BILATERAL BARGAINING AS A DOUBLE AUCTION: THE CASE OF FIRMS AND WORKERS IN DENMARK*

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Abstract

We employ a simple two-person bargaining model to interpret wage data—demands (offers) by workers (firms) and acceptances by firms (workers)—as the equilibrium outcome at a double auction. Under two polar-extreme bargaining solutions, we develop a strategy to recover estimates of the marginal-productivity and the opportunity-cost distributions. We then implement this framework using particularly rich data from a sample of Danish firms and workers. Subsequently, we use our estimates to measure the cost of the inefficiencies that arise from the bilateral-monopoly problem under the two alterative bargaining solutions.

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1. Introduction and Motivation

A quarter of a century ago, a substantial portion of economic research concerned the nature of the employment contract, specifically how the existence of asymmetric information affected the properties of contracts and how these properties could be used to explain outcomes in labour markets. For example, Hall and Lazear (1984) studied the excess sensitivity of layoffs and quits to shocks in labour demand.[1] The main idea of Hall and Lazear was that, under asymmetric information, a layoff may occur when an employer learns of a disppointment in demand which is not public knowledge and which cannot be made a contingency in the employment contract. Alternatively, a quit may occur when a worker learns of favourable developments in the outside market, again in a way that is not public knowledge and for which no contingencies exist in his contract. Thus, after a labour contract has been written, new information held by one party, but unknown to the other, creates a bilateral-monopoly problem, so inefficiencies can obtain. Documenting the incidence and magnitude of such inefficiences is, however, difficult to do under asymmetric information because, in general, it is very difficult to estimate the marginal productivity of a worker or the opportunity cost of his time. In this paper, we develop an empirical framework that allows us to recover estimates of the distributions of worker marginal productivity and opportunity cost as well as to gain some insights concerning factors that influence these distributions.

Following Hall and Lazear (1984), we employ a simple two-person bargaining model, due to Chatterjee and Samuelson (1983), to interpret wage data—demands (offers) by workers (firms) and acceptances by firms (workers)—as the equilbrium outcome at a double auction. Subsequently, we apply methods from the literature concerned with the structural econometric analysis of field data from auctions to develop an estimation strategy. We then implement this framework using particularly rich data from a sample of Danish firms and workers. Because the asymptotic distribution of these estimates is nonstandard, we calculate confidence intervals for our parameter estimates via the parametric bootstrap.

[1] In their paper, Hall and Lazear discussed carefully much of the earlier research concerned with the bilateral-monopoly problem encountered by the firm and the worker, once an employment contract has obtained.

Within the European context, the Danish labour market is characterized by considerable flexibility both in wage setting and regarding employment separations. Specifically, in Denmark, collective wage bargaining was abandoned by most labour unions in the early 1990s, being replaced by individual wage bargaining at the firm level, especially for white-collar workers. This relatively recent development suggests that the bargaining process may be well-approximated by the double-auction model we develop below.

Our empirical work is based on a particularly rich data source, the "Ever-Private-Sector" database created by Statistics Denmark and administered by the Center for Corporate Performance at the Aarhus School of Business. This database links employees to their employers and contains detailed wage information as well as a host of variables related to demographic characteristics and labour-market outcomes.

We use this data set to implement the empirical framework that we have developed. Based on our estimates of the distributions of marginal productivity and the opportunity cost of time in alternative uses, we estimate that the inefficiencies that obtain are on the order of ten percent. We also investigate how sensitive our inefficiency measures are to outliers, which appear important in these data.

Our empirical framework is closely related to, but different from, the research reported by Elyakime, Laffont, Loisel, and Vuong (1997). In that paper, data from first-price, sealed-bid auctions of timber in the southwest of France were examined. Elyakime *et al.* noted that, in their application, bargaining occurred between the seller and the n potential buyers *after* a first-price, sealed-bid auction. Specifically, the highest bidder won the right to bargain with the seller. In that case, the offers of the potential buyers, their bids, were observed as was the reserve price of the seller. In our application, neither the offer of the firm (the buyer) nor the demand of the worker (the seller) are directly observed. Moreover, under our assumptions concerning the bargaining solution, a wage is observed at the boundary of the Pareto set only if a bargain can be struck. On the other hand, Elyakime *et al.* observed an interior trade and the price of that trade. Thus, some major differences exist between our research and theirs.

We also investigate the importance of firm- and worker-specific heterogeneity in determing wage heterogeneity, which is related to research by Postel-Vinay and Robin (2002) who have studied wage heterogeneity using an equilibrium search model. Postel-Vinay and Robin assumed that firms make take-it-or-leave-it offers to workers, conditional on the characteristics of workers, but that firms can respond to outside offers received by employees. The model of Postel-Vinay and Robin is one of complete information; ours is a model of incomplete, asymmetric information.

Our paper has five more parts: in the next section, we define a notation and then develop a simple model. Subsequently, in section 3, we derive an empirical specification and propose an estimation strategy. In section 4, we present the empirical results obtained from applying our estimation strategy to a sample of Danish firms and workers using particular econometric specifications, while in section 5, we present estimates of the cost of asymmetric information in the Danish labour market. In section 6, we summarize our results and conclude the paper, and in an appendix to the paper we document the development of results too cumbersome or too detailed to be included in the text of the paper.

2. A Simple Model of Bilateral Bargaining

We first define a notation and then develop a simple model. This model was applied by Hall and Lazear (1984) to the case of labour, but was first developed by Chatterjee and Samuelson (1983) in a more general setting. An elaborate survey of these models has been undertaken by Ausubel, Cramton, and Deneckere (2002). In our application of the model, a buyer is a firm and a seller is a worker. We assume that the firm knows the value of the marginal product of the worker at that firm; we denote this B. (In what follows, we shall often just refer to B as the marginal productivity of the worker, for short.) We also assume that the worker knows the opportunity cost of his outside alternative; we denote this S. The letters B and S are pnemonic for the values to the *buyer* and the *seller*.[2] In the model, an asymmetry of information exists: each party knows his own valuation, but regards the other party's valuation as an independent draw from an atomless distribution. Denote by $F_B(b)$ the cumulative distribution function of B which has support on $[\underline{b}, \bar{b}]$ where

[2] Note that, with minor modification, this framework can be used in any situation that has a single seller of an indivisible good who faces a single potential buyer.

$f_B(b)$ denotes the corresponding probability density function. Similarly, denote by $F_S(s)$ the cumulative distribution function of S which has support on $[\underline{s}, \bar{s}]$ where $f_S(s)$ denotes the corresponding probability density function. Realizations of B and S are denoted by b and s, respectively. We assume that $F_B(\cdot)$ and $F_S(\cdot)$ are common knowledge. In addition, we assume that the firm is risk neutral, while the worker is risk averse, having a von Neumann–Morgenstern utility function U over prospect x which exhibits constant absolute risk aversion (CARA) α, so

$$U(x) = 1 - \exp(-\alpha x) \quad \alpha > 0.$$

We restrict ourselves to two extreme cases. First, the worker sets the wage by announcing a wage demand; the firm then decides whether to employ the worker at that wage. Second, the firm sets the wage by announcing a wage offer; the worker then decides whether to work at that wage.[3] Hereafter, we refer to the first case as Case 1, and the second as Case 2.

Consider Case 1 first. Here, the worker makes his wage demand and the firm must then decide whether to accept this demand. The expected utility of the worker, whose alternative value of time is s and who demands w, is then

$$U(w)[1 - F_B(w)] + U(s)F_B(w).$$

The first term is the utility when the wage demand w is accepted multiplied by the probability of that event, while the second term is the utility of the alternative value of time s when the demand is not accepted multiplied by the probability of that event. From the objective function, we see that a trade-off exists between a higher wage and the probability that the firm rejects that wage demand. The first-order condition is given by

$$U(w) = U(s) + \frac{[1 - F_B(w)]U'(w)}{f_B(w)}.$$

Inserting the CARA utility function, we obtain

$$[1 - \exp(-\alpha w)] = [1 - \exp(-\alpha s)] + \frac{[1 - F_B(w)]\alpha \exp(-\alpha w)]}{f_B(w)}$$

[3] These cases correspond to k equal zero and k equal one in Chatterjee and Samuelson (1983).

or

$$s = w - \frac{\log\left\{1 + \frac{\alpha[1-F_B(w)]}{f_B(w)}\right\}}{\alpha}.$$

In general,

$$W = \omega_1(S).$$

One can think of $\omega_1(\cdot)$ as a function which maps unobserved S into sometimes observed W. Here, the subscript 1 indicates that we are dealing with Case 1, while later we shall examine Case 2 and, then, $\omega_2(\cdot)$ will be used.

Note that the worker always demands more than the opportunity cost of his outside alternatives, so inefficiencies can obtain when the firm refuses some of these demands. When $\{[1 - F_B(b)]/f_B(b)\}$ satisfies the Monotone Hazard-Rate Property (MHRP), this first-order condition has a unique solution. The firm will accept the wage demand if it is below the worker's marginal product and reject when the wage demand is above it. The optimal strategy of a firm at which the worker has marginal productivity b is truth-telling, to reveal b.

To illustrate the mechanics of the solution to this problem, consider an example where B is exponentially distributed, having mean parameter β, so

$$F_B(b) = [1 - \exp(-b/\beta)] \quad b > 0,\ \beta > 0.$$

In this example, the wage-demand function can be solved in closed-form to be

$$W = \omega_1(S) = S + \frac{\log(1 + \alpha\beta)}{\alpha}.$$

The wage-demand function is a constant added to the value of the outside option S. Note, too, that as α gets large, the wage demand gets close to the "full-information" solution where the worker reveals his true value of the outside option.

In Case 2, the firm makes its wage offer w and the worker must then decide whether to accept this offer. The firm at which the worker's marginal productivity is b seeks to maximize expected profit, which is

$$(b - w)F_S(w),$$

the first-order condition for which is

$$w = b - \frac{F_S(w)}{f_S(w)}.$$

When $[F_S(s)/f_S(s)]$ satisfies the Monotone Likelihood-Ratio Property (MLRP), a unique solution to this first-order conditions obtains. The worker must decide whether to accept this offer. If the worker rejects the offer, then he receives the utility of the alternative value of his time $U(s)$, while if he accepts the offer, then he receives the utility of the wage $U(w)$. Hence, the worker will accept the wage offer w when it is above the value of his outside option s. Consequently, truth-telling, revealing the opportunity cost of his time, is the optimal strategy of a worker having an outside option with value s.

To illustrate the mechanics of the solution in this case, suppose $\log S$ is distributed according to a Gumbel distribution, having location parameter μ and scale parameter σ, so

$$F_{\log S}(\log s) = \exp\left\{-\exp\left[-(\log s - \mu)/\sigma\right]\right\}.$$

Now,

$$\begin{aligned} b &= w + \frac{F_S(w)}{f_S(w)} \\ &= w + \frac{wF_{\log S}(\log w)}{f_{\log S}(\log w)} \\ &= w\left[1 + \frac{\sigma}{\exp\left(\frac{\mu - \log w}{\sigma}\right)}\right] \\ &= \omega_2^{-1}(w). \end{aligned}$$

In general, we denote the solution by

$$W = \omega_2(B) \leq B.$$

Note that the firm always offers less than the marginal product of the worker, so inefficiencies can obtain when the worker refuses some of these offers.[4]

[4] Chatterjee and Samuelson (1983) have derived a more general bargaining rule than the ones

3. Econometric Specification and Estimation Strategy

In what follows, we assume that, as researchers, we do not observe the offers made by either party. Instead, we only observe the wage when both parties agree. In the event that the two parties fail to reach an agreement, we assume that we observe a separation and that the worker gets his outside option, either through unemployment benefits or from the wage at his new job.

Below, we describe only our econometric analysis of Case 1—when the worker makes a wage demand and the firm either accepts or rejects this demand—because an analysis of Case 2—when the firm makes an offer and the worker either accepts or rejects this offer—is virtually identical. Of course, the actual bargain may obtain somewhere between these two cases. Later, we present empirical evidence regarding Case 2, while in a section of the appendix we provide a complete derivation of the likelihood function in Case 2.

As mentioned, we only observe the wage when both parties agree. In Case 1, the probability of this event is

$$\Pr[B \geq \omega_1(S)] = \Pr[B - \omega_1(S) \geq 0]$$

where

$$W = \omega_1(S).$$

Of course, the probability of a separation is

$$\Pr[B < \omega_1(S)] = \Pr[B - \omega_1(S) < 0].$$

considered above. Under their rule, the buyer and the seller submit offers, b and s. If b is weakly more than s, then bargaining obtains and the good is sold at a price p equal to $[k(b-s)+s]$ where $k \in [0,1]$. When b is less than s, no sale obtains. Within this general framework, the set of well-behaved equilibrium strategies is characterized by the following pair of linked differential equations:

$$kF_B(y)\sigma'(y) + f_B(y)\sigma(y) = \beta^{-1}([\sigma(y)])f_B(y)$$
$$(1-k)[1-F_S(x)]\beta'(x) - f_S(x)\beta(x) = -\sigma^{-1}[\beta(x)]f_S(x)$$

along with a pair of boundary conditions. Because the current model appears unidentified when $k \in (0,1)$, we do not examine these cases.

Essentially, in the vocabulary of Amemiya (1985), this is a *Type-5 Tobit* model. To see this, let

$$Y = B - \omega_1(S).$$

We can then rewrite the data-generating process (DGP) as

$$Y = B - \omega_1(S);$$

$$W = \begin{cases} \omega_1(S) & \text{if } Y \geq 0 \\ 0 & \text{otherwise;} \end{cases}$$

$$S = \begin{cases} S & \text{if } Y < 0 \\ 0 & \text{otherwise.} \end{cases}$$

Suppose $F_S(\cdot)$ belongs to a parametric family $F_S(\cdot|\boldsymbol{\varphi})$ indexed by the unknown vector $\boldsymbol{\varphi}$ and $F_B(\cdot)$ belongs to another parametric family $F_B(\cdot|\boldsymbol{\lambda})$ indexed by the unknown vector $\boldsymbol{\lambda}$.

Introducing $f_{SY}(s, y)$ to denote the joint density function of S and Y and $f_{WY}(w, y)$ to denote the joint density function of W and Y, the likelihood function is then given by

$$\mathcal{L}_1 = \prod_0 \int_{-\infty}^{0} f_{SY}(s_t, y)\ dy \prod_1 \int_0^{\infty} f_{WY}(w_t, y)\ dy$$

where $\prod_0$ denotes those observations corresponding to separations, while $\prod_1$ denotes those observations corresponding to employed workers. Now, since

$$f_{SY}(s, y) = f_S(s) f_B[y + \omega_1(s)],$$

we have

$$\begin{aligned} \int_{-\infty}^{0} f_{SY}(s, y)\ dy &= \int_{-\infty}^{0} f_S(s) f_B[y + \omega_1(s)]\ dy \\ &= f_S(s) \int_{-\infty}^{0} f_B[y + \omega_1(s)]\ dy \\ &= f_S(s) F_B[\omega_1(s)]. \end{aligned}$$

Consequently,

$$\begin{aligned} \mathcal{L}_1 &= \prod_0 \int_{-\infty}^{0} f_{SY}(s_t, y)\ dy \prod_1 \int_0^{\infty} f_{WY}(w_t, y)\ dy \\ &= \prod_0 f_S(s_t) F_B[\omega_1(s_t)] \prod_1 f_W(w_t)[1 - F_B(w_t)]. \end{aligned} \tag{3.1}$$

Thus far, we have ignored the fact that the support of the observed random variable may depend on the parameters of interest. In the structural econometrics of auctions literature, building on the work of Flinn and Heckman (1982) in the search literature, Donald and Paarsch (1996) solved a similar problem when analyzing data from Dutch or first-price, sealed-bid auctions; we apply an approach similar in spirit to theirs here.

The main idea is to maximize the likelihood function defined by (3.1) subject to the constraints that the observed data could actually have been generated from the estimated DGP. To cast this problem, first collect $\boldsymbol{\varphi}$ and $\boldsymbol{\lambda}$ in the vector $\boldsymbol{\theta}$ which equals $(\boldsymbol{\varphi}^\top, \boldsymbol{\lambda}^\top)^\top$. Characterize the set of feasible values of $\boldsymbol{\theta}$, that are consistent with the data, by

$$\boldsymbol{\Theta}_T^* = \{\boldsymbol{\theta} \in \boldsymbol{\Theta}^0 | \underline{w}(\boldsymbol{\theta}, \boldsymbol{X}_t) \leq w_t \leq \bar{w}(\boldsymbol{\theta}, \boldsymbol{X}_t), t = 1, \ldots, T\}$$

where $\underline{w}(\boldsymbol{\theta}, \boldsymbol{X}_t)$ and $\bar{w}(\boldsymbol{\theta}, \boldsymbol{X}_t)$ denote the lower and the upper support for the distribution of observed wages, respectively. Here, $\boldsymbol{X}_t$ denotes an observation-specific covariate vector that can affect the lower and upper bounds of support through $f_B(\cdot|\boldsymbol{\lambda})$ and $f_S(\cdot|\boldsymbol{\varphi})$. The solution of the maximization problem is then given by

$$\max_{\boldsymbol{\theta}} \ \log \mathcal{L}(\boldsymbol{\theta}) \quad \text{subject to} \quad \boldsymbol{\theta} \in \boldsymbol{\Theta}_T^*.$$

We do not discuss here the asymptotic distribution of this maximum-likelihood estimator, which is nonstandard. However, details concerning it can be found in Donald and Paarsch (1996), Hong (1998), and Chernozhukov and Hong (2004). Suffice it to say that we use simulation methods, specifically the parametric bootstrap, to calculate confidence intervals for our parameter estimates.

4. An Application to the Danish Labour Market

Within the European context, the Danish labour market is characterized by considerable flexibility both in wage setting and regarding employment separations. This makes the Danish labour market a useful "test-site" to examine various theories concerning labour markets. Lay-offs of workers in Denmark are subject to severance payments, *Fratrædelsesgodtgørelse*, which are one month's salary for job tenure less than or equal to twelve years, and up to three months' salary for job tenure above eighteen years. Firms must give reasons for lay-offs, but these reasons are often vague and the procedure is perfunctory. Collective wage bargaining was abandoned by almost all occupations in the late 1980s, having been replaced in the 1990s by individual bargaining at the firm level, especially for white-collar workers. This recent development suggests that the bargaining process may be well-approximated by the double-auction model outlined above.

4.1. Data

Our empirical work is based on the "Ever-Private-Sector" database, created by Statistics Denmark and administered by the Center for Corporate Performance at the Aarhus School of Business. These data are a subsample of the "Integrated Database for Labour Market Research (IDA)," which contains matched employer-employee data generated by Statistics Denmark from various registries; it covers the entire private sector of the Danish economy. The Ever-Private-Sector database involves data from the entire population employed in the private sector during the period 1980 to 2000. Data are collated each November from a variety of private and public sources. In our research, we considered the transition of employees from 1999 to 2000.

In the data, a distinction is made between primary and secondary jobs. A worker's main job is determined by the job which had the highest cumulative wage in a given year. We defined a separation as the end of a job between an employee and an employer. Because minor changes in relative wage income across different jobs for the same person may occur, to avoid misclassifications, we considered only "main job" relationships. Thus, we focused on employees with only one employer-employee relationship in 1999.

Another objective was data reliability. Thus, we focused on full-time employees. By eliminating part-time employees, we hoped to filter out reporting errors that might occur when jobs are changed. We also eliminated any individuals who reported any time devoted to schooling, reasoning that these individuals might have a tenuous connection to the labour market. In addition, we eliminated those who reported themselves as self-employed. Self-employed individuals present a problem for the model developed in section 2—who is the employer and who is the employee? Also, in an effort to decrease heterogeneity in the sample, we eliminated workers who were not members of any labour union, or for whom union membership was not identified in the data because unemployment benefits are conditional on union membership. Finally, in an effort to avoid mismeasuring wages, we also eliminated workers who took temporary leaves. To wit, those workers who did not separate from the firm, but who did receive unemployment benefits or who were registered as unemployed in 2000.

Despite our care in "cleaning" the data, some unusual observations remained. For example, a worker existed who earned just 10DKK per year, less than \$2US, even though he was eligible for unemployment benefits on the order of about 143, 500DKK. To avoid the potential contimination introduced by these observations, which we consider to be recording errors, we trimmed the bottom and top one percent of the earnings distributions.

In Table 4.1, we present descriptive statistics concerning the final data sets that we used with no trimming and one percent trimming.

4.2. Empirical Implementation

We consider Case 1 first. Later, for the purposes of comparison, we examine Case 2. In what follows, we assume that the value of the marginal product of a worker B follows the exponential law, having mean parameter β_t which varies across workers. We assume that

$$\beta_t = \exp(\boldsymbol{x}_t \boldsymbol{\gamma})$$

where $\boldsymbol{x}_t$ is a vector of covariates that is conformable to the unknown parameter vector $\boldsymbol{\gamma}$. We chose the exponential distribution for several reasons. First, simplicity—it is fairly easy to interpret the equilibrium wage demand. Second, computational parsimony. Under the exponential assumption, the cumulative distribution function of B_t, conditional on $\boldsymbol{X}$, is given by

$$F_{B|\boldsymbol{X}}(b_t|\boldsymbol{x}_t) = 1 - \exp\left(b_t/\beta_t\right) \quad 0 < \beta_t.$$

Table 4.1
Sample Descriptive Statistics

	Full Sample 120,336 Observations				One-percent Trimming 117,922 Observations			
	Mean	S.D.	Min.	Max.	Mean	S.D.	Min.	Max.
Demographic Information								
Age	39.69	5.90	30	50	39.68	5.91	30	50
Female	0.47	0.50	0	1	0.48	0.50	0	1
Number of Kids	1.16	1.03	0	8	1.16	1.03	0	8
Residence in Copenhagen	0.31	0.46	0	1	0.31	0.46	0	1
Labor Market Experience	15.96	7.29	0	36	16.02	7.27	0	36
Tenure at Current Firm	7.37	6.44	1	33	7.41	6.45	1	33
Education								
< 12 Years	0.07	0.25	0	1	0.07	0.25	0	1
High School	0.06	0.23	0	1	0.05	0.23	0	1
Vocational	0.38	0.48	0	1	0.38	0.49	0	1
College, BA	0.23	0.42	0	1	0.23	0.42	0	1
University Degree	0.26	0.44	0	1	0.26	0.44	0	1
Industry of Employer								
Manufacturing	0.06	0.23	0	1	0.06	0.23	0	1
Trade	0.10	0.30	0	1	0.10	0.30	0	1
Transport	0.05	0.22	0	1	0.05	0.22	0	1
Service	0.25	0.44	0	1	0.25	0.44	0	1
Education	0.19	0.39	0	1	0.19	0.39	0	1
Other	0.38	0.56	0	2	0.38	0.56	0	2
Separation Rate	0.22	0.42	0	1	0.21	0.41	0	1
Annual Wage in 1000s DKK	347.71	145.27	7	5526	343.85	117.80	52	798

Finally, we assume that the logarithm of the opportunity cost of the outside option is distributed according to a Gumbel distribution, having location parameter μ_t and scale parameter σ, so

$$F_{\log S|\boldsymbol{X}}(\log s_t|\boldsymbol{x}_t) = \exp\left\{-\exp\left[-(\log s_t - \mu_t)/\sigma\right]\right\}$$

where

$$\mu_t = \exp(\boldsymbol{x}_t\boldsymbol{\delta}).$$

Here, $\boldsymbol{x}_t$ is a vector of covariates that is conformable to the unknown parameter vector $\boldsymbol{\delta}$. We also allow α to vary with covariates according to

$$\alpha_t = \exp(\boldsymbol{x}_t\boldsymbol{\psi}).$$

Under these assumptions, the first-order condition for observation t and Case 1 is given by

$$s_t = w_t - \frac{\log\left(1+\alpha_t\beta_t\right)}{\alpha_t} \geq 0,$$

which yields the restrictions on the parameters—the lower bound of support of W_t conditional on $\boldsymbol{x}_t$ depends on both $\boldsymbol{\gamma}$ and $\boldsymbol{\psi}$.

The first-order condition in Case 2 reduces to

$$b_t = w_t\left[1 + \frac{\sigma}{\exp\left(\frac{\mu_t - \log w_t}{\sigma}\right)}\right].$$

4.3. Empirical Results

Using the data set described above, which contains nearly 120 thousand observations, we estimated the exponential/Gumbel specification discussed above for both Case 1 and Case 2 using before-tax wages measured in $100,000$s DKK. Our results for Case 1 are presented in Table 4.2, while those for Case 2 are presented in Table 4.3. The ninety-five percent confidence intervals were calculated using the parametric bootstrap and 100 replicates.

In general, the parameter estimates have the expected signs, the exception being the effect of education on the risk-aversion parameter. A few other things also warrant comment: First, our parameter estimate for the indicator covariate for females is

Table 4.2
Case 1: Parameter Estimates and 95-Percent Confidence Intervals

	Location Parameter: of Outside Option μ	Location Parameter: of Marginal Product λ	Risk-Aversion Parameter α
Covariate			
Constant	-0.322 $(-0.355, -0.292)$	1.317 $(1.228, 1.443)$	1.316 $(1.232, 1.373)$
Age	-0.005 $(-0.006, -0.005)$	0.012 $(0.009, 0.014)$	0.005 $(0.004, 0.008)$
Female	-0.405 $(-0.415, -0.395)$	-0.284 $(-0.312, -0.258)$	-0.115 $(-0.134, -0.099)$
Number of Children	0.039 $(0.035, 0.043)$	0.017 $(0.005, 0.029)$	0.034 $(0.027, 0.041)$
Copenhagen Area	0.123 $(0.116, 0.132)$	-0.052 $(-0.075, -0.022)$	-0.035 $(-0.051, -0.018)$
Labor-Market Experience	0.014 $(0.013, 0.014)$	0.033 $(0.031, 0.036)$	-0.001 $(-0.003, 0.000)$
Tenure at Current Firm	-0.002 $(-0.003, -0.002)$	0.060 $(0.057, 0.063)$	0.015 $(0.014, 0.016)$
Vocational	-0.070 $(-0.084, -0.054)$	0.020 $(-0.018, 0.050)$	0.018 $(-0.007, 0.045)$
College, BA	0.233 $(0.219, 0.247)$	0.173 $(0.113, 0.220)$	-0.160 $(-0.187, -0.133)$
University Degree	0.273 $(0.258, 0.287)$	0.275 $(0.230, 0.332)$	-0.552 $(-0.573, -0.520)$
Manufacturing	0.121 $(0.105, 0.137)$	0.252 $(0.193, 0.311)$	0.214 $(0.167, 0.260)$
Trade	0.119 $(0.104, 0.134)$	0.141 $(0.088, 0.184)$	0.556 $(0.511, 0.597)$
Transport	-0.033 $(-0.055, -0.007)$	0.208 $(0.131, 0.272)$	0.192 $(0.151, 0.233)$
Education	-0.499 $(-0.526, -0.477)$	0.097 $(0.063, 0.131)$	-0.121 $(-0.143, -0.097)$
Other	0.057 $(0.051, 0.067)$	-0.019 $(-0.047, 0.009)$	0.101 $(0.085, 0.120)$
Logarithm of Variance Parameter for Outside Option	-0.957 $(-0.965, -0.951)$		

negative, both on the value of the outside option and on the marginal product. In addition, we estimate a negative relationship on the risk-aversion parameter. Second, to the extent that tenure at the firm measures firm-specific human capital, we find a positive relationship with the marginal product, but hardly any effect on the value of the outside option. Tenure also seems to have a positive relationship with the risk-aversion parameter. Third, living in the Copenhagen area appears to have a

Table 4.3
Case 2: Parameter Estimates and 95-Percent Confidence Intervals

	Location Parameter: of Outside Option μ		Location Parameter: of Marginal Product λ	
Covariate				
Constant	-0.288	$(-0.371, -0.225)$	1.687	$(1.618, 1.783)$
Age	-0.017	$(-0.019, -0.015)$	0.025	$(0.023, 0.027)$
Female	-0.335	$(-0.355, -0.310)$	-0.486	$(-0.507, -0.468)$
Number of Children	0.045	$(0.037, 0.053)$	0.016	$(0.008, 0.027)$
Copenhagen Area	0.265	$(0.248, 0.279)$	0.001	$(-0.015, 0.018)$
Labor Market Experience	0.009	$(0.008, 0.011)$	0.013	$(0.012, 0.015)$
Tenure at Current Firm	-0.027	$(-0.029, -0.024)$	0.017	$(0.015, 0.018)$
Vocational	-0.176	$(-0.211, -0.146)$	-0.138	$(-0.164, -0.110)$
College, BA	0.284	$(0.257, 0.314)$	0.003	$(-0.035, 0.039)$
University Degree	0.176	$(0.146, 0.210)$	0.563	$(0.526, 0.595)$
Manufacturing	-0.101	$(-0.141, -0.063)$	0.148	$(0.098, 0.188)$
Trade	0.052	$(0.019, 0.085)$	0.042	$(0.006, 0.076)$
Transport	-0.424	$(-0.488, -0.373)$	0.325	$(0.282, 0.381)$
Education	-0.923	$(-0.976, -0.867)$	0.019	$(-0.014, 0.048)$
Other	0.015	$(0.000, 0.031)$	-0.038	$(-0.058, -0.019)$
Logarithm of Variance Parameter for Outside Option	-0.985	$(-0.992, -0.980)$		

positive relationship on the value of the outside option. This is, perhaps, not all that surprising since Copenhagen is by far the largest city in Denmark; one would expect that a higher concentration of jobs exists in Copenhagen than elsewhere in Denmark.

5. Estimating the Cost of Asymmetric Information

How important is asymmetric information in this market? The costs are directly linked to "bid shading." One consequence of bid shading is that some efficient trades are not made. Therefore, one measure of the cost is the expected value of the difference between the full-information solution and the solution under asymmetric information. Thus, in Case 1,

$$\begin{aligned}\text{Cost(Case 1)} &= \mathcal{E}_S\left[\omega_1(S) - S\right] \\ &= \int_0^\infty \left[\omega_1(s) - s\right] f_S(s)\ ds\end{aligned}$$

while, in Case 2,

$$\begin{aligned}\text{Cost(Case 2)} &= \mathcal{E}_B\left[B - \omega_2(B)\right] \\ &= \int_0^\infty \left[b - \omega_2(b)\right] f_B(b)\ db.\end{aligned}$$

We estimated these using

$$\int_0^\infty \left[\hat{\omega}_1(s) - s\right] \hat{f}_S(s)\ ds$$

and

$$\int_0^\infty \left[b - \hat{\omega}_2(b)\right] \hat{f}_B(b)\ db.$$

For one percent trimming, our estimated costs, by case, are as follows:

Case 1: 0.9493;

Case 2: 10.0170.

The first thing to note is that the estimated cost of asymmetric information is very high in Case 2, over $1,000,000$DKK, about \$160,000 US per year. We find this cost to be unrealistic, suggesting that our estimated parameters for Case 2 are also unrealistic.

For Case 1, our estimate is lower, around $95,000$DKK, about \$15,000 US per year. This latter number may seem quite high too, but one must remember that this is based on before-tax income, and the majority of the workers in this sample face a marginal tax rate of around sixty percent, so this is about \$6,000 US per year, perhaps ten percent of average total wage income.

Thus, we believe that this market is best described by Case 1. To wit, the worker has all of the power. One explanation for this could be that in 1999 the Danish economy was booming and had very low unemployment. Of course, further analysis of this conjecture is warranted.

6. Summary and Conclusions

We have employed a simple two-person bargaining model to interpret wage data—demands (offers) by workers (firms) and acceptances by firms (workers)—as the equilibrium outcome at a double auction. Under two polar-extreme bargaining solutions, we developed a strategy to recover estimates of the marginal-productivity and the opportunity-cost distributions. We then implemented this framework using particularly-rich data from a sample of Danish firms and workers. Subsequently, we used our estimates to measure the cost of the inefficiencies that arise from the bilateral-monopoly problem under the two alterative bargaining solutions. Our estimates suggest nontrivial costs, about ten percent, arising from the presence of asymmetric information.

A. Appendix

In this appendix, among other things, we document the construction of the likelihood function under the assumptions of section 4 for Case 2.

A.1. Derivation of Likelihood Function

Under the assumptions made in section 4, we can reduce this to the following:

$$Y = \omega_2(B) - S;$$

$$W = \begin{cases} \omega_2(B) & \text{if } Y \geq 0 \\ 0 & \text{otherwise;} \end{cases}$$

$$S = \begin{cases} S & \text{if } Y < 0 \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the likelihood function is then given by

$$\mathcal{L}_2 = \prod_0 \int_{-\infty}^{0} f_{SY}(s_t, y)\ dy \prod_1 \int_0^{\infty} f_{WY}(w_t, y)\ dy.$$

where f_{SY} is the joint density function of S and Y. Now,

$$f_{SY}(s, y) = f_S(s) f_B[\omega_2^{-1}(s + y)].$$

Also, f_{WY} is the joint density function of W and Y, which is given by

$$f_{WY}(s, y) = f_S(w - y) f_W(w).$$

Now,

$$\begin{aligned} \int_{-\infty}^{0} f_{SY}(s, y)\ dy &= \int_{-\infty}^{0} f_S(s) f_B[\omega_2^{-1}(s + y)]\ dy \\ &= f_S(s) \int_{-\infty}^{0} f_B[\omega_2^{-1}(s + y)]\ dy \\ &= f_S(s) F_B[\omega_2^{-1}(s)]. \end{aligned}$$

and

$$\begin{aligned} \int_0^{\infty} f_{WY}(w, y)\ dy &= \int_0^{\infty} f_S(w - y) f_W(w)\ dy \\ &= f_W(w) \int_0^{\infty} f_S(w - y)\ dy \\ &= f_W(w) F_S(w), \end{aligned}$$

so substituting these density functions into the likelihood function above, we obtain

$$\mathcal{L}_2 = \prod_0 f_S(s) F_B[\omega_2^{-1}(s)] \prod_1 f_W(s) F_S(w).$$

B. Bibliography

Amemiya, T.. *Advanced Econometrics.* Cambridge, Massachusetts: Harvard University Press, 1985.

Ausubel, L. P. Cramton, and R. Deneckere. "Bargaining with Incomplete Information," in *Handbook of Econometrics*, Volume 3, edited by R. Aumann and S. Hart. New York: Elsevier, 2002.

Chatterjee, K. and W. Samuelson. "Bargaining under Incomplete Information," *Operations Research*, 31 (1981), 835–851.

Chernozhukov, V. and H. Hong. "Likelihood Estimation and Inference in a Class of Nonregular Econometric Models," *Econometrica*, 72 (2004), 1445–1480.

Donald, S. and H. Paarsch. "Identification, Estimation, and Testing in Parametric Empirical Models of Auctions within the Independent Private Values Paradigm," *Econometric Theory*, 12 (1996), 517–567.

Elyakime, B., J.-J. Laffont, P. Loisel, and Q. Vuong. "Auctions and Bargaining: An Econometric Study of Timber Auctions with Secret Reservation Prices," *Journal of Business Economics and Statistics*, 15 (1997), 161–205.

Flinn, C. and J. Heckman. "New Methods for Analyzing Structural Models of Labor Force Dynamics," *Journal of Econometrics*, 18 (1982), 115–168.

Hall, R. and E. Lazear. "The Excess Sensitivity to Layoffs and Quits to Demand," *Journal of Labor Economics*, 2 (1984), 233–257.

Hong, H.. *Econometric Models of Asymmetric Ascending Auctions.* Ph.D. dissertation, Department of Economics, Stanford University, 1998.

Postel-Vinay, F. and J.-M. Robin. "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70 (2002), 2295–2350.