

Individual decisions under risk, risk sharing and asset prices with regret¹

Christian Gollier
University of Toulouse

Bernard Salanié
Columbia University

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Abstract

We consider an Arrow-Debreu economy in which expected-utility-maximizing agents are sensitive to regret. According to regret theory, the marginal utility of their consumption is increasing in the maximum payoff that they could have obtained if they would have made another choice ex ante. We show that regret biases the optimal portfolio allocation towards assets that perform particularly well in low probability states. The competitive asset pricing kernel is convexified by regret if the distribution of the macroeconomic risk is logconcave. Regret also reduces the equity premium when the macro risk is positively skewed. We characterize the competitive allocation of risk when consumers have heterogenous preferences, and we show how to aggregate individual intensities of regret.

Keywords: regret theory, portfolio choice, complete markets, long odds bias.

1 Introduction

The pleasure extracted from a favorable event is often mitigated by the feeling that this event could have been even more favorable if other choices had been made. For example, I am happy to have bet on a horse which won its race, but my pleasure is limited by the feeling that I could have earned a larger payoff by betting more on that horse. This feeling is particularly acute for winning horses with large odds. Similarly, investors who invested 50% of their financial wealth in stocks are hopeful in a bullish market, but they anticipate that they will regret not having invested more in stocks if stock prices surge. Quoting Bell (1985), regret is a psychological reaction to making a wrong decision, where the quality of decision is predicated on the basis of actual outcomes rather than on the information available at the time of the decision. Bell (1982, 1983) and Loomes and Sugden (1982), have convincingly argued that many people are likely to feel regret, and that the expectation of such future regret should have an impact on the optimal behavior in the face of risk. Regret theory may explain some of the puzzles coming from the confrontation of the expected utility theory with the market and experimental data. For example, Bell (1982) has shown that regret theory can explain why it may be optimal to purchase insurance at an actuarially unfair price and, at the same time, gambling on a zero-mean risk: the former appeals to risk-averse agents and the latter to agents who fear that they will regret missing the chance of a big win.

In this paper, we assume that agents are subject to regret, and we examine the consequences of this assumption on the optimal decisions under risk, the allocation of risk in the economy, and asset prices. To do this, we consider a static model with complete markets for Arrow-Debreu securities. Following the authors mentioned above, we assume that agents maximize their expected utility, where each individual's ex-post utility is a function of two variables: the actual level of consumption x , and the largest level of consumption y that would have been attainable if another decision would have been made ex ante. This second variable will hereafter be called the "forgone best alternative". Two alternative properties of this utility function can be considered to define regret. The first candidate for a definition of regret is that the utility function is decreasing in y . In this paper, we use a more behavioral approach in defining regret. We start from the intuitive notion that consumption x should be a substitute for regret: the latter should be per-

ceived less acutely if the consumption level actually achieved is larger. This translates into the positiveness of the cross-derivative of the utility function. Said in other words, the marginal utility of actual consumption is increasing in the forgone best alternative. As explained by Eeckhoudt, Rey and Schlesinger (2005) in a different context, this definition of regret is equivalent to a preference for a positive correlation between the actual payoff and the forgone best alternative. A more restrictive specification of the utility function that satisfies this assumption has been proposed by Bell, Loomes and Sugden.

This definition of regret has intuitive consequences, as the so-called long odds bias that is observed in racetrack betting and national lotteries.¹ When the budget available for gambling is fixed, the forgone best alternative *ex post* is of course to bet the player's entire budget on the winning horse, yielding a best payoff that is proportional to its odds. When prices are actuarially fair, this means that the forgone best alternative is inversely proportional to the corresponding state probability. Thus, the feeling of regret is relatively more pronounced for long-shots than for favorites. The preference for a positive correlation between the actual payoff and the forgone best alternative implies that the player biases his betting strategy towards longshots. Similarly, consumers who are sensitive to regret bias their insurance demand towards low probability events, and they bias their portfolio allocation towards assets that perform particularly well in low probability states, i.e., highly skewed assets.

More generally, we characterize the competitive price of any contingent claim. In a regret-free economy, the competitive state price per unit of probability is a decreasing function of the aggregate consumption in that state. This function is usually referred to as the pricing kernel. The equity premium is proportional to the absolute value of its derivative, which is the absolute risk aversion of the representative agent in the classical model. The characterization of the pricing kernel is more complex when the representative agent is sensitive to regret. Indeed, the competitive state price per unit of probability also is a function of the state probability in that case. The increased demand for contingent claims associated to unlikely states that is generated by regret tends to raise their competitive price. Therefore, state prices will be abnormally large in the tails of the distribution of aggregate

¹Where it is often called the favourite-longshot-bias

consumption. In other words, regret convexifies the pricing kernel. This is compatible with the observation made by Rosenberg and Engle (2002) who showed that the empirical pricing kernel at the lower tail is much steeper than what would be obtained in the classical model with a regret-free model. However, Rosenberg and Engle (2002) do not observe a similar pattern at the upper tail of the distribution.

We also describe the preferences of the representative agent when consumers have heterogeneous preferences. Wilson (1968) already showed that the representative agent has an absolute risk tolerance which is the mean absolute risk tolerance in the population. In a similar fashion, we show that the representative agent has an intensity of regret that is the mean of the intensities of regret in the population.

Muerman, Mitchell and Volkman (2005) and Michenaud and Solnik (2005) have examined a portfolio choice problem when the investor anticipates regret. Our work differs much from their analyses in at least three essential dimensions. First, we derive our results by just assuming that the cross-derivative of the bivariate utility function $U(x, y)$ is nonnegative, whereas they assume that $U(x, y) = v(x) + f(v(x) - v(y))$. Second, we go beyond the portfolio choice problem to derive an asset pricing model and a preference aggregation formula. Last but not least, these authors consider a two-asset model, whereas we assume the existence of a complete set of markets for Arrow-Debreu securities. Because regret theory is very sensitive to the definition of the investment opportunity set, this alternative assumption has a deep impact on the characterization of the optimal portfolio. For example, both Muerman et al. (2005) and Michenaud et al. (2005) obtain as a central result that the demand for the risky asset is positive when the price of the risky asset is fair. Our finding is radically different under complete markets, since we show that, under actuarially fair prices, the risk free position is still optimal when all states are equally likely. More generally, we show that the sign of the demand for the risky asset depends upon the skewness of the distribution of its returns.

2 Regret and binary lottery choices

As initially suggested by Bell (1982), we assume that agents are sensitive to regret in the sense that their utility U in any state is a function of two

variables: (1) the actual consumption x achieved in that state, and (2) the maximal consumption y that could have been attained in that state if another feasible choice had been made at the beginning of the period. Of course, by definition, we always have that y is larger or equal to x . We make the following assumption on this utility function U . First, it is assumed to be increasing and concave in x . Agents always prefer to consume more to less ($U_x > 0$), and are averse to risk on actual consumption ($U_{xx} \leq 0$). Second, we assume that agents are sensitive to regret.² As explained in the introduction, we take this to mean that the marginal utility of actual consumption is increasing in the forgone alternative y : $U_{xy} \geq 0$.

Contrary to the existing literature on regret initiated by Bell (1982) and Loomes and Sugden (1982), we do not advocate any particular specification of our bivariate utility function U . In the remaining of this section, we justify why the cross-derivative of this function characterizes the agent's attitude to regret. Following Bell (1982), consider the possibility to bet an amount b on a horse whose objective probability of winning the race is $p > 0$. Suppose moreover that the bets are fair, i.e., for a bet of size b euros, the bookmaker pays b/p euros if the horse wins and zero otherwise. In this section, the only alternative to bet b is not to bet at all. Confronted with this binary choice, it is optimal to bet if and only if

$$pU\left(b\frac{1-p}{p}, b\frac{1-p}{p}\right) + (1-p)U(-b, 0) \geq pU\left(0, b\frac{1-p}{p}\right) + (1-p)U(0, 0). \quad (1)$$

There are two situations in which the agent feels regret: (1) he bets, and the horse loses, and (2) he does not bet and the horse wins. These are the two terms in (1) in which the two variables in U differ. When $U_{xx} < 0$ and $U_{xy} > 0$, we see two contradictory effects in (1). First, risk aversion discourages taking this fair bet, since x is riskier in the left-hand side of (1) than in its right-hand side. Second, anticipated regret encourages risk taking, since it is equivalent to correlation loving between x and y . Indeed, x and y are positively correlated when the lottery is accepted, whereas they are independent when no risk is taken. These contradictory effects can be made more explicit for small risks. When the size of the bet b is small enough, it

²The feeling of regret corresponds to $U_y < 0$. However, we don't use this assumption in this paper.

is indeed easy to check that the above inequality holds if and only if

$$U_{xx}(0, 0) + 2(1 - p)U_{xy}(0, 0) \geq 0. \quad (2)$$

Thus, betting may be optimal if the anticipation of regret is strong enough to overcome risk aversion.

We see from (2) that the willingness to accept the fair lottery due to the anticipation of regret depends upon the probability p of winning. When p is close to unity, the covariance between x and y , which is equal to $b^2(1 - p)^2/p$ when the lottery is accepted, tends to zero. The effect of regret – or xy -correlation loving – is almost inexistent in that case. When p is reduced, the covariance increases, and the effect of regret increases. The effect of regret is maximum for very low-probability events. This could explain the long-odds bias. This bias is well-documented in the literature. For example, Kahneman and Tversky (1979) showed that 72% of their subjects in a laboratory experiment preferred the lottery (5000, 1/1000; 0, 999/1000) over the sure gain of 5, which can be interpreted equivalently as accepting a fair bet of 5 on a horse that has a probability of winning of 1/1000. Jullien and Salanié (2000) provide information on the unfavorable equilibrium prices observed for long shots in horse races in the U.K.. The tendency of air travellers to buy plane crash insurance at prices that are large multiples of the actuarially fair premium is also well documented.(...)

Bell (1982) and Loomes and Sugden (1982) suggested the following specification:

$$U(x, y) = v(x) + f(v(x) - v(y)), \quad (3)$$

where functions f and v are increasing and concave. This is clearly a special case of our model. Another possible specification for U exhibits "multiplicative" regret:

$$U(x, y) = \frac{x^{1-\gamma}y^\alpha}{1-\gamma}, \quad (4)$$

where γ and α are two positive scalars. Whereas we consider these particular specifications as quite intuitive, we claim that they are unnecessarily restrictive. We will derive most of our results for the general bivariate utility model.

3 Optimal portfolio for regret-sensitive agents

Contrary to the classical expected utility model, the evaluation of a specific decision is not independent of the set of possible alternative decisions when the decision maker is sensitive to regret. Contrary to the binary decision model that we considered in the previous section, we assume here that the agent has a continuum of choice. This is typical of a portfolio choice problem. The uncertainty prevailing at the end of the period is described by the S possible states of nature, indexed $s = 1, \dots, S$. There is an agreed-upon objective probability distribution of the states given by a vector $(p_1, \dots, p_S) > 0$. We assume that markets are complete. For each state s , there is a tradable Arrow-Debreu security that yields one unit of the single consumption good to its owner if state s occurs, and that yields nothing otherwise. The price of this asset at the beginning of the period is denoted $\Pi_s > 0$. When $\Pi_s = p_s$ for all s , we say that state prices are actuarially fair.

The investor's endowment in the consumption good is state-contingent, with ω_s denoting the endowment in state s . Let x_s denote the actual consumption in state s . It is financed by the state endowment ω_s plus the purchased amount $x_s - \omega_s$ of the Arrow-Debreu security associated to that state. This consumption plan must satisfy the budget constraint, which is written as:

$$\sum_{s=1}^S \Pi_s x_s = w =_{def} \sum_{s=1}^S \Pi_s \omega_s, \quad (5)$$

where w is the market value of agent i 's state-contingent endowment. We prohibit personal bankruptcy by imposing constraint $x_s \geq 0$ for all s . The preferences of the decision maker are characterized by the bivariate utility function $U(x, y)$ as defined in the previous section.

It is obvious that, given the no-bankruptcy constraint, the maximal level of consumption in state s is $y_s = w/\Pi_s$. This would be the level of consumption attained by that agent in that state if her entire wealth w would have been invested in the Arrow-Debreu security associated to that state. Given this observation, the portfolio decision problem can be written as

$$(x_1, \dots, x_S) \in \arg \max_{x_1, \dots, x_S \geq 0} \sum_{s=1}^S p_s U(x_s, \frac{w}{\Pi_s}) \quad s.t. \quad (5). \quad (6)$$

As U is concave in its first argument, the objective function is concave in the

vector of decision variables, whereas the budget constraint is linear in it. This implies that the solution to this program is unique. We assume that the utility function satisfies the Inada condition that states that the marginal utility of actual consumption tends to infinity when actual consumption tends to zero; then no-bankruptcy constraint is never binding. The necessary and sufficient condition characterizing the optimal portfolio of Arrow-Debreu securities is therefore written as

$$U_x(x_s, \frac{w}{\Pi_s}) = \lambda \frac{\Pi_s}{p_s}, \quad (7)$$

where λ is the Lagrange multiplier associated to the budget constraint. Let $\pi_s = \Pi_s/p_s$ denote the state price per unit of probability. For any given pair (π, Π) , let X denote the unique solution of the following equation:

$$U_x(X, \frac{w}{\Pi}) = \lambda \pi. \quad (8)$$

For a given marginal utility of (expected) wealth λ , this equation characterizes a function $X(\pi, \Pi)$. Comparing equation (7) and (8) implies that the optimal portfolio is such that $x_s = X(\pi_s, \Pi_s)$. The optimal risk exposure is measured by the size of the differences in state consumption. By equation (8), these differences in states consumption may be due either to differences in π_s or differences in Π_s across states. We now compute them by examining the variation of x_s across states. Totally differentiating equation (8) yields

$$U_{xx}dX - U_{xy}\frac{w}{\Pi^2}d\Pi = \lambda d\pi,$$

where the derivatives of U are evaluated at $(X(\pi, \Pi), w/\Pi)$. Replacing λ by U_x/π , we obtain the following result.

Proposition 1 *The demand x_s for the claim contingent to state s is a function X of the state price per unit of probability $\pi_s = \Pi_s/p_s$ and of the state price Π_s . This function satisfies the following property:*

$$dX = -T(X, w/\Pi)\frac{d\pi}{\pi} - \Gamma(X, w/\Pi)\frac{d\Pi}{\Pi}, \quad (9)$$

where $T(x, y) = -U_x(x, y)/U_{xx}(x, y)$ is the Arrow-Pratt index of absolute tolerance towards actual consumption, and $\Gamma(x, y) = -yU_{xy}(x, y)/U_{xx}(x, y)$ measures the intensity of regret.

No regret. In the absence of regret ($\Gamma_{xy} = 0$), we know from (7) that the optimal demand for the Arrow-Debreu security associated to state s depends upon the state only through the corresponding state price per unit of probability $\pi = \Pi/p$. This property of the classical model is essential, as it means that all risks that can be diversified at an actuarially fair price will be fully diversified away in individual portfolios. This property is central to derive the CAPM and APT pricing formulas. At the limit, when all assets are actuarially priced ($\Pi_s = p_s$ for all s), full insurance is optimal, i.e., agents purchase a risk free portfolio ($X \equiv w$). When state prices are not actuarially fair, the optimal risk exposure in the absence of regret is such that $-\pi dX/d\pi = T$. In words, the demand for Arrow-Debreu securities is larger for states with a smaller state price per unit of probability. This sensitivity is proportional to the investor's tolerance to risk. Seen from ex ante, this sensitivity is a measurement of the riskiness of the portfolio selected by investor i , which is optimally proportional to the agent's risk tolerance.

Regret with fair prices and equally likely states. Proposition 1 characterizes the optimal portfolio of agent i in the more general case with regret. The index Γ of regret measures the increase in actual consumption that preserves the marginal utility of actual consumption when the forgone alternative is increased by one percent. Because Γ is positive under regret aversion, equation (9) states that the demand for state consumption is decreasing in the corresponding state price, since regret is inversely related to it. The simplest case with regret arises when all states are equally likely and state prices are actuarially fair, i.e., $\Pi_s = p_s = 1/S$ for all s . In that case, the riskless portfolio is still optimal, with $x_s = X(1, 1/S) = w$ for all s . This is a case where risk aversion is the driving force behind this full insurance result. Because the forgone best alternative is the same in all states, regret affects the marginal utility of consumption in the same way in all states. Therefore, it does not affect the willingness to fully insure risk.

Regret with fair prices and heterogeneous state probabilities. The next step is to maintain the assumption of actuarially fair prices, but to relax the assumption that states are all equally likely: $\Pi_s = p_s$ for all s , but $\exists(s, s') : \Pi_s = p_s > p_{s'} = \Pi_{s'}$. In that case, equation (9) tells us that, in spite of the fairness of asset prices, the demand for the contingent claim associated to the more likely state s is smaller than the contingent claim associated to the less likely state s' . As explained above, the reason is the larger forgone best alternative in state s' . The difference in demand is proportional to the

intensity of regret measured by Γ , and to the relative increase in probability dp/p . This is the mechanism at work in the observation by Bell (1982) that an agent can at the same time have a positive demand for gambling and for insurance, when prices are actuarially fair.³ Risk-averse agents are willing to insure a 50-50 risk of accident, and, at the same time, they want to bet on low probability events, both at unfair prices. We summarize these findings in the following proposition.

Proposition 2 *Suppose that state prices are actuarially fair: $\pi_s = \pi_{s'}$ for all (s, s') . Then, the risk free position is optimal only if all states are equally likely ($p_s = p$ for all s), or if the agent is insensitive to regret ($U_{xy} \equiv 0$). More generally, under actuarially fair prices, the demand for Arrow-Debreu securities is decreasing in the corresponding state probability (long odds bias).*

This implies for example that, when insurance prices are fair, regret-averse agents will purchase partial insurance – or no insurance at all – against a binary risk of loss when the loss probability is larger than 1/2. Observe that the long odds bias would be reversed if we would have assumed that U_{xy} be negative. Moreover, observe that the notion of regret ($U_y < 0$) is irrelevant to this problem. This confirms that the key notion for the behavioural effects of regret-sensitiveness is regret-aversion, not regret.

Regret with unfair prices. We have just shown that regret tends to induce investors to accept risk in a situation where the risk free position would have been optimal in the absence of regret. However, when prices are not fair, it is not true in general that the expectation of regret ex post induces people to take more risk ex ante. To illustrate this point, suppose that vectors (π_1, \dots, π_S) and (Π_1, \dots, Π_S) are anti-comonotone in the sense that for all (s, s') , $(\pi_s - \pi_{s'})(\Pi_s - \Pi_{s'})$ is nonpositive. This is a situation in which the two terms in the right-hand side of equation (9) have opposite signs. The effect of regret works against the effect of risk tolerance. From the point of view of her risk tolerance, the agent would like to purchase less contingent claims associated to states with a larger state price per unit of probability. But these are the states where the effect of regret is stronger, thereby yielding an increase in demand for these claims. Thus, in that case, regret reduces the differences of the demands for Arrow-Debreu securities, i.e., it reduces

³However, Bell (1982) does not consider explicitly the joint insurance and gambling decision.

the optimal portfolio risk. We can formalize this result by defining a function q such that $\Pi_s = q(\pi_s)$ for all s . We can then rewrite equation (9) as

$$dX = - \left[T(X) + \frac{q'(\pi)}{q(\pi)} \Gamma(X, \frac{w}{q(\pi)}) \right] \frac{d\pi}{\pi}. \quad (10)$$

When vectors (π_1, \dots, π_s) and (Π_1, \dots, Π_s) are (anti-)comonotonic, q' is positive (negative). Assuming that the bracketed term in (10) is positive, the optimal risk exposure is increasing (decreasing) in the intensity Γ of regret.

To illustrate, let us consider a simple insurance problem. There are two states of nature, a no-loss state $s = 1$ and a loss state $s = 2$ in which the agent loses a fraction L of his wealth. The endowment of the agent is $\omega_1 = 1$ in state $s = 1$, whereas it is only $\omega_2 = 1 - L$ in state $s = 2$. The probability of loss is denoted p . For each euro of indemnity paid in case of loss, the policyholder must pay a premium $(1 + k)p$, where $k > 0$ is the loading factor of the insurance premium. This is compatible with the following state prices:

$$\begin{aligned} \Pi_1 &= 1 - p(1 + k); & \pi_1 &= 1 - k \frac{p}{1-p} \\ \Pi_2 &= p(1 + k); & \pi_2 &= 1 + k \end{aligned}$$

Because obviously π_1 is less than π_2 under the assumption that k is positive, the vectors of the π and the Π are comonotonic when $\Pi_1 \leq \Pi_2$, i.e., when p is larger than $\hat{p} = 0.5(1 + k)^{-1} < 0.5$. In our calibration exercise, we assume that $L = 0.5$ and $k = 0.3$, so that the threshold probability \hat{p} equals 0.38. We also assume that the utility function is $U(x, y) = x^{1-\gamma}y^\alpha/(1 - \gamma)$. Note that because $\Gamma(x, y) = \alpha x/\gamma$, parameter α is a measure of the intensity of regret aversion. In this context, the optimal indemnity equals

$$I = \frac{1 - \mu(1 - L)}{(1 + k)(1 - \mu)p + \mu} \text{ with } \mu = \left[\frac{1 - k \frac{p}{1-p}}{(1 + k)^{1+\alpha}} \right]^{-\frac{1}{\gamma}} \left[\frac{1 - p(1 + k)}{p} \right]^{-\frac{\alpha}{\gamma}}. \quad (11)$$

In Figure 1, we depicted the optimal indemnity as a function of the probability of loss, for $\gamma = 2$ and $\alpha = 0, 0.1, \dots, 0.5$. We see that when the probability of loss is larger than $\hat{p} = 0.38$, the optimal indemnity is decreasing in regret aversion. The expectation of future regret raises the optimal risk exposure because π and Π are comonotone for these values of the loss probability. This figure and this argument may explain why we do not observe active insurance markets for high-frequency risks. Note that when the

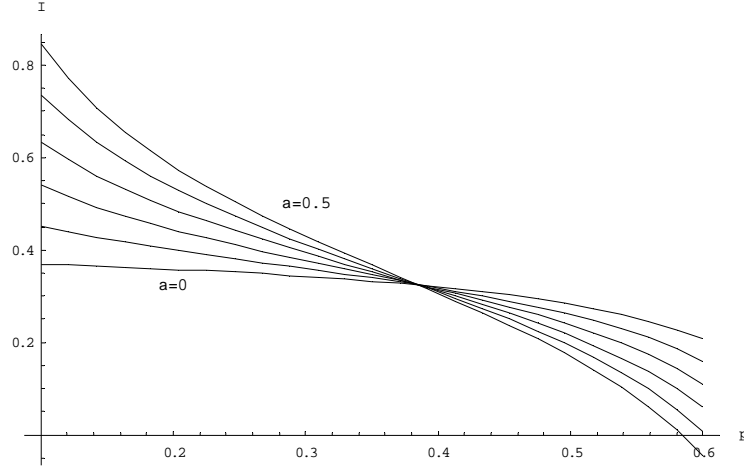


Figure 1: Optimal indemnity as a function of the probability of loss for a regret factor $\alpha = 0, 0.1, 0.2, 0.3, 0.4$ and 0.5 .

intensity of regret is large, it may be optimal to overinsure the risk, i.e. to select an insurance contract promising an indemnity larger than the loss.⁴

4 Asset prices

From our analysis of the characteristics of the optimal individual portfolios, we can easily derive an asset pricing formula by assuming that all consumers in the economy have the same utility function U and the same state-dependent endowment $(\omega_1, \dots, \omega_S)$. In that case, autarky must be a competitive equilibrium. Plugging the market-clearing conditions $x_s = \omega_s$ for all s

⁴There is no asymmetric information in our model. This implies that we do not restrict the indemnity to be nonnegative and smaller than the loss. If we imposed this constraint, the maximum consumption in the no-loss state would be the initial wealth, whereas it would be the initial wealth minus the full insurance premium in the loss state. Because the second is larger than the first, regret would always work in favor of partial insurance under this constraint, as shown by Braun and Muermann (2004).

in the first-order condition (7) yields

$$U_x(\omega_s, \frac{w}{p_s \pi_s}) = \lambda \pi_s. \quad (12)$$

The competitive price per unit of probability of the Arrow-Debreu security associated to state s is given by $\pi_s = \pi(\omega_s, p_s)$, where function π solves the following equation

$$U_x(\omega, \frac{w}{p\pi(\omega, p)}) = \lambda \pi(\omega, p) \quad (13)$$

for all (ω, p) . The left-hand side of this equation is decreasing in π whereas its right-hand side is increasing in π . Thus the solution of this equation is unique. Function π is usually referred to as the pricing kernel, as it allows to price any asset in the economy. Notice also that $1/\pi_s$ measures the expected return of the investment in the Arrow-Debreu security s , whereas $1/\Pi_s$ measures the return of the Arrow-Debreu security conditional to the occurrence of state s .

Without regret ($U_{xy} \equiv 0$), it is well-known that diversifiable risks are actuarially priced at equilibrium, a property of asset prices that induces investors to eliminate diversifiable risks from their portfolio. Technically, this means that, without regret, if there are two states (s, s') such that $\omega_s = \omega_{s'}$, then $\pi_s = \pi_{s'}$. Thus, without regret, the price kernel is independent of the state probability. This property does not hold in general when investors are sensitive to regret. Indeed, totally differentiating equation (13) yields

$$U_{xx}d\omega - U_{xy}\frac{w}{p\pi}\frac{d\pi}{\pi} - U_{xy}\frac{w}{p\pi}\frac{dp}{p} = \lambda d\pi.$$

Eliminating λ from this equation by using (13) and dividing by U_{xx} yields the following result.

Proposition 3 *The equilibrium price per unit of probability π_s of the Arrow-Debreu security associated to state s equals $\pi(\omega_s, p_s)$, where the function π is defined by equation (13). It satisfies the following property:*

$$-\frac{d\pi}{\pi} = [T + \Gamma]^{-1} \left(d\omega + \Gamma \frac{dp}{p} \right), \quad (14)$$

where T and Γ are respectively the absolute risk tolerance and the intensity of regret, evaluated at $(\omega, w/p\pi(\omega, p))$. As a consequence, we obtain that

1. *the price kernel π is independent of the state probability only if the representative agent is insensitive to regret;*
2. *the price kernel π is decreasing in the state probability when the representative agent is sensitive to regret: regret increases the price of assets whose returns are skewed towards low-probability states.*

When the representative agent feels regret, diversifiable risks are actuarially priced only if the involved states of nature are equally likely. States that are more likely to occur have a smaller state price per unit of probability. Because the forgone alternative $w/p\pi$ is larger in less likely states, the increased demand for consumption in these states must be compensated at equilibrium by an increase in the corresponding state prices. Longshots get a negative risk premium. By equation (14), an increase in the state probability by 1% has an effect on π that is equivalent to an increase in the state wealth by 0.01Γ . In the special case with $U(x, y) = x^{1-\gamma}y^\alpha/(1-\gamma)$, yielding $\Gamma(x, y) = \alpha x/\gamma$, an increase in the state probability by 1% has an effect on π equivalent to an increase in wealth ω by $\alpha/\gamma\%$.

4.1 The equity premium when the macroeconomic risk is small

Let us now compute the equity premium in such an economy. Equity is a right on the wealth per capita. By a standard arbitrage argument, the price of equity must equal $E\omega\pi T$. Similarly, the gross risk-free rate must equal $(E\pi)^{-1}$. The equity premium is thus equal to $EP = (E\omega E\pi/E\omega\pi) - 1$. Let $p(\omega)$ denote the probability that the wealth per capita be equal to ω . Suppose that $\omega = 1 + \varepsilon$, where ε is a zero-mean small risk whose support is in a small neighborhood of 0. Using a first-order approximation for π , we obtain the following standard approximation of the equity premium $EP \simeq -\pi^{-1}(d\pi/d\omega)Var(\omega)$, where π and its derivative is evaluated at the expected final wealth $\omega = 1$. Using (14), the derivative of the log state price with respect to wealth equals $-(1 + \Gamma p'/p)/(T + \Gamma)$, which implies in turn that

$$EP = \frac{1 + \Gamma \frac{p'(\omega)}{p(\omega)}}{T + \Gamma} Var(\omega), \quad (15)$$

where T and Γ are evaluated at $(1, w/p(\omega)\pi)$. In the classical case where the representative agent is insensitive to regret ($\Gamma = 0$), the equity premium is proportional to the absolute risk aversion $1/T$. When the representative agent is sensitive to regret, the right-hand side of equality (15) is decreasing in the intensity of regret Γ when p'/p is smaller than the index of absolute risk aversion $1/T$.

Proposition 4 *Consider two economies with the same distribution p of wealth per capita, one of which has a representative agent who is insensitive to regret. Consider a state ω for which the degrees of risk tolerance T are the same in the two economies. If $p'(\omega)/p(\omega)$ is smaller (larger) than absolute risk aversion $1/T$, then*

1. *the log state price per unit of probability is less (more) sensitive to differences in state wealth in the economy with regret .*
2. *if the macroeconomic risk is small, the equity premium is smaller (larger) in the economy with regret.*

This implies in particular that the equity premium is reduced by regret if state probabilities are monotonically decreasing in state wealth. The intuition of this result is simple. When wealthy states are relatively unlikely, their state prices Π must be low. This raises the forgone best alternative in these states, thereby raising the demand for these contingent claims. As a consequence, the corresponding state prices do not need to be reduced as much as they must be in a regret-free economy to induce people to consume more in these states. This reduces the equity premium. This is in fact another illustration of the long odds bias. It thus appears that the skewness of the distribution of ω is important to determine the equity premium. When the distribution of wealth is positively skewed, then its mode is smaller than its expectation. Therefore $p(\omega)$ decreases at the expected wealth level, and the result above applied directly: regret reduces the equity premium.

This intuition on the role of skewness is confirmed in the special case of the multiplicatively CRRA separable utility function when the state wealth per capita is lognormally distributed. An analytical solution for the equity premium is obtained in this case.

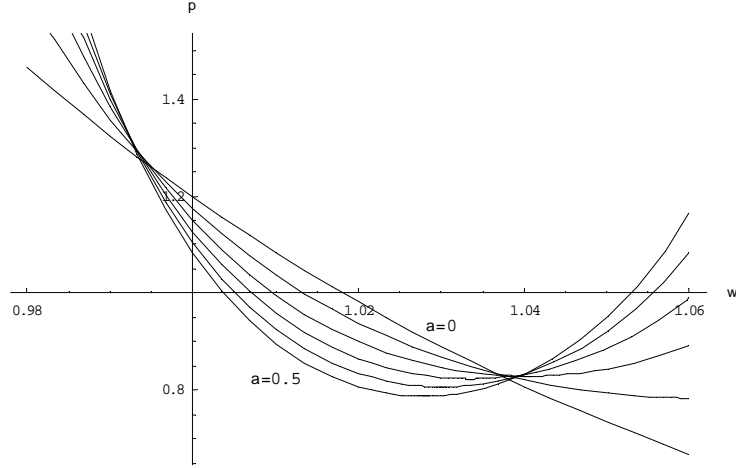


Figure 2: The pricing kernel when the log consumption is normally distributed with $\mu = \sigma = 2\%$, and $U(x, y) = -x^{-1}y^\alpha$, for $\alpha = 0, 0.1, 0.2, 0.3, 0.4$, and 0.5 .

Proposition 5 *Suppose that the representative agent has a multiplicatively separable CRRA utility function $U(x, y) = x^{1-\gamma}y^\alpha/(1-\gamma)$, and that the aggregate wealth in the economy is lognormally distributed: $\log \omega \sim N(\mu, \sigma^2)$. The equity premium in such an economy is such that $\ln(1 + EP) = (\gamma - 1.5\alpha)\sigma^2$.*

Proof: See the Appendix.

This extends the well-known result that in such an economy, the equity premium equals the product of relative risk aversion γ by the variance of log consumption σ^2 when there is no feeling of regret. The increase in the measure of regret α by Δ has the same effect on the equity premium than a reduction in relative risk aversion by $3\Delta/2$. This reduction in the equity premium is not due to a uniform reduction in the derivative of $-\log \pi$. As seen in Figure 2 where the pricing kernel is drawn for different values of α . In this figure, we assumed that $\mu = \sigma = 2\%$, and $\gamma = 2$.

Note that positive skewness is not necessary for regret to reduce the equity premium, since the same mechanism is still at play when states are equally likely. Because of risk aversion, state prices Π remain decreasing in

the state wealth in that case. Regret then raises the demand for wealthier states, thereby reducing the equity premium. The state probabilities must be sufficiently increasing in the state wealth to compensate this effect in order to reverse the result. Because $\log \pi$ decreases at rate $1/T$ without regret, it must be that $\log p$ increases at least at that rate to guarantee that $\Pi = p\pi$ be increasing in state wealth.

4.2 The equity premium with a logconcave macro risk

In Figure 2, we see that regret convexifies the pricing kernel. This will always be the case when the distribution of growth is logconcave, and absolute risk aversion is non-decreasing. This is stated in the following proposition.

Proposition 6 *Suppose that the consumption per capita is logconcave distributed. Suppose also that the index of absolute risk tolerance is independent of the forgone alternative and is non-increasing. Then, regret convexifies the price kernel in the sense that there exists a critical consumption $\bar{\omega}$ such that $-\pi^{-1}d\pi/d\omega$ is increased by regret if $\omega < \bar{\omega}$, and it is reduced by regret if $\omega > \bar{\omega}$.*

Proof: In the absence of regret, the slope of the price kernel is given by

$$-\frac{d\pi}{\pi d\omega} = \frac{1}{T(\omega)}$$

Consider an economy with regret, but with the same regret-independent absolute risk tolerance function $T(\omega)$. By equation (14), the price kernel in such an economy is such that

$$-\frac{d\pi}{\pi d\omega} = [T(\omega) + \Gamma]^{-1} \left(1 + \Gamma \frac{p'(\omega)}{p(\omega)} \right),$$

where Γ is evaluated at $(\omega, w/p\pi(\omega, p))$. When comparing these two equations, we see that regret raises the absolute value of the slope $-\pi^{-1}d\pi/d\omega$ if $p'(\omega)/p(\omega)$ is larger than $1/T(\omega)$. Because p'/p is assumed to be decreasing in ω and $1/T$ is assumed to be nondecreasing in ω , there exists a critical $\bar{\omega}$ such that $p'(\omega)/p(\omega) \geq 1/T(\omega)$ for all $\omega \leq \bar{\omega}$, and $p'(\omega)/p(\omega) \leq 1/T(\omega)$ otherwise. ■

When the distribution of state consumption is not logconcave, when the utility function U is not multiplicatively separable, or when absolute risk aversion is decreasing (as is usually assumed), more complex transformations of the pricing kernel due regret are possible. Rosenberg and Engle (2002) empirically estimated the pricing kernel by using financial data from options markets. They observed that the slope of the empirical price kernel at the lower tail is much steeper than what would be obtained in the classical model with a CRRA regret-free model. This is compatible with the idea that investors feel much regret in these low-wealth small-probability states. However, the oscillation of the price kernel observed by these authors is compatible with our model only if absolute risk aversion is sufficiently decreasing, a condition that would require an unrealistically large γ .

5 Efficient risk sharing and aggregation of heterogeneous preferences

In this section, we characterize the equilibrium price kernel when agents have heterogeneous preferences. This raises the questions of the preferences of the representative agent and of the allocation of risks in the economy. Wilson (1968) and Constantinides (1982) performed such task in the classical case of expected utility. The problem is made more complex in our model because preferences are affected by prices.

There are N agents in the economy. Agent i , $i = 1, \dots, N$, is characterized by a bivariate utility function U^i and a state-dependent endowment $(\omega_1^i, \dots, \omega_S^i)$. The competitive equilibrium is characterized by a pricing kernel $\pi(\omega, p)$ and N portfolio choice functions (x^1, \dots, x^N) such that $\pi_s = \pi(\omega_s, p_s)$ for all s , and $x_s^i = x^i(\omega_s, p_s)$. It is defined by the standard optimality conditions of the price-taker investors, together with the following market-clearing conditions:

$$\frac{1}{N} \sum_{i=1}^N x_s^i = \omega_s =_{def} \frac{1}{N} \sum_{i=1}^N \omega_s^i, \quad (16)$$

for $s = 1, \dots, S$. Totally differentiating equation (7) and eliminating λ^i yields

$$dx^i = - (T^i + \Gamma^i) \frac{d\pi}{\pi} - \Gamma^i \frac{dp}{p}, \quad (17)$$

where $T^i = -U_x^i/U_{xx}^i$ and $\Gamma^i = -yU_{xy}^i/U_{xx}^i$ are evaluated at $(x^i(\omega, p), w^i/p\pi(\omega, p))$. Totally differentiating the market-clearing condition (16) yields

$$\frac{1}{N} \sum_{i=1}^N dx^i = d\omega.$$

Combining these two condition directly proves the following proposition.

Proposition 7 *The competitive equilibrium is characterized by functions (π, x^1, \dots, x^N) such that $\pi_s = \pi(\omega_s, p_s)$ and $x_s^i = x^i(\omega_s, p_s)$ for all $s = 1, \dots, S$ and all $i = 1, \dots, N$. These functions satisfy the following conditions:*

$$-\frac{d\pi}{\pi} = [\bar{T} + \bar{\Gamma}]^{-1} \left(d\omega + \bar{\Gamma} \frac{dp}{p} \right) \quad (18)$$

and

$$dx^i = \frac{T^i + \Gamma^i}{\bar{T} + \bar{\Gamma}} d\omega + \frac{T^i \bar{\Gamma} - \bar{T} \Gamma^i}{\bar{T} + \bar{\Gamma}} \frac{dp}{p}, \quad (19)$$

where T^i and Γ^i are evaluated at $(x^i(\omega, p), w^i/p\pi(\omega, p))$ and

$$\bar{T}(\omega, p) = \frac{1}{N} \sum_{i=1}^N T^i \left(x^i(\omega, p), \frac{w^i}{p\pi(\omega, p)} \right) \quad (20)$$

$$\bar{\Gamma}(\omega, p) = \frac{1}{N} \sum_{i=1}^N \Gamma^i \left(x^i(\omega, p), \frac{w^i}{p\pi(\omega, p)} \right). \quad (21)$$

Equation (18) generalizes the asset pricing formula (14) to heterogenous preferences. This generalization is particularly straightforward since we just need to replace the absolute risk tolerance T and the intensity of regret Γ by their means \bar{T} and $\bar{\Gamma}$ defined by equations (20) and (21). In other words, the asset prices of this heterogenous economy can be duplicated in an economy with identical agents endowed with absolute risk tolerance \bar{T} and intensity of regret $\bar{\Gamma}$. Wilson (1968) already observed that the absolute risk tolerance of a group equals the mean absolute risk tolerance of its members. The above proposition also states that the intensity of regret of a group equals the mean intensity of regret of its members.

Equation (19) characterizes the competitive sharing of risks in the economy. When all states are equally likely, the share of the macroeconomic risk that is borne by agent i is proportional to $T^i + \Gamma^i$, the sum of this agent's risk tolerance and intensity of regret. Because state prices are decreasing with ω (equation (18)), regret is increasing with ω . Therefore, consumers that are relatively more sensitive to regret are more willing to accept the macroeconomic risk.

The second term in the right-hand side of equation (19) tells us who in the population will bet on long shots. The demand x^i for contingent claim s is decreasing in p if $T^i\bar{\Gamma} - \bar{T}\Gamma^i$ is negative, i.e., if Γ^i/T^i is larger than $\bar{\Gamma}/\bar{T}$. The bettors on long shots are those whose intensity of regret Γ^i expressed as a percentage of their risk tolerance T^i is larger than the mean intensity of regret expressed as a percentage of the mean risk tolerance in the economy. Seen from a different angle, equation (19) tells us that regret-free insurers will supply insurance coverage for low-probability event, and they will get a premium for that.

Notice that in general T^i , Γ^i , \bar{T} and $\bar{\Gamma}$ are state-dependent, which means that the above analysis must be interpreted locally. The only case where they are state-independent is when $U^i(x, y) = -y^{\alpha^i} \exp(-x/t^i)$, implying

$$T^i = t^i, \quad \Gamma^i = \alpha^i t^i, \quad \bar{T} = \frac{1}{N} \sum_{i=1}^N t^i, \quad \text{and} \quad \bar{\Gamma} = \frac{1}{N} \sum_{i=1}^N \alpha^i t^i.$$

In that case, the share of the macroeconomic risk borne by agent i is proportional to $t^i(1 + \alpha^i)$. Agents with an α^i larger than $\Sigma_j \alpha^j t^j / \Sigma_j t^j$ bet on longshots, whereas the others bet on favorites.

6 Conclusion

The starting point of our analysis is that most decision makers find it difficult to evaluate the benefit of an outcome ex post without taking into account what they could have obtained if another decision would have been made ex ante. This is taken into account in our model by assuming that the von Neumann-Morgenstern utility is a function of the forgone best alternative. We say that an agent is sensitive to regret if the marginal utility of his consumption is increasing in the forgone best alternative. We showed

that regret-sensitive investors tend to bias their portfolio towards assets that perform particularly well in low-probability states. In terms of asset prices, this implies that these assets yield risk premia that are smaller than in the classical asset pricing formula.

The effect of regret on the equity premium depends upon the distribution of the macroeconomic risk. If the density function is decreasing with aggregate wealth, the equity premium is unambiguously decreasing with the intensity of regret of the representative agent. When the distribution of the macroeconomic risk is logconcave, the price kernel is convexified by regret. We get an explicit formula for the equity premium when it is lognormally distributed and the bivariate utility is a power multiplicatively separable function. The equity premium is also decreasing with the intensity of regret in that case, thereby suggesting that positively skewed risks have a smaller risk premium at equilibrium.

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Appendix: Proof of Proposition 5

We first prove the following Lemma.

Lemma 1 *Suppose that ω is lognormally distributed: $\log \omega \sim N(\mu, \sigma^2)$. It implies that*

$$\int_0^\infty \omega^a p(\omega)^b d\omega = \frac{(2\pi)^{0.5(1-b)} \sigma^{1-b}}{\sqrt{b}} \exp \left[\mu(a+1-b) + \frac{(a+1-b)^2 \sigma^2}{2b} \right], \quad (22)$$

where $p(\omega) = (\omega \sigma \sqrt{2\pi})^{-1} \exp [-(\log \omega - \mu)^2 / 2\sigma^2]$ is the density function of ω .

Proof: We have that

$$\int_0^\infty \omega^a p(\omega)^b d\omega = \left(\sigma \sqrt{2\pi} \right)^{-b} \int_0^\infty \omega^{a+1-b} \exp \left[-\frac{b(\log \omega - \mu)^2}{2\sigma^2} \right] \frac{d\omega}{\omega}.$$

If $W = \log \omega$ and $\hat{\sigma}^2 = \sigma^2/b$, this is equivalent to

$$\int_0^\infty \omega^a p(\omega)^b d\omega = \left(\sigma \sqrt{2\pi} \right)^{-b} \int_0^\infty \exp \left[(a+1-b)W - \frac{(W-\mu)^2}{2\hat{\sigma}^2} \right] dW,$$

or, equivalently,

$$\begin{aligned} \int_0^\infty \omega^a p(\omega)^b d\omega &= \left(\sigma \sqrt{2\pi} \right)^{-b} \times \\ &\int_0^\infty \exp \left[-\frac{(W-\mu-\hat{\sigma}^2(a+1-b))^2}{2\hat{\sigma}^2} + \mu(a-1+b) - 0.5\hat{\sigma}^2(a+1-b)^2 \right] dW. \end{aligned}$$

Now, observe that

$$\int_0^\infty \exp \left[-\frac{(W-\mu-\hat{\sigma}^2(a+1-b))^2}{2\hat{\sigma}^2} \right] dW = \hat{\sigma} \sqrt{2\pi}. \quad (23)$$

Combining the last two equations yields (22). ■

This Lemma implies that

$$\int_0^\infty \omega p(\omega) d\omega = \exp \left[\mu + \frac{\sigma^2}{2} \right],$$

$$\int_0 \omega^{\frac{-\gamma}{1+\alpha}} p(\omega)^{\frac{1}{1+\alpha}} d\omega = (\sigma\sqrt{2\pi})^{\frac{\alpha}{(1+\alpha)}} \sqrt{1+\alpha} \exp \left[\mu \frac{\alpha - \gamma}{1+\alpha} + \frac{(\alpha - \gamma)^2 \sigma^2}{2(1+\alpha)} \right],$$

and

$$\int_0 \omega^{1-\frac{\gamma}{1+\alpha}} p(\omega)^{\frac{1}{1+\alpha}} d\omega = (\sigma\sqrt{2\pi})^{\frac{\alpha}{(1+\alpha)}} \sqrt{1+\alpha} \exp \left[\mu \frac{1+2\alpha-\gamma}{1+\alpha} + \frac{(1+2\alpha-\gamma)^2 \sigma^2}{2(1+\alpha)} \right].$$

The equity premium equals

$$EP = \frac{\left[\int_0 \omega p(\omega) d\omega \right] \left[\int_0 \omega^{\frac{-\gamma}{1+\alpha}} p(\omega)^{\frac{1}{1+\alpha}} d\omega \right]}{\int_0 \omega^{1-\frac{\gamma}{1+\alpha}} p(\omega)^{\frac{1}{1+\alpha}} d\omega} - 1.$$

Combining the last 4 equations yields

$$\ln(EP+1) = \mu + \frac{\sigma^2}{2} + \mu \frac{\alpha - \gamma}{1+\alpha} + \frac{(\alpha - \gamma)^2 \sigma^2}{2(1+\alpha)} - \mu \frac{1+2\alpha-\gamma}{1+\alpha} - \frac{(1+2\alpha-\gamma)^2 \sigma^2}{2(1+\alpha)}.$$

After some tedious simplifications, we obtain that

$$\ln(EP+1) = \sigma^2 \left[\gamma - \frac{3}{2}\alpha \right].$$

This concludes the proof of Proposition 5. ■