',c'')}}>0$, (compare expression (5)).

And for the second derivative with respect to capacity choice we obtain:

$$\frac{d^2\pi_i(x,q_{FB})}{d\left(x^{(\overline{c}_i)}\right)^2}=\int_{\theta^{\overline{c}}_{FB}}^{\overline{\theta}}P_q\left(\overline{X},\theta\right)\;dF\left(\theta\right)<0. \tag{22}$$

Notice: Since the integrand is continuous at $\theta^{\overline{c}}_{FB}$ (equals zero), the derivative with respect to this lower limit drops out according to Leibnitz rule.

Proof of lemma 3

According to lemma 2, the overall capacity bound $\overline{X}^*_{FB}$ under perfect competition at technology $\overline{c}^*$ needs to satisfy the following to conditions:

$$\begin{aligned}(i)\qquad & F(\overline{\theta}^*)-(k_c(\overline{c}^*)+1)=0\\ (ii)\qquad & \overline{X}^*_{FB}:\;\int_{\theta^{\overline{c}^*}_{FB}}^{\overline{\theta}}P(\overline{X}^*_{FB},\theta)-\overline{c}^*=k(\overline{c}^*)\end{aligned}$$

We rewrite the integrand of (ii) in terms of the critical demand realization $\theta^{\overline{c}^*}_{FB}$ by making use of its definition (5) and obtain:

$$P(\overline{X}^*_{FB},\theta)-\overline{c}^*=\theta-B(\overline{X}^*_{FB})-\overline{c}^*=\theta-\theta^{\overline{c}^*}_{FB}$$

We obtain expression (8) as given in lemma 3:

$$\begin{aligned}(i)\qquad & F(\overline{\theta}^*)-(k_c(\overline{c}^*)+1)=0\\ (ii)\qquad & \int_{\overline{\theta}^*}^{\overline{\theta}}\theta-\overline{\theta}^*dF(\theta)=k(\overline{c}^*)\end{aligned}$$

Where $\overline{\theta}^*$ denotes the critical demand realization where firms start to become capacity constrained and $\overline{c}^*$ is the corresponding technology where firms start to be capacity constrained. Denote by $\theta^{cap}(c)$ the locus of all points satisfying (ii) and remember that the locus of all points satisfying (i) is denoted by θ^{c*}_{FB}. In the following we show that these two lines intersect exactly once whenever firms are active on the market, i.e. $(\overline{\theta}^*, \overline{c}^*)$ exists and is uniquely defined by the above system:

- **existence:** If we have $\int_{\underline{c}^*}^{\overline{\theta}} \theta - \underline{c}^* dF(\theta) > k(\underline{c}^*)$, then indeed the capacity condition (ii) is solved for some $\theta^{cap}(\underline{c}^*) > \underline{c}^*$, whereas condition (i) is solved by $\theta^{c*}_{FB} = \underline{c}^*$. Thus $\theta^{cap}(c)$ lies strictly above θ^{c*}_{FB} at $\underline{c}^*$. If on the other hand we consider some sufficiently high c^H, condition (i) will be solved at $\overline{\theta}$. Since we assume however $k(c) > 0$ for all c (see assumption 2 (i)) condition (ii) can only be satisfied for $\theta^{cap}(c^H) < \overline{\theta}$ (if not, the LHS of (ii) would drop to zero). Thus $\theta^{cap}(c)$ lies strictly below θ^{c*}_{FB} at c^H, which proves that at least one solution to the above system of two equations must exist.

- **Uniqueness:** Deriving the slope of $\theta^{cap}(c)$ we obtain by implicit function theorem:

$$\theta^{cap'}(c) = \frac{-k_c(c)}{1 - F(\theta^{cap}(c))}$$

  Whenever $\theta^{cap}(c)$ intersects θ^{c*}_{FB}, we obtain: $\theta^{cap'}(\overline{c}^*) = \frac{-k_c(\overline{c}^*)}{1-F(\overline{\theta}^*)} = 1$. Since all interior solutions satisfying condition (i) exhibit positive slope (i.e. $\theta^*_{FB}(c) > 1$ we observe that whenever the locus where condition (ii) is satisfied intersects the locus where (i) is satisfied, then $\theta^{cap}(c)$ intersects necessarily from above (i.e. at $(\overline{\theta}^*, \overline{c}^*)$, the locus $\theta^{cap}(c)$ is less steep than the locus θ^{c*}_{FB}), which proves uniqueness.

Proof lemma 4

The argument with respect to the cross derivatives is analogous to the proof of lemma 2 in appendix 8. The second derivatives with respect to the same technologies are obtained as follows:

$$\frac{d^2\pi_i(x, q_{FB})}{d\left(x^{(c',c'')}\right)^2} = \int_{c'}^{c''} \left(-f(\theta^c_{FB})\, \frac{\theta^c_{FB}}{dx^{(c',c'')}}\right)\, dc < 0. \tag{23}$$

This expression is negative since $\frac{\theta^c_M}{dx^{(c',c'')}} > 0$, (compare expression (10)). For the second derivative with respect to capacity choice we obtain:

$$\frac{d^2\pi_i(x, q^{FB})}{d\left(x^{(\overline{c}_i)}\right)^2} = \int_{\theta^{\overline{c}}_{FB}}^{\overline{\theta}} P_q\left(\overline{X}\right) + P_{qq}\left(\overline{X}\right)\overline{X}\ dF\left(\theta\right) < 0. \tag{24}$$

Due to assumption 1(ii) the integrand is negative. Notice: Since the integrand of the first order condition is continuous at $\theta^{\overline{c}}_{FB}$ (equals zero), the derivative with respect to this lower limit drops out according to Leibnitz rule.

Proof of Lemma 5 Preliminaries: Properties of the Spot–market Equilibrium

In order to proof lemma 5, we need to precisely characterize the Cournot-spot market equilibrium and it's reaction to changed investment of firms. In section 5 we characterized the spot market equilibrium somehow unusually without explicitly making use of marginal cost functions $C^j_q(q)$ but only in terms of the investments $x_j(c)$ made by each firm. Only throughout appendix 8 we will make use of the usual notation in terms of marginal cost $C^j_q(q)$, as already emphasized marginal cost $C^j_q(q)$ are just the inverse of the investment function $x_j(c)$. For $q^0 = x_j(c^0)$ we obtain thus the well known relationship: $x'_j(c^0) = \frac{1}{C^j_{qq}(q^0)}$.

(i) Properties of the spot market equilibrium For fixed θ:

Derive the reaction of the spot market equilibrium for fixed values θ to a change in investment level of firm i at some specific marginal cost c (denoted by $dx^{(c)}$). The spot market equilibrium for given marginal cost functions $C^j_q(q_j)$, for $j = 1, \ldots, n$ is characterized by the usual equilibrium conditions for an asymmetric Cournot-equilibrium:

$$\begin{array}{lll} j: & P(Q^{EQ},\theta) + P_q(Q^{EQ},\theta)q_j^{EQ} & = C^j_q(q_j^{EQ}) \\ i: & P(Q^{EQ},\theta) + P_q(Q^{EQ},\theta)q_i^{EQ} & = C^i_q(q_i^{EQ} - x_i^c) \end{array} \tag{25}$$

Thus investment of the amount x_i^c will allow firm i to produce not at $C^i_q(q)$ but at lower marginal cost given by $C^i_q(q - x_i^c)$ (where x_i^c is small, with $x_i^c \searrow 0$). Differentiation of expression (25) with respect to $dx^{(c)}$ yields:[39]

$$\begin{array}{ll} j: & (P_q(Q,\theta) + P_{qq}(Q,\theta)q_j)\,\frac{dQ}{dx^{(c)}} + P_q(Q,\theta)\frac{dq_j}{dx^{(c)}} = C^j_{qq}(q_j)\frac{dq_j}{dx^{(c)}} \\ i: & (P_q(Q,\theta) + P_{qq}(Q,\theta)q_i)\,\frac{dQ}{dx^{(c)}} + P_q(Q,\theta)\frac{dq_i}{dx^{(c)}} = C^i_{qq}(q_i)\frac{dq_i}{dx^{(c)}} - C^i_{qq}(q_i) \end{array}$$

Solving for $\frac{dq_i}{dx^{(c)}}$, the reaction of spot market output of firm i to it's change in the cost function, and $\frac{dq_j}{dx^{(c)}}$ the reaction of spot market output of the other firms j:

$$\begin{array}{ll} j: & \frac{dq_j}{dx^{(c)}} = \left(\frac{P_q(Q,\theta)+P_{qq}(Q,\theta)q_j}{C^j_{qq}(q_j)-P_q(Q,\theta)}\right)\frac{dQ}{dx^{(c)}} \\ i: & \frac{dq_i}{dx^{(c)}} = \left(\frac{(P_q(Q,\theta)+P_{qq}(Q,\theta)q_i)}{C^i_{qq}(q_i)-P_q(Q,\theta)}\right)\frac{dQ}{dx^{(c)}} + \frac{C^i_{qq}(q_i)}{C^i_{qq}(q_i)-P_q(Q,\theta)} \end{array}$$

[39] From here on we drop the superscript ^{EQ}, in order to save notation. In what follows we always refer to equilibrium outputs of stage 2.

Now we introduce the following notation for all $j = 1, ..., n$,

$$R_j := \left(\frac{-P_q(Q,\theta) - P_{qq}(Q,\theta) q_j}{C^j_{qq}(q_j) - P_q(Q,\theta)} \right). \tag{26}$$

Note that R_j corresponds to the share that firm j has on the total change in Q. We can now solve the equation system by summing up over all reactions $\sum_{j=1}^n \frac{dq_j}{dx^{(c)}}$, and obtain:

$$\frac{dQ}{dx^{(c)}} = \frac{C^i_{qq}}{C^i_{qq} - P_q} - \frac{dQ}{dx^{(c)}} \left(\sum_{j=1}^n R_j \right) \quad \Leftrightarrow \quad \frac{dQ}{dx^{(c)}} = \frac{C^i_{qq}}{(C^i_{qq} - P_q)\left(1 + \sum_{j=1}^n R_j\right)}$$

We finally obtain for $\frac{dQ_{-i}}{dx^{(c)}}$:

$$\frac{dQ_{-i}}{dx^{(c)}} = (-1) \left(\sum_{j \neq i} R_j \right) \frac{dQ}{dx^{(c)}} = \frac{(-1) C^i_{qq} \left(\sum_{j \neq i} R_j \right)}{(C^i_{qq} - P_q)\left(1 + \sum_{j=1}^n R_j\right)} \in (-1, 0). \tag{27}$$

(ii) Properties of the spot–market equilibrium at $\theta^c_{EQ,i}$:

Now we derive the reaction of the spot market equilibrium at the critical demand realization $\theta^c_{EQ,i}(x^c_i)$ to a change in investment level of firm i at some specific marginal cost c_0. The critical $\theta^c_{EQ,i}$ is characterized through the following equation system:

$$\theta^c_{EQ,i}: \begin{array}{ll} j: & P(q^{EQ}_i + Q^{EQ}_{-i}, \theta^{c0}_{EQ,i}) + P_q q^{EQ}_j - C^j_q(q^{EQ}_j) = 0 \\ i: & P(q^{EQ}_i + Q^{EQ}_{-i}, \theta^{c0}_{EQ,i}) + P_q q^{EQ}_i - c = 0 \end{array} \tag{28}$$

Differentiation wrt x^c_i yields:

$$\begin{array}{ll} j: & P_q \left(1 + \frac{dQ^{EQ}_{-i}}{dx^{(c)}}\right) + P_q \frac{dq^{EQ}_j}{dx^{(c)}} - C^j_{qq} \frac{dq^{EQ}_j}{dx^{(c)}} + \frac{d\theta^c_{EQ,i}}{dx^{(c)}} = 0 \\ i: & P_q \left(1 + \frac{dQ^{EQ}_{-i}}{dx^{(c)}}\right) + P_q + \frac{d\theta^c_{EQ,i}}{dx^{(c)}} = 0 \end{array}$$

Now we need to recover $\frac{dQ^{EQ}_{-i}}{dx^{(c)}}$ from equations $j \neq i$. We obtain by summing up and making use of R_j as defined above:

$$\frac{dq^{EQ}_j}{dx^{(c)}} = -R_j \frac{dQ^{EQ}_{-i}}{dx^{(c)}} - R_j \left(1 + \frac{\frac{d\theta^c_{EQ,i}}{dx^{(c)}}}{P_q}\right)$$

Summing up over all $\frac{dq^{EQ}_j}{dx^{(c)}}$ and solving for $\frac{dQ^{EQ}_{-i}}{dx^{(c)}}$ yields the following system of two equations:

$$\begin{array}{ll} j: & \frac{dQ^{EQ}_{-i}}{dx^{(c)}} = \frac{(-1)\left(\sum_{j \neq i} R_j\right)\left(1 + \frac{\frac{d\theta^c_{EQ,i}}{dx^{(c)}}}{P_q}\right)}{1 + \left(\sum_{j \neq i} R_j\right)} \\ i: & P_q \left(1 + \frac{dQ^{EQ}_{-i}}{dx^{(c)}}\right) + P_q + \frac{d\theta^c_{EQ,i}}{dx^{(c)}} = 0 \end{array}$$

Plugging the first equation into the second yields:

$$(-P_q)\left(\frac{\left(\sum_{j\neq i}R_j\right)\left(1+\frac{\frac{d\theta^c_{EQ,i}}{dx^{(c)}}}{P_q}\right)}{1+\left(\sum_{j\neq i}R_j\right)}-1\right)+P_q+\frac{d\theta^c_{EQ,i}}{dx^{(c)}} \quad =0$$

And solving for $\frac{d\theta^c_{EQ,i}}{dx^{(c)}}$ we finally obtain:

$$\frac{2+\sum_{j\neq i}R_j}{1+\sum_{j\neq i}R_j}P_q+\left(\frac{(-1)\sum_{j\neq i}R_j}{1+\sum_{j\neq i}R_j}+1\right)\frac{d\theta^c_{EQ,i}}{dx^{(c)}}=0$$

$$\frac{d\theta^c_{EQ,i}}{dx^{(c)}}=(-P_q)\left(2+\sum_{j\neq i}R_j\right)>0 \tag{29}$$

And solving for $\frac{dQ^{EQ}_{-i}(\theta^c_{EQ,i})}{dx^{(c)}}$ we obtain:

$$\frac{dQ^{EQ}_{-i}(\theta^c_{EQ,i})}{dx^{(c)}}=\sum_{j\neq i}R_j \quad \text{and} \quad \frac{dq^{EQ}_j}{dx^{(c)}}=R_j>0 \tag{30}$$

Finally we derive an important property of $R_j(\theta^c_{EQ,i})$ which will be needed later in order to prove the second order conditions. We obtain for all $j\neq i$:

$$\frac{dR_j(\theta^c_{EQ,i})}{dy^{c0}_i}=\frac{P_q}{(C^j_{qq}-P_q)^2}\underbrace{C^j_{qqq}}_{>0}(\cdot)\underbrace{\frac{dq^{EQ}_j(\theta^c_i)}{dx^{(c)}}}_{>0 \text{ see (30)}}<0 \tag{31}$$

We can thus conclude, that whenever $(C^j_{qqq}\geq 0)\Leftrightarrow(x_j''(c)\leq 0)$, we obtain $\frac{dR_j(\theta^c_{EQ,i})}{dx^{(c)}}\leq 0$.

(iii) Properties of the spot–market equilibrium at $\theta^c_{EQ,i}$, derive $\frac{d\theta^c_{EQ,i}}{dc}$:

Remember $\theta^c_{EQ,i}$ is defined by the equation system given by (28).

Differentiation wrt c yields:

$$\begin{aligned}
j:&\quad \tfrac{d\theta^c_{EQ,i}}{dc}+P_q\tfrac{dQ^{EQ}_{-i}(c)}{dc}+\left(P_q-C^j_{qq}\right)\tfrac{dq^{EQ}_j}{dc}+P_q\tfrac{dq^{EQ}_i}{dc} &&=0\\
&\ldots\\
i:&\quad \tfrac{d\theta^c_{EQ,i}}{dc}+P_q\tfrac{dQ^{EQ}_{-i}(c)}{dc}+2P_q\tfrac{dq^{EQ}_i}{dc}-1 &&=0
\end{aligned}$$

Solving for $\frac{dq^{EQ}_j}{dc}$, then summing up and solving for $\frac{dQ^{EQ}_i}{dc}$ yields:

$$\begin{aligned}
\frac{dq^{EQ}_j}{dc} &= (-1)R_j\left(\frac{dQ^{EQ}_i}{dc}+\frac{dq^{EQ}_i}{dc}+\frac{1}{P_q}\frac{d\theta^c_{EQ,i}}{dc}\right)\Leftrightarrow\\
\frac{dQ^{EQ}_{-i}}{dc} &= (-1)\frac{\frac{dq^{EQ}_i}{dc}\sum_{j\neq i}R_j+\frac{1}{P_q}\frac{d\theta^c_{EQ,i}}{dc}\sum_{j\neq i}R_j}{1+\sum_{j\neq i}R_j}
\end{aligned}$$

Now plug in into the equation of firm i. It is important to notice that $\frac{dq_i^{EQ}}{dc} = x_i'(c) = \frac{1}{C_{qq}^i\left(q_i^{EQ}\right)}$. In total we obtain:

$$\begin{aligned}
&\frac{d\theta_{EQ,i}^c}{dc} + (-P_q)\frac{\frac{dq_i^{EQ}}{dc}\sum_{j\neq i} R_j + \frac{1}{P_q}\frac{d\theta_{EQ,i}^c}{dc}\sum_{j\neq i} R_j}{1+\sum_{j\neq i} R_j} + 2P_q\frac{dq_i^{EQ}}{dc} - 1 = 0 \Leftrightarrow \\
&\frac{d\theta_{EQ,i}^c}{dc}\left(1 - \frac{\sum_{j\neq i} R_j}{1+\sum_{j\neq i} R_j}\right) = 1 + (-P_q)\frac{dq_i^{EQ}}{dc}\left(2 - \frac{\sum_{j\neq i} R_j}{1+\sum_{j\neq i} R_j}\right) \Leftrightarrow \\
&\frac{d\theta_{EQ,i}^c}{dc}\left(\frac{1}{1+\sum_{j\neq i} R_j}\right) = 1 + \frac{(-P_q)}{C_{qq}^i}\left(\frac{2+\sum_{j\neq i} R_j}{1+\sum_{j\neq i} R_j}\right) \Leftrightarrow \\
&\frac{d\theta_{EQ,i}^c}{dc} = 1 + \sum_{j\neq i} R_j + \frac{(-P_q)}{C_{qq}^i}\left(2 + \sum_{j\neq i} R_j\right) > 0
\end{aligned}$$

It can be verified by simple algebraic manipulation that the above can also be rewritten:

$$\frac{d\theta_{EQ,i}^c}{dc} = \frac{C_{qq}^i - P_q}{C_{qq}^i}\left(1 + \sum_{j\neq i} R_j + R_i\right) = \frac{C_{qq}^i - P_q}{C_{qq}^i}\left(1 + \sum_{j=1}^{n} R_j\right) \tag{32}$$

(iv) Properties of the spot–market equilibrium, approximate $q_i^{EQ}(\theta_{EQ,i}^c)$

Later on we need to have an upper bound on $q_i^{EQ}(\theta_{EQ,i}^c)$. This is obtained as follows. Notice in the linear case the spot market equilibrium (characterized in (28)) can also be written as follows:

$$\begin{aligned}
j: \quad & \theta_{EQ,i}^c + P_q\cdot(q_i^{EQ} + Q_{-i}^{EQ}) + P_q q_j^{EQ} - C_{qq}^j q_j^{EQ} + pos_j &= 0 \\
i: \quad & \theta_{EQ,i}^c + P_q\cdot(2q_i^{EQ} + Q_{-i}^{EQ}) - c &= 0
\end{aligned}$$

Notice if $C_{qqq}(q) > 0$, then $pos_j \geq 0$.

Solve for q_j^{EQ}, then summing up and solving for $Q_{-i}^{EQ}((\theta_{EQ,i}^c))$ yields:

$$\begin{aligned}
q_j^{EQ} &= R_j\left(-Q_{-i}^{EQ} + \frac{\theta_{EQ,i}^c + pos_j}{(-P_q)} - y_i^{c0}\right) \\
Q_{-i}^{EQ} &= \frac{\sum_{j\neq i} R_j}{1+\sum_{j\neq i} R_j}\left(\frac{\theta_{EQ,i}^c + pos_j}{(-P_q)} - y_i^{c0}\right)
\end{aligned}$$

Plugging into the first order condition of firm i and solving for q_i^{EQ} yields:

$$q_i^{EQ} = \frac{1}{-P_q}\left(\frac{\theta_{EQ,i}^c - c\left(1+\sum_{j\neq i} R_j\right)}{2+\sum_{j\neq i} R_j} - pos\frac{\sum_{j\neq i} R_j}{2+\sum_{j\neq i} R_j}\right) \tag{33}$$

Proof of Lemma 5

Part (i): First order conditions

Throughout section 5 we already have shown how expressions (18) and (19) are derived from the general first order conditions stated in lemma 1.

In appendix 8 we have derived expression (27), which is given by: $\frac{dQ_{-i}^{EQ}}{dx^{(c',c'')}} = \left(\sum_{j\neq i} R_j\right) \frac{(-1)C_{qq}^i}{(C_{qq}^i - P_q)\left(1+\sum_{j=1}^n R_j\right)}$ and expression (32) is given by: $\frac{d\theta_{EQ,i}^c}{dc} = \frac{(C_{qq}^i - P_q)\left(1+\sum_{j=1}^n R_j\right)}{C_{qq}^i}$. After multiplication we obtain:

$$\frac{d\theta_{EQ,i}^c}{dc}\frac{dQ_{-i}^{EQ}}{dx^{(c',c'')}} = (-1)\sum_{j\neq i} R_j = \sum_{j\neq i} \frac{\left(P_q(\cdot) + P_{qq}x_j(c)\right) x_j'(c_j^{EQ}(\theta))}{1 - P_q x_j'(c_j^{EQ}(\theta))}$$

This is due to the following two observations: R_j is defined in (26) by $R_j := \frac{-P_q - P_{qq}q_j}{C_{qq}^j - P_q}$. Since $C_q^j(q)$ is the inverse of investment $x_j(c)$ of firm j, for any $q^0 = x_j(c^0)$ the following well known relationship is satisfied: $C_{qq}^j(q^0) = \frac{1}{x_j'(c^0)}$.

We thus obtain slightly rewriting (18) for the interior first order condition:

$$\frac{d\pi_i(x, q^{EQ})}{dx^{(c',c'')}} = \int_{c'}^{c''} \left(1 - F(\theta_{EQ,i}^c) + k_c(c) + f(\theta_{EQ,i}^c)\left(\sum_{j\neq i} R_j\right)(-P_q)\, x_i(c)\right) dc \qquad (34)$$

Part (ii): Second order conditions

In order to verify second order conditions we first analyze the cross partial derivatives with respect to different technologies and observe that they equal to zero: $\frac{d}{dx^{c^1,c^2}}\left[\frac{d\pi_i(x,q^{EQ})}{dx^{(c',c'')}}\right] = 0$, for all $[c^1, c^2]$ different from $[c', c'']$. Formally this is due to the fact that the critical demand realization θ_{EQ}^c for $c \in [c', c'']$ as defined by (16) and the resulting Spot market equilibrium $Q^{EQ}(\theta_{EQ}^c)$ does only depend on $x_i(c)$ for $c \in [c', c'']$, but not on $x_i(c)$ for $c \in [c^1, c^2]$, where $[c', c'']$ an $[c^1, c^2]$ are arbitrary non-overlapping intervals. Thus expression (18) is not a function of $X(c)$ for $c \in [c^1, c^2]$ and the cross derivative equals zero. The argument is analogous for $\frac{d}{dx^{(\bar{c}_i)}}\left[\frac{d\pi_i(x,q^{EQ})}{dx^{(c',c'')}}\right] = 0$

We now focus on the second derivatives with respect to the same technologies, first we focus on the interior case, i.e we need to show $\frac{d^2\pi_i(x,q^{EQ})}{d\left(x^{(c',c'')}\right)^2} < 0$. In order to prove concavity of the interior solution, we differentiate expression (34) with respect to $dx^{(c'c'')}$:

$$\frac{d\pi_i^2(x, q^{EQ})}{d\left(x^{(c',c'')}\right)^2} = \int_{c'}^{c''} \frac{d}{dx^{(c',c'')}}\left(1 - F(\theta_{EQ,i}^c) + k_c(c) + f(\theta_{EQ,i}^c)\left(\sum_{j\neq i} R_j\right)(-P_q)\, x_i(c)\right) dc$$

The derivative of the Integrand reads as follows:

$$\left(\left(\sum_{j \neq i} \frac{dR_j^{\diamond}}{dx^{(c)}} \right) \left[-P_q q_i^{EQ} \right] + \left(\sum_{j \neq i} R_j \right) (-P_q) \right) f(\theta_{EQ,i}^c) +$$
$$\left(\left(\sum_{j \neq i} R_j \right) (-P_q) \, q_i^{EQ}(\theta_{EQ,i}^c) f'(\theta_{EQ,i}^c) - f(\theta_{EQ,i}^c) \right) \frac{d\theta_{EQ,i}^c}{dx^{(c)}} < 0?$$

For the case of linear demand (condition (a) of the lemma) and concave marginal cost functions of the rivals, i.e. $x_j''(c) < 0$ (condition (b) of the lemma), we obtain $\frac{dR_j^{\diamond}}{dx^{(c)}} \leq 0$ as derived in appendix 8 expression (30). Thus whenever conditions (a) and (c) of lemma 5 are satisfied, this term is negative and can be omitted.

We now check only the remaining terms. After plugging in for $\frac{d\theta_{EQ,i}^c}{dy_i^c} = -P_q \left(2 + \sum_{j \neq i} R_j\right)$, as derived in appendix 8 expression (29) we obtain:

$$f(\theta_{EQ,i}^c) \left[(-P_q) \left(\sum_{j \neq i} R_j \right) - (-P_q) \left(2 + \left(\sum_{j \neq i} R_j \right) \right) \right] +$$
$$f'(\theta_{EQ,i}^c) P_q^2 q_i^{EQ} \left(\sum_{j \neq i} R_j \right) \left(2 + \left(\sum_{j \neq i} R_j \right) \right) < 0 \ ?$$

and after slight simplification:

$$f(\theta_{EQ,i}^c) 2P_q + f'(\theta_{EQ,i}^c) P_q^2 q_i^{EQ} \left(\sum_{j \neq i} R_j \right) \left(2 + \left(\sum_{j \neq i} R_j \right) \right) < 0 \ ?$$

Notice that this is always satisfied for $f'(\theta) < 0$. However, whenever $f'(\theta) > 0$ we could get problems. Rearranging under the assumption $f' > 0$ the following should hold true:

$$\frac{f(\theta_{EQ,i}^c)}{f'(\theta_{EQ,i}^c)} > \frac{\left(\sum_{j \neq i} R_j \right)}{2} (-P_q) \left(2 + \sum_{j \neq i} R_j \right) q_i^{EQ} \ ? \tag{35}$$

In appendix 8 expression (33) we obtain an upper bound for q_i^{EQ}. By making use of (33)[40] we can find an upper bound on the right hand side of (35) and obtain:

$$\frac{\left(\sum_{j \neq i} R_j \right)}{2} (-P_q) \left(2 + \sum_{j \neq i} R_j \right) q_i^{EQ} \leq \frac{\left(\sum_{j \neq i} R_j \right)}{2} \left(\theta_{EQ,i}^c - c \left(1 + \sum_{j \neq i} R_j \right) \right)$$

[40] This is given as follows:

$$q_i^{EQ} \leq \frac{\theta_{EQ,i}^c - c \left(1 + \sum_{j \neq i} R_j \right)}{(-P_q) \left(2 + \sum_{j \neq i} R_j \right)}$$

Finally under the assumption $f''(\theta) < 0$, whenever $f'(\theta) > 0$, as stated in lemma 5 condition (b) we obtain $\frac{f(\theta^{c,*}_{EQ,i})}{f'(\theta^{c,*}_{EQ,i})} > \theta^{c,*}_{EQ,i}$ and thus (35) is satisfied. Concavity of profits for the interior case can consequently be guaranteed, provided conditions (a), (b), and (c) as specified in lemma 5 are satisfied.

We now show that also for the boundary case second derivatives are negative, i.e. $\frac{d^2\pi_i(x,q^{EQ})}{d\left(x^{(c',c'')}\right)^2} < 0$. In order to do so we need to differentiate expression (19) with respect to $dx^{(\overline{c}_i)}$, which yields:

$$\frac{d\pi_i^2(x,q^{EQ})}{d\left(x^{(\overline{c}_i)}\right)^2} = \int_{\theta^{\overline{c}_i}_{EQ,i}}^{\overline{\theta}} \frac{d}{dx^{(\overline{c}_i)}} \left(P\left(Q^{EQ},\theta\right) + P_q\left(Q^{EQ}\right)\overline{x}\left(1 + \frac{dQ^{EQ}_{-i}}{dx^{(\overline{c}_i)}}\right) - \overline{c}_i \right) dF(\theta) < 0 \ ?$$

Notice that according to Leibnitz rule we need to consider only the derivative of the integrand, the derivative wrt to the lower border cancels out, since the integrand evaluated at $\theta^{c}_{EQ,i}$ equals zero.

The derivative of the integrand obtains as follows:

$$\left(1 + \frac{dQ^{EQ}_{-i}}{dx^{(\overline{c}_i)}}\right)\left(2P_q\left(Q^{EQ}\right) + P_{qq}\left(Q^{EQ}\right)\overline{x}\left(1 + \frac{dQ^{EQ}_{-i}}{dx^{(\overline{c}_i)}}\right)\right) + P_q\left(Q^{EQ}\right)\overline{x}\left(\frac{d^2Q^{EQ}_{-i}}{d\left(x^{(\overline{c}_i)}\right)^2}\right) < 0 \ ?$$

When analyzing the sign of $\frac{d^2Q^{EQ}_{-i}}{d\left(x^{(\overline{c}_i)}\right)^2}$, again assumption (c) as stated in the lemma is crucial: the steeper the marginal cost functions of the other firms j, the less pronounced are their reactions $\frac{dQ^{EQ}_{-i}}{dx^{(\overline{c}_i)}}$ to changed cost function of firm i. In the limit, whenever firm j is capacity constrained (marginal cost vertical) it will not react at all to changed cost functions of firm i. We thus obtain $\frac{d^2Q^{EQ}_{-i}}{d\left(x^{(\overline{c}_i)}\right)^2} > 0$. Furthermore due to assumption 1, also the first summand is negative (both for the case of linear and nonlinear demand). Concavity of profits for the boundary case can consequently be guaranteed, provided conditions (a), and (c) as specified in lemma 5 are satisfied.

Proof of Theorem 3

The proof of the theorem is in three parts. First we derive the symmetric equilibrium candidate, second we show that deviation from that candidate is not profitable and third we prove that an asymmetric equilibrium cannot exist.

Part I) Derive the Symmetric equilibrium candidate:
A symmetric equilibrium candidate needs to satisfy the first order conditions given in lemma 5. Since firms can choose to invest at any level of marginal cost $c \geq 0$, we directly consider the integrand of expression (18), which yields for the case of symmetry:

$$1 - F\left(\theta_{EQ}^{c}\right) + k_c(c) + f\left(\theta_{EQ}^{c}\right)(n-1)\frac{\left(P_q + P_{qq}x\right)x'}{1 - P_q x'}P_q x = 0 \tag{36}$$

For $x(c)$ close to zero, the second summand drops out and we simply obtain[41] $1-F(c)+k_c(c)$. Equilibrium investment can thus not be desirable whenever the first derivative is negative, which is always the case for c small enough (compare assumption 2 (i)). Thus as in the case of perfect competition and monopoly investment is profitable only whenever $c > \underline{c}^*$, where $\underline{c}^*$ has been characterized in lemma 3.

Whenever $c > \underline{c}^*$, the interior solution has to satisfy the following differential equation which is obtained directly from (36):

$$\begin{aligned} x'(c) &= \frac{F\left(\theta_{EQ}^{c}\right) - 1 - k_c(c)}{f\left(\theta_{EQ}^{c}\right)(n-1)\left(P_q^2 x + P_q P_{qq} x^2\right) + P_q\left(F\left(\theta_{EQ}^{c}\right) - 1 - k_c(c)\right)} \\ \text{with} \quad & \theta_{EQ}^{c} = c + B(nx) + P_q x \end{aligned} \tag{37}$$

For the linear case where $P_{qq} = 0$ and $P_q = -b$ this yields expression (20) of theorem 3. Solutions of the differential equation (37) are illustrated as dotted lines in figure 9. The locus of all pairs (x, c) where (37) equals 0 is denoted by $x_0^*(c)$, all solutions of differential equation (37) pass through $x_0^*(c)$ with slope 0. $x_0^*(c)$ is given by:

$$\begin{aligned} & F\left(\theta_{EQ}^{c}\right) - 1 - k_c(c) = 0 \\ \text{with} \quad & \theta_{EQ}^{c} = c + B(nx) + P_q x \end{aligned}$$

Formally the solutions of the differential equations exhibit negative slope above the $x_0^*(c)$ - locus, since firms cannot make negative investment, however, they find it optimal to make no further investment above the $x_0^*(c)$ - locus. In other words, for all pairs (x, c) below

[41] Notice for $x = 0$ we obtain $\theta_{EQ}^{c} = c$, compare expression (17), since $B(0) - P_q * 0 = 0$.

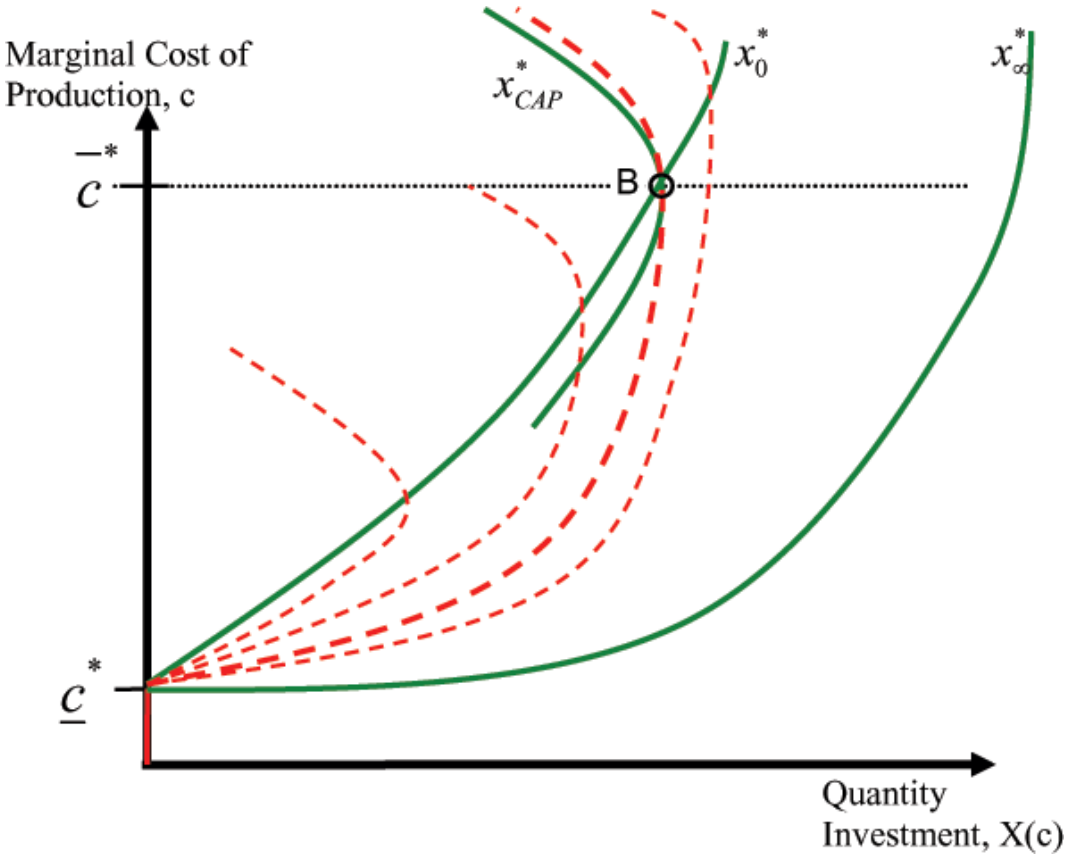

Figure 9: Derive the symmetric equilibrium candidate, solutions of differential equation (37) and boundary condition given by point "B".

the locus $x_0^*(c)$, firms will never find it optimal to remain capacity constrained but will always choose to slightly increase their investment (notice all optimal trajectories exhibit $x'(c) > 0$) and for all pairs (x, c) above the locus $x_0^*(c)$ firms will not find it optimal to increase investment, but will stop to invest choosing to be capacity constrained (which de facto implies $x'(c) = 0$). We can conclude that in the candidate equilibrium the pair $(\overline{x}, \overline{c})$ where firms start to be capacity constrained necessarily must lie on the $x_0^*(c)$ locus.

Finally the candidate solution must not only satisfy the interior optimality condition given by expression (18) of lemma 5, but also the optimality condition for optimal overall capacity choice given by expression (19) of lemma 5. For the symmetric case, expression (19) yields all pairs $(\overline{x}, \overline{c})$ which satisfy the optimal total capacity choice condition. We denote their locus by $x_{CAP}^*(c)$ which is given as follows:

$$x_{CAP}^*(c) : \left\{ (\overline{x}, \overline{c}) : \int_{\theta_{EQ}^{\overline{c}}}^{\overline{\theta}} P(n\overline{x}, \theta) + P_q \overline{x} - \overline{c} dF(\theta) = 0 \right\} \tag{38}$$

The locus of the optimal capacity choice condition is illustrated in figure 9. The candidate equilibrium has to satisfy differential equation (37) and pass through the intersection of the locus $x_0^*(c)$ with the locus $x_{CAP}^*(c)$ (point "B" in figure 9) as boundary condition.

In order to show uniqueness of the symmetric candidate equilibrium it remains to show that the boundary condition is indeed unique, i.e. that the intersection of the locus $x_0^*(c)$ with the locus $x_{CAP}^*(c)$ is unique. In order to do so we observe that the integrand of expression (38) can be rewritten in terms of the critical demand realization $\theta_{EQ}^{\overline{c}}$, by making use of expression (17):

$$P(n\overline{x},\theta) + P_q\overline{x} - \overline{c} = \theta - (B(n\overline{x}) - P_q(n\overline{x})\overline{x} + \overline{c}) = \theta - \theta_{EQ}^{\overline{c}}$$

The pair $(\overline{x}^*, \overline{c}^*)$ which has to lie both on the locus $x_0(c)$ and the locus $x_{CAP}(c)$ can thus be characterized by the following two conditions:

$$\begin{aligned} a) \qquad & F(\overline{\theta}^*) - (k_c(\overline{c}^*) + 1) = 0 \\ b) \qquad & \int_{\overline{\theta}^*}^{\overline{\theta}} \theta - \overline{\theta}^* dF(\theta) = k(\overline{c}^*) \end{aligned}$$

Where $\overline{x}^* = \left\{ x : P(x, \overline{\theta}^*) + P_q(x)x - \overline{c}^* = 0 \right\}$, $\overline{\theta}^*$ denotes the critical demand realization where firms start to become capacity constrained and $\overline{c}^*$ is the corresponding technology where firms start to be capacity constrained. Existence and uniqueness of $(\overline{\theta}^*, \overline{c}^*)$ have already been established in lemma 3. We can conclude that theorem 3 characterizes a unique symmetric equilibrium candidate.

Part II) Show that deviation from the candidate equilibrium is not profitable

We now show that deviation of firm $i = 1, \ldots, n$ is not profitable if all other firms stick to the candidate equilibrium $x_{EQ,-i}^*$. By construction of the symmetric candidate equilibrium the first order conditions of firm i both for an interior change (expression (18) of lemma 5) and for a change of total capacity are satisfied (expression (19) of lemma 5). Moreover whenever firm i is capacity constrained (i.e. $x'_{EQ,i} = 0$, $x_{EQ}^*(c) = \overline{x}^*$ for all $c \geq \overline{c}^*$, "above point S in figure 9") then the first order condition for an interior change (expression (18) of lemma 5) is negative, which implies that firm i indeed finds it optimal to be capacity constrained for all $c \geq \overline{c}^*$ (Remember, this is how the unique symmetric equilibrium candidate has been determined in Part I) of the current proof).

In lemma 5 (ii) we furthermore show sufficiency of the first order conditions of firm i. That is, when considering the second derivative with respect to changing investment for a

given interval of marginal cost of production $[c', c'']$ we show concavity. Furthermore, most importantly, all cross derivatives with respect to changing investment at different level of production cost (i.e. the two disjunct intervals $[c', c'']$ and $[c^1, c^2]$) equal zero. This implies that optimality of increasing or reducing investment for a given interval of marginal cost of production $[c', c'']$ is independent of changes in investment made at any other level of marginal cost of production, i.e. $[c^1, c^2]$.

This is illustrated in figure 10. Whenever cumulative investment $x_i^{DEVb}(c)$ is in the

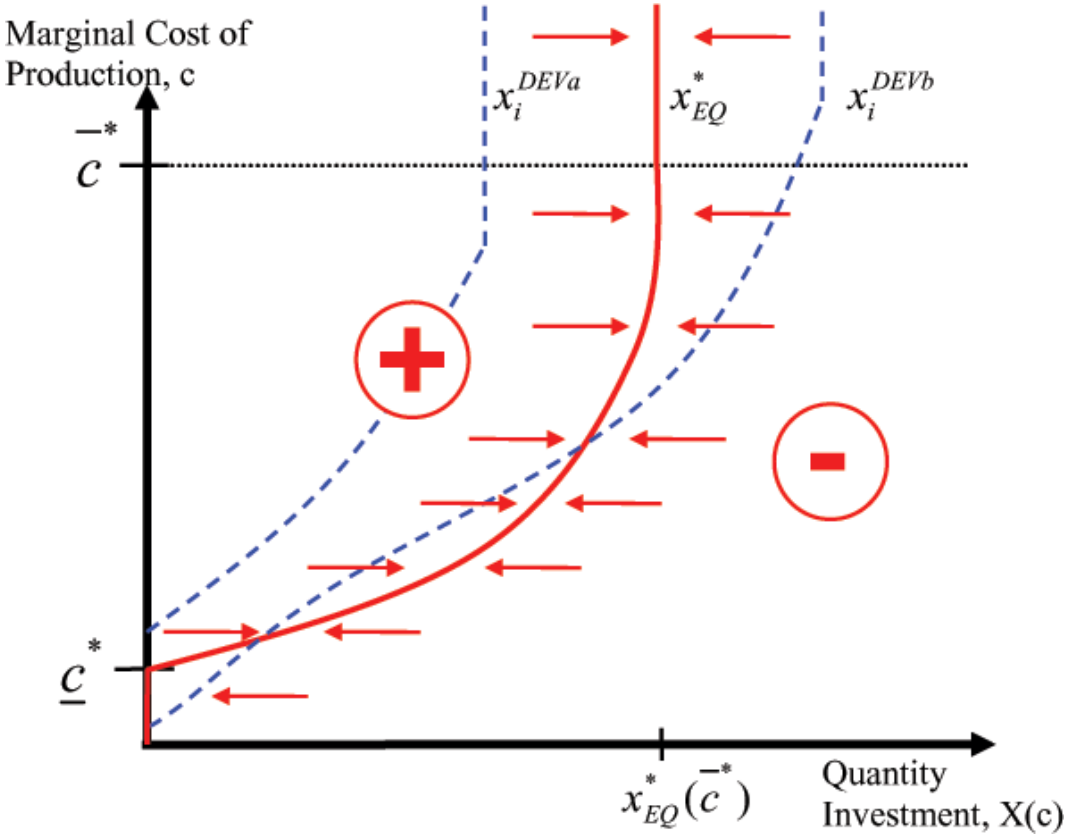

Figure 10: Deviation from the symmetric equilibrium candidate is not profitable.

region below $x^*_{EQ,i}(c)$ for some $c \in [c', c'']$, then firm i can increase it's profits by increasing investment in $c \in [c', c'']$. Since cross derivatives equal zero this does not interfere with changing investment for other levels of marginal cost of production. We can thus conclude that firm i will always reduce it's profits when deviating from the symmetric equilibrium candidate x^*_{EQ}. This proves existence of the symmetric equilibrium.

Part III) Show that an asymmetric equilibrium cannot exist:

We finally show that an asymmetric equilibrium of the investment market game cannot

exist. The first order conditions for the optimal investment decision in an asymmetric equilibrium are the following system of differential equations:

$$x_i'(c) = \frac{F(\theta^c_{EQ,j}) - 1 - k_c(c)}{f(\theta^c_{EQ,j})b^2 x_j - b(F(\theta^c_{EQ,j}) - 1 - k_c(c))} \tag{39}$$

$$x_j'(c) = \frac{F(\theta^c_{EQ,i}) - 1 - k_c(c)}{f(\theta^c_{EQ,i})b^2 x_i - b(F(\theta^c_{EQ,i}) - 1 - k_c(c))} \tag{40}$$

The proof is in two parts:

(i) Take a potential equilibrium candidate x_0 and denote by $\underline{c}_i^0$ the technology where firm i starts to have positive investment. Wlg sort firms such that firm 1 is the one investing into the lowest technology according to the candidate equilibrium x_0 (i.e. $\underline{c}_1^0 = \min[\underline{c}_i^0]$).

We first show now that x_0 cannot be an equilibrium if $\underline{c}_1^0 < \underline{c}^*$: Notice that $x_1^0(c) > 0$ for all $c > \underline{c}_1^0$ can only be a solution if:

$$1 - F(0 + \underline{c}_1^0) + k_c(\underline{c}_1^0) = 0 \tag{41}$$

Since the above equation is solved by $\underline{c}^*$ and since $k_{cc}(c) > f(c)$ for all c (see assumption 2 (iii)) we necessarily have $1 - F(0 + \underline{c}_1^0) + k_c(\underline{c}_1^0) < 0$, contradicting (41).

Since furthermore $x_i(c)$ cannot become negative, we can conclude that all (potentially asymmetric) equilibrium candidates pass through $(0, \underline{c}^*)$.

(ii) We now derive the slopes of the solutions of the above equation system (given by expressions (39) and (39)) at the point $(0, \underline{c}^*)$. Since the right hand sides of the above equation system yield "$\frac{0}{0}$", the slopes can be determined by applying the rule of l'Hopital, the result is given in expression (43), and we obtain $x_i'(\underline{c}^*) = x_j'(\underline{c}^*)$.

Now suppose there exists an asymmetric equilibrium $x_i(c)$ and $x_j(c)$. As shown in part (i), both necessarily pass through $(0, \underline{c}^*)$. Then for any $c > \underline{c}^*$, such that $x_i(c) < x_j(c)$, the equations (39) and (39) imply that $x_i(c)' < x_j(c)'$. However this is inconsistent with the above statement (i.e. $x_i(c) < x_j(c)$).

We can thus conclude that an asymmetric equilibrium cannot exist.

Proof of Lemma 6

First determine $x^{*'}_{EQ}(\underline{c}^*)$: Since $x^*_{EQ}(\underline{c}^*) = 0$ both numerator and denominator of (37) equal to zero at $\underline{c}^*$. In order to determine $x^{*'}_{EQ}(\underline{c}^*)$ we apply the rule of l'Hopital and obtain:

$$\lim_{c\to\underline{c}^*} x^{*'}_{EQ}(c) \;=\; \lim_{c\to\underline{c}^*} \frac{F\left(\theta^c_{EQ}\right) - 1 - k_c(c)}{f\left(\theta^c_{EQ}\right)(n-1)\left(P_q^2 x^*_{EQ} + P_q P_{qq}\left(x^*_{EQ}\right)^2\right) + P_q\left(F\left(\theta^c_{EQ}\right) - 1 - k_c(c)\right)}$$

Differentiation of numerator and denominator with respect to c yields[42]:

$$x^{*'}_{EQ}(\underline{c}^*) \;=\; \frac{f(\underline{c}^*)\theta^{*'}_{EQ}(\underline{c}^*) - k_{cc}(\underline{c}^*)}{f\left(\underline{c}^*\right)(n-1)P_q^2 x^{*'}_{EQ} + P_q\left(f(\underline{c}^*)\theta^{*'}_{EQ}(\underline{c}^*) - k_{cc}(\underline{c}^*)\right)} \tag{42}$$

Where according to expression (37), the critical demand realization θ^c_{EQ} is given by:

$$\theta^c_{EQ}(c) = B(nx^*_{EQ}) - P_q(nx^*_{EQ})x^*_{EQ} + c$$

and differentiation wrt c yields $\theta^{*'}_{EQ}(c) = x^{*'}_{EQ}\left(-P_q(n+1) - P_{qq}nx^*_{EQ}\right) + 1$. We can thus replace $\theta^{*'}_{EQ}$ in (42) and then solve the resulting quadratic form for $x^{*'}_{EQ}(\underline{c}^*)$. This yields the following unique positive solution:

$$x^{*'}_{EQ}(\underline{c}^*) = \frac{k_{cc}(\underline{c}^*) - (n+2)f(\underline{c}^*) + \sqrt{\left(k_{cc}(\underline{c}^*) - (n+2)f(\underline{c}^*)\right)^2 + 8f(\underline{c}^*)(k_{cc}(\underline{c}^*) - f(\underline{c}^*))}}{-P_q(0)\;\; f(\underline{c}^*)\;\; 4} \tag{43}$$

In order to prove the lemma we now compare $x^{*'}_{EQ}(\underline{c}^*)$ to $x^{*'}_{FB}(\underline{c}^*)$. Remember in section 3 we obtain: $x^{*'}_{FB}(\underline{c}^*) = \frac{1}{n}\;\; \frac{k_{cc}(\underline{c}^*) - f(\underline{c}^*)}{-P_q(0)\;\; f(\underline{c}^*)}$. Direct comparison of both results reveals now:

$$x^{*'}_{EQ}(\underline{c}^*) - x^{*'}_{FB}(\underline{c}^*) = \frac{1}{-P_q(0)\;\; f(\underline{c}^*)\;\; 4\;\; n}$$

$$\left(k_{cc}(\underline{c}^*)(n-4) - (n(n+2)-4)f(\underline{c}^*) + n\sqrt{\left(k_{cc}(\underline{c}^*) - (n+2)f(\underline{c}^*)\right)^2 + 8f(\underline{c}^*)(k_{cc}(\underline{c}^*) - f(\underline{c}^*))}\right) > 0\,?$$

$$\Leftrightarrow \qquad \left(k_{cc}(\underline{c}^*) - f(\underline{c}^*)\right)\left(k_{cc}(\underline{c}^*) - 2\,f(\underline{c}^*)\frac{n-1}{n-2}\right) > 0\,?$$

Since by assumption 2 (iii) we have $k_{cc}(\underline{c}^*) - f(\underline{c}^*) > 0$, we observe over–investment (with respect to first best investment) in efficient production technologies if and only if $k_{cc}(\underline{c}^*) > 2f(\underline{c}^*)\frac{n-1}{n-2}$, which proves the lemma.

[42]Notice as $c \to \underline{c}^*$, we obtain $x^*_{EQ} \to 0$ and $\theta^{\underline{c}^*}_{EQ} \to \underline{c}^*$, after differentiation these values can directly be plugged in.