# *The Cake-Eating problem: Non-linear sharing rules*

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# Outline of the talk

1. *The model*

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2. *The stories*

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2. *The stories*
3. *The results*

# The Model

## A program P with identical utility

$$\max_{\mathbf{x}} \sum_{i=1}^{n} a_i v(x_i)$$
$$\text{s.t. } \mathbf{p}'\mathbf{x} = y. \quad (1)$$

- $v : \mathbb{R}_+ \to \mathbb{R}$ strictly increasing and concave, satisfies "Inada conditions" and is the same for each attribute;
- The goods are ranked such that the "kernel prices" $\frac{p_i}{a_i}$ are decreasing with $i$

# The aim of the paper

## The FOC

$$\frac{v'(x_i^*)}{v'(x_j^*)} = \frac{p_i a_j}{p_j a_i} = \pi_{ij} \; \forall i, j$$
$$x_i^* < x_j^* \text{ iff } \pi_{ij} > 1 \Leftrightarrow i < j \tag{2}$$

- Exploring integrability conditions
- How is the shape of the demand of the least demanded good related to the properties of the utility function ?

# Stories

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| time | weights | prices |
|---|---|---|
| 1 | $a = \frac{1}{1+\beta}$ | 1 |
| 2 | $1 - a = \frac{\beta}{1+\beta}$ | $\frac{1}{1+r}$ |

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# Group decision-making

- **Intra-household allocation:** No prices, Samuelson's household welfare function.

$$\max_{x_1, x_2} av(x_1) + (1-a)v(x_2)$$
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- **Risk-sharing:** $\theta \in \Theta$ states of the world, risk: $F : \Theta \longrightarrow [0,1]$,while $v(x)$ are the identical vNM utility of the two individuals.

$$\max_{x_1,x_2} a \int_{\Theta} v(x_1(\theta))dF(\theta) + (1-a)\int_{\Theta} v(x_2(\theta))dF(\theta), \; with \; a \in (0, \frac{1}{2}]$$

$$\text{s.t.} \;\; z_1(\theta) + z_2(\theta) = y(\theta) = x_1(\theta) + x_2(\theta), \; \forall\theta \in \Theta; \quad x_1 \geq 0; \quad x_2 \geq 0.$$

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- Borch (1960): the consumption in each state of the world only depends on the total wealth in that state. Wealth is not transferable from one state to another.
- Solving the risk-sharing problem then reduces to solve the intra-household allocation for any feasible $y$.

# Integrability conditions for any n (exposition for n=2)

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- We normalize $a = 1/2$, $p_1 = p > 1$ and $p_2 = 1$.Then $x_1^*(y, \mathbf{p}, a) \equiv x(y, p)$.

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- Let $h(x, p)$ be the demand of good 2 as a function of good 1 and $p$.
- $h(x, p) = g(x, p) - px$, where $g(x, p)$ is the inverse function of $x(y, p)$ wrt $y$ using the fact that the two goods are normal.

# Integrability conditions

## Proposition

*A function* $x(y, \pi)$*, strictly increasing with* $y$ *and decreasing with* $p$ *is a solution of program* **P** *for all* $y \in \mathbb{R}_+$ *and for all* $p > 1$*, iff there exist a positive function* $A(x)$ *such that:*

$$\frac{h_x(x,p)}{h_p(x,p)} = A(x)p \tag{3}$$

*Then* $A$ *represents the Arrow-Pratt absolute risk aversion coefficient, that is* $v'(x) = \exp \int_0^x A(s)ds$ *.*

# Integrability conditions: examples

- $x_1^*(y,p) = \frac{1}{2p} y^\gamma$, for $\gamma < 1$ does not satisfy the integrability conditions.

- If $h(x,p) = (1+x)^p - 1$ , we get $\dfrac{h_x}{h_p} = \dfrac{p}{(1+x)\ln(1+x)}$. Then $h$ is the solution of **P** with the log-integral utility function $v(x) = \int_0^x \frac{1}{\ln(1+s)} ds$

- If $h(x,p) = \ln(1+e^x - p) - \ln p$, we get $\dfrac{h_x}{h_p} = \dfrac{e^x}{1+e^x} p$ , solution of **P** under the linex utility function $v(x) = x - e^{-x}$.

# Integrability without prices

## The "group" case

- Group decision-making set-up: prices are fixed (eventually equal to 1) and weights are fixed

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**Proposition**

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### Proposition

- *For all* $f(y)$ *and* $a \in (0, 1/2)$, *there exists a continuous differentiable utility function* $v$ *such that, for all* $y \in \mathbb{R}_+$, *from Program (1) we get* $x_1^*(y; a)) = f(y)$.

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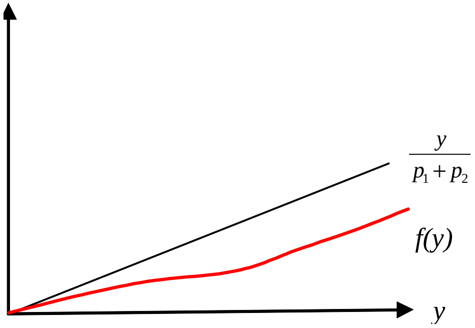
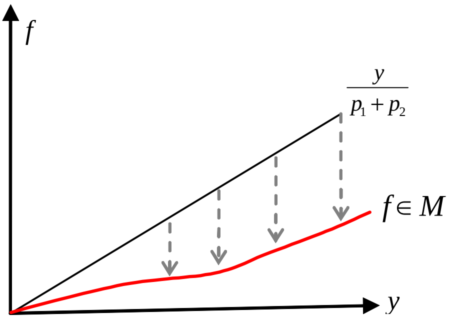
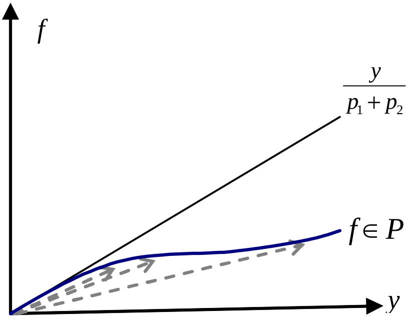
- From $p_1 x_1 + p_2 x_2 = y$ we know $x_1 = x_2 \implies x_1 = \frac{y}{p_1 + p_2}$



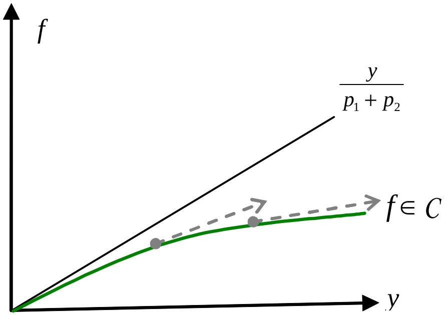
**Type 1: Class $\mathcal{M}$, or "Moving Away" sharing functions**



## Type 2: Class $\mathcal{P}$, or "progressive" sharing functions



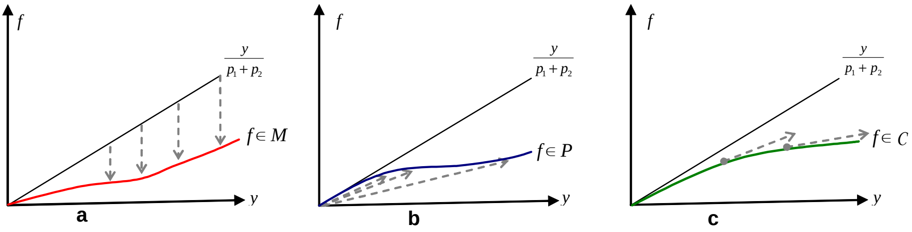
## Type 3: Class $\mathcal{C}$, or "concave"



# 2.d Remark

**The classes are nested**

$$\mathcal{C} \subset \mathcal{P} \subset \mathcal{M}$$



# A characterization result for the first good

## Proposition

- *Suppose that $x_1^*(y;\cdot)$ is twice continuously differentiable. Then:*

$i)\ v \in DARA \Longleftrightarrow x_1^*(y;\cdot) \in M \quad \text{for all } \pi \geq 1$
$ii)\ v \in DRRA \Longleftrightarrow x_1^*(y;\cdot) \in P \quad \text{for all } \pi \geq 1.$
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Where

- DARA = Decreasing Absolute Risk Aversion
- DRRA = Decreasing Relative Risk Aversion
- CT = Convex Tolerance

# An extension to a "sequential" setting

## Proposition (3bis)

*Let* **P** *represent an intertemporal consumption choice, with* $n = T$ *periods and initial wealth* $y$*. Let us consider the associated dynamic programming problem where at time* $t$ *the consumer chooses the optimal consumption pattern* $c_t, c_{t+1}, ..., c_T$ *of the remaining* $T - t$ *periods as a function of the current wealth* $y_t$*. Then the conditions of the previous proposition apply to the sharing function linking the current consumption* $c_t$ *to the current wealth* $y_t$ *for each period* $t = 1...T - 1$.

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  - *linlog* utility function $v(x) = \alpha x + \beta \log x$.

# Applications: Individual Choice

## 1- Arrow Debreu contingency claims

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  - $v \in DRRA \iff \frac{p_1 x_1^*}{y}$ is decreasing with $y$
  - $v \in CT \iff x_1^*$ is concave in $y$ (the marginal share of the less demanded attribute decreases with wealth)

# Individual choice

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# Individual choice

## Insurance

- Initial wealth $Y$ ; risk of a loss $-X$ in state 1 with probability $a$.
- Insurance contract where $0 \leq C \leq X$.
- The premium $\beta C$ is proportional to the coverage, with $\beta < 1$
- 

| states | prob. | final wealth |
|---|---|---|
| 1 | $a$ | $x_1 = Y - X + (1 - \beta)C$ |
| 2 | $1 - a$ | $x_2 = Y - \beta C$ |

- Uninsured loss $z_1 = x_2 - x_1$
- Results:
  - $v \in DARA \iff z_1^*$ is increasing with $y$
  - $v \in DRRA \iff$ proportion of uninsured wealth is increasing with $y$
  - $v \in CT \iff$ uinsured wealth is concave with $y$

- **Given the model**

| time | wheights | prices |
|---|---|---|
| 1 | $a = \frac{1}{1+\beta}$ | 1 |
| 2 | $1 - a = \frac{\beta}{1+\beta}$ | $\frac{1}{1+r}$ |

- **Given the model**

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- The initial condition $\lambda = \frac{p_1(1-a)}{p_2 a} \geq 1$ becomes $\beta \geq \frac{1}{1+r}$. The marginal opportunity cost of saving is lower than the intertemporal MRS $\Longrightarrow$ lower consumption in the first period.

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  - $v \in CT \iff x_1^*$ is concave with $y$

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- If individual 1 is the "weaker" individual ($a \leq \frac{1}{2}$) then $x_1^*(y, a) \leq \frac{1}{2}y$.
- Immediate interpretation of the Proposition 1, for the risk-sharing too.