Arrow's theorem of the deductible: moral hazard and stop-loss in health insurance

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Arrow's theorem of the deductible

Theorem

"If an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy's actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100% coverage above a deductible minimum" (Arrow, 1963).

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Logic is obvious (and robust): since it is better for the consumer to insure expenditures when disposable income is low rather than high, insurance funds should be spent on the highest expenditures.

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- Real world insurance policies often feature explicit deductibles (the Netherlands, Switzerland), or a stop-loss (Belgian maximum billing system). Partial first-dollar insurance and stop loss in RAND-experiment.

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4. Ex ante moral hazard.

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- Functions f_s and (state-independent) g are continuously differentiable and strictly concave.
- Resources are state-independent: $W_s = W_t = W$ for all $s, t = 1, ..., S$.
- Individual may buy insurance at a premium

$$\pi = (1 + \lambda) \sum_s p_s \alpha_s M_s$$

Optimal policy

Optimal policy problem

$$\max_{\alpha_1,\ldots,\alpha_S,M_1,\ldots,M_S} V(M,C) = \sum_s p_s \left[f_s(M_s) + g(W - \pi - (1-\alpha_s)M_s)\right]$$

subject to $\pi = (1+\lambda)\sum_s p_s \alpha_s M_s$.

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First-order conditions:

$$\frac{dV}{dM_s} = p_s\left[f_s' - (1-\alpha_s)g_s'\right] - (1+\lambda)p_s\alpha_s \sum_t p_t g_t' = 0,$$

$$\frac{dV}{d\alpha_s} = p_s M_s \left[g_s' - (1+\lambda)\sum_t p_t g_t'\right] \leqslant 0, \qquad \alpha_s \frac{dV}{d\alpha_s} = 0.$$

Arrow's result

(1) **Level of medical expenditures is set optimally:**

$$\text{for all } s = 1, ..., S,\ f_s' = g_s'$$

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(1) **Level of medical expenditures is set optimally:**

$$\text{for all } s = 1, ..., S,\ f'_s = g'_s$$

(2) **Optimality of the deductible:**

$$\text{either } \alpha_s = 0 \text{ or } g'_s = (1+\lambda)\sum_t p_t g'_t := (1+\lambda)\overline{g}'.$$

or (with the deductible $D := (1-\alpha_s)M_s$ and g'_D for marginal utility of wealth at $C = W - \pi - D$),

$$\alpha_s = \max(0, \frac{M_s - D}{M_s}), \qquad g'_D = (1+\lambda)\overline{g}'.$$

SECOND BEST: ex post-moral hazard

Choice of treatment after observing the state (without regard for the impact of M_s on premium π):

$$\max_{M_s} f_s(M_s) + g(W - \pi - (1 - \alpha_s)M_s)$$

leading to "overconsumption",

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Define

$$\eta_s = \frac{\alpha_s}{M_s}\frac{dM_s}{d\alpha_s} > 0$$

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$$\max_{\alpha_1,\ldots,\alpha_S} \Lambda = \sum_s p_s \left[f_s(M_s(\alpha_s)) + g(W - \pi - (1-\alpha_s)M_s(\alpha_s)) \right]$$

subject to $\pi = (1+\lambda) \sum_s p_s \alpha_s M_s(\alpha_s)$.

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First-order conditions

$$\frac{\partial \Lambda}{\partial \alpha_s} = p_s M_s \left[g'_s - \overline{g}' (1+\lambda)(1+\eta_s) \right]$$

$$\alpha_s \frac{\partial \Lambda}{\partial \alpha_s} = 0.$$

"Implicit deductible" property

Rewriting, we obtain

$$\text{either } \alpha_s = 0 \text{ or } g_s' = (1+\lambda)\overline{g}'(1+\eta_s)$$

Proposition. If resources are state-independent, preferences are separable with state-independent consumption preferences and the probabilities of the different states cannot be influenced by the consumer, the optimal insurance contract results in the same indemnities as a contract with 100% insurance above a variable deductible positively related to η_s, the elasticity of medical expenditures with respect to the insurance rate α_s.

Interpretation

- Special case $\eta_s = \overline{\eta}$ for all s: Arrow's result, but with the loading factor blown up by the moral hazard factor $(1 + \overline{\eta})$.

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- Policy implemented through variable insurance rates α_s, NOT through the explicit announcement of a deductible D. Assumption of state-specific insurance rates is unrealistic.
- Qualitative finding 1: our results validate the practice of higher insurance rates (not only indemnities) for major medical expenses. (If $\eta_s = \eta_t$, then $(1 - \alpha_s)M_s = (1 - \alpha_t)M_t$).

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- Qualitative finding 1: our results validate the practice of higher insurance rates (not only indemnities) for major medical expenses. (If $\eta_s = \eta_t$, then $(1 - \alpha_s)M_s = (1 - \alpha_t)M_t$).
- Qualitative finding 2: optimal medical insurance scheme will in general be nonlinear. Our vector of insurance rates $(\alpha_1, ..., \alpha_S)$ can be seen as discrete approximation of non-linear model of Blomqvist (1997).

THIRD BEST: *explicit stop-loss arrangement*

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$$\max_{\alpha_s, D} \Lambda = \sum_{M_s < D} p_s \left[f_s(M_s(\alpha_s)) + g(W - \pi - (1-\alpha_s) M_s(\alpha_s)) \right]$$

$$+ \sum_{M_s \geqslant D} p_s \left[f_s(M_s) + g\left(W - \pi - D\right) \right]$$

under the constraints

$$\pi = (1+\lambda) \left[\sum_{M_s < D} p_s \alpha_s M_s(\alpha_s) + \sum_{M_s \geqslant D} p_s (M_s - D) \right]$$

$$f_s' = (1-\alpha_s) g_s' \text{ if } M_s < D, \qquad f_s' = 0 \text{ if } M_s \geqslant D.$$

Solution

First-order conditions for α_s (states with $M_s < D$)

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$$\text{either } \alpha_s = 0 \text{ or } g'_s = (1+\lambda)\overline{g}'(1+\eta_s).$$

First-order condition for D

$$\frac{\partial \Lambda}{\partial D} = -\sum_{M_s \geqslant D} p_s \left[g'_s - (1+\lambda) \sum_t p_t g'_t \right] \leqslant 0, \qquad D \frac{\partial \Lambda}{\partial D} = 0.$$

Writing g'_D for $g'(W - \pi - D)$, this gives

$$\text{either } D = 0 \text{ or } g'_D = \overline{g}'(1+\lambda). \tag{1}$$

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Combining

$$\text{if } \alpha_s D > 0, \text{ then } g'_s = g'_D(1+\eta_s) > g'_D.$$

Result

Conclusion: if $D > 0$, then $\alpha_s = 0$.

Proposition If resources are state-independent, preferences are separable with state-independent consumption preferences and the probabilities of the different states cannot be influenced by the consumer, an optimal stop-loss insurance policy takes the form of a deductible, i.e. there is no reimbursement for expenses below the stop-loss amount and full reimbursement of the excess of expenses over the deductible.

EX ANTE MORAL HAZARD: *treatment as prevention*

General preventive behavior (lowering probability of expensive states) should be subsidized.
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- Consulting GP in state s may lead to early detection of severe diseases and may help avoiding severe complications: $p_t = p_t(M_s)$ with $dp_t/dM_s < 0$.
- Preventive and curative aspects from regular doctor visits cannot be distinguished.

Optimal policy

Policy problem:

$$\max_{\alpha_s, D} \Lambda = (1 - p_t(M_s)) \left[f_s(M_s(\alpha_s)) + g(W - \pi - (1 - \alpha_s) M_s(\alpha_s)) \right]$$

$$+ p_t(M_s) \left[f_t(M_t) + g(W - \pi - D) \right]$$

subject to

$$\pi = (1 + \lambda) \left[(1 - p_t(M_s)) \alpha_s M_s(\alpha_s) + p_t(M_s)(M_t - D) \right].$$

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$$\pi = (1 + \lambda) \left[(1 - p_t(M_s)) \alpha_s M_s(\alpha_s) + p_t(M_s)(M_t - D) \right].$$

Define the elasticity of p_s with respect to M_s:

$$\eta_{p_s M_s} = \frac{M_s dp_s}{p_s dM_s} > 0$$

Optimality conditions

Behavior insured patient, who disregards the impact of $M_t - D$ on the premium π:

$$\frac{\partial \Lambda}{\partial M_s}\Big|_{\pi} = (1 - p_t)\left[f_s' - g_s'(1 - \alpha_s)\right] + \frac{dp_t}{dM_s}\left[f_t + g_t - (f_s + g_s)\right] = 0.$$

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Condition defining a **socially efficient level of** M_s:

$$\frac{\partial \Lambda}{\partial M_s} = \frac{\partial \Lambda}{\partial M_s}|_\pi - \overline{g}'\,(1+\lambda)\left[(1-p_t)\alpha_s + \frac{dp_t}{dM_s}\,(M_t - D - \alpha_s M_s)\right] = 0.$$

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Optimal α_s :

$$\alpha_s = \frac{\eta_{p_s M_s}}{1 + \eta_{p_s M_s}} \frac{(M_t - D)}{M_s}$$

Result

Proposition If resources are state-independent and preferences are separable with state-independent consumption preferences, the desirability of preventive behaviour (lowering the probability of the expensive health states) justifies some insurance below the deductible (i.e. $\alpha_s > 0$) if health care expenditures in a state of standard health have a negative effect on the probability of getting into a state with large medical expenses, but the preventive component of these expenditures cannot be identified as such.

Result

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Strong analogy with literature on complementarity/substitution relationships between different health care commodities (e.g. Goldman and Philipson, 2007): subsidizing medicines to lower hospital expenditures.

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- Common practice of first-dollar insurance in a model with stop-loss is not optimal in standard model: a straight deductible is optimal.

Conclusion

- Logic of Arrow's theorem of the deductible remains at work in a model with ex post moral hazard. Strong arguments in favour of stop-loss arrangement.
- Common practice of first-dollar insurance in a model with stop-loss is not optimal in standard model: a straight deductible is optimal.
- However, some insurance below deductible is optimal if health care expenditures in relatively healthy states have a negative effect on the probability of getting into a state with large medical expenses.

Important open issues

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- Redistributive considerations in public health insurance schemes. Relationship with other redistributive instruments (e.g. nonlinear income tax).