The ambiguous effect of contracts on competition and prices in restructured electricity markets

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Abstract

Some argue that contracts reduce the pricing power of firms and hence are beneficial for the efficiency of the market; others explain that they foreclose the market and hence restrict entry of more efficient competitors. We construct a stylized model where two firms, each specializing in some technology, invest in a first stage, contract part of their production in the second stage and sell the rest in the spot market in the third stage. Working with two contracts (peak and off peak), we find cases where the contracts change neither capacity nor prices, where the foreclosing effect can increase investments and reduce prices, and where the opportunity to foreclose the market can incentivize one firm to reduce its investment in order to foreclose the contract market to the other firm and increase pricing power to the detriment of consumers. The model relies on the simplest possible assumptions of imperfect competition (subgame perfect equilibria with Cournot agents). The different results are obtained by changing one single parameter, the height of the off-peak time segment, which can result from different degrees of wind penetration. Our conclusion is also very simple: if it is impossible to characterize the consequences of contracts in a simple example it is not clear how regulators or competition authorities can assess the benefits or drawbacks of contracts in the complexity of real world restructured electricity markets.

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1 Introduction

The potential of contracts concluded by dominant firms to foreclose markets has long been recognized in the economics literature. The problem has mainly been considered in the context of vertical integration, such as when access to essential facilities can be denied to competitors. The treatment of horizontal foreclosure is more limited (see Tirole and Rey (2009) for a survey of the different situations of foreclosure). Horizontal foreclosure deals with situations where dominant agents conclude contracts that both extract rents from customers and limit the entry of possibly more efficient competitors (Aghion and Boltron (1987)). The rent is extracted by setting a high sales price, the barrier to entry by imposing a high penalty for breaching the contract. A related question is to find efficiency defenses that justify the contract. Even though we find little application of the results of that literature for our model, we effectively treat a problem of this type. We consider the impact of contracts on investments in generation capacity in markets with dominant agents. The contracts assumed in this paper always remain in force and hence imply an infinite penalty for breaking the contract. The intuition derived from the existing literature is that contracts would limit entry and be detrimental to welfare. To the best of our knowledge there is no literature on the problem of horizontal foreclosure in the context of power markets.

The literature on the restructuring of the electricity industry has not paid much attention to exclusionary effects of contracts because the literature investments and entrance is relatively new. In contrast, considerable attention has been devoted to the impact of contracts on abusive prices in markets with given capacities. Green (1992) was probably the first one to note that contracts mitigate the pricing power of generators. Newbery (1998) and Green and Newbery (1992) further developed the idea, which was then taken up by several authors (e.g. Wolfram (1999), Wolak (2000) and more recently Bushnell et al (2008)). This literature has resulted in a general consensus that forward positions decrease pricing power.

Harvey and Hogan (2000) asked why parties enter forward markets that decrease their pricing power. Allaz and Vila's (1993) seminal paper had provided an explanation. Leaving aside the role of contracts as hedging instruments, Allaz-Vila developed a two-stage game in which Cournot players take positions in the forward market in the first stage and act on the spot market in the second stage. Assuming arbitrage between the spot and contract markets and affine demand and cost curves, they showed that competing generators have an incentive to enter forward positions for purely strategic reasons that they identify as a prisoner's dilemma game. This result added to the interest in forward contracts and an extensive literature developed looking at variants of their paper. We do not review this literature here but simply note that we rely on Allaz Vila's result in this paper. Specifically, we consider contracts of the forward type that we insert in a static capacity expansion model. By construction these contracts cannot be breached and are in force for an infinite period of time. Competition law would see them as the most damaging contracts that one can think of.

Referring to Allaz-Vila (1993) immediately suggests looking at Vila's (????) extension of the model to risk and hedging. Contracts can indeed be used to hedge risks and hence facilitate investments. Finon and Perez (2008), and Finon and Roques (2008) offer informal arguments that explain that contracts are of the essence to facilitate investments in the risky environment of merchant plants. More recently Argenton and Willems (2009) extend the Aghion and Bolton (1987) model to invalidate risk hedging as an efficiency defense for long-term contracts.

This brief discussion of some of the different roles of contracts suggests that it is difficult to ascertain the global effect of contracts on competition. The reality is much worse. de Hautecloque and Glachant (2008) and de Hautecloque (2009) review the economic literature on contracts that they find relevant for the application of competition law in the power and gas industries. They discover, for each of the three subjects mentioned above, a whole range of literature with sometimes contradictory results: changing economic assumptions modifies the conclusion on the benefits and damages of contracts. The authors conclude that economically based actions by competition authorities could lead to considerable legal uncertainty. de Hautecloque and Glachant (2008) then analyze the recent rulings of these authorities and argue that the procedural approach adopted in the law considerably limits this legal uncertainty. We examine this claim and find it is impossible to ascertain to overall impact of contracts on competition, even in the simplest models of competition.

We have limited the above discussion to three important properties of contracts, foreclosure, pricing power, and risk hedging. See de Hautecloque and Glachant (2008) for other properties. We further simplify our analysis and concentrate on foreclosure and the mitigation of pricing power in a risk free world, dropping all considerations of hedging.

Our contribution to the literature can be summarized as follows. First, the literature reveals a wide set of assumptions and results on the efficiency of contracts; in contrast with this literature we remain within a single model to obtain different conclusions on contract efficiency. Second, while foreclosure is generally seen as detrimental to market efficiency, we see that it can increase capacity, decrease prices and enhance welfare. Third, this apparently counter-intuitive result is obtained because our model is specialized to electricity: the decomposition of demand into time segments without any possibility to store electricity between them is instrumental to getting these results. Our model is simple: we cast the problem in an investment context where agents compete in capacity development, contracts, and supplies to the wholesale market. This model expresses in the simplest possible form the basic ingredients of the investment process in generation capacities. The common wisdom is that generators extract monopoly rents from customers. We thus use the simplest possible model of market power, namely the Cournot model and we assume it applies to all stages of the market.

Even with these simplifying assumptions, the variety of outcomes obtained is striking: contracts can benefit, hurt or have no effect; their impact is totally undetermined. We do not have to do anything complicated to illustrate our theoretical results in an example. We change one coefficient of the model, the demand intercept in the base-period demand curve. This coefficient is standard: it can usefully be interpreted as resulting from more or less penetration of wind but many other interpretations are possible. We reason that if it is impossible to ascertain the impact of contracts in such a simple model, it is unlikely that a more complex treatment will give additional insight. As mentioned above, we did not look at the impact of risk. Because we rely on an investment model and invoke extensively Allaz and Vila's model, our model can be extended to risk, albeit at the cost of considerable technicalities.

We begin by elaborating on an investment model discussed in Murphy-Smeers (2005). In contrast to the extensive focus on market power in the wholesale and contracts markets, interest in the impact of market power on investments is relatively new, but the literature is rapidly growing. This investment model takes a stylized view of an electricity system with two generators, each specializing in one technology. The model describes a two stage game where generators invest in a first stage and operate on the spot market in the second stage. Based on that model, we show

- that an equilibrium of the capacity game does not necessarily exist because there are jumps in the profit of player $-i$ when it increases capacity and forecloses player i from some time segment (foreclosure in the energy market);
- but when the equilibrium exists, it leads to higher capacity than the one where all capacity is sold forward at the time of investment.

Our methodology consists of comparing the results from that model with those obtained by including a third stage in the game where contracts are signed between construction and the spot market.

The paper is organized as follows: we first present a summary of our results in section 2. Section 3 looks at the main features of the spot and capacity games without contracts and derives the necessary conditions for an equilibrium in the capacity market in that game. Section 4 introduces contracts in the investment model: agents invest in the first stage, contract part of their sales in the second stage and participate in the spot market in the third stage. We characterize the equilibrium of the contract market in section 4 and discuss equilibrium conditions in the capacity game in section 5. Section 6 illustrates these results withthe small example introduced in section 2. Section 7 concludes with our main result: we do not know the impact of contracts on competition and prices; it all depends on the particular case in hand. In order to facilitate the reading of the paper, all technical developments are given in appendices.

2 A summary of the argument

2.1 General economic context

As in Murphy and Smeers (2005) we assume two technologies, each operated by a different generator. We make the simplest (and most tractable) assumption of market power, namely Cournot competition, where we try to identify and compare subgame perfect equilibria. The economic problem is as follows. Assume a merchant organization of the market where each operator invests in the first stage and operates on a spot market in the second stage. Operators behave as Cournot players in both stages. We search for a subgame perfect equilibrium of that game as our benchmark. In order to assess the impact of contracts, we extend this reference case in the simplest possible way, by moving from the two-stage game where one invests in the first stage and operates in the second stage to a three stage game where one invests in the first stage, contracts in the second stage and operates in the third stage. Again, we are looking for subgame perfect equilibrium and are interested in assessing the impact of contracts on total capacity and prices.

2.2 A simplified model

We summarize our analysis with a simplified model and illustrate the different behaviors of the model with numerical results in Section 6. In this section we assume demand is decomposed into two time segments, peak (p) and off peak (b) of equal duration; we let the inverse demand function be $p^s = \alpha^s - q^s$, $s = p, b$ in each segment. In our theoretical development we assume S load segments. We assume two technologies indexed by $i = 1, 2$. Let k_i be the investment cost and ν_i the operating cost and player i specializes in technology i. Capacities are denoted by $x_i, i = 1, 2$ and contractual positions in time segment s are denoted by $y_i^s, i = 1, 2; s = p, b$. Actual generation is written as $z_i^s, i = 1, 2; s = p, b$. Assume an equilibrium exists when there are no contracts. Then both players are at capacity in the peak segment. In contrast three different situations can prevail in the off-peak segment. Both players can remain at capacity off peak (case a), both can be below capacity in the off-peak period (case b), and one of the players can operate at capacity while the other player is below capacity (case c). These different cases lead to different contributions to the value of the capacity investment and hence have different impacts on competition. Our analysis of contracts elaborates on their effect in this off-peak period to derive global results on both the peak and off-peak segments. We summarize this analysis of incentive to invest in the rest of this section and fully develop the argument in the rest of the paper. We illustrate these results numerically in Section 6.

2.3 Investment and prices when both players are at capacity in both time segments (case (a))

Let base and peak demands be identical or close (e.g., $\alpha^p \sim \alpha^b$). Then both players operate at full capacity in peak and off peak when there is no contract,

$$z_i^s = x_i,\ i = 1, 2;\ s = p, b.$$

Later in the paper we show that each time segment contributes a marginal revenue of $\alpha^s - 2x_i - x_{-i} - \nu_i$ to the investment cost k_i. This marginal revenue accrues half of the time because both periods have equal duration. At equilibrium, capacities x_i and x_{-i}, therefore, satisfy

$$\frac{1}{2}(\alpha^p - 2x_i - x_i - \nu_i) + \frac{1}{2}(\alpha^b - 2x_i - x_{-i} - \nu_i) = k_i,\ i = 1, 2.$$

In this case introducing contracts has no effect on the market equilibrium. Both players remain at capacity and prices are not changed.

2.4 Investment and prices when both players are at capacity in peak and only one player is at capacity off peak (case (c))

Consider now the case where both players operate at full capacity in peak,

$$z_i^p = x_i,\ i = 1, 2.$$

Assume that $-i$ is at capacity off peak while i remains below capacity in peak,

$$z_i^b < x_i,\ z_{-i}^b = x_{-i}.$$

Because player i is at capacity only during the peak period, the marginal revenue of its capacity is limited to that period. This revenue is $\alpha^s - 2x_i - x_{-i} - \nu_i$. At equilibrium

$$\frac{1}{2}(\alpha^p - 2x_i - x_{-i} - \nu_i) = k_i.$$

Player $-i$ is at capacity in both periods. Our results show that the marginal revenue accruing to its capacity is different in peak and off peak . The marginal revenue is $\alpha^p - x_i - 2x_{-i} - \nu_{-i}$ in peak and $\frac{1}{2}(\alpha^b - 2x_{-i} - 2\nu_{-i} + \nu_i) > (\alpha^b - x_i - 2x_{-i} - \nu_{-i})$ off peak. At equilibrium, the incentive to invest satisfies

$$\frac{1}{2}(\alpha^p - x_i - 2x_{-i} - \nu_{-i}) + \frac{1}{4}(\alpha^b - 2x_{-i} - 2\nu_{-i} + \nu_i) = k_{-i}.$$

We are able to prove that player i does not enter into contracts and the incentives to invest are unchanged.

2.5 Investment and prices when both players are at capacity in peak and below capacity in off peak (case (b))

This is the case where contracts play a key role. Let base and peak demand be sufficiently different (e.g., $\alpha^p \gg \alpha^b$) and assume that both players operate at full capacity in peak

$$z_i^p = x_i,\ i = 1, 2$$

but remain below capacity off peak

$$0 < z_i^b < x_i,\ i = 1, 2$$

(recall: Cournot equilibria allow for both players to operate in base even though $\nu_i > \nu_{-i}$). Because both operate below capacity in the off peak segment, only the peak segment makes a contribution to cover the investment cost. At equilibrium, the investment criterion without contracts is then

$$\frac{1}{2}\left(\alpha^p - 2x_i - x_{-i} - \nu_i\right) = k_i,\ i = 1, 2.$$

Introducing contracts leads to three possible cases.

2.5.1 Both players increase their off peak production after introducing contracts and remain below capacity. Investment does not change but the exercise of market power decreases off peak

This is the case where the Allaz-Vila effect applies without modification. Contracts increase generation off peak, but not to the point of reaching capacity, and decrease market power. The marginal contributions to investment and capacities remain unchanged and market power does not change in the peak period.

2.5.2 One player finds it profitable to foreclose the contract market. Investment increases, reducing the exercise of market power

Contracts can induce one agent to increase generation and fully use its capacity off peak. Both players keep operating at full capacity in peak

$$z_i^p = x_i, \quad i = 1, 2$$

and player $-i$ operates at capacity off peak, while player i operates below capacity

$$z_{-i}^b = x_{-i}, \; z_i^b < x_i.$$

Our results show that player $-i$ forecloses the contract market. The incentives to invest then take the same form as case (b) above. The marginal contribution to capacity of player i is unchanged

$$\frac{1}{2}\left(\alpha^p - 2x_i - x_{-i} - \nu_i\right) = k_i, \; i = 1, 2.$$

For player $-i$ it changes to

$$\frac{1}{2}(\alpha^p - x_i - 2x_{-i} - \nu_{-i}) + \frac{1}{4}(\alpha^b - 2x_{-i} - 2\nu_{-i} + \nu_i) = k_{-i}.$$

The consequence is that total investment increases over the case without contract and market power decreases in base and peak.

2.5.3 One player reduces capacity in order to be in position to foreclose the contract market: this decreases total capacity and increases market power

A third possibility exists. The underlying argument for a player to foreclose the contract market in section 2.5.2 is that operating at capacity increases its profit. This also leads to an incentive for this player to increase investment. In contrast to this case, player $-i$ may be induced to decrease capacity in order to make it easier (in fact possible) to operate at capacity and thus foreclose the contract market. This situation is a mixture of the two preceding cases in the following sense. Assume an equilibrium x_i^*, x_{-i}^* from the capacity game without contracts and that $z_i^b < x_i^*$, $i = 1, 2$ but $z_{-i}^b = x_{-i}$ for some $x_{-i} < x_{-i}^*$. The foreclosure effect applies when player $-i$ decreases capacity to some point below x_{-i}^* (and operates at capacity in base). Yet, the standard Allaz-Vila result applies (this player operates below capacity) with capacity at x_{-i}^*. At the largest capacity for $-i$ that allows for a foreclosure, there is a discrete change in player $-i$ profits. The profits are higher with lower capacity because player i drops out of the contract market from foreclosure and its production makes a discrete drop. If the profits for $-i$ are higher with the lower capacity associated with foreclosure, then there may exist an equilibrium where total capacity decreases. We elaborate on this mechanism in section 5 and show in section 6 that the likelihood of it occurring depends on the shape of the load curve and the cost parameters.

2.6 Summary conclusions

These three cases show that the impact of contracts is not predictable even in the simplest possible long-term model of a generation system. Contracts can leave capacities unchanged and keep market power unchanged in the peak period and decrease it during the off peak periods. Contracts can also foreclose the market and increase capacities, benefiting consumers. Alternatively, contracts can foreclose the market and decrease capacities, harming consumers. In short the effect of contracts is ambiguous. We now present the results leading to this conclusion.

3 The game without contracts

3.1 Introductory intuition

We extend our example by increasing the number of time segments, denoted by $s = 1, \cdots, S$. The segments are of duration π^s. The signs of the expressions

$$\alpha^s - 2z_i - z_{-i} - \nu_i \text{ and } x_i - z_i,\ i = 1, 2$$

fully characterize time segment s when there are no contracts. Specifically, given capacities (x_i, x_{-i}), time segments can be characterized as follows.

$$\alpha^s - 2x_i - x_{-i} - \nu_i > 0,\ x_i - z_i = 0 \quad i = 1, 2 \qquad \text{(e.g. peak load)}$$

$$\alpha^s - 2z_i - z_{-i} - \nu_i = 0,\ x_i - z_i > 0 \quad \text{and } \alpha^s - z_i - 2z_{-i} - \nu_{-i} > 0,\ x_{-i} - z_{-i} = 0 \qquad \text{(e.g. shoulder load)}$$

$$\alpha^s - 2z_i - z_{-i} - \nu_i = 0,\ x_i - z_i > 0 \quad i = 1, 2 \qquad \text{(e.g. off peak load).}$$

When

$$\alpha^s - 2x_i - x_{-i} - \nu_i > 0, x_i - z_i = 0$$

player i can profitably use all its capacity in the spot market and would increase production if it could do so. Conversely,

$$\alpha^s - 2z_i - z_{-i} - \nu_i = 0,\ x_i - z_i > 0, i = 1, 2$$

signals a case where none of the players desire to operate at full capacity. Lastly,

$$\alpha^s - 2z_i - z_{-i} - \nu_i = 0, x_i - z_i > 0 \text{ and } \alpha^s - z_i - 2z_{-i} - \nu_{-i} > 0, x_{-i} - z_{-i} = 0$$

indicates a time segment where player i does not want to use all of its capacity in the spot market if the other player is at capacity.

Section 3.2 develops this idea by characterizing the spot game equilibrium.

3.2 The spot market in time segments s

Let x_i be the capacities resulting from the investment stage. The equilibrium conditions of the spot market are obtained when each agent solves the following optimization problem,

$$\max_{0 \leq z_i^s \leq x_i} [\alpha^s - z_i^s - z_{-i}^s] z_i^s - \nu_i z_i^s. \tag{1}$$

The existence and uniqueness of the equilibrium in the spot market are easily established. They result from the solution of the following complementarity problem

$$\begin{array}{llll} \alpha^s - 2z_i^s - z_{-i}^s - \nu_i + \omega_i^s = \lambda_i^s & & i = 1, 2 \\ x_i - z_i^s \geq 0 & \lambda_i^s \geq 0 & (x_i - z_i^s)\lambda_i^s = 0 \\ z_i^s \geq 0 & \omega_i^s \geq 0 & z_i^s \omega_i^s = 0 \end{array} \tag{2}$$

Using n to indicate the game without a contract market, let $z^{ns}(x)$ be the solution to these equilibrium conditions as a function of the capacities x inherited from the first investment stage. Note that $z^{ns}(x)$ is single valued and continuous in x. Also, $z^{ns}(x)$ is not continuously differentiable in x. The point where $z^{ns}(x)$ is not differentiable is where $z^{ns}(x) = x_i$ and $\lambda_i^s = 0$. At this point both the left and right derivatives exist and are the limits as $\epsilon \to 0$ of the derivatives at $z^{ns}(x) = x_i - \epsilon$ and $z^{ns}(x) = x_i + \epsilon$ respectively.

As in Murphy and Smeers (2005), we treat a set of cases that characterize the solution for the different steps of the load duration curve. In order to simplify the presentation, we focus on the load steps where the equilibrium satisfies $0 < z_i^s \le x_i$, that is, the two producers are active at the equilibrium. Including a discussion of cases with one $z_i^s = 0$ adds complications without offering insight or affecting our conclusion on the ambiguity of the impact of contracts. See Murphy and Smeers (2005) for a discussion of these cases. The equilibria in the spot market that we consider satisfy one of three conditions.

(a)

$$\alpha^s - 2x_i - x_{-i} - \nu_i - \lambda_i^s = 0, \tag{3}$$

$$0 < z_i^{ns}(x) = x_i, \lambda_i^s \ge 0, i = 1, 2 \tag{4}$$

(b)

$$\alpha^s - 2z_i^{ns}(x) - z_{-i}^{ns}(x) - \nu_i = 0, \tag{5}$$

$$0 < z_i^{ns}(x) < x_i, \lambda_i^s = 0, i = 1, 2 \tag{6}$$

(c)

$$\alpha^s - 2x_i - z_{-i}^{ns}(x) - \nu_i = 0, \tag{7}$$

$$0 < z_i^{ns}(x) < x_i, \lambda_i^s = 0 \tag{8}$$

$$\alpha^s - x_i^s - 2z_{-i}^{ns}(x) - \nu_{-i} - \lambda_{-i}^s = 0, \tag{9}$$

$$0 < z_{-i}^{ns}(x) = x_{-i}, \lambda_{-i}^s \ge 0 \tag{10}$$

Proposition 1 *Let x_i and x_{-i} be given capacities (not necessarily an equilibrium in the capacity game). Then the equilibrium on the spot game can be of one of the three following types*

(a) $\alpha^s - 2x_i - x_{-i} - \nu_i \ge 0, i = 1, 2$*;* $x_i = z_i^s, i = 1, 2$

(b) $\alpha^s - 2z_i^s - z_{-i}^s - \nu_i = 0, i = 1, 2$*;* $x_i > z_i^s, i = 1, 2$

(c) $\alpha^s - 2z_i - x_{-i} - \nu_i = 0, \alpha^s - z_i - 2x_{-i} - \nu_{-i} \ge 0$*;* $x_i > z_i^s$, $x_{-i} = z_{-i}^s$.

Proof. The result follows directly from the KKT conditions of each player in Nash Cournot.

We refer to $z_i^s = x_i$, $i = 1, 2$; $z_i^s < x_i, i = 1, 2$ and $z_i^s < x_i$ and $z_{-i}^s = x_{-i}$ (and conversely $z_i^s = x_i$ and $z_{-i}^s < x_{-i}$) respectively as full capacity, interior, and corner equilibria (in the spot market).

3.3 The capacity game without a forward market

We now find the equilibrium in the capacity game that accounts for the behavior of the players in the spot market. This is commonly referred to as a subgame-perfect equilibrium or closed-loop equilibrium (Fudenberg and Tirole (2000)). The optimization that each player solves can be stated as follows:

$$\max_{x_i \geq 0} \Pi_i^n(x_i, x_{-i}) = \max_{x_i \geq 0} \sum_{s=1}^{S} [\alpha^s - z_i^{ns}(x) - z_{-i}^{ns}(x) - \nu_i] z_i^{ns}(x) - k_i x_i. \tag{11}$$

Assuming each player invests a positive amount and a point x_i, x_{-i} where the $z(x)$ are differentiable, the necessary equilibrium condition for each player is

$$\sum_{s=1}^{S} \left\{ [\alpha^s - 2z_i^{ns}(x) - z_{-i}^{ns}(x) - \nu_i] \frac{\partial z_i^{ns}(x)}{\partial x_i}(x) - \frac{\partial z_{-i}^{ns}(x)}{\partial x_i} z_i^{ns}(x) \right\} - k_i = 0. \tag{12}$$

Note that the partial derivatives in (12) exist when the solutions for all the load steps in (7) to (10) satisfy strict complementarity. If we do not have strict complementarity in some load segment, the right and left derivatives exist and are either 0, 1, or $-\frac{1}{2}$, and the arguments we develop for the differentiable case apply to the right and left derivatives in the relevant cases.

Note that the partial derivative $-\frac{1}{2}$ is the partial of z_i^{ns} with respect to x_{-i} when $z_i^{ns} < x_i$ and

$$\alpha^s - 2z_i^{ns} - x_{-i} - \nu_i = 0. \tag{13}$$

Using the partial derivatives of z_i and z_{-i} with respect to x_i given above to compute the contribution of each segment type (the term in brackets $\{\}$) in (12), we obtain:

- for load segments s with equilibria of type (a) ($s \in S^n(a)$),

$$\alpha^s - 2x_i - x_{-i} - \nu_i = \lambda_i \qquad i = 1, 2; \tag{14}$$

- for segments of type (b) ($s \in S^n(b)$),

$$\alpha^s - 2z_i^{ns}(x) - z_{-i}^{ns}(x) - \nu_i = 0 \qquad i = 1, 2; \tag{15}$$

- and for segments of type (c) ($s \in S^n(c)$), we have after replacing $\frac{\partial z_i^{ns}(x)}{\partial x_i}$ and $\frac{\partial z_{-i}^{ns}(x)}{\partial x_i}$ by 1 and $-\frac{1}{2}$ respectively

$$\alpha^s - \frac{3}{2}x_{-i} - z_i^{ns}(x) - \nu_{-i} = \mu_{-i}^s \tag{16}$$

$$\alpha^s - x_{-i}^s - 2z_i^{ns}(x) - \nu_i = 0. \tag{17}$$

Notice that we use μ_{-i}^s rather than λ_{-i}^s in load segments of type (c) because the perceived marginal value of capacity is different in the capacity game (relation (12) versus the spot game (relations (3) to (10)) since in the capacity game player $-i$ sees that an increase in capacity decreases the generation by player i in the spot game as expressed by $\dfrac{\partial z_i^{ns}(x)}{\partial x_{-i}}$, whereas under the Cournot assumption, it does not see this response in the spot game. From (12) we can conclude that the necessary equilibrium conditions for each player are

$$\begin{aligned} &\textstyle\sum_{s\in S^n(a)} \pi^s \lambda_{-i}^s + \sum_{s\in S^n(c)} \pi^s \mu_{-i}^s = k_{-i}, \\ &\textstyle\sum_{s\in S^n(a)} \pi^s \lambda_i^s = k_i. \end{aligned} \tag{18}$$

Note for further reference that substituting $z_{-i}^{ns}(x)$ in (16) by its expression taken from (17) gives

$$\mu_{-i}^s = \frac{1}{2}(\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i)$$

which will play a crucial role in further characterizing time segments when discussing the impact of contracts. Because $\mu_{-i}^s > \lambda_{-i}^s$ from the complementarity conditions (7) to (10) in segments of type (c), we were able to conclude in Murphy and Smeers (2005) that the closed-loop game leads to higher capacity than in the single-stage open-loop game. Also, Murphy and Smeers (2005) show that only the player with the base load cost structure has $\mu_{-i}^s > 0$, for $s \in S^n(c)$.

Proposition 2 *An equilibrium x_i, x_{-i} of the capacity game, if it exists, satisfies*

$$\begin{aligned} &\sum_{s\in S(a)} \pi^s(\alpha^s - 2x_i - x_{-i} - \nu_i) = 2k_i \\ &\sum_{s\in S(a)} \pi^s(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) + \sum_{s\in S(c)} \pi^s \tfrac{1}{2}(\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i) = 2k_{-i} \end{aligned}$$

Proof. The proof derives from relations (18) after replacing μ_i^s by its value.

The expression $\mu_{-i}^s = \frac{1}{2}(\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i)$ appearing in the investment objective is crucial for the rest of the discussion. We complete this section by emphasizing its interpretation in relation (18). Assume again that all functions $z(x)$ are differentiable, then we have the following proposition.

Proposition 3 *Let s be a time segment with a corner equilibrium in the spot market. Assume $-i$ is at capacity in that corner equilibrium. Then $\frac{1}{2}(\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i)$ is the marginal revenue of player $-i$ accruing from the spot market in that time segment.*

Proof. See the above discussion.

4 The game with contracts

4.1 Introductory intuition

We now turn to the more complex case of a game where investors in merchant plants can contract part of their production forward, trading the rest in the spot market. The definitions of the (local) closed-loop equilibrium of the two-stage game can be readily extended to the three-stage game after introducing additional notation.

Assuming that forward contracts are linked to the spot market by an arbitrage argument, Allaz and Vila show that contracts incentivize agents to increase production and hence mitigate market power. Making their assumptions and trying to apply their results, we immediately encounter difficulties in cases (a) and (c) when one or both players are at capacity off peak. Case (b) poses a different type of difficulty. The Allaz-Vila result applies directly if the increased generation induced by the contract does not hit capacity. The question is what happens if this increased generation exceeds capacity? We show that this introduces a foreclosure effect in the contract market that can benefit or hurt the consumer.

We now examine the role of contracts in the different types of time segments and successively look at (a) (full capacity equilibrium), (c) (corner equilibrium), and (b) (interior equilibrium).

4.2 Spot and contract games with given capacities

4.2.1 Equilibrium conditions in the spot game when there is a contract game

The discussion here summarizes results in Murphy and Smeers (2009) where we develop results for forward markets with uncertain demand without a load curve. The three-stage game can be solved starting from the spot game as follows. For a given x and y_i^s the spot market optimization is

$$\max_{0 \le z_i^s \le x_i} \Pi_i^s(x, y^s; z_i^s, z_{-i}^s) = [\alpha^s - (z_i^s + z_{-i}^s)](z_i^s - y_i^s) - \nu_i z_i^s \qquad \text{for } i = 1, 2. \tag{19}$$

The difference between the optimizations with and without a contract market is that the contract positions y_i^s are subtracted from the production levels, as the revenues from these positions are accounted for in the contract game.

The equilibrium conditions in the spot game can be written as

$$\alpha^s - 2z_i^s - z_{-i}^s - \nu_i + y_i^s + \omega_i = \lambda_i^s \qquad i = 1, 2$$

$$(x_i - z_i^s) \geq 0 \qquad \lambda_i^s \geq 0 \qquad (x_i - z_i^s)\lambda_i^s = 0 \qquad i = 1, 2 \tag{20}$$
$$z_i \geq 0 \qquad \omega_i \geq 0 \qquad z_i^s \omega_i = 0 \qquad i = 1, 2.$$

Note that y_i can be either positive or negative, corresponding to selling or buying in the contract market. Needless to say, the results are more easily interpreted in terms of long-term contracts if y_i is positive, which we show is always the case. Given values for x and y^s, the spot market equilibrium exists. When both players have positive production, as in the game without a contract market, the equilibrium $z^s(x,y) = (z_i^s(x,y), z_{-i}^s(x,y))$ of the spot market satisfies one of the following conditions:

$$\begin{array}{lll} \text{(a)} & 0 < z_i^s(x,y) = x_i; & i = 1,2 \\ \text{(b)} & 0 < z_i^s(x,y) < x_i; & i = 1,2 \\ \text{(c)} & 0 < z_i^s(x,y) < x_i; & 0 < z_{-i}^s(x,y) = x_{-i}. \end{array} \tag{21}$$

In case (a) (a full capacity solution), we have

$$\alpha^s - 2x_i - x_{-i} - \nu_i + y_i^s = \lambda_i^s \geq 0 \qquad i = 1, 2. \tag{22}$$

In case (b), (an interior solution), we can solve for z_i^s without including the capacity constraints, since they are not binding. The equilibrium conditions are

$$\begin{aligned} \alpha^s - 2z_i^s(x,y) - z_{-i}^s(x,y) - \nu_i + y_i^s &= 0 \\ \alpha^s - z_i^s(x,y) - 2z_{-i}^s(x,y) - \nu_{-i} + y_{-i}^s &= 0 \\ 0 < z_i^s(x,y) < x_i \qquad i &= 1,2 \end{aligned} \tag{23}$$

or after solving

$$z_i^s(x, y^s) = \frac{1}{3}[\alpha^s - 2(\nu_i - y_i^s) + (\nu_{-i} - y_{-i}^s)]. \tag{24}$$

We need to verify that $0 < z_i^s(x,y) < x_i$ for this solution to be interior.

In case (c), (a corner solution), the equilibrium conditions are

$$\begin{array}{ll} \alpha^s - 2z_i^s(x,y) - x_{-i} - \nu_{-i} + y_{-i}^s = 0 & 0 < z_i^s < x_i^* \\ \alpha^s - z_{-i}^s(x,y) - 2x_{-i} - \nu_i + y_i^s = \lambda_i^s & 0 < z_{-i}^s = x_{-i}^*. \end{array} \tag{25}$$

From them we find the following solution to the system

$$\begin{aligned} z_i^s(x,y) &= \frac{1}{2}(\alpha^s - x_{-i} - \nu_i + y_i^s) \\ \lambda_{-i} &= \alpha^s - 2x_{-i} - \nu_{-i} + y_{-i}^s - \frac{1}{2}(\alpha^s - x_{-i} - \nu_i + y_i^s) \\ &= \frac{\alpha^s}{2} - \frac{3}{2}x_{-i} - \frac{1}{2}(2\nu_{-i} - \nu_i) + \frac{1}{2}(2y_{-i}^s - y_i^s) \end{aligned} \tag{26}$$

where we again need to verify that $0 < z_i^s(x,y) < x_i$.

4.2.2 Necessary equilibrium conditions in the contract game

We can write the contract market optimization using the solution in (19) as follows:

$$\max_{y_i^s} \Pi_i^{fs}(x, y^s) = \max_{y_i^s} \Pi_i^{fs}(x, y^s, z^s(x, y^s)) \tag{27}$$

where

$$\Pi_i^{fs}((x,y), z^s(x,y^s)) = [\alpha^s - z_i^s(x,y^s) - z_{-i}^s(x,y_i^s) - \nu_i] z_i^s(x,y^s).$$

To characterize the solution, we have to look at the three cases (21).

In case (b) where $z_i^s < x_i$ $i = 1,2$, (24) applies and we solve for y_i^s. Inserting the solutions for $z^s(x,y^s)$ from (24) into the forward-market profit function $\Pi_i^{fs}(x^*, y, z^s(x,y^s))$, we get

$$\Pi_i^{fs} = \frac{1}{9}\big[\big(\alpha^s - 2\nu_i + \nu_{-i} - y_{-i}\big)^2 + \big(\alpha^s - 2\nu_i + \nu_{-i} - y_{-i}\big)y_i^s - 2{y_i^s}^2\big]. \tag{28}$$

Using the first order equilibrium conditions from the profit function and solving the system, we find that

$$y_i^s(x) = \frac{1}{5}[\alpha^s - (3\nu_i - 2\nu_{-i})] \qquad i = 1,2. \tag{29}$$

Letting $y^s(x) = (y_i^s(x), y_{-i}^s(x))$ we have

$$z_i^s(x, y^s(x)) = \frac{2}{5}[\alpha^s - (3\nu_i - 2\nu_{-i})] \qquad i = 1,2. \tag{30}$$

The computation of $y_i^s(x)$ in (29) does not guarantee that these values are an equilibrium in the contract game. If $z_i^s(x, y^s(x)) < x_i$ for $i = 1,2$, we have an interior contract equilibrium. In other words, an interior contract equilibrium is a pair (y_i^s, y_{-i}^s) that is an equilibrium in the contract game and induces a type (b) equilibrium in the spot game. In this case the spot and contract games have unique equilibria of type (b) given the capacities. The discussion of Section 5 and the example of Section 6.5 show that one can have both an interior and a corner equilibrium. Note that the effect of adding a contract market in this case is to increase production, as the standard Allaz-Vila's result implies.

We now turn to case (c) where $z_{-i}^s(x, y^s(x)) = x_{-i}$ and $z_i^s(x, y^s(x)) < x_i$. The contract optimization for player i is to choose the y_i^s that maximizes profits subject to the equilibrium conditions in the spot market, equations (25). From (26) we know the value of $z_i^s(x, y^s(x))$ as a function of y_i^s and can calculate the profit as a function of y_i^s as follows.

$$\begin{aligned} & \Pi_i^{fs}(x; y_i, y_{-i}) \\ = \quad & [\alpha^s - x_{-i} - \tfrac{1}{2}(\alpha^s - x_{-i} - \nu_i - y_i^s) - \nu_i]\tfrac{1}{2}(\alpha^s - x_{-i} - \nu_i - y_i^s) \\ = \quad & \tfrac{1}{4}\big[(\alpha^s - x_{-i} - \nu_i)^2 - y_i^2\big]. \end{aligned} \tag{31}$$

Clearly, the profit for player i is maximized when $y_i^s = 0$ and z_i^s has the following solution.

$$z_i^s = \frac{1}{2}(\alpha^s - x_i - \nu_i). \tag{32}$$

For player $-i$, from (25), we see that y_{-i}^s must be chosen to guarantee that $\lambda_{-i}^s \geq 0$ for $z_{-i}^s = x_{-i}$ for all potential values of z_i^s. The following condition on y_{-i}^s ensures that $\lambda_{-i}^s \geq 0$,

$$y_{-i}^s \geq -(\alpha^s - 2x_{-i} - z_i^s - \nu_{-i}), \tag{33}$$

and the contract equilibrium y^s, if it exists, turns out to be a vector-valued point-to-set map $y^s(x)$ with a unique solution in the spot market associated with this set of contract positions. Again the set $y_i^s = 0$, $y_{-i}^s \geq -(\alpha^s - 2x_{-i} - z_i^s - \nu_{-i})$ is not yet proven to be an equilibrium on the contract game. If it is, we refer to it as a corner contract equilibrium. In other words, a corner contract solution is a pair (y_i^s, y_{-i}^s) that is an equilibrium in the contract game and induces a type (c) equilibrium on the spot game.

We now turn to the last case (a) where $z_i^s(x, y^s(x)) = x_i, i = 1, 2$. The contract optimization for each player i is to choose y_i^s that maximizes profit subject to the equilibrium on the spot game. One can easily check that

$$y_i^s \geq -(\alpha^s - 2x_i - x_{-i} - \nu_i) \quad i = 1, 2$$

guarantees $\lambda_i^s \geq 0$ and hence $z_i^s(x, y^s(x)) = x_i$, $i = 1, 2$. If a point (y_1^s, y_2^s) satisfying these conditions can be shown to be an equilibrium on the contract game, we refer to it as a full capacity contract equilibrium, or in other words an equilibrium on the contract game that induces a type (a) equilibrium on the spot game.

4.2.3 Sufficient equilibrium conditions in the contract game: cases (a) and (c)

Lemma 1 *Let $\alpha^s - 2x_i - x_{-i} - \nu_i > 0$, $i = 1, 2$. Then any position*

$$y_i^s \geq -(\alpha - 2x_i - x_{-i} - \nu_i) < 0, i = 1, 2$$

is an equilibrium of the contract game. All these equilibria lead to the same full capacity equilibrium.

Proof. See Lemma A1 in Appendix 1.

The proposition is not surprising and the proof amounts to showing that there is no incentive to reduce generation by making y_i^s sufficiently negative (something that could in principle occur since, in contrast with the standard Alla-Vila reasoning, the other player cannot respond by increasing generation when it is at capacity). Note that any non-negative y_i satisfies this condition; non-negative y_i can easily be interpreted as long term contracts.

Corollary 1 *If $\alpha^s - 2x_i - x_{-i} - \nu_{-i} \geq 0$, $i = 1, 2$, then $z_i = x_i$, $i = 1, 2$ at the equilibrium of the capacity game with and without the contract game.*

Lemma 2 *Let $\alpha^s - 2z_i - x_{-i} - \nu_i = 0$, $z_i^s < x_i$, $\alpha^s - z_i - 2x_{-i} - \nu_{-i} > 0$ and $z_{-i}^s = x_{-i}$ in the no-contracts game. Then any position $y_{-i}^s \geq -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) < 0$; $y_i^s = 0$ is an equilibrium of the contract game. All these equilibria lead to the same corner equilibrium $x_i > z_i^s$, $x_{-i} = z_{-i}^s$. There is no other equilibrium in the contract game.*

Proof. See Lemma A2.1 and A2.2 in Appendix 2.

The proof is technical; its structure is as follows. Lemma A2.1 of Appendix 2 shows that $y_i = 0$ is the best response of player i to any contract position of player $-i$ that is sufficiently large, as we see from (31). The idea behind the proof is that player i cannot cause z_{-i} to decrease by acting on the contract market: this is one major difference between our result and the result of Allaz and Vila. Lemma A2.2 of Appendix 2 also shows that the best response of player $-i$ to $y_i = 0$ is $y_{-i}^s > -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) > 0$. Thus, given (x_i, x_{-i}), we have a set of equilibria in the contract market that leads to a unique spot-market equilibrium (even though the x may not be an equilibrium in the capacity market).

Note that player $-i$ forecloses the contract market as player i is crowded out of the forward market: player $-i$ can sell forward ($y_{-i}^s > -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) > 0$) but player i does not ($y_i^s = 0$) (recall that $\alpha^s - 2x_i - x_{-i} - \nu_i < 0$ implies in some sense that player i has too much capacity for this time segment). Corollary 2 states that foreclosure has no impact on competition in this case: foreclosure has no effect on capacity or price.

Corollary 2 *Let $\alpha^s - 2z_i - x_{-i} - x_{-i} - \nu_i = 0$, $z_i^s < x_i$, $\alpha_i^s - z_i - 2x_{-i} - \nu_{-i} > 0$ and $z_{-i}^s = x_{-i}$ in the no-contracts game. Then $z_i < x_i$, $z_{-i} = x_{-i}$ at the equilibrium of the spot game with and without a contract market.*

The above discussion makes it clear that time segments s that give corner and full capacity equilibria in the game without a contract market have the same property with the addition of a contract market: using the same capacities, the equilibria on the spot markets are identical in the two games. This situation is not covered by Allaz-Vila's theory since capacity is binding. Note however that Allaz-Vila's result is not contradicted in the sense that the contract market does not decrease spot production.

Corollary 3 *Time segments s that lead to full capacity and corner equilibria without a contract market still lead to full capacity and corner equilibria with a contract market and production in the spot market is also the same.*

The Allaz-Vila effect is thus inoperant (or trivially satisfied) for time segments of type (a) and (c). Contracts have no effect on investment and the exercise of market power if there are only time segments of type (a) and (c). Note that we have assumed differentiability. The same results hold when working with right and left derivatives as the lower bounds on the equilibrium y_i^s lead to the points of nondifferentiability where the left and right derivatives must be used. We now turn to time segments of type (b) which are more involved.

4.2.4 Sufficient equilibrium conditions in the contract game: case (b)

The case of the interior equilibrium is more complex. We note in passing that time segments of type (b) are those where observation of real markets suggest competitive behaviors because of unused capacity. Nevertheless, we continue with the Cournot paradigm in these time segments.

Recall that $\alpha - 2z_i - z_{-i} - \nu_i = 0$, $x_i - z_i > 0$, $i = 1, 2$ implies an interior spot equilibrium in the game without a contract market. Allaz-Vila's theory can apply here as capacities are not binding in the game without a contract market. With α^s sufficiently small, capacity is not binding and the introduction of contracts increases production in the spot market, leading to the interior solution obtained in (30) in Section 4.1,

$$z_i^s(x, y^s(x)) = \frac{2}{5}[\alpha^s - (3\nu_i - 2\nu_{-i})], \ i = 1, 2.$$

The following lemma formalizes this finding.

Lemma 3 *Let $\alpha^s - 2z_i - z_{-i} - \nu_i = 0$, $z_i^s < x_i$, $i = 1, 2$ and*

$$z_i = \frac{2}{5}[\alpha^s - (3\nu_i - 2\nu_{-i})] < x_i, \ i = 1, 2$$

then $y_i = \frac{1}{2}[\alpha^s - (3\nu_i - 2\nu_{-i})], i = 1, 2$ is the unique interior equilibrium in the contract game.

Proof. The profit function (28) is strictly concave. Thus, decreasing y_i while y_{-i} remains constant decreases the profit of player i. Conversely, increasing y_i also decreases this profit until z_i reaches x_i and the profit remains constant at that lower level.

It is, however, possible that one or both of the capacity constraints on z are violated, and a time segment that leads to an interior equilibrium in the problem without a contract market has a corner equilibrium with contracts. It is also possible that, even though there is a unique interior solution with contracts, there can also exist other corner or full capacity equilibria. These are the questions that we turn to now. Note that since $y_i^s = 0$ when $z_{-i}^s = x_{-i}$ for $i = 1, 2$, we have $\alpha^s - 2z_i^s - z_{-i}^s - \nu_i < 0$, $i = 1, 2$. Thus, there can be no full-capacity equilibrium resulting from contracts if we are in a time segment of type (b) when there are no contracts.

We first present the following result.

Lemma 4 *Let* (x_i, x_{-i}) *imply*

$$\alpha^s - 2z_i^s - z_{-i}^s - \nu_i = 0, z_i^s < x_i, \ i = 1, 2.$$

Then $y_i^s = 0$ *is the optimal response of player* i *to any* $y_{-i}^s \geq \widetilde{y}_{-i}^s(x) = -(\alpha^s - x_i - 2x_{-i} - \nu_{-i})$.

Proof. With the assumed choice of $y_{-i}^s, z_{-i}^s = x_{-i}$. By (31) the result holds locally. In order to show that it applies globally, note that the assumption $\alpha^s - z_i - 2x_{-i} - \nu_{-i} > 0$ never plays a role in the proof of Lemma A2.1 in Appendix 2 and this lemma applies to this case.

Lemma 4 shows that player $-i$ can foreclose the contract market ($y_i = 0$) by selling forward. The question is whether it is in player $-i$'s interest to do so, that is, whether $y_{-i} \geq -(\alpha^s - x_i - 2x_{-i} - \nu_i) > 0$ is the best response of player $-i$ to $y_i = 0$. Lemma A2.2 allows us to prove this type of result for a time segment of type (c). The proof of Lemma A2.2 effectively involves the condition $\alpha^s - z_i - 2x_{-i} - \nu_{-i} > 0$, in its parts (ii) and (iii). It therefore cannot be used here. We adapt the proof by delving into the conditions that guarantee an interior equilibrium.

Lemma 5 *Assume that there exists no corner equilibrium in time segment s of type (b) in the contract game. Then there exists an interior equilibrium with the Allaz-Vila solution remaining within the capacities.*

Proof. See Lemma A3.1 in Appendix 3

We now show a somewhat symmetric result.

Lemma 6 *Assume that there is no interior equilibrium in time segment s of type (b) in the forward game. Then there exists a corner equilibrium.*

Proof. See Lemma A3.2 in Appendix 3.

Lemma 7 *Let (x_i, x_{-i}) and the Allaz-Vila solution (z_i^{av}, z_{-i}^{av}) satisfy*

$$\alpha - 2z_i^{av} - z_{-i}^{av} - \nu_i = 0, \; i = 1, 2, \; z_i^{av} < x_i, z_{-i}^{av} > x_i$$

(z_{-i} in the solution of the Allaz-Vila exceeds its bound x_{-i}). Then

$$y_i = 0, \; y_{-i} > \widetilde{y}_{-i}(x)$$

is a corner solution.

Proof. From Lemma 4, $y_i = 0$ is the optimal response to any $y_{-i} \geq \widetilde{y}_{-i}(x)$. From Lemma 6 we must have a corner solution. Thus, $y_{-i} \geq \widetilde{y}_{-i}(x)$ is the optimal response of player $-i$ to $y_i = 0$.

Recall that $\mu_{-i}^s = \frac{1}{2}(\alpha^s - 2x_{-i} + \nu_i - 2\nu_i)$ is the marginal revenue accruing to player $-i$ from its capacity at a corner equilibrium; the following corollary is a byproduct of the above discussion.

Corollary 4 $\alpha^s - 2x_{-i} + \nu_i - 2\nu_{-i} > 0$ *is a necessary condition for a corner equilibrium with player $-i$ at capacity in the contract game.*

Proof. This result is embedded in the proof of Lemma 5.

The interpretation of this proposition is that a positive incentive to invest ($\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i > 0$) in segment s is also an incentive to foreclose the contract market in that time segment.

Corollary 5 *Assume the solution to the Allaz-Vila problem violates capacity bounds for both players, then there are two corner equilibria in the contract market.*

Note however, as mentioned before, that the violation of the two bounds of the Allaz-Vila solution does not imply a full capacity equilibrium. Indeed both players operating at capacity would imply a negative marginal profit for both of them, which cannot be an equilibrium in the contract game. The following proposition summarizes this discussion:

Proposition 4 *There always exists a pure strategy equilibrium of the contract game.*

We characterize these equilibria for a given time segment s and given capacities as follows:

- a full capacity equilibrium (type (a)) when there is no contract game is a full capacity equilibrium when there is a contract game,
- a corner equilibrium (type (c)) when there is no contract game is also a corner equilibrium when there is a contract game,
- an unconstrained equilibrium (type (b)) when there is no contract game can give rise to an unconstrained equilibrium in the contract game if

  $$\frac{2}{5}(\alpha - (3\nu_i - 2\nu_{-i})) < x_i, \; i = 1, 2.$$

  It can also give rise to a corner equilibrium $y_i = 0, y_{-i} \geq \widetilde{y}_{-i}(x)$ but only if

  $$\alpha + \nu_i - 2\nu_{-i} > 2x_i.$$

  There can also be two corner equilibria

  $$y_i = 0, \; y_{-i} \geq \widetilde{y}_{-i}(x) \text{ and } y_i \geq \widetilde{y}_i(x), \; y_{-i} = 0$$

  if and only if

  $$\alpha + \nu_i - 2\nu_{-i} > 2x_i \text{ and } \alpha + \nu_{-i} - 2\nu_i > 2x_{-i}.$$

It is also possible to have simultaneously

$$\begin{array}{ll} \frac{2}{5}(\alpha - (3\nu_i - 2\nu_{-i})) < x_i, & i = 1, 2 \\ \alpha + \nu_i - 2\nu_{-i} > 2x_i & \text{for } i = 1 \text{ or } 2 \text{ or } 1 \text{ and } 2 \end{array}$$

in which case one has both interior and corner solutions (see the example of Section 6.5).

4.2.5 Summing up

We summarize the preceding discussion in the following theorem. Let $S^c(a)$, $S^c(b)$, and $S^c(c)$ be the sets of load segments with spot solutions of type (a), (b), and (c), respectively when there is a contract game. Recall that we denoted these sets as $S^n(a), S^n(b)$ and $S^n(c)$ in the no contract game.

Theorem 1 *Given the same capacities in the games without and with contracts,*

- *(a) the sets $S^n(a) = S^c(a)$ are identical and both models have the same equilibrium conditions:*

$$\alpha^s - 2x_i - x_{-i} - \nu_i = \lambda_i \qquad i = 1, 2. \tag{34}$$

  in the games with and without contracts;

- *(b) $S^n(b) \supseteq S^c(b)$ and production is higher with a contract game for $s \in S^n(b)$. Note that $S^n(b)$ can be empty if there are only full capacity or corner equilibria in the no contract case;*

- *(c) $S^n(c) \subseteq S^c(c)$.*

5 Necessary equilibrium conditions in the capacity game

The above analysis of equilibria in the contract game allows us to write necessary conditions for an equilibrium in the capacity game with contracts. Recalling the discussion of the capacity game without contracts, note that the necessary equilibrium conditions are obtained under the assumption that the $z_i(x)$ are differentiable in x. We also briefly mentioned in passing that the conditions should be slightly modified in case of non-differentiability. This occurs when one of the time segments changes categories, that is, at a value of x where z_i is equal to x_i and $\lambda_i = 0$. This lack of differentiability was extensively discussed in Murphy-Smeers (2005); it leads to discontinuities in the players' profit functions, which can cause the pure strategy equilibrium to not exist. Discontinuities also appear here, resulting from changes of equilibria in the contract game. They are of a different nature and hence warrant some preliminary discussion.

We have seen in Corollary 4 that $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i > 0$ signals a corner equilibrium for the game with contracts and is equal to the marginal revenue of capacity $-i$ in time segment s at that corner equilibrium. We have also seen that it is possible to have both a corner and an interior equilibrium at a point with $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i > 0$, implying that players's payoffs are ambiguously defined. Conversely, $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i < 0$ for both players always implies a single interior equilibrium. An interior equilibrium in some time segment implies a zero marginal revenue of the capacity in that time segment. The revenue is thus non-differentiable when $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i = 0$ and one moves from an interior to a corner solution. Also we can state necessary conditions for an equilibrium in the capacity game if the payoff of each player is well defined and hence only if there is a single equilibrium in the contract market. The discussion of section 4 reveals that this is not always the case. All this requires adapting the necessary equilibrium conditions. All these restrictions are assembled in the following proposition that states necessary conditions for an equilibrium at a point where revenues are continuous for variations in capacities around the equilibrium. This is the case if the sets $S^c(a), S^c(b)$ and $S^c(c)$ are well defined (there is a single equilibrium in the contract game) and remain unchanged around the equilibrium. These necessary conditions are obtained by adapting the necessary conditions for an equilibrium in the capacity game without contracts: it suffices to replace $S^n(a), S^n(b)$ and $S^n(c)$ by $S^c(a), S^c(b)$ and $S^c(c)$. This is stated in Proposition 5, which is similar to Proposition 2 for the no contract game.

Proposition 5 *Let (x_i, x_{-i}) be a point such that $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i \neq 0$ for all s and a unique corner or interior equilibrium prevails in the contract market. If this point is an equilibrium of the game with contracts, it satisfies*

$$\sum_{s\in S^c(a)} \pi^s(\alpha^s - 2x_i - x_{-i} - \nu_i) = 2k_i$$

$$\sum_{s\in S^c(a)} \pi^s(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) + \sum_{s\in S^c(c)} \pi^s \tfrac{1}{2}(\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i) = 2k_{-i}$$

The following intuitive comments can help to understand the content of Proposition 5. Consider the move from the capacity game without contracts to the game with contracts. Suppose one remains at the equilibrium capacities of the no contract game, then the introduction of contracts can (but does not necessarily) enlarge the set of time segments with a corner equilibrium. This happens when a generator exceeds its generation capacity in some time segment in the Allaz-Vila solution. Lemmas 4 to 6 indicate that this player is induced to increase its contract position in order to fully utilize its generation capacity. This increases the marginal revenue accruing from capacity and hence the incentive to invest. This in turn leads to a global increase of capacity and a reduction of market power. This phenomenon was uncovered in Murphy-Smeers (2005) when moving from the open to the closed loop equilibrium in the capacity market without contracts. In short, in this model as well as in Murphy-Smeers (2005) the possibility of foreclosing the market (here offered by contracts) in some off peak time segments can lead to an increase of capacity and a decrease of market power.

The introduction of contracts with given capacities does not necessarily enlarge the set of time segments with corner contract equilibria. Suppose that neither player at the capacity equilibrium of the game without contracts finds its Allaz-Vila generation exceeding capacity and there is no incentive for these generators to reach a corner solution at these capacity levels. Then the capacities of the game without contracts may still be in equilibrium (and are in any case a local equilibrium). Alternatively, there can be an incentive to decrease capacity in order to create the possibility of a corner equilibrium.

This introduces a discontinuity in the profit functions. We illustrate this phenomenon in Figure 1 which shows the profit of both players as a function of their capacities in a game with two load segments, peak and off peak. Assume again that we start from the equilibrium capacities (x_i^*, x_{-i}^*) of the game with no contracts and there is an interior equilibrium for these capacities in the off peak segment. Suppose also that the Allaz Vila solution is also an interior equilibrium in the off peak segment in the contract game and there is no corner equilibrium at that point. We then depict the profit function of player $-i$ when it decreases capacity with the other player remaining at x_i^*. We first find an interval where the interior equilibrium is the sole equilibrium in the contract market $(\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i < 0)$. The profit function splits into two components at x_{-i}^f, where $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i$ goes from a negative value (interior equilibrium only) to zero, where there is both an interior and a corner equilibrium in the contract market. We analyze what is taking place at this point in more detail.

Recall that one can always write

$$\begin{aligned} 0 = \left(\alpha - 2x_{-i}^f + \nu_i - 2\nu_{-i}\right) &= \left(\frac{4}{5}\alpha - 2x_{-i}^f - \frac{12}{5}\nu_{-i} + \frac{8}{5}\nu_i\right) \\ &+ \frac{1}{5}\left(\alpha + 2\nu_{-i} - 3\nu_i\right). \end{aligned}$$

Because $\frac{2}{5}(\alpha + 2\nu_{-i} - 3\nu_i)$ is the value of z_i in the spot market of the no contract game, it is positive. This implies that the first term in the right hand side is negative or that the Allaz-Vila solution z_{-i}^{av} is smaller than x_{-i} and hence that there is both a corner and an interior equilibrium in the contract market at (x_{-i}^f, x_i^*). There is thus only an interior equilibrium in the contract market when $x_{-i} > x_{-i}^f$ (because $\alpha - 2x_{-i} + \nu_i - 2\nu_{-i} < 0$), and both interior and corner equilibrium in some interval for $x_{-i}^f > x_{-i}$. The upper function in the left part of the graph ($x_{-i}^f > x_{-i}$) is the profit of player $-i$ when it forecloses the contract market. We include two profit functions for the corner solution to illustrate two possible outcomes. The bottom function in the left part (that extends the function in the right part $x_{-i} > x_{-i}^f$) is the profit of player $-i$ with the interior solution in the off-peak period. The profit of player $-i$ jumps when it forecloses the market because player i has a discrete drop in production at the point where it is foreclosed from the contract market, when $-i$ is at capacity. With the interior-solution profit function, profit is maximized in the figure at x_{-i}^*. As drawn in the example, player $-i$ can achieve a higher profit with capacity x_{-i}^f, when player i remains at x_i^*. But there are two contract-market equilibria at (x_{-i}^f, x_i^*), both a corner and an interior, implying that player $-i$ cannot guarantee a corner solution in the contract market at x_{-i}^f.

Decreasing x_{-i} further, there is a region with only a corner equilibrium in the contract market. Let

x^u_{-i} be this point such that there is a unique corner equilibrium in the off peak segment of the contract game for all x_{-i} such that $x^u_{-i} > x_{-i}$. Player $-i$ can set its capacity at this level and impose the corner equilibrium on player i because it can always marginally decrease its capacity to a region where there is only a corner equilibrium. The top curve in Figure 1 illustrates a case where player $-i$'s profits are higher at x^u_{-i}. However, this may not always be the case as seen in the middle curve. In the examples, we show cases where the highest profits are at the interior solution. Note that profits are always higher at x^f_{-i} than at x^u_{-i} when foreclosure happens. However, any capacity between these two points allows for two equilibria on the contract market, which precludes finding a capacity equilibrium in such a region. For illustrative purposes, in the next section, we present examples where x^f_{-i} increases and decreases capacity. However, we always show a decrease of capacity at x^u_{-i} when the Allaz-Vila solution is interior with the capacities from the game without futures and foreclosure is profitable.

The above reasoning assumes that player i remains at the no-contract equilibrium capacity x^*_i. The same reasoning can be applied for any other capacity adopted by player i, assuming that player $-i$ manages to secure the profit function of the corner equilibrium. When player $-i$ decreases capacity to foreclose the market,player i increases capacity. We present one possibility in Figure 2. Here capacity and profit increase, illustrating that foreclosure by player $-i$ does not necessarily hurt player i. We have the following lemma on total capacity.

Lemma 8 *Assume that there is an equilibrium (x^u_{-i}, x^u_i) where player $-i$ can always enforce the corner equilibrium by marginally decreasing its capacity at x^u_{-i} to foreclose the contract market while increasing its profits above the profits at the interior equilibrium of the contract game for those capacities. Then total capacity decreases compared to the equilibrium of the no contract game*

Proof. Player $-i$ has to decrease capacity to x^u_{-i} in order to guarantee foreclosure. Player i increases capacity in response. Thus, we can use results in Murphy and Smeers (2005) that in the reaction curve of player i, i increases capacity at a lower rate than $-i$ decreases capacity. Thus, total capacity decreases.

6 Finding an equilibrium in the capacity game

Based on the necessary conditions. We can say:

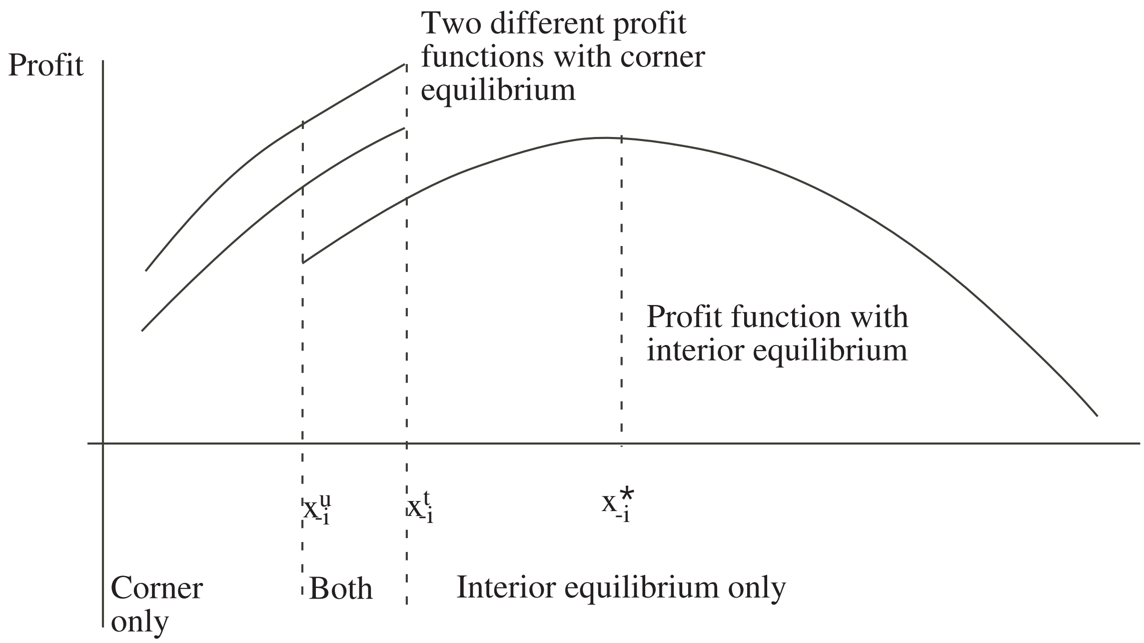

Figure 1: Profit for player $-i$ as a function of capacity given x_i^*

(i) Adding contracts in time segments of type (a) and (c) does not change the incentive to invest

(ii) Adding contracts in time segments of type (b) can produce different outcomes

- unconstrained equilibrium on the contract market, which do not change the incentive to invest
- a single corner equilibrium on the contract market that increases the incentive to invest
- a multiplicity of equilibria on the contract markets, which complicates the analysis of the equilibrium in the capacity game.
- an incentive to decrease capacity in order to make it possible to guarantee a corner equilibrium, which decreases total capacity

These results are obtained by only considering necessary conditions and are thus inconclusive: they do not tell us anything positive about the outcome of the introduction of contract market but are indicative that everything can happen. We confirm this conclusion numerically. We do this on a simple numerical example: consider the situation with two time segments, peak and off peak, where $\alpha^p = 300$ and α^b

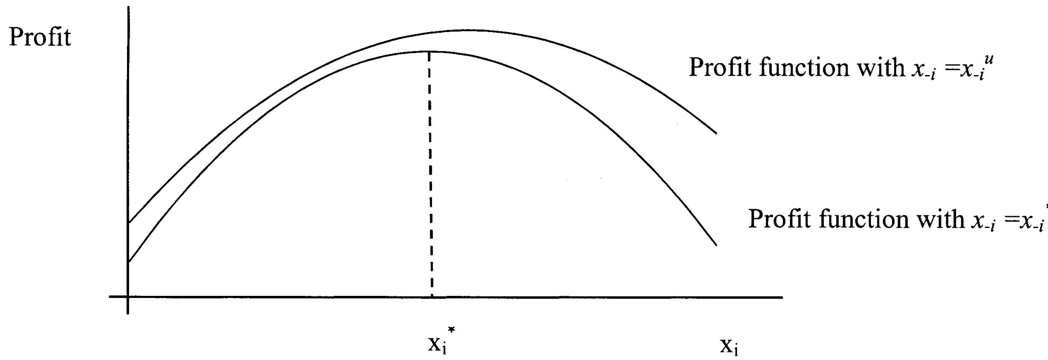

Figure 2: Profit for player i as a function of capacity given x^*_{-i} and x^u_{-i}

is varied. Suppose that the technologies have the costs $k_1 = 25, \nu_1 = 30$ and $k_2 = 15, \nu_2 = 50$. The corresponding generation costs in base load would then be 55 and 65.

6.1 Case 1– $\alpha^b = 100$: low off peak demand: contract markets leave capacity unchanged

With $\alpha^b = 100$ the demand level is low relative to the 300 of time segment 1 and there is a unique equilibrium for all games with the production in the second time segment falling below capacity with and without a contract market. Because the capacity constraint is binding only in the first time segment with a contract market, the capacity level is unchanged with the addition of this market. The production in the off peak segment increases as in Allaz Villa (1993). We know the equilibrium is unique by testing the potential solution with z_1^b equal to capacity through calculating the left partial derivative of the futures profit function with respect to z_1^b at this point. It is negative, given the optimal response, z_2^b, at this point. We conclude in this case that contracts have no effect on investments, they do not foreclose markets; they mitigate market power in off peak but have no effect in peak load.

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s = p$	Price $s = b$
W/O forward	73.3	73.3	146.6	30	10	40	153.3	60
W/ forward	73.3	73.3	146.6	44	4	44	153.3	52

Table 1: *An example with only an interior equilibrium,* $\alpha^b = 100$

6.2 Case 2 – $\alpha^b = 200$: medium base demand: contract markets increase capacity

We now change α^b to 200. This higher intercept increases total production in the second time segment. If there were no capacity limit, player 1 would produce beyond the capacity limit. As proved before, this means there is no interior solution and adding contracts leads to a corner solution where player 1 forecloses the contract market. We tested the alternative equilibrium with player 2 at capacity in the second step. That solution is not an equilibrium. Thus, we have another unique equilibrium. Even though player 2 is excluded from the contract market, total capacity and production increase in both load steps.

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s = p$	Price $s = b$
W/O forward	73.3	73.3	146.6	63.3	43.3	106.6	153.3	93.4
W/ forward	82	69	151	82	34	116	149	84

Table 2: *An example with one corner equilibrium,* $\alpha^b = 200$

6.3 Case 3 – $\alpha^b = 225$: medium base demand: multiple corner solutions

Increasing α^b to 225 leads to two corner equilibria in the contract market where each of the players can move to capacity in the second load step and foreclose the contract market, even though without contracts neither player operates at capacity in the second load step. As in the previous example, production increases with the addition of a contract market no matter which player forecloses the contract market. The results are as follows.

Because of the multiplicity of corner equilibrium, one cannot concluded that (87, 66.5) or (72.5, 75) are capacity equilibria.

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s = p$	Price $s = b$
W/O forward	73.3	73.3	146.6	71.7	51.7	123.4	153.3	101.3
W/ player 1 corner	87	66.5	153.5	87	44	131	146.5	94
W/ player 2 corner	72.5	75	147.5	60	75	135	152.5	90

Table 3: *An example with two corner equilibria,* $\alpha^b = 225$

6.4 Case 4 – $\alpha^b = 155$ and 132: the forward market decreases capacity and the corner equilibrium is guaranteed with $\alpha^b = 155$ but is not guaranteed with $\alpha^b = 132$

So far in the examples we have presented either total capacity does not change or it increases after introducing the contract market. We begin with the case where $\alpha^b = 155$. In Table 4 a case where capacity would decrease after introducing the contract market if player 1 could guarantee the corner equilibrium in the off peak period. We indeed see that there exist regions of capacities for which there exist two equilibria in the contract market. The profit of player 1 is higher at the corner equilibrium, but there is no guarantee that it can secure this profit because of the other interior equilibrium. Starting from capacities in the no contract equilibrium ($x_1 = x_2 = 73.3$) the profit of player 1 first falls when it decreases its capacity as long as the corresponding contract equilibrium remains interior. When player 1 reduces capacity below its Allaz Vila production, there is no interior equilibrium and the corner solution is guaranteed in the game with contracts. Before reaching that capacity level, however, there is a region where there is both a corner and an interior equilibrium in the contract market. The capacity of 72.5 is the point of discontinuity in the profit function of player 1 in Figure 1 (it is the point where $\alpha + \nu_i - 2\nu_{-i} = 2x_i$). At this point the optimal y_b^2 goes from a positive number to 0 if player 1 can achieve the corner solution.

The question is whether this would be profitable, and if so whether player 1 can guarantee that solution. We now solve for the necessary conditions, assuming that player 1 can secure the corner solution in the contract market. First, the equilibrium solution at the corner is $x_1 = 72.5, x_2 = 73.75$. To verify whether this is indeed an equilibrium, we look at the sequence of moves the players would have to take to go from the interior solution to this corner solution. Player 1 moves first. If player 1 sets its capacity to 72.5 with player 2 unchanged, its profit drops from 3778 with the starting interior equilibrium with capacities of (73.333,73.333) to 3777 with capacities of (72.5,73.333). This shows that player 1 would not make the unilateral move to the lower capacity if it retains the interior equilibrium in the contract market. At the corner equilibrium in the contracts market with capacities (72.5,73.333) the profit for player 1 is 4003.

If player 1 can guarantee the corner solution, it is profitable to move from the interior equilibrium to the corner solution in the capacity game. Since player 2 increases capacity in response to player 1's decrease in capacity, the corner equilibrium capacities are (72.5,73.75) and profit of player 1 is 3987.5, an increase over the interior solution. But there are still two contract equilibria (the corner and the interior) at those capacities, implying that player 1 is taking the risk of not insuring the corner equilibrium in the contract game. To guarantee that that the corner equilibrium obtains, player 1 has to drop its capacity to 66 ($\alpha - (3\nu_i - 2\nu_{-i})) = x_i$ where the Allaz Vila solution is at capacity), resulting in a profit of 3965.5 for player 1 with capacities (66,73.333). Thus, it is profitable for player 1 to cut capacity to the point where it guarantees the corner equilibrium when $\alpha^b = 155$. The optimal response by player 2 is to increase capacity, leading to an equilibrium of (66,77) and a profit of 3844.5, which is higher than the profit of 3778 at the interior solution. Table 4 gives the numerical results for the interior solution and the guaranteed corner equilibrium. Results with $\alpha^b = 155$, therefore, show that at this reduced capacity level player 1's profit is higher than at the interior solution, which actually is just a local equilibrium at (73.333,73.333).

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s = p$	Price $s = b$
W/O forward	73.3	73.3	146.6	48.33	28.33	76.67	153.3	78.3
W/ interior	73.33	73.33	146.67	66	26	92	153.3	63
W/ corner	66	77	143	66	19.5	85	157	70

Table 4: *An example where total capacity decreases with the addition of contract markets and the equilibrium is unique,* $\alpha^b = 155$

We repeat the moves of the players in the game with $\alpha^b = 132$ (Table 5). The interior equilibrium again has capacities (73.333,73.333) and the profit of player 1 is 3495.4. Player 1 looks at decreasing its capacity to the point where there are both corner and interior equilibria in the contracts game and finds its profit at the corner solution with capacities of (61,73.333)to be 3543.1, which is a profitable move if the corner equilibrium can be guaranteed. With the player 2 response, we reach the equilibrium of (61,79.5) and the profit of player 1 is 3355. That is, the profit is actually lower than with the interior local equilibrium. Nevertheless, this is an equilibrium if the corner equilibrium can be guaranteed in the contracts game because once can calculate that 61 is the best response of player 1 to 79.5. To guarantee the corner solution, player 1 would have to cut its capacity to 56.8. With capacities (56.8,73.333) the profit drops to 3478.1 and it is not profitable for player 1 to guarantee the corner solution. The following table gives the results for the interior solution and the corner solution that is not necessarily guaranteed.

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s = p$	Price $s = b$
W/O forward	73.3	73.3	146.7	40.7	20.7	61.4	153.3	70.6
W/ interior	73.3	73.3	146.7	56.8	16.8	73.6	153.3	58.4
W/ corner	61	79.5	140.5	61	10.5	71.5	159.5	60.5

Table 5: *An example where total capacity decreases with the addition of contract markets,* $\alpha^b = 132$

6.5 Case 5 – $\alpha^b = 160$: the contract market increases capacity with the corner equilibrium. However, there is also an interior equilibrium

This case rounds out the possible solutions to the game (Table 6). Capacity increases with the corner equilibrium. However, at that capacity player 1 cannot guarantee the corner equilibrium. If it reduces capacity to 68 to guarantee a corner equilibrium, then capacity decreases. At this reduced capacity the profits for player 1 are higher than with the interior equilibrium but lower than they would be if player 1 could enforce the corner equilibrium with the interior capacities.

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s = p$	Price $s = b$
W/O forward	73.3	73.3	146.7	50	30	80	153.3	80
W/ interior	73.3	73.3	146.7	68	28	96	153.3	64
W/ corner	74	73	147	74	18	94	155	62

Table 6: *An example where total capacity decreases with the addition of contract markets,* $\alpha^b = 160$

7 Conclusion

Market power remains a subject of intense interest in the restructured electricity industry. It has been claimed that firms commonly exercise market power during periods of high demand while markets can be quite competitive during low demand. Contracts are commonly advocated by economists to mitigate this exercise of market power. Competition authorities have taken another perspective. They see contracts concluded by firms in dominant positions as an instrument to foreclose the market and hence reinforce their position. We examine these different points through a simple model. There are two agents each operating a different technology. Firms can invest, contract part of their production forward, and sell the rest on the spot market. All of this is driven by the standard Cournot behavior. We assume that their incentive to contract is driven by an Allaz-Vila representation of contract markets that we adapt to a game with capacities. We assess the situation by comparing subgame perfect equilibrium in markets with and without contracts.

Our results are not in line with the common wisdom. We first find that contracts have essentially no effect in periods of high demand. Firms that exercise market power without contracts find no advantage in these contracts. In contrast we find that the role of contracts is mainly felt during periods of low demand, where observation indicates that firms do not exercise substantial market power. But assuming that they still behave as Cournot players in low-demand periods, our results cast some doubt on the validity of the conclusion of both regulatory economists and competition authorities. We indeed find that contracts allow dominant firms to foreclose the market as foreseen by competition authorities. But the effects of that foreclosure are ambiguous. If the capacities inherited from a market without contracts are high compared to demand in off peak periods, the foreclosure effect increases the incentive to invest and hence mitigates market power. In contrast, if capacities inherited from the market without contracts are relatively low compared to demand in these low periods, it can be profitable to reduce investments and increase market power. In short, the impact of contracts is totally unpredictable in this small model. The question becomes how can it be more predictable the complexities of real world electricity markets?

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Appendix 1: Proof of Lemma 1

Lemma A1 *Let* (x_i, x_{-i}) *satisfy*

$$\alpha - 2x_i - x_{-i} - \nu_i > 0 \qquad i = 1, 2$$

then

$$y_i \geq \tilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) < 0, \quad i = 1, 2$$

is an equilibrium of the contract game.

Proof. Take x given and let $\widetilde{y}_i = \widetilde{y}_i(x)$, $i = 1, 2$ for this given x. One has

$$\alpha - 2x_i - x_{-i} - \nu_i + \widetilde{y}_i = 0, \quad i = 1, 2$$

and hence $z_i = x_i$ is an equilibrium on the spot game.

We want to prove that any $y_i \geq \widetilde{y}_i$ is the best response of player i to a contract position $y_{-i} \geq \widetilde{y}_{-i}$ of player $-i$. Suppose $y_i > \widetilde{y}_i$, one has

$$\begin{aligned} \alpha - 2x_i - x_{-i} - \nu_i + y_i &= \lambda_i > 0 \\ \alpha - x_i - 2x_{-i} - \nu_{-i} + y_{-i} &= \lambda_{-i} \geq 0 \end{aligned}$$

and $z_i = x_i$ remains an equilibrium on the spot game. Taking $y_i > \tilde{y}_i$, therefore, maintains the profit of player i, whatever $y_{-i} \geq \widetilde{y}_{-i}$ is selected by player $-i$.

Let $y_i < \widetilde{y}_i$, then $y_{-i} \geq \widetilde{y}_{-i}$. z_i becomes smaller than x_i and one can write the equilibrium conditions of the spot game as

$$\begin{aligned} \alpha - 2z_i - x_{-i} - \nu_i + y_i &= 0 \\ \alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} &= \lambda_{-i} > 0. \end{aligned}$$

This implies

$$z_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i + y_i)$$

and

$$\Pi_i^f(x; y_i, y_{-i}) = \frac{1}{4}\left[(\alpha - x_{-i} - \nu_i)^2 - y_i^2\right].$$

The optimum of the profit of player i is achieved for $y_i = 0$ with a payoff equal to $\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2$. This is the global optimum of player i if and only if

$$0 = y_i < \widetilde{y}_i = -(\alpha - 2x_i - x_{-i} - \nu_i) < 0 \quad ,which is a contradiction.$$

Therefore, $y_i < \widetilde{y}_i$ cannot be the best response of player i to $y_{-i} \geq \tilde{y}_{-i}$. Thus $\tilde{y}_i(x)$, $i = 1, 2$ is a sub-game perfect equilibrium of the contract game and any $y_i \geq \tilde{y}_i(x)$, $i = 1, 2$ is also a sub-game perfect equilibrium of the contract game.

Appendix 2: Proof of Lemma 2

Lemma A2.1 *For a given (x_i, x_{-i}), if $\alpha - 2z_i - x_{-i} - \nu_i = 0$, $\alpha - z_i - 2x_{-i} - \nu_{-i} > 0$, $z_i < x_i$ and $z_{-i} = x_{-i}$, then $y_i = 0$ is the optimal response of player i to any $y_{-i} \geq \widetilde{y}_{-i}(x) = -(\alpha - x_i - 2x_{-i} - \nu_{-i})$.*

Proof. Suppose player $-i$ takes a position $y_{-i} \geq \widetilde{y}_{-i}(x)$ and $y_i = 0$. Consider the relations

$$\begin{aligned} &\alpha - 2z_i - x_{-i} - \nu_i = 0 \\ &\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} = \lambda_{-i} \geq 0. \end{aligned}$$

Because the definition of $\tilde{y}_{-i}(x)$ implies $\alpha - x_i - 2x_{-i} - \nu_{-i} + \widetilde{y}_{-i}(x) = 0$, any $z_i < x_i$ and $y_{-i} > \widetilde{y}_{-i}(x)$ satisfies $\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} = \lambda_{-i} \geq 0$, which shows that $z_i < x_i$ and $z_{-i} = x_{-i}$ is the equilibrium on the spot market.

Consider the reaction of player i to $y_{-i} > \widetilde{y}_{-i}(x)$. Because $y_{-i} \geq \widetilde{y}_{-i}(x)$, $\alpha - x_i - 2x_{-i} - \nu_{-i} + y_{-i} \geq 0$, and $\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} \geq 0$ for all $z_i < x_i$. Therefore, $z_{-i} = x_{-i}$ whenever $y_{-i} \geq \widetilde{y}_{-i}(x)$, whatever the position of player i on the contract market. Consider the following strategies of player i, keeping in mind that $y_{-i} \geq \widetilde{y}_{-i}(x)$ implies $z_{-i} = x_{-i}$, whatever i does on the forward market. Because the shape of the objective function depends on the value of y_i, we treat two cases:

(i) $y_i \geq \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$

(ii) $y_i \leq \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$.

Note first that player i's payoff in case (i) remains constant at $(\alpha - x_i - x_{-i} - \nu_i)x_i$ (which is lower than its payoff at $y_i = 0$) for all $y_i \geq \widetilde{y}_i(x) > 0$. Therefore player i cannot improve its payoff by selecting $y_i \geq \widetilde{y}_i(x)$ and the optimum in case (i) is a global optimum.

Player i's payoff in case (ii) can be computed as follows. Because $y_i \leq \widetilde{y}_i(x)$, $z_i \leq x_i$ and z_i solves

$$\alpha - 2z_i - x_{-i} - \nu_i + y_i = 0$$
$$\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} = \lambda_{-i} > 0.$$

As in Lemma A1, the optimal response of player i is

$$z_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i + y_i) < x_i$$

and

$$\Pi_i^f(x; y_i, y_{-i}) = \frac{1}{4}\left[(\alpha - x_{-i} - \nu_i)^2 - y_i^2\right].$$

The maximum profit is achieved for $y_i = 0$ with the player i payoff equal to $\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2$. This is the global optimum of player i's payoff if one has both

$$0 = y_i < \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$$

and

$$\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2 > (\alpha - x_i - x_{-i} - \nu_i)x_i.$$

The first condition is true by assumption. To verify the second condition, first note that it can be rewritten as

$$(\alpha - x_{-i} - \nu_i)^2 - 4(\alpha - x_{-i} - \nu_i)x_i + 4x_i^2 > 0$$

or

$$(\alpha - 2x_i - x_{-i} - \nu_i)^2 > 0$$

which is always satisfied.

The optimal reaction of player i is thus $y_i = 0$ when player $-i$ selects $y_{-i} \geq \bar{y}_{-i}$ and $\alpha - 2x_i - x_{-i} - \nu_i < 0$. Note that this solution is unique by the strict concavity of the objective function in this range. This proves the lemma.

Lemma A2.2 *Let $\alpha - 2z_i - x_{-i} - \nu_i = 0$, $\alpha - z_i - 2x_{-i} - \nu_{-i} > 0$, $z_i < x_i$ and $z_{-i} = x_{-i}$. Then $y_{-i} \geq \widetilde{y}_{-i}(x)$ is the optimal reaction of player $-i$ to $y_i = 0$.*

Proof. With $y_i = 0$, define $\widetilde{z}_i$ such that $\alpha - 2\widetilde{z}_i - x_{-i} - \nu_i = 0$. By assumption, $\widetilde{z}_i$ is smaller than x_i. We consider three cases that reflect the shape of the objective function of player $-i$, depending on whether the spot decisions of players i and $-i$ are at capacity. We examine the following strategies of player $-i$ on the forward market.

(i) y_{-i} is selected to guarantee $z_{-i} = x_{-i}$.

(ii) y_{-i} is selected to optimize the payoff in the range where $z_i < x_i$, $z_{-i} < x_{-i}$.

(iii) y_{-i} is selected to optimize the payoff in the range where $z_i = x_i$, $z_{-i} < x_{-i}$.

We successively consider these three cases and compute the resulting payoff for player $-i$.

(i) Player $-i$ uses the contract market to guarantee the full utilization of its capacity and it takes $y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x)$ where $\widetilde{\widetilde{y}}_{-i}(x)$ is defined by

$$\alpha - \widetilde{z}_i - 2x_{-i} - \nu_{-i} + \widetilde{\widetilde{y}}_{-i}(x) = 0.$$

Because z_i is equal to

$$\widetilde{z}_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i).$$

One has

$$\widetilde{\widetilde{y}}_{-i} = -(\alpha - 2x_{-i} - \nu_{-i} - \widetilde{z}_i) = \left(\frac{\alpha}{2} - \frac{3}{2}x_{-i} - \nu_{-i} + \frac{1}{2}\nu_i\right)$$

The equilibrium on the spot market associated with $y_i = 0$, $y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x)$ is $z_i = \widetilde{z}_i$ and $z_{-i} = x_{-i}$. The payoff for player $-i$ is

$$(\alpha - \widetilde{z}_i - x_{-i} - \nu_{-i})x_{-i} = \frac{1}{2}(\alpha - x_{-i} + 2\nu_i - \nu_{-i})x_{-i}.$$

(ii) Let $y_{-i} = \widetilde{\widetilde{y}}_{-i}(x) - \varepsilon_{-i}$ where ε_{-i} is small enough to guarantee that z_i does reach x_i and z_{-i} does not hit zero. z_i and z_{-i} then solve the system

$$\alpha - 2z_i - z_{-i} - \nu_i = 0$$
$$\alpha - z_i - 2z_{-i} - \nu_{-i} + y_{-i} = 0$$

We can solve for z_i and z_{-i} as a function of y_{-i} to obtain as in relation (23) after setting $y_i = 0$

$$\begin{aligned} z_i &= \tfrac{1}{3}\big[\alpha - 2\nu_i + (\nu_{-i} - y_{-i})\big] \\ z_{-i} &= \tfrac{1}{3}\big[\alpha - 2(\nu_{-i} - y_{-i}) + \nu_i\big]. \end{aligned}$$

The price of the market is then

$$\alpha - z_i - z_{-i} = \frac{1}{3}(\alpha + \nu_i + \nu_{-i} - y_{-i})$$

and the profit of the player $-i$

$$\begin{aligned} & \tfrac{1}{3}\big(\alpha + \nu_i - 2\nu_{-i} - y_{-i}\big)\tfrac{1}{3}(\alpha - 2\nu_{-i} + \nu_i + 2y_{-i}) \\ \text{or} \quad & \tfrac{1}{9}\Big[\big(\alpha + \nu_i - 2\nu_{-i}\big)^2 + y_{-i}\big(\alpha + \nu_i - 2\nu_{-i}\big) - 2y_{-i}^2\Big]. \end{aligned}$$

The derivative of this profit function with respect to y_{-i} is

$$\alpha + \nu_i - 2\nu_{-i} - 4y_{-i}.$$

Let $\widetilde{\widetilde{y}}_{-i}$ be the value of y_{-i} where z_i is equal to

$$\widetilde{z}_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i).$$

We have

$$\widetilde{\widetilde{y}}_{-i} = -(\alpha - 2x_{-i} - \nu_{-i} - \widetilde{z}_i) = \Big(\frac{\alpha}{2} - \frac{3}{2}x_{-i} - \nu_{-i} + \frac{1}{2}\nu_i)$$

. Computing this derivative of the profit function of player $-i$ at $\widetilde{\widetilde{y}}_{-i}$, we have

$$\begin{aligned} & \alpha + \nu_i - 2\nu_{-i} + 2\alpha - 6x_{-i} + 4\nu_{-i} + 2\nu_i \\ = \quad & 3\alpha - 6x_{-i} + 3\nu_i - 6\nu_{-i} = 3(\alpha - 2x_{-i} + \nu_i - 2\nu_{-i}). \end{aligned}$$

The derivative is positive (and hence $y_{-i} = \widetilde{\widetilde{y}}_{-i}(x) - \varepsilon_{-i}$ is not optimal) if $\alpha + \nu_i - 2\nu_{-i} > 2x_{-i}$. We now verify that this is true under the current assumptions.

We have seen that $\alpha - 2z_i - x_{-i} - \nu_i = 0$, $z_i < x_i$ and $\alpha - z_i - 2x_{-i} - \nu_{-i} > 0$ implies $z_{-i} = x_{-i}$ or that the unconstrained spot equilibrium $z^c_{-i} > x_{-i}$.

By relation (23) we have

$$z^c_{-i} = \frac{1}{3}\big(\alpha - 2\nu_{-i} + \nu_{+i}\big) > x_{-i}$$

and hence

$$\alpha - 2\nu_{-i} + \nu_{-i} > 3x_{-i} > 2x_{-i}$$

which proves the desired inequality.

(iii) The following elaborates on the same concavity argument to prove that decreasing y_{-i} to the level where z_i reaches x_i or z_{-i} reaches 0 cannot maximize $\Pi^f_{-i}[x; 0, y_{-i}]$. There is obviously no gain for player $-i$ to further decrease y_{-i} if z_{-i} hits zero before z_i reaches x_i since its payoff is then exactly zero. Consider the alternative case where z_i hits x_i and z_{-i} is still positive. This occurs for some $\overline{z}_{-i}$ that satisfies

$$\begin{aligned} &\alpha - 2x_i - \overline{z}_{-i} - \nu_i = 0 \\ \text{or} \quad &\overline{z}_{-i} = \alpha - 2x_i - \nu_i. \end{aligned}$$

Consider decreasing y_{-i} further to check the possibility of the resulting price increasing profits. We show that this cannot happen. Let $z_{-i} = \overline{z}_{-i} + \varepsilon$. The corresponding profit of player $-i$ is

$$(\alpha - x_i - \overline{z}_{-i} - \varepsilon - \nu_{-i})(\overline{z}_{-i} + \varepsilon).$$

The derivative of this expression at $\varepsilon = 0$ (for $z_{-i} = \overline{z}_{-i}$) is equal to $3x_i + (2\nu_i - \nu_{-i}) - \alpha$. This expression is positive because it is greater than $3z_i + (2\nu_i - \nu_{-i}) - \alpha$, which is equal to

$$2(-\alpha + 2z_i + x_{-i} + \nu_i) + (\alpha_i - z_i - 2x_{-i} - \nu_{-i})$$

which is positive by assumption.

The conclusion is that it cannot pay to further decrease y_i beyond the point where $z_i = x_i$. $y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x)$ thus guaranteeing the maximum profit of player $-i$ when $y_i = 0$. Because $-(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) > -(\alpha^s - z_i - 2x_{-i} - \nu_{-i}) = \widetilde{\widetilde{y}}_{-i}(x)$, we have proved that any $y^s_{-i} \geq -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) = \widetilde{y}_{-i}(x)$ is an optimal reaction of player $-i$ to $y_i = 0$.

Appendix 3: Proofs of Lemmas 5 and 6

Lemma A3.1 *Assume that there exists no corner equilibrium in time segments of type (b) in the contract game. Then there exists an interior equilibrium (the Allaz-Vila solution remains within capacities).*

Proof. Corner equilibria, if they existed would be given by

$$\begin{aligned} & y_{-i}=0, \quad y_i \geq \widetilde{y}_i(x)=-(\alpha-2x_i-x_{-i}-\nu_i)>0 \\ \textit{and} \quad & y_i=0, \quad\ \ y_{-i} \geq \widetilde{y}_{-i}(x)=-(\alpha-x_i-2x_{-i}-\nu_{-i})>0. \end{aligned}$$

Take $(y_i=0, y_{-i} \geq \widetilde{y}_{-i}(x))$ and assume it is not a corner equilibrium. We know from the proof of part (i) of Lemma A.2.2 that the objective function of player $-i$ does not increase when increasing y_{-i} beyond $\widetilde{y}_{-i}(x) > \widetilde{\widetilde{y}}_{-i}(x)$. Therefore, this point not being an equilibrium implies that player $-i$ has an incentive to decrease y_{-i} below $\widetilde{y}_{-i}(x)$ or more precisely below $\widetilde{\widetilde{y}}_{-i}(x)$ such that

$$\begin{aligned} & \alpha-2\widetilde{z}_i-x_{-i}-\nu_i=0 \\ & \alpha-\widetilde{z}_i-2x_{-i}-\nu_{-i}+\widetilde{\widetilde{y}}_{-i}(x)=0 \end{aligned}$$

In order to explore this incentive, consider values of $y_{-i}=\widetilde{\widetilde{y}}_{-i}-\varepsilon$. Following the same reasoning as in part (ii) of Lemma A.2.2 we can show that the profit function of player $-i$ for $y_{-i}=\widetilde{\widetilde{y}}_{-i}(x)-\varepsilon$, $y_i=0$ is quadratic and its left derivative at $\widetilde{\widetilde{y}}_{-i}(x)$ is

$$\alpha+\nu_i-2\nu_{-i}-4\widetilde{\widetilde{y}}_{-i}=3(\alpha-2x_{-i}+\nu_i-2\nu_{-i}).$$

Departing at this stage from the proof of part (ii) of Lemma A.2.2, we note that player $-i$ has an incentive to decrease y_{-i} below $\widetilde{\widetilde{y}}_{-i}$ if

$$\alpha-2x_{-i}+\nu_i-2\nu_{-i}<0.$$

We want to show that this implies that the solution of the Allaz-Vila problem is bounded away by x_{-i} or

$$\frac{2}{5}\big(\alpha-3\nu_{-i}+2\nu_i\big)<x_{-i}$$

or equivalently

$$\begin{aligned} 0 \quad & \geq \quad \tfrac{4}{5}\alpha-\frac{12\nu_{-i}}{5}+\tfrac{8}{5}\nu_i-2x_{-i} \\ & = \quad \big(\alpha-2x_{-i}-2\nu_{-i}+\nu_i\big)-\big(\tfrac{1}{5}\alpha+\tfrac{2}{5}\nu_{-i}-\tfrac{3}{5}\nu_i\big) \end{aligned}$$

This relation is true because the first term of the right-hand side is negative as assumed and the second term is a multiple of $\alpha+2\nu_{-i}-3\nu_i$ which is the assumed positive value of z_i in the solution of the Allaz Vila problem.

Assuming $\alpha-2x_{-i}+\nu_i-2\nu_{-i}<0$ is a sufficient by not necessary condition to incentivize player $-i$ to decrease y_{-i}. One can indeed have a positive derivative of the profit of player $-i$ at $\widetilde{\widetilde{y}}_{-i}(x)$, but an incentive to decrease $y_{-i}(x)$ to an extent sufficient to push z_i to x_i. Part (iii) of the proof of Lemma A.2.2 shows that his happens for a value of $\overline{z}_{-i}$ that satisfies

$$\alpha-2x_i-\overline{z}_{-i}-\nu_i=0 \text{ or } \overline{z}_{-i}=\alpha-2x_i-\nu_i.$$

We follow again the reasoning of the proof of (iii) of Lemma A.2.2 to show that the derivative of the profit function of player $-i$ for $z_{-i} = \overline{z}_{-i} + \varepsilon$ is

$$(\alpha - x_i - \overline{z}_{-i} - \varepsilon - \nu_{-i})(\overline{z}_{-i} + \varepsilon)$$

with derivative of $\varepsilon = 0$ equal to $3x_i + (2\nu_i - \nu_{-i}) - \alpha$. We now depart from the proof of (iii) of Lemma A.2.2 to note the following. The case discussed here is of type (b), that is the solution of the no-forward-markets case remains below capacity, that is

$$z_i = \frac{1}{3}\big(\alpha - 2\nu_i + \nu_{-i}\big) < x_i$$

or

$$3x_i - \alpha + 2\nu_i - \nu_{-i} > 0.$$

This shows that the incentive of $-i$ to decrease y_{-i} cannot go beyond pushing it below $\overline{z}_{-i}$. Summing up, $y_i = 0$, $y_{-i} \geq \widetilde{y}_{-i}(x)$ not being a corner equilibrium implies that the z_{-i} that is the solution of the Allaz Vila solution remain away from x_{-i}. The same reasoning applied to

$$y_{-i} = 0 \qquad y_i \geq -y_i(x)$$

implies that the solution z_i of the Allaz Vila problem remains away from x_i. The absence of a corner solution therefore implies that the Allaz Vila solution remains away from the bounds and hence is an unconstrained equilibrium.

Lemma A3.2 *Assume that there is no interior equilibrium in time segment s of type (b) in the contract game. Then there exists a corner equilibrium.*

Proof. The non-existence of the interior equilibrium implies that the solution of the Allaz-Vila problem exceeds one of the bounds. Suppose

$$\frac{2}{5}\big(\alpha - 3\nu_{-i} + 2\nu_i\big) > x_{-i}.$$

we want to show that part (ii) of the proof of Lemma A.2.2 holds. That is, we want to show that the derivative of the profit function of player $-i$ at $\widetilde{y}_{-i}$ is positive. Using the argument of the proof of part (ii) of Lemma A.2.2, we know that this derivative is equal to

$$\alpha - 2x_{-i} + \nu_i - 2\nu_{-i}.$$

We then write

$$\begin{aligned}\big(\alpha - 2x_{-i} + \nu_i - 2\nu_{-i}\big) &= \big(\tfrac{4}{5}\alpha - 2x_{-i} - \tfrac{12}{5}\nu_{-i} + \tfrac{8}{5}\nu_i\big) \\ &+ \tfrac{1}{5}\big(\alpha + 2\nu_{-i} - 3\nu_i\big)\end{aligned}$$

The first term of the righthand side is positive by assumption and the second term is positive because it is the value of the i component of the Allaz-Vila solution.

Using the same reasoning as in Lemma 5, we prove that player $-i$ cannot have an incentive to lower y_{-i} to the point where z_{-i} becomes lower than $\overline{z}_{-i}$. Therefore $y_i = 0$, $y_{-i} \geq \widetilde{y}_{-i}(x)$ is a corner equilibrium.