Individual decisions under risk, risk sharing and asset prices with regret

"Risk and Choice: A Conference in Honor of Louis Eeckhoudt"

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Regret: An hypothetical example

- An urn contains 60 balls, with 59 black balls and 1 white ball.
- A ball is drawn from the urn. If its color is white, Louis gets 60 bottles of Belgian beer.
- Suppose that Louis has the possibility to exchange this lottery by a contract in which he gets 1 bottle with certainty.

Regret

- Bell (1982) and Loomes and Sugden (1982).
- Regret is a psychological reaction to making a wrong decision, where "wrong" or "good" is determined by using the ex post information, rather than the ex ante one.
- Translated in utility theory: Bivariate utility function:

$$U(x, y)$$

Utility
in state s

Actual payoff
in state s

Maximum payoff that one could have
obtained in state s
“forgone best alternative”

We need a notion of regret *aversion*: I apply Eeckhoudt-Rey-Schlesinger’s notion of correlation loving: I prefer to have a larger actual payoff when the forgone best alternative is larger.

A tiny bibliography

- Braun and Muermann (2004): insurance demand.
- Muermann, Mitchell and Volkman (2005), and Michenaud and Solnik (2006): assets demand.
- *Typically using very specific bivariate utility functions.*
- Regret aversion can either mitigate or exacerbate risk aversion.
- Regret aversion adjusts away from the extreme choices: no/full insurance, 100% equity/100% bonds.
- *Importance of the choice set.*

Expected utility with regret

- We assume that ex ante, the agent selects the action that maximizes his expected utility.
- $U(x,y)$: utility level when the actual payoff is x and the "forgone best alternative" is y.
- $U_{xx}<0$: Aversion to risk: U_x sensitive to x.
- $U_y<0$: Sensitiveness to regret.
- $U_{xy}>0$: Aversion to regret: U_x sensitive to y.
- Index of relative regret aversion: When y is increased by 1%, by how much should I increase x to maintain U_x unchanged?

$$\Gamma(x,y) = -\frac{yU_{xy}(x,y)}{U_{xx}(x,y)}$$

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A simple example

- Fair bet b on a horse whose winning probability is p.
- Binary choice: bet b or nothing. It is optimal to bet if and only if

$$pU\left(b\tfrac{1-p}{p}, b\tfrac{1-p}{p}\right) + (1-p)U(-b,0) \geq pU\left(0, b\tfrac{1-p}{p}\right) + (1-p)U(0,0).$$

- Two states with regret.
- If p is small, regret is particularly severe if not betting and horse wins.
- b small: betting is optimal iff $\Gamma(0,0) \geq 1/2(1-p)$, i.e., if p is small.
- The effect of regret aversion is maximum for a longshot horse.

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The Arrow-Debreu portfolio problem

$$\max_{x_1,\ldots,x_S} \sum\nolimits_{s=1}^{S} p_s U\left(x_s, \frac{w}{\Pi_s}\right) \quad s.t. \quad \sum\nolimits_{s=1}^{S} \Pi_s (x_s - \omega_s) = 0$$

- Some results:
  - If prices are fair,
    - full insurance is optimal if and only if all states are equally likely;
    - the demand for AD securities is inversely related to the state probability (longshot bias);
  - Preference for skewed lotteries. Optimality of the joint purchase of
    - insurance of (very) unlikely events at an unfair price;
    - an unfavorable lottery ticket with a low probability of a high payoff .

In honor of Louis:
A simple insurance model with 2 states!

- Two states, $\omega_1=1$ and $\omega_2=1-L$.
- Loading factor: k.

$$\begin{aligned} \Pi_1 &= 1-p(1+k); & \pi_1 &= 1-k\tfrac{p}{1-p} \\ \Pi_2 &= p(1+k); & \pi_2 &= 1+k \end{aligned}$$

- Utility function:

$$U(x,y)=\frac{x^{1-\gamma}y^{\alpha}}{1-\gamma} \quad \Rightarrow \quad \Gamma(x,y)=\frac{\alpha x}{\gamma}$$

- Case $\alpha=0$: Eeckhoudt-Gollier (1999): the demand for insurance is decreasing in the probability of loss.

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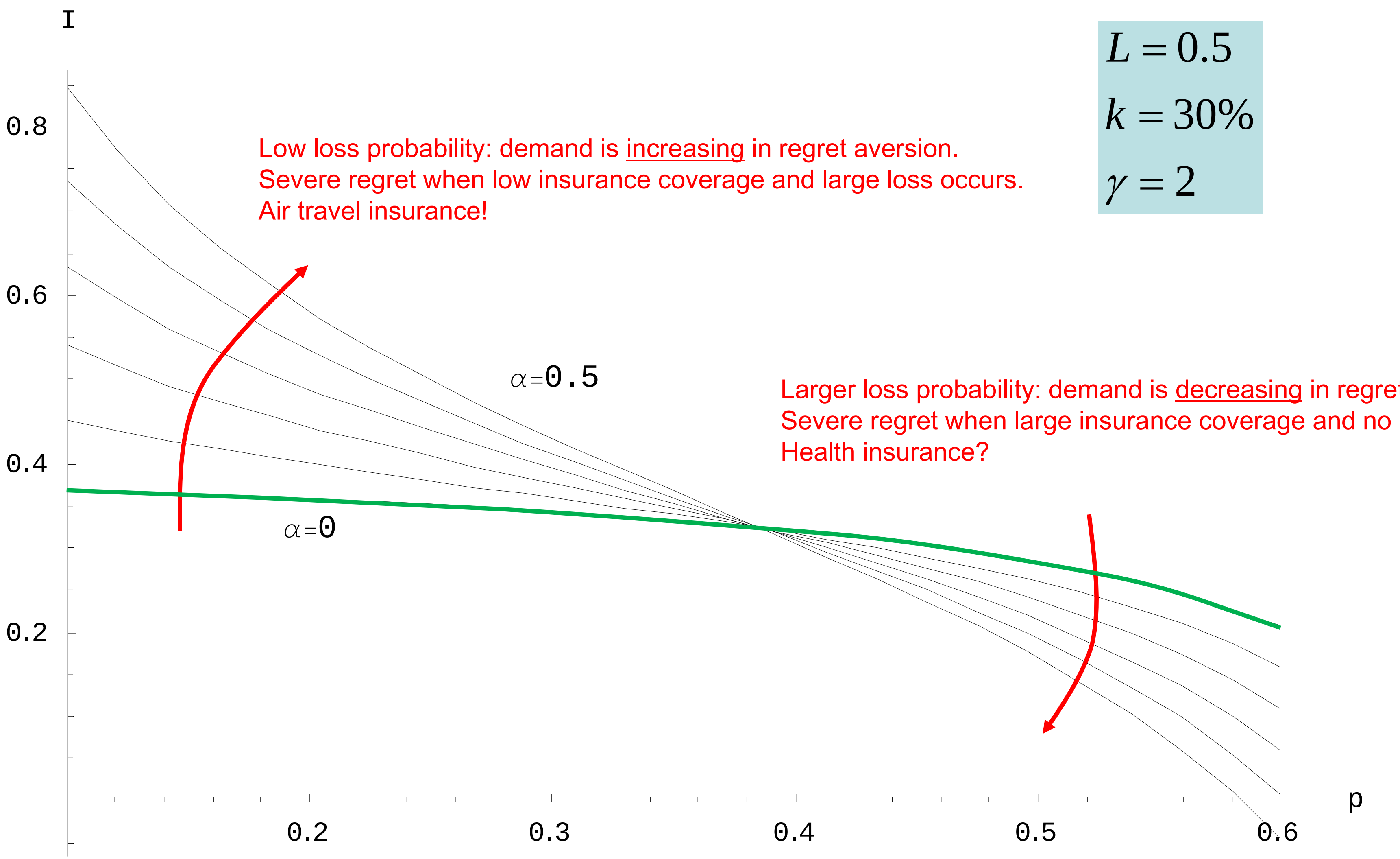
An example



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The equity premium with lognormal growth of GDP

- In order to explain the equity premium puzzle, we need that equity returns be negatively skewed.
- Suppose that *log(w)* is $N(\mu,\sigma^2)$. This is positively skewed.
- Suppose also that:

$$U(x,y)=\frac{x^{1-\gamma}y^{\alpha}}{1-\gamma} \quad \Rightarrow \quad \Gamma(x,y)=\frac{\alpha x}{\gamma}$$

- This is a case where an analytical solution exists:

$$EP=(\gamma-1.5\alpha)\sigma^2$$

- Regret aversion reduces the equity premium.

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Take-home messages

- Regret aversion means preference for a positive correlation between actual payoffs and forgone best alternatives.
- Regret aversion induce risk taking in favor of longshots and positively skewed risks.
- Regret aversion could explain the equity premium puzzle only if the macro risk is negatively skewed.
- Other results in the paper: explicit formula for assets demand, risk sharing, aggregation of preferences, …