Electricity Production with Intermittent Sources of Energy

Stefan Ambec*
Claude Crampes†

October 2009

Abstract

The paper analyzes the interaction between a reliable source of electricity production and an intermittent source, available only in some states of nature (for example wind power). Depending on the cost parameters, the marginal surplus function and the probability of having intermittent energy we specify the first-best dispatch and investment in the two types of energy. We also show that first best is implementable by market mechanisms under the condition that contingent energy prices are feasible. By contrast, the second-best choices constrained by uniform pricing are not feasible without cross subsidies as they do not allow the producers using fossil fuel plants to balance their budget.

Keywords: windpower, electricity, state-contingent prices, renewable resources

JEL codes: D24, D61, Q27, Q32, Q42

*Toulouse School of Economics (INRA-LERNA)
†Toulouse School of Economics (Gremaq and IDEI)

1 Introduction

The non storability of electricity is an essential feature of the energy industry. However, some of the energy services produced from electricity (e.g. heating) as well as most of the primary fuels (e.g. coal, natural gas) are storable. This obviously facilitates the task of the operators of electricity systems when they have to dispatch production plants in order to balance production and a highly variable demand. Since Directive 2001/77/EC system operators all around the European Union must also take into account a momentum favouring renewable sources of energy as an alternative to fossil fuels. The objective is not only to reduce greenhouse gas emissions but also to reduce the dependence on imports of fossil fuels.[1]

In a Communication of 10 January 2007[2] the European Commission has fixed a minimum target of a 20 % share of energy from renewable sources in the overall energy mix for the year 2020. It also has recalled that if all Member States could achieve their national targets fixed in 2001, 21 % of overall electricity consumption in the EU would be produced from renewable energy sources by 2010. But "The majority of Member States are still significantly lagging behind in their efforts to achieve the agreed targets", the EC says. Therefore, the promotion of wind power, solar power (thermal and photovoltaic), hydro-electric power, tidal power, geothermal energy and biomass in the production of electricity will be accelerated. These energy sources have very heterogeneous characteristics. The paper is devoted to wind power and photovoltaic power that are intermittent sources. Even though meteorological forecasts can provide average data about the availability of wind and solar energies, they remain intrinsically non storable and random. The question we are mainly interested in is the compatibility between these energy sources and market mechanisms. In effect, the EC is permanently pushing the liberalisation process in the energy industry. Does it make sense to substitute intermittent sources for non-intermittent sources whereas *i)* electricity is an essential good for consumers and politicians and *ii)* decentralized decisions to consume and produce are supposed to be based on price signals freely determined on markets? A second important question is the compatibility between these intermittent sources and the transport and distribution grids that have been designed for non random energy sources.

The economics of intermittent sources of electricity production are still in their infancy. Most papers on the subject are empirical and country specific. For example, Neuhoff *et al.* (2006 and 2007) develop a linear programming model to capture the effects of the regional variation of wind output on

[1] Directive of the European Parliament and of the Council of 27 September 2001 on the promotion of electricity from renewable energy sources in the internal electricity market.

[2] "Renewable Energy Road Map. Renewable energies in the 21st century: building a more sustainable future"; available at europa.eu/scadplus/leg/en/lvb/l27065.htm.

investment planning and on dispatching in the UK when transport is constrained. Kennedy (2005) estimates the social benefit of large-scale wind power production (taking account of the avoidance of environnement damages) and applies it to the development of this technology in the South of Long Island. N. Boccard (2008) computes the social cost of wind power as the difference between its actual cost and the cost of replacing the produced energy. He divides the social cost into technological and adequacy components and applies the decomposition to Denmark, France, Germany, Ireland, Portugal and Spain. Müsgens and Neuhoff (2006) build an engineering model representing inter-temporal constraints in electricity generation with uncertain wind output. They provide numerical results for the German power system. Coulomb and Neuhoff (2005) focus on the cost of wind turbines in relation with changes in their size using data on German prices. Papers like Butler and Neuhoff (2004) and Menanteau *et al.* (2003) are closer to ours than the former ones. In effect they consider the variety of tools available for public intervention in the development of renewable energy in general, and intermittent sources in particular.[3] Our analysis is upstream the latter papers as it provides a microeconomic framework for the study of optimal investment and dispatching of wind or solar plants. It also allows to determine by how much market mechanisms depart from the outcome of optimal decisions.

The paper is organised as follows. In section 2, we set up a model with two sources of energy, one fully controlled (for example burning fossil fuel) and the other using an intermittent source. There are two states of nature, one where the intermittent energy is available, the other where it is not. In this framework, we determine the first-best dispatch and the first-best production capacities for the two energy sources as functions of the costs, willingness-to-pay and the availability probability of intermittent sources. In section 3, we show that first best can be decentralized provided that there exist state-dependent markets and we analyse the distortion created by a constraint of non-contingent pricing mainly due to the traditional meters installed at consumption nodes unable to signal varying prices. In section 4, we consider an extension of the model with different intermittent energy sources (either correlated or not). Section 5 proposes several other extensions and concludes.

2 First best production

We consider an industry where consumers derive gross utility $S(q)$ from the consumption of q kWh of electricity. This function is unchanged along the period considered. It is a continuous derivable function with $S' > 0$ and

[3] All these papers are devoted to wind power. Borenstein (2008) proposes a deep economic analysis of solar photovoltaic electricity production with a focus on California.

$S'' < 0$.

Electricity can be produced by means of two technologies. First a fully controlled technology (e.g. coal, oil, gas, nuclear, ...) allows to produce q_f at unit cost c as long as production does not exceed the installed capacity, K_f. The unit cost of capacity is r_f. This source of electricity is referred as the "fossil" source. We assume $S'(0) > c + r_f$: producing electricity from fossil energy is efficient when it is the only source.

The second technology relies on an intermittent source of energy such as solar energy or wind. It allows to produce q_i at 0 cost as long as the energy is available. Also, production from intermittent energy cannot be larger than the installed capacity K_i, whose unit cost is r_i. We assume two states of nature: "with" and "without" intermittent energy (i.e. wind, sun). The state of nature with (respectively without) intermittent energy occurs with probability ν (respectively $1 - \nu$) and is denoted by the superscript w (respectively $\overline{w}$).

The first-best problem to solve is twofold. First, the central planer needs to determine the capacities K_i, K_f to install, which correspond to a long run decision. Second, it chooses how to dispatch the capacities in each state of nature q_i^w, q_f^w and $q_i^{\overline{w}}$, $q_f^{\overline{w}}$, depending on the availability of intermittent energy. It is a short run decision constrained by the size of the installed capacities.

Although the problem a priori counts six decision variables, three of them can be easily determined, leaving us with only three unknowns to be determined. First $q_i^{\overline{w}} \equiv 0$: windmills cannot produce if there is no wind and solar batteries cannot produce absent any sun ray. Second $q_i^w \equiv K_i$: since the installation cost of the capacity for producing with the intermittent source is positive, it would be inefficient to install idle capacity.[4] Third $q_f^{\overline{w}} = K_f$: without intermittent energy, since demand is the same but the available capacity is reduced from $K_f + K_i$ to K_f, it would be inefficient to leave idle some production capacity.[5]

For the three remaining decision variables K_i, K_f and q_f^w, the planner's program can be written as follows:

$$(P1) \qquad \max_{K_i, K_f} \nu \left[\max_{q_f^w} S(K_i + q_f^w) - c q_f^w \right] + (1 - \nu)[S(K_f) - cK_f] - r_f K_f - r_i K_i$$

$$s.t. \qquad q_f^w \geq 0 \quad , \quad q_f^w \leq K_f \quad , \quad K_i \geq 0$$

[4]We discard the necessary maintenance operations, for example assuming that they can be performed during type $\overline{w}$ periods.

[5]Here again, we discard maintenance operations, for example by assuming that capacity is measured in terms of available plants.

Note that it is not necessary to write explicitly the constraint $K_f \geq 0$ because $K_f > 0$ is granted by the assumption $S'(0) > c + r_f$.

As proven in the Appendix, we can establish the following:

Proposition 1 *First best capacities and outputs are such that*

a) *for* $\frac{r_i}{\nu} > c + r_f$

$$q_f^w = q_f^{\overline{w}} = K_f = S'^{-1}(c + r_f), \qquad q_i^w = K_i = 0$$

b) *for* $c > \frac{r_i}{\nu}$

$$q_f^w = 0 < q_f^{\overline{w}} = K_f = S'^{-1}\left(c + \frac{r_f}{1-\nu}\right),\ q_i^w = K_i = S'^{-1}\left(\frac{r_i}{\nu}\right)$$

c) *for* $c + r_f > \frac{r_i}{\nu} > c$

$$q_f^w = q_f^{\overline{w}} = K_f = S'^{-1}\left(\frac{c+r_f-r_i}{1-\nu}\right),\ \ q_i^w = K_i = S'^{-1}\left(\frac{r_i}{\nu}\right) - S'^{-1}\left(\frac{c+r_f-r_i}{1-\nu}\right)$$

In case a) the intermittent energy is so scarce (small ν) and/or the technology using this energy is so costly (high r_i) that no plant using intermittent energy should be installed. In case b) intermittent energy is so cheap that it totally replaces fossil energy in state of nature w. The long run marginal cost of th e intermittent energy is thus the capacity cost r_i discounted by the probability of using it ν. Fossil energy capacity is only used in state of nature $\overline{w}$. Its long run marginal cost is c plus the capacity cost r_f discounted by the probability of using it $1 - \nu$ since it is dispatched only when the intermittent source is not available. In the intermediary case c), fossil energy is used at full capacity jointly with intermittent energy. This case is illustrated in Figure 1. The merit order in state of nature $\overline{w}$ just consists in dispatching fossil energy up to K_f determined by the equality between marginal utility and long run marginal cost. The latter is equal to the cost of the marginal technology in state $\overline{w}$ that is $\frac{c+r_f}{1-\nu}$ reduced by the saving on the cost of developing the other technology. This is because, at periods w, f is the marginal technology to dispatch (since $c > 0$) but i is the one to develop (since $\frac{r_i}{\nu} < c + r_f$). Then $\frac{r_i}{\nu}$ is the long run marginal cost of the whole system and it determines $K_i + K_f$. An increase in K_f is compensated by a decrease in K_i.

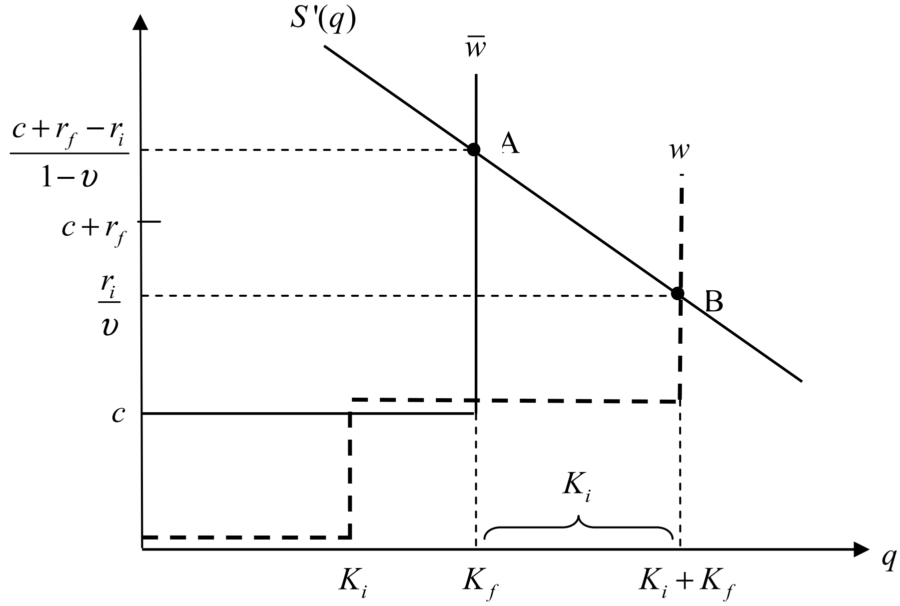

Figure 1: First best when the two technologies are used at type-w hours

In Figure 2, we have depicted the capacities K_i , K_f and the sum K_i+K_f as functions of $\frac{r_i}{\nu}$. The graph clearly shows that when the intermittent technology i becomes profitable (that is when $\frac{r_i}{\nu} \leq c + r_f$) it is not simply substituted for fossil energy f. As $\frac{r_i}{\nu}$ decreases, it is true that there occurs some substitution since K_f decreases but the total capacity $K_f + K_i$ increases. Substitution cannot be done on a one-to-one basis since nothing can be produced with technology i in state of nature $\overline{w}$. Nevertheless there is some substitution with the consequence that, as compared with a world without technology i, there is less energy available in state $\overline{w}$ than in state w hours.

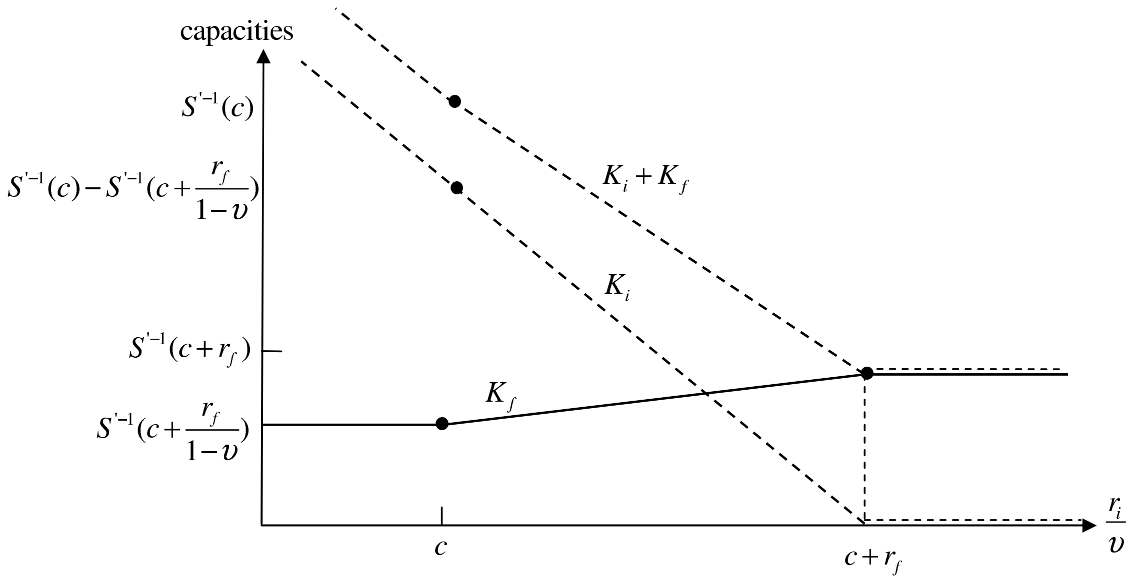

Figure 2: Capacities as a function of the development cost of type-i

3 Decentralization

The European authorities simultaneously want to promote renewable sources of electricity production and the liberalization of the industry. Therefore, we now consider the implementation of first best by market mechanisms taking account of the reactivity of consumers to price variations (3.1), then their lack of reactivity (3.2).

3.1 Market implementation with reactive consumers

Assume that consumers and firms are price-takers. Suppose also that they are equipped to be price sensitive. It is easy to show that the optimal outcome can be decentralized with prices contingent on states of nature p^w and $p^{\overline{w}}$. In practice, it means that electricity prices should depend on the presence or the absence of the intermittent source of energy.

In each state of nature $s \in \{w, \overline{w}\}$, consumers facing price p^s solve $\max_q S(q^s) - p^s q$. They demand q^s kWh in state s where $S'(q^s) = p^s$ (marginal utility equals price) for $s = w, \overline{w}$.

First consider case a) whereby $\frac{r_i}{\nu} > c + r_f$. The prices that decentralize the optimal outcome are $p^w = p^{\overline{w}} = c + r_f$. Consumers react to that prices by consuming the efficient productions $q^w = q^{\overline{w}} = S'^{-1}(c + r_f)$. Producers owning the intermittent technology i invest nothing since the long term marginal cost r_i of each kWh exceeds the expected unit benefit $p^w\nu$. Producers endowed with the fossil technology f invest up to supply all consumers $K_f = q^w = q^{\overline{w}}$. Since the long run marginal cost of each kWh $c + r_f$ equals the market price in both states of nature $p^w = p^{\overline{w}}$, they make zero profit. Clearly, the prices that decentralize first-best are unique. With lower prices, fossil electricity producers would not recoup their investment and thus would invest nothing. Symmetrically, with higher prices, more fossil fuel capacities would be installed and perfect competition would reduce prices to the long term marginal cost.

Second, in case b) where $c > \frac{r_i}{\nu}$, the prices that decentralize the optimum are $p^w = \frac{r_i}{\nu}$ and $p^{\overline{w}} = c + \frac{r_f}{1-\nu}$ per kWh. As before, consumers react to those prices by consuming $q^w = S'^{-1}(\frac{r_i}{\nu})$ in the state of nature with intermittent source of energy w and $q^{\overline{w}} = S'^{-1}(c + \frac{r_f}{1-\nu})$ in state of nature $\overline{w}$. In state of nature w, firms producing energy from fossil sources cannot compete with those producing from intermittent sources. They therefore specialize in producing only during state of nature $\overline{w}$. Their expected return on each unit of capacity is thus $(1-\nu)(p^{\overline{w}} - c) = r_f$. Since it exactly balances the marginal cost of capacities, the plants using fossil source have a zero expected profit. Similarly, firms with intermittent technology obtain an expected return $\nu p^w = r_i$ per unit of investment and thus zero profit on average. In other words, under those prices, each type of producer recoups exactly its long term marginal cost, taking into account the probability of using capacities.

Third, in case c) whereby $c + r_f > \frac{r_i}{\nu} > c$, with prices $p^w = \frac{r_i}{\nu}$ and $p^{\overline{w}} = \frac{c+r_f-r_i}{1-\nu}$ the market quantities also are at first-best levels. Again, consumers' demand is $q_f^{\overline{w}} = S'^{-1}(\frac{c+r_f-r_i}{1-\nu})$ when the wind is not blowing and $q_f^w + q_i^w = S'^{-1}(\frac{r_i}{\nu})$ when it is. In state w, competing producers are ordered on the basis of their short run marginal cost, that is 0 for i-producers and c for f-producers. The investment in capacity depends on expected returns and long run marginal costs. Fossil electricity firms produce in both states of nature. The return per unit of capacity is thus $\nu p^w + (1-\nu)p^{\overline{w}} - c$ which matches exactly the capacity marginal cost r_f. Thus f-producers make zero profit. On the other hand, i producers get in expectation νp^w per unit of investment which also matches exactly their costs r_i. They therefore make zero profit as well which is the equilibrium under free entry condition. Therefore, we have established the following:

Proposition 2 *State contingent prices p^w and $p^{\bar{w}}$ with $p^{\bar{w}} \geq p^w$ and free entry allow market mechanisms to reach first best. When it is efficient to install windmills and/or solar batteries, $p^{\bar{w}} > p^w$.*

3.2 Uniform pricing constraint

The implementation of this perfect competition mechanism faces a serious hurdle. The first best dispatch and investment can be driven with state contingent prices only if consumers have smart meters signaling scarcity values and if they are able to adapt to price signals. Most consumers, particularly among households, are equipped with traditional meters. Consequently they will be billed at uniform price, whatever the state of nature.

To assess the consequences of a uniform pricing constraint, we determine the efficient production and investment levels constrained by uniform delivery. Indeed, with a stationary surplus function as assumed here, consumers reacts to uniform prices by consuming the same amount of electricity in both states of nature. Formally, it translates into the constraint $q_i^w + q_f^w = q_f^{\overline{w}}$. Yet, since the intermittent (resp. fossil) technology is used under full capacity in state w (resp. $\overline{w}$) the later constraint lead to $K_i + q_f^w = K_f$. To distinguish with the first-best solution, we denote the solution of the (second-best) uniform pricing constrained program by $(\tilde{q}_i^w, \tilde{q}_f^w, \tilde{K}_i\ \tilde{K}_f$.

As shown in the Appendix, the main consequence of this restriction is that the intermittent source of energy will never be used in complement to fossil energy in state w. More precisely, case c) of Proposition 1 where both technologies are operated in state w (namely for $c < \frac{r_i}{\nu} < c+r_f$) disappears. This is because the constraint of uniform provision $K_i + q_f^w = K_f$ makes the two technologies perfect substitutes in state w. It results in a bang-bang solution. If $c < \frac{r_i}{\nu}$ only technology f is installed and $S'(K_f) = c + r_f = \tilde{p}^w = \tilde{p}^{\overline{w}}$. The uniform price just matches the long term marginal cost of the f technology. On the other hand, if $c > \frac{r_i}{\nu}$, both technologies are installed but only technology i is used when possible, i.e. in state w with $S'(K_f) = S'(K_i) = (1-\nu)c + r_f + r_i = \tilde{p}^w = \tilde{p}^{\overline{w}}$. The uniform price equals the long term marginal cost of each kWh namely $(1-\nu)c+r_f+r_i$, taking into account that the two technology are developed to insure uniform delivery and c is incurred only in state $\overline{w}$ which arises with probability $1-\nu$.

We therefore can assert the following:

Proposition 3 *When prices cannot be state contingent, second best capacities and outputs are such that:*

a) for $\frac{r_i}{\nu} > c$

$$\tilde{q}_f^w = \tilde{q}_f^{\overline{w}} = \tilde{K}_f = S'^{-1}(c + r_f), \quad \tilde{q}_i^w = \tilde{K}_i = 0$$

b) for $\frac{r_i}{\nu} < c$*,*

$$\tilde{q}_f^w = 0 < \tilde{q}_f^{\overline{w}} = \tilde{K}_f = \tilde{q}_i^w = \tilde{K}_i = S'^{-1}((1-\nu)c + r_f + r_i)$$

The disappearance of the joint operation of the two technologies in state w can be illustrated using Figure 1. If consumers are weakly price-sensitive, the marginal surplus curve $S'(q)$ is more vertical. Consequently the horizontal difference between A and B is smaller and smaller, which means that K_i converges to zero.

An important drawback of the second-best solution with a mix of the two technologies (that is when $c > \frac{r_i}{\nu}$) is that it requires a subsidy from technology i to technology f to secure non-negative profits. It thus can be decentralized only under certain conditions, for example a regulated electricity monopoly or competitive firms owning the two technologies. Indeed the expected unit profit of a firm using the two technologies is:

$$\nu\tilde{p}^{w} - r_i + (1-\nu)(\tilde{p}^{\overline{w}} - c) - r_f = 0$$

Thus the division operating technology i obtains positive cash flows

$$\nu\tilde{p}^{w} - r_i = \nu\left[(1-\nu)(c - \frac{r_i}{\nu}) + r_f\right] > 0$$

whereas the fossil energy f division incurs financial losses $(1-\nu)(\tilde{p}^{\overline{w}} - c) - r_f < 0$. Transfers from division i towards division f are therefore necessary to sustain this second-best.

What occurs when the two technologies are owned by separate operators and transfers are not allowed? In a competitive industry with free-entry, the fossil energy-based electricity producers would exit the market under the second-best electricity price. This would reduce the supply of energy in state $\overline{w}$ and, therefore, increase the price of electricity in both states of nature above the second best level. The free entry equilibrium price in a competitive industry with a unique price in the two states of nature is such that firms with fossil technology make zero profit. It thus matches the fossil energy producer's long term marginal cost $c + r_f$. The firms with intermittent energy technology i enjoy strictly positive profits. They free-ride on the uniform price constraint.

Finally, note that since $p^{\overline{w}*} = c + \frac{r_f}{1-\nu} > \tilde{p}^{w} = \tilde{p}^{\overline{w}} > p^{w*} = \frac{r_i}{\nu}$, and prices signal investment opportunities, the capacity of intermittent energy installed under uniform price is smaller than at first-best whereas the opposite stands for fossil energy, i.e. $\tilde{K}_i < K_i^*$ and $\tilde{K}_f > K_f^*$. This is true when $\tilde{K}_i = \tilde{K}_f$, that is when $\frac{r_i}{\nu} < c$, but it is also obviously true when $c < \frac{r_i}{\nu} < c + r_f$ since $\tilde{K}_i = 0 < K_i^*$ and $\tilde{K}_f < K_f^* + K_i^*$.

We therefore can assert the following:

Proposition 4 *When the price cannot be state contingent, second best is implementable only if the two technologies are owned by the same financial entity or the government transfers revenues from intermittent sources to reliable sources. Otherwise, market mechanisms with free entry result in over-investment and zero profit in the f-technology and under-investment and positive profit in the i-technology.*

To summarize, this section the decentralization of first-best production levels of electricity calls for a lower price when intermittent sources of energy are available. But consumers do not react to price variability so that at second best uniform pricing distort first best prices by increasing the price of intermittent energy p^w and reducing the price of fossil energy (when intermittent energy in not available) $p^{\overline{w}}$. A uniform price leads to under-investment into intermittent sources of energy and over-investment into fossil power-plants. This is because prices do not reflect state-of-nature marginal costs and, therefore, consumers tend to over-consume electricity when it is costly to produce (in state $\overline{w}$) and under-consume it when it is cheap (in state w). Compared to first-best, this increases demand for fossil energy and reduces it for intermittent energy. Long run supply through investment in capacities is adapted accordingly.

4 Two sources of intermittent energy

We now examine the investment decisions when two sources of intermittent energy are available. Let us label them 1 and 2. The two sources can be of different kind, e.g. wind and solar. They also can be of the same kind but at different locations e.g. turbines facing different wind conditions. As a consequence, the two sources differ potentially both on their occurrence and on the energy produced when available. For instance, they might face different dominant winds (north versus south), one being stronger on average than the other.

The results of the former sections can be extended to the multiplicity of sources by increasing the number of states of nature. For example, with two turbines located at different places, we have four states of nature: in state 1 only the intermittent source of energy 1 is available, in state 2 only the intermittent source of energy 2 is available, in state 12 both are available and, as before, in state $\overline{w}$ none of them are available (and therefore electricity can only be produced with fossil energy). These states of nature occur with probabilities ν_1, ν_2, ν_{12} and $1-\nu$ respectively where $\nu = \nu_1+\nu_2+\nu_{12}$. Let us denote by K_i the investment into intermittent source of energy i for $i = 1, 2$. The long term marginal cost of source i is denoted $r_i > 0$ for $i = 1, 2$ where $r_2 > r_1$. For instance, if wind turbines are at different locations and the mean wind is stronger[6] at location 1 than at 2 when it is windy, then with a smaller number of wind turbines one can to produce the same amount of electricity at location 1 and at location 2. Yet, the occurrence of the two sources of intermittent energy might make location 2 attractive.

The planner must determine the capacity of the two intermittent sources of energy K_1 and K_2 in addition to the fossil source K_f. It also has to dispatch the capacities in each state of nature among the different sources

[6] Recall that the wind should not be "too strong" because windmills could not resist.

of energy f, 1 and 2 given their availability. More precisely the planner chooses the production levels q_f^i, q_1^i and q_2^i in states $i = \overline{w}, 1, 2$ and 12 given that no intermittent source of energy is available in state $\overline{w}$, only source i in state i for $i = 1, 2$ and both sources in state 12. Using notations similar to the former section's, we can easily determine the following decision variables: $q_i^{\overline{w}} \equiv 0$ for $i = 1, 2$, $q_i^i = q_i^{12} = K_i$ for $i = 1, 2$ and $q_f^{\overline{w}} = K_f$. For the remaining decision variables K_1, K_2, K_f, q_f^1, q_f^2 and q_f^{12} the planner's program can be written as follows:

$$
\begin{aligned}
(P2) \max_{K_1, K_2, K_f} \quad & \nu_1 \max_{q_f^1} \left[S(K_1 + q_f^1) - c q_f^1 \right] + \nu_2 \max_{q_f^2} \left[S(K_2 + q_f^2) - c q_f^2 \right] \\
& + \nu_{12} \max_{q_f^{12}} \left[S(K_1 + K_2 + q_f^{12}) - c q_f^{12} \right] + (1 - \nu)[S(K_f) - cK_f] \\
& - r_f K_f - r_1 K_1 - r_2 K_2
\end{aligned}
$$

subject to

$$0 \leq q_f^i \leq K_f \text{ for } i = 1, 2, 12; \ K_i \geq 0 \text{ for } i = 1, 2.$$

In the appendix, we establish the following proposition.

Proposition 5 *First best capacities in the intermittent sources of energy* 1 *and* 2 *are such that*

a) *For* $\nu_1 > 0$ *and* $\nu_2 > 0$, $K_1 = K_2 = 0$ *if and only if* $c + r_f < \dfrac{r_i}{\nu_i + \nu_{12}}$ *for* $i = 1, 2$.

b) *For* $c + r_f > \dfrac{r_i}{\nu_i + \nu_{12}}$ *for* $i = 1, 2$, *we have:*

   1) $K_1 > 0$ *and* $K_2 = 0$ *if* $\nu_1 = \nu_2 = 0$ *and* $\nu_{12} > 0$,

   2) $K_1 > 0$ *and* $K_2 > 0$ *if* $\nu_1 > 0$, $\nu_2 > 0$ *and* $\nu_{12} = 0$.

As for investment portfolios, the decision to invest in various intermittent technologies does not only depend on the return on investment but also on the risk associated to each return. According to *a)* in Proposition 5, a necessary and sufficient condition for investing in an intermittent source of energy is $\dfrac{r_i}{\nu_i + \nu_{12}} < c + r_f$ for one $i \in \{1, 2\}$ at least: the long run marginal cost of electricity produced from source i discounted by the probability of its availability $\nu_i + \nu_{12}$ must be lower than the long run marginal cost of electricity produced from fossil energy. Depending on the value of the parameters, in some cases, the two sources of intermittent energy are installed and in other cases only one is installed. For instance, consider the extreme cases *b.1)* and *b.2)* of perfectly positive and negative correlations respectively. If sources 1 and 2 are always available at the same time only (perfect positive correlation), we have $\nu_1 = \nu_2 = 0$. Then only the more efficient source of intermittent energy is installed. Formally, even if $\frac{r_i}{\nu_{12}} < c + r_f$ for $i = 1, 2$ so that the two sources of intermittent energy have lower discounted marginal cost than fossil energy, only source 1 is installed since we have assumed $r_1 < r_2$. On the contrary, if sources 1 and 2 are never available at the same time (perfect negative correlation), which translates formally into $\nu_{12} = 0$, then as long as $\frac{r_i}{\nu_i} < c + r_f$ for $i = 1, 2$ both sources of intermittent energy are installed. In particular, source 2 is installed even if it is more costly ($r_2 > r_1$) and/or less frequent ($\nu_2 < \nu_1$). Concretely, if wind turbines can be developed at two different locations, one being superior in terms of wind speed and frequencies, it is efficient to install turbines at both locations to exploit the complementarity of the two sources of energy provided that the discounted long run marginal cost are lower than with fossil energy.

In any case, as $\nu_1 + \nu_2 + \nu_{12} < 1$, it is necessary to install reliable capacity to replace the intermittent technologies in "bad" states of nature. This cost should be internalized by the builders and operators of the plants using intermittent sources.

5 Concluding remarks

The development of intermittent sources of energy to produce electricity creates a series of difficulties to accommodate the installed markets and networks to the characteristics of these sources. Hereafter are some of these questions.

i) day ahead commitment:

Most wholesale markets are organized on a day-ahead basis where producers commit to inject a given quantity provided that the price is above a given value. Every day, there is a separate auction for each of the 24 hours of the day ahead. One of the defaults of intermittent sources is that they are hardly reliable. Except for the theoretical case of perfect negative correlations between sources, no operator of electricity plant using intermittent energy can commit to inject a given quantity at a given hour. The consequence is that intermittent sources should not be included into the merit order used by the dispatcher... except if the operator of the intermittent plan manages his own spinning reserves. Intermittent energy should rather be viewed as negative demand since demand forecast are made by the system operator and transmitted to the market operator. In terms of our model, this means that only type f operators can commit to deliver energy and the demand they have to supply can be either $S'(q - K_i)$ with probability ν or $S'(q)$ with probability $(1 - \nu)$. First best optimization would give the same results but the implementation would transfer the load of "demand" forecast to the operator with a meteorological expertise.

ii) public aids

By lack of internalization, for most intermittent technologies it is true that $\frac{r_i}{\nu} > c + r_f$ where r_i, r_f and c are market values. Therefore under pure market mechanisms no intermittent sources would be installed. Actually taking into account the damages to the environment and the depletion of fossil resources, the type-f technology actually costs $c + \Delta c + r_f > \frac{r_i}{\nu}$ and maybe even $c + \Delta c > \frac{r_i}{\nu}$. Under this ranking of costs, there is a gap between market outcomes and socially efficient outcomes. Public intervention is necessary. It can take the form of taxes, feed-in tariffs, green certificates or investment planning to approximate first best. The relative mix and the absolute values to fix for the different types of renewable energies and particularly for intermittent ones have been mainly determined on political grounds up to now. The normative economic analysis of the tools used to correct inefficient market outcomes as regards intermittent sources of energy remains to be done.

iii) market power

Big operators can exert market power and distort the use of the different types of technologies. This is true on the energy markets but also upstream and downstream in the equipment markets. One way to distort prices consists in withdrawing capacities in order to push prices up. This anticompetitive behavior is closely monitored by competition authorities but it is obviously easier to play strategically when the energy source is intermittent. Turbines availability is then a private information and this facilitates strategic withdrawal.

iv) network

Unlike traditional type-f energy sources, intermittent energy is scattered on a given territory. This has two consequences. First, connection requires large investment in small scale lines, transformers and meters. This obviously makes coordination necessary between producers, transmitters and system and market operators. Second, random local injections radically modify the business model of distributors since they now have to equilibrate the flows on the grid under their responsibility and maybe to install new lines to guarantee the reliability of the local system under the constraint to accept all injections by authorized plants. The adaptation of networks to the development of intermittent sources has been underestimated. In many countries it is now evident that the grid must be made "smart", which means embedding information technology allowing and measuring (before billing) bi-directional flows of energy.[7]

[7] See for example the website www.smartgrids.eu.

References

Boccard, Nicolas (2008), "Economic Properties of Wind Power: A European Assessment", October, available at ssrn.com/abstract=1285462

Borenstein S. (2008) "The market value and cost of solar photovoltaic electricity production", Technical report, University of California Energy Institute (UCEI).

Butler L. and K. Neuhoff (2004) "Comparison of Feed in Tariff, Quota and Auction Mechanisms to Support Wind Power Development", MIT Center for Energy and Environmental Policy Research, CMI Working Paper 70, CWPE 0503

Coulomb L. and K. Neuhoff (2006), "Learning Curves and Changing Product Attributes: the Case of Wind Turbines", February CWPE 0618 and EPRG 0601, University of Cambridge.

Kennedy S. (2005), "Wind power planning: assessing long-term costs and benefits", *Energy Policy*, 33, 16611675

Menanteau P., D. Finon and M.L. Lamy (2003) "Prices versus quantities: choosing policies for promoting the development of renewable energy", *Energy Policy*, 31 799-812;

Müsgens F. and K. Neuhoff (2006) "Modelling Dynamic Constraints in Electricity Markets and the Costs of Uncertain Wind Output", February, CWPE 06010 and EPRG 0514, University of Cambridge.

Neuhoff K., A. Ehrenmann, L. Butler, J. Cust, H. Hoexter, K. Keats, A. Kreczko and G. Sinden (2006), "Space and Time: Wind in an Investment Planning Model", February, CWPE 0620 and EPRG 0603, University of Cambridge.

Neuhoff K., J. Cust and K. Keats (2007), "Implications of intermittency and transmission constraints for renewables deployment", February, CWPE 0711 and EPRG 0702, University of Cambridge.

A First best production and capacity

The Lagrange function corresponding to problem (P1) in the text is[8]

$$\begin{aligned}\mathcal{L} &= \nu \left[S(K_i + q_f^w) - cq_f^w + \xi_f^w q_f^w + \eta_f^w (K_f - q_f^w) + \xi_i K_i\right] \\ &\quad +(1-\nu)\left[S(K_f) - cK_f\right] - r_f K_f - r_i K_i\end{aligned}$$

Given the linearity of technologies and the concavity of the surplus function, the following first order conditions are sufficient to determine the first best allocation:

$$\frac{\partial \mathcal{L}}{\partial q_f^w} = \nu \left[S'(K_i + q_f^w) - c + \xi_f^w - \eta_f^w\right] = 0 \tag{A1}$$

$$\frac{\partial \mathcal{L}}{\partial K_f} = \nu \eta_f^w + (1-\nu)\left[S'(K_f) - c\right] - r_f = 0 \tag{A2}$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \nu \left[S'(K_i + q_f^w) + \xi_i\right] - r_i = 0 \tag{A3}$$

plus the complementary slackness conditions derived from the three constraint of (P1).

We first identify conditions for $K_i > 0$.

¿From (A3), if $K_i > 0$, $S'(K_i + q_f^w) = \frac{r_i}{\nu}$ and we can write from (A1) $\frac{r_i}{\nu} - c = \eta_f^w - \xi_f^w$. Then, we face two possibilities:

- if $\frac{r_i}{\nu} > c$, $\eta_f^w > 0$ so that $q_f^w = K_f > 0$ and $\xi_f^w = 0$.
  Plugging $\eta_f^w = S'(K_i + K_f) - c$ into (A2) we obtain

$$\nu S'(K_i + K_f) + (1-\nu)S'(K_f) = c + r_f$$

¿From $K_i > 0$ and $S'' < 0$, we have $S'(K_i + K_f) < S'(K_f)$ so that

$$S'(K_i + K_f) = \frac{r_i}{\nu} < c + r_f$$

[8] $\xi_f^w \geq 0, \eta_f^w \geq 0$ and $\xi_i \geq 0$ are the multipliers respectively associated to $q_f^w \geq 0, q_f^w \leq K_f$ and $K_i \geq 0$.

- in the second possibility, $\frac{r_i}{\nu} < c$, the condition $\frac{r_i}{\nu} < c + r_f$ is obviously satisfied.

  We conclude that $\frac{r_i}{\nu} > c + r_f$ is sufficient for $K_i = 0$.

  We can therefore partition the set of parameters as follows:

a) for $\frac{r_i}{\nu} > c+r_f$, $K_i = 0$. As regards the output of the reliable technology in the state of nature w, we have $q_f^w = K_f$. Indeed, assume $q_f^w < K_f$. Then $\eta_f^w = 0$ and from (A1) $S'(q_f^w) - c = -\xi_f^w \leq 0$. Similarly from (A2) $S'(K_f) - c = \frac{r_f}{1-\nu} > 0$. But since $S'' < 0$ these two inequalities are not compatible. It results that $q_f^w = q_f^{\overline{w}} = K_f$ and combining (A1) and (A2) , $S'(K_f) = c + r_f$.

b) for $c > \frac{r_i}{\nu}$, since from (A1) $\frac{r_i}{\nu} - c = \eta_f^w - \xi_f^w$, we have $\xi_f^w > 0$ so that $q_f^w = 0$. Knowing that $K_f > 0$, this implies $\eta_f^w = 0$. Then, from (A2) we have

$$S'(K_f) = c + \frac{r_f}{1-\nu}$$

and from (A3) and $K_i > 0$

$$S'(K_i) = \frac{r_i}{\nu}.$$

c) for the intermediary case $c + r_f > \frac{r_i}{\nu} > c$, we saw formerly that $K_i > 0$ and $q_f^w = K_f$. From equation (A3).

$$S'(K_i + K_f) = \frac{r_i}{\nu}$$

and combining (A1) and (A2)

$$\nu S'(K_i + K_f) + (1-\nu)S'(K_f) = c + r_f$$

Plugging the first equation into the second,

$$S'(K_f) = \frac{c + r_f - r_i}{1-\nu}.$$

B Second best problem (uniform provision)

Adding the constraint $K_i + q_f^w = K_f$ and the multiplier γ to the Lagrange function of first best, the first order conditions become

$$\frac{\partial \mathcal{L}}{\partial q_f^w} = \nu[S'(K_i + q_f^w) - c + \xi_f^w - \eta_f^w + \gamma] = 0 \tag{B1}$$

$$\frac{\partial \mathcal{L}}{\partial K_f} = \nu(\eta_f^w - \gamma) + (1-\nu)\left[S'(K_f) - c\right] - r_f = 0 \tag{B2}$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \nu\left[S'(K_i + q_f^w) + \xi_i + \gamma\right] - r_i = 0 \tag{B3}$$

We already know that $K_i = 0$ when $\frac{r_i}{\nu} > c + r_f$. We then focus on $\frac{r_i}{\nu} < c + r_f$. Combining (B1) and (B3) we have that

$$\frac{r_i}{\nu} - c = \eta_f^w + \xi_i - \xi_f^w$$

- If $\frac{r_i}{\nu} < c$, $\xi_f^w > 0$ so that $q_f^w = 0$ and $K_i = K_f$. Because $K_f > q_f^w = 0$, $\eta_f^w = 0$ and $\xi_i = 0$. Consequently we can combine (B2) and (B3) to get

$$S'(K_i = K_f) = (1-\nu)c + r_f + r_i.$$

- If $\frac{r_i}{\nu} > c$, $\eta_f^w > 0$ so that $q_f^w = K_f$ and $\xi_i > 0$ so that $K_i = 0$. In effect, we cannot have $K_i > 0$ because, if so, $\xi_i = 0$ and $\eta_f^w > 0$ so that $q_f^w = K_f$. The uniform delivery constraint becomes $K_i + K_f = K_f$ which cannot be true for $K_i > 0$. Then, second best commands $S'(K_f) = c + r_f$ like for $\frac{r_i}{\nu} > c + r_f$.

C Two sources of intermittent energy

Using the same notation as before for the multipliers, the first-order conditions yield:

$$\frac{\partial \mathcal{L}}{\partial q_f^i} = \nu_i\left[S'(K_i + q_f^i) - c + \xi_f^i - \eta_f^i\right] = 0 \text{ for } i = 1, 2 \tag{C1-2}$$

$$\frac{\partial \mathcal{L}}{\partial q_f^{12}} = \nu_{12}\left[S'(K_1 + K_2 + q_f^{12}) - c + \xi_f^{12} - \eta_f^{12}\right] = 0 \tag{C3}$$

$$\frac{\partial \mathcal{L}}{\partial K_f} = (1-\nu)\left[S'(K_f) - c\right] + \nu_1\eta_f^1 + \nu_2\eta_f^2 + \nu_{12}\eta_f^{12} - r_f = 0 \tag{C4}$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \nu_i \left[S'(K_i + q_f^i) + \xi_i\right] + \nu_{12}\left[S'(K_1 + K_2 + q_f^{12}) + \xi_i\right] - r_i = 0 \text{ for } i = 1,2 \tag{C5-6}$$

plus the complementary slackness conditions derived from constraint of (P2).

Since in states of nature with intermittent energy, one can always use the fossil fuel equipment, production cannot be lower: $K_1 + K_2 + q_{12}^f \geq K_i + q_i^f \geq K_f$ for $i = 1,2$. Since S' is decreasing, these inequalities imply $S'(K_1 + K_2 + q_{12}^f) \leq S'(K_i + q_i^f) \leq S'(K_f)$ for $i = 1,2$.

Proof of *a)*. We show that $K_i = 0$ if and only if $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1,2$. Suppose that $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1,2$. By $(C5-6)$,

$$\frac{r_i}{\nu_i + \nu_{12}} = \frac{\nu_i S'(K_i + q_i^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12}}{\nu_i + \nu_{12}} + \xi_i \tag{1}$$

Moreover, by $(C4)$,

$$c + r_f = (1-\nu)S'(K_f) + \nu c + \nu_1 \eta_f^1 + \nu_2 \eta_f^2 + \nu_{12} \eta_f^{12}$$

Using $(C1-3)$, we substitute for η_f^j $(j = 1,2,12)$ to obtain:

$$c + r_f = E[S'(K_i + q_i^f)] + \nu_1 \xi_f^1 + \nu_2 \xi_f^2 + \nu_{12} \xi_f^{12}, \tag{2}$$

where $E[S'(K_i + q_i^f)] \equiv \nu_1 S'(K_1 + q_1^f) + \nu_2 S'(K_2 + q_2^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12}) + (1-\nu)S'(K_f)$ is the expected marginal surplus. The assumption $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1,2$ combined with (2), (1) and the non-negativity of ξ_f^j for $j = 1,2,12$ leads to

$$E[S'(K_i + q_i^f)] < \frac{\nu_i S'(K_i + q_i^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12})}{\nu_i + \nu_{12}} + \xi_i, \tag{3}$$

for $i = 1,2$. Suppose first that $K_1 + q_1^f \leq K_2 + q_2^f$ then $S'(K_1 + K_2 + q_{12}^f) \leq S'(K_2 + q_2^f) \leq S'(K_1 + q_1^f) \leq S'(K_f)$ which implies:

$$E[S'(K_i + q_i^f)] \geq \frac{\nu_2 S'(K_2 + q_2^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12})}{\nu_2 + \nu_{12}},$$

for $i = 1,2$. For the last inequality to be consistent with (3) for $i = 2$, it must be that $\xi_2 > 0$ which implies $K_2 = 0$. Since by assumption $K_1 + q_1^f \leq K_2 + q_2^f = q_2^f = K_f$ and then we must have $K_1 = 0$ and $q_1^f = K_f$.

Suppose now that $K_i = 0$ for $i = 1,2$ then $\xi_i > 0$ for $i = 1,2$ in (C5-6) which leads to

$$(\nu_i + \nu_{12})S'(K_f) < r_i \tag{4}$$

for $i=1,2$. Moreover by (C1-3), $\eta_f^j = S'(K_f) - c$ for $j = 1,2,12$ which combined with (C4) leads to $S'(K_f) = c + r_f$. The last equality joint with (4) leads to $c + r_f < \dfrac{r_i}{\nu_i + \nu_{12}}$ for $i = 1,2$.

Proof of *b.1)*. Suppose $\nu_1 = \nu_2 = 0$, $\nu_{12} > 0$ and $c + r_f > \frac{r_i}{\nu_{12}}$ for $i = 1,2$. The first-order conditions simplify to:

$$S'(K_1 + K_2 + q_f^{12}) = c - \xi_f^{12} + \eta_f^{12} \tag{C'3}$$

$$(1-\nu)\left[S'(K_f) - c\right] = r_f - \nu_{12}\eta_f^{12} \tag{C'4}$$

$$S'(K_1 + K_2 + q_f^{12}) = \frac{r_i}{\nu_{12}} - \xi_i \;\text{ for }\; i = 1,2 \tag{C'5-6}$$

Conditions (C'5-6) lead to $\xi_2 - \xi_1 = \frac{r_2}{\nu_{12}} - \frac{r_1}{\nu_{12}} > 0$ where the last inequality is due to the assumption $r_2 > r_1$. Therefore $\xi_2 > 0$ which implies $K_2 = 0$. Since there are only two states of nature with only one source of intermittent energy in one state like in Section 2, Proposition 1 holds. In particular, with our notation we have $K_1 > 0$ for $c + r_f > \frac{r_1}{\nu_{12}}$.

Proof of *b.2)*. Suppose that $\nu_1 > 0$, $\nu_2 > 0$ and $\nu_{12} = 0$ and $c + r_f > \frac{r_i}{\nu_i}$ for $i = 1,2$. The first-order conditions simplify to:

$$S'(K_i + q_f^i) = c - \xi_f^i + \eta_f^i \text{ for } i = 1,2 \tag{C"1-2}$$

$$(1-\nu)\left[S'(K_f) - c\right] = r_f - \nu_1\eta_f^1 - \nu_2\eta_f^2 \tag{C"4}$$

$$S'(K_i + q_f^i) = \frac{r_i}{\nu_i} - \xi_i \;\text{ for }\; i = 1,2 \tag{C"5-6}$$

Case 1: $\frac{r_i}{\nu_i} < c$ for one $i \in \{1,2\}$. The conditions (C"1-2) and (C"5-6) imply $c - \frac{r_i}{\nu_i} = \xi_f^i - \eta_f^i - \xi_i > 0$ which implies $\xi_f^i > 0$ and therefore $q_f^i = 0$, i.e. no fossil power in state i. As long as $\nu_i > 0$ and $S'(0) = +\infty$, $q_f^i = 0$ is optimal only if $K_i > 0$.

Case 2: $\frac{r_i}{\nu_i} > c$ for $i = 1,2$. Suppose first that $K_1 = K_2 = 0$. Then $K_i + q_i^f = K_f$ for $i = 1,2,12$ (use of fossil power under full capacity in all states of nature). Moreover, $K_i = 0$ implies $\xi_i > 0$ and therefore $S'(K_f) < \frac{r_i}{\nu_i}$ by (C"5-6). The first-order conditions (C"1-2) and (C"4) imply $S'(K_f) = c + r_f$ which combined with the last inequality contradicts the assumption $c + r_f > \frac{r_i}{\nu_i}$. Suppose now that $K_1 > 0$ and $K_2 = 0$ which implies $K_2 + q_2^f = K_f$ and $\xi_2 > 0$. The first-order conditions (C"4) and (C"5-6) imply respectively $(1-\nu_1)\left[S'(K_f) - c\right] = r_f - \nu_1\eta_f^1$ and $S'(K_f) < \frac{r_2}{\nu_2}$. The two last relations lead to $\nu_1\eta_f^1 > r_f + c - \frac{r_2}{\nu_2} + \nu_1\left[\frac{r_2}{\nu_2} - c\right]$. Since by assumption $r_f + c > \frac{r_2}{\nu_2} > c$, $\eta_f^1 > 0$ and therefore $q_1^f = K_f$. The first-order conditions imply $E[S'(K_i + q_i^f)] = r_f + c$. Since $S'(K_1 + K_f) < E[S'(K_i + q_i^f)] < S'(K_f)$, the last equality combined with $S'(K_f) < \frac{r_2}{\nu_2}$ contradicts our starting assumption $c + r_f > \frac{r_2}{\nu_2}$.