Discrete Bids and Empirical Inference in Divisible Good Auctions

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What is a divisible good auction?

- Auction of large quantities, where bidders can be thought of as bidding for a "share" of the quantity offered for sale rather than the actual number of units
- "Share" of the good is a continuous choice variable
- Bidders submit whole demand (bid) functions
- Government securities, electricity, IPOs, emission permits

Literature

- Theory:
  Wilson (1979), Ausubel and Cramton (2002)
- Reduced form estimation:
  Umlauf (1993), Nyborg and Sundaresan (1996)
- Structural approach:
  Hortacsu (2002), Wolak (2002, 2005)

Objectives of this paper

- Wilson's model is too restrictive to fit the data: Need a model yielding equilibria in step functions
- Investigate the extent to which the restrictions of the model with continuous bid functions matter
- Structural estimation using proper link between the primitives of the model and the data
- Provide a method for evaluating the performance of an auction mechanism based on data on individual bids

Why a different model?

Figure:

Why a different model?

- Step functions observed since upper bound on number of bidpoints imposed by auctioneers. (Exogenous restriction of the strategy set)
- Moreover: bidders never approach this bound, and hence intentionally submit step functions.
- *How to explain the data generating process if the equilibrium of the assumed underlying model is in continuously differentiable functions?*
- No rationing in the traditional model. (with step functions it occurs with probability one)

Basic divisible good auction model w/ private info and values

- Marginal valuation function for bidder i: $v_i(q, s_i)$ where $s_i \sim F_i(s_i)$; $s_i \in \Re$
- Total quantity normalized: $Q = 1$
- Submit a bid function: $y_i(p|s_i)$ specifying a share $y_i \in [0, 1]$ that type s_i requests at price p
- Auctioneer aggregates the bids to determine the aggregate bid function $\sum_i y_i(p|s_i)$
- Market clearing price is determined, and payments and allocations are made according to auction rules

Bidder's problem

Uniform price auctions: Wilson's Approach

- BNE (bidders strategies restricted to be continuously differentiable functions) characterized by:

$$v_i(y_i(p|s_i), s_i) = p + \text{Nonnegative Shading Term}$$

- Notice: a UPA in which bidders bid truthfully their marginal valuation schedules constitutes an upper bound on the revenue of any equilibrium of a UPA characterized above - p is the bid for share $y_i\left(p|s_i\right)$

K-step equilibrium of a UPA with private values

- BNE: $y_i(p|s_i, t_i) : \mathcal{S} \times \mathcal{T} \to \mathcal{Y}^{K_i}$
  $\mathcal{Y}^K$ set of left-continuous step functions with at most K steps
- K_i can differ across bidders
- Cost $c\left(K, t\right)$ where $t \sim G\left(t|s\right)$ is private info
- Rationing rule: pro-rata on the margin
  If excess demand, marginal bidders with bids exactly at the market clearing price will be rationed proportionally. (Bids above market clearing price are given priority.)

K-step equilibrium characterization

- Theorem (Characterization): In a UPA with private values, and rationing pro-rata on the margin, in any K-step equilibrium the quantity requested at k^{th} bidpoint has to satisfy:

$$v_i(q_k, s_i) = E\left[p|p_k > p > p_{k+1}\right] + \frac{q_k}{\Pr\left(p_k > p > p_{k+1}\right)} \frac{\partial E\left[p; p_k \geq p \geq p_{k+1}\right]}{\partial q_k}$$

- Bidder is (almost) like an oligopolist facing a random (residual) demand who has to commit to one quantity

$$MC = \mathbb{E}\left(MR\right) = \mathbb{E}\left[P\left(q\right) + qP'\left(q\right)\right]$$

Intuition

Major differences from the model with continuous bids

- Different optimality condition, which is used for empirical identification
- Ex post revenue in a UPA is NOT bounded by the revenue from a UPA in which each bidder bids his true marginal valuation schedule:

$$v_i(q_k, s_i) = E[p|p_k > p > p_{k+1}] + \frac{q_k}{\Pr(p_k > p > p_{k+1})} \frac{\partial E[p; p_k \geq p \geq p_{k+1}]}{\partial q_k}$$

$$v_i(q_k, s_i) = p_k + \text{Nonnegative Shading Term}$$

Estimation Strategy

- optimality condition approach - obtain consistent estimates of all pieces to obtain an estimate of marginal valuation at the submitted bid

$$v_i(q_k, s_i) = E[p|p_k > p > p_{k+1}] + \frac{q_k}{\Pr(p_k > p > p_{k+1})} \frac{\partial E[p; p_k \geq p \geq p_{k+1}]}{\partial q_k}$$

- Suppose we observe $\{y(p|s_1, t_1), ..., y(p|s_N, t_N)\}$ for each auction and data are generated by a symmetric K-step equilibrium behavior.
- A bid function $y(p|s_1, t_1)$ is the same bid function any bidder would submit had he drawn the type (s_1, t_1).
- Estimate $\mathbb{E}[p|p_k > p > p_{k+1}]$ by resampling bid functions

Resampling Procedure

- fix a bidder
- draw $N - 1$ bid functions (with replacement)
- construct the residual supply and obtain the corresponding market clearing price

Distribution of the market clearing price

Figure:

Auctions of Czech T-bills: Rules

- uniform price auctions
- rationing rule: pro-rata on-the-margin
- maximum number of bidpoints allowed is 10
- noncompetitive bids allowed (explained later)
- face value: 1,000,000 CZK
- auctions conducted every Wednesday
- auction plan announced quarterly

Bidders

- bidders (mostly banks) must be registered with the auctioneer before the auction
- banking or broker license within EU required for registration
- one bidder can aquire at most 50% of the supply in any given auction
- minimum buying limit for a calendar year - never binding, satisfied much earlier

Private values?

- major reason for interest in the T-bill auctions: reserve requirements on risky investment
- banks buy T-bills for their portfolios to obtain some return on their cash reserves
- secondary market for T-bills virtually nonexistent
- banks have private information about their liquidity positions and investment opportunities
- if there is a common value component, it is (almost) perfectly known
- Independent private values? - I test for affiliation

Data

- Data on 28 uniform price auctions of 3-month T-bills of the Czech government from 11/25/1999 until 12/14/2000
- on average 13 active bidders
- each bidder submits on average 2.3 bidpoints (maximum is 9)
- bid in terms of the annual yield varies from 4.99% to 5.65%
- market clearing yield varies from 5.22% to 5.54%
- for estimation using resampling we need larger number of bidders - I will assume 4 neighboring auctions are repetitions of the same experiment and group the bids from those together (test for potential problems)

Data

- bidders seem to be asymmetric and can be split into two groups according to size

Table: Data Summary - Large vs Small Bidders

	Mean Large	Mean Small
Active Bidders in an Auction	8	5
Number of Submitted Bidpoints	2.88	1.59
Price Bids[a]	5.30	5.30
Quantity Bids[b]	0.077	0.02

[a] In terms of the annual yield of T-bills
[b] As a share of total quantity offered for sale

Noncompetitive bids

- commitment to buy q at the market clearing price (supply reduction)
- not used by regular bidders
- auctioneer reserves the right to buy some or all of the T-bills offered for sale for his own portfolio
- actual noncompetitive bid ranges from 0 to 75% of T-bills offered (on average 36%)
- for estimation purposes, I will treat the bids on behalf of the auctioneer as a separate bidder group (might be problematic if signals affiliated - I test for this)

Example of the estimation results for individual bidders

Figure:

valuations

Truthful bidding

- suppose we run a uniform price auction and bidders submit their marginal valuation schedules as bids
- use the upper envelope of marginal valuations

Table: Comparison with truthful bidding - market clearing yield

Auction	Actual yield	Highest yield[a]	Lowest yield[b]
94*	5.26	5.31	5.30
95*	5.28	5.34	5.34
107	5.41	5.40	5.40
108	5.40	5.40	5.38
Mean (all)	5.31	5.32	5.30

* Ex post revenue higher than under truthful bidding
[a] Achieved by bidding the lower envelope of marginal valuations
[b] Achieved by bidding the upper envelope of marginal valuations

Truthful bidding

- Results: in 7 out of 28 auctions, ex post revenue is higher than the revenue achieved under truthful bidding
- Therefore using the indirect comparison to argue that discriminatory auction performs better in terms of revenue may not be correct.

Performance of the mechanism

- ideal mechanism (1^{st} best world): 1) extracts all value given an allocation, 2) implements the efficient allocation
- if efficiency ≈ 1 and bidders' interim profits ≈ 0, then the mechanism performs well
- use estimated distribution of the market clearing price and marginal valuation to estimate bidders' interim profits
- estimate efficiency: (realized surplus)/(efficient surplus)

Performance of the mechanism

Table: Interim profit of bidders per T-bill for sale

Auction — Int. Profit[b]	Average	Max	Min	Total	Efficiency[c]
94*	2.83	14.86	-0.01	28.27	0.99993
	(1.47)	(14.33)	(0.41)	(14.73)	($6*10^{-7}$)
95*	8.88	70.76	-0.82	88.81	0.99999
	(2.51)	(22.15)	(1.97)	(25.08)	($2*10^{-7}$)
107	0.54	4.27	-0.01	6.98	0.99998
	(0.18)	(1.98)	(0.21)	(2.32)	($2*10^{-6}$)
108	4.13	32.62	-0.21	53.64	0.99995
	(14.22)	(181.4)	(0.02)	(184.83)	($2*10^{-6}$)
Mean (Auctions 52-108)	5.07	43.24	-0.20	66.14	0.9999

* Ex post revenue was higher than under truthful bidding
[a] Standard errors in parentheses
[b] Using the upper envelope of marginal valuations
[c] Defined as (Actual surplus)/(Surplus from the efficient allocation)

Performance of the mechanism

- employed mechanism performs quite well: loss versus the ideal mechanism is less than 8 basis points
- 223,000 T-bills sold to regular bidders during the sample period $\Rightarrow$ total loss worth less than 22 T-bills (0.01%)

Bidder Asymmetry

Figure:

- small and large bidders are asymmetric also in term of the distribution of their private information

Effect of quantity won on signals

- relationship between quantity won in auction t and value for units in auction $t+1$

Table: Testing dependence of signals and quantities won earlier

Auctions:	$\{52-60\}$	All
Constant	986,477.4	990,817.5
	(49.93)	(1625.5)
q_{t-1}	885.6	-13576.3
	(514.8)	(15638.4)
R^2	0.09	0.003
N	33	250

[a] Std. errors in parentheses
[b] Dependent variable: s_t

Affiliation of signals

- affiliation of signals - if signals affiliated, then the distribution of the noncompetitive bid would differ depending on the signal received

Table: Wilcoxon Rank Sum Test of Equality of Distributions $F_{s_{-1}|s_1}$

Auctions — Sample split	$\{1,2\},\{3,4\}$	$\{1\},\{2\}$
$\{52, 55, 56, 60\}$	0.85	0.45
$\{61, 64, 65, 67\}$	0.78	0.88
$\{69, 72, 73, 75\}$	0.25	0.38
$\{76, 81, 82, 85\}$	0.12	0.05
$\{86, 87, 91, 92\}$	0.30	0.83
$\{94, 95, 99, 100\}$	0.40	0.85
$\{103, 104, 107, 108\}$	0.82	0.72

[a] p-values of H_0: Samples are from the same continuous distribution.

Important questions

- What role does the possibility of noncompetitive bids play in policing bidding behavior?
  - in electricity markets - withholding part of the demand is impossible - what to do then?
  - maybe the auctioneer should sign private (option) contracts with some generators before the auction, so that the possibility of withholding is there
- Are 3 points "enough" to capture "almost all" of the surplus from a multiunit auction?
  - Is there an analogy between this simple bidding and simple linear pricing menus?

Conclusion

- Steps matter! (otherwise overestimating marginal valuations, and thus results biased against the uniform price auction)
- Bids can exceed marginal valuations in a uniform price auction!
- Empirical:
  - a method for evaluation of the employed mechanism
  - uniform price auctions of Czech T-bills perform well
  - unextracted total surplus of the bidders is less than 3 basis points, while efficiency loss is less than 5 basis points