Structural Econometric Approach to Bidding in the Main Refinancing Operations of the Eurosystem

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Abstract

This paper contributes to the existing literature on central bank repo auctions. It is based on a structural econometric approach, whereby the primitives of bidding behavior (individual bid schedules and bid-shading components) are directly estimated. With the estimated parameters we calibrate a theoretical model in order to illustrate some comparative static results. This exercise sheds light on the debate about the reversed winner's curse found in the empirical literature on ECB auctions by showing that it may be related to an identification problem. Overall the results suggest that strategic and optimal behavior is prevalent in ECB tenders. We find evidence of a statistically significant bid-shading component, even though the number of bidders is very large. Bid-shading increases with liquidity uncertainty and decreases with the number of participants and with price uncertainty. We argue that a sufficient condition for the latter effect to appear in the data is that the residual supply facing an individual bidder does not change much ex-post when very short-term market rates increase.

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1. Introduction

The banking system in the euro area[1] is in a structural deficit position vis-à-vis the Eurosystem.[2] In fact, according to the consolidated financial statement of the Eurosystem[3] on 1 July 2005, on the asset side, the refinancing of the ECB provided to the banking system via open market operations and recourse to the marginal lending facility amounted to EUR 398 billion, of which EUR 308 billion corresponded to liquidity provided through the regular (weekly) main refinancing operations. The latter are executed in the form of tender procedures.[4]

Central bank operations and government auctions of treasury securities look like similar means of allocating a good. In particular, both take place in the environment of a secondary market which in principle allows potential buyers to arbitrage away any potential difference in prices between the primary and the secondary markets. However, the central bank auctions like those conducted by the ECB differ from Treasury auctions in several important dimensions. Firstly, central bank refinancing is provided against collateral. To the extent that low opportunity cost collateral is used first, the marginal valuation of liquidity should be declining as collateral of better quality must be increasingly provided. Second, in the euro area banks have to fulfill reserve requirements and this, rather then reselling in the secondary market, is the main motive for banks to bid in the regular open market operations of the ECB. Third, unlike T-bills, there are only imperfect substitutes to ECB refinancing. For example banks face credit limits and may not be able to borrow the full extent of their liquidity needs, or they may not be willing to extend their own credit limits. Thus, borrowing in the primary market with the objective of reselling in the secondary market is not as prevalent as in the Treasury bond market. Fourth, there is little uncertainty about the

[1] The euro area refers to the 12 European Union (EU) Member States that share a single currency - the euro. These countries are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxemburg, the Nederlands, Portugal and Spain.

[2] The Eurosystem refers to the European Central Bank (ECB) and the 12 National Central Banks (NCBs) of the participating EU Member States.

[3] The consolidated balance sheet of the Eurosystem shows that, on the liabilities side, the main liquidity absorbing factor is banknotes in circulation followed by current account holdings of credit institutions with the Eurosystem, where the latter cater essentially for the minimum reserve system. The consolidated balance sheet of the Eurosystem is published regularly in the Euro Area Statistics Annex of the ECB Monthly Bulletin.

[4] For details on the operational framework of the Eurosystem see "The implementation of monetary policy in the euro area: general documentation on Eurosystem monetary policy instruments and procedures", ECB, February 2005, downloadable from www.ecb.int.

(common) value of the good auctioned.[5] In fact, refinancing is provided for very short-term (overnight in the case of the marginal lending facility or one-week in the case of the main refinancing operations of the ECB) for which there is little price risk and, besides, a very liquid derivatives (swap) market exists, revealing the common value of the good.[6] Moreover, the announcement of the outcome of the main refinancing operations of the ECB has, in general, no additional informational content for market participants.[7] This means that banks do not change the private value attached to the good they receive after knowing the tender results.

The combined features of declining marginal valuations, low uncertainty about the market value of the good and reserve requirements should be taken into account when modelling ECB tenders. In this paper we empirically test a model of optimal bidding in variable rate tenders using data from ECB auctions. Existing empirical work on the ECB main refinancing operations has relied exclusively on panel data analysis without any underlying structural model (see Nyborg et al., 2002 and Scalia et al., 2005). Both papers conclude that bid shading by participants to ECB tenders decreases with interest rate uncertainty, which is against the prediction of standard single-unit, common value auction theory (winner's curse). This paper contributes to the existing literature on central bank auctions in so far as it is based on a structural econometric approach, whereby the primitives of bidding behavior (individual bid schedules and bid-shading components) are directly estimated. With the estimated parameters we calibrate a theoretical multi-unit private values auction model, in order to illustrate some comparative static results. This exercise allows us to shed some light on the debate about

[5]Except on the final day of the reserve maintenance period. In the euro area the reserve maintenance period has a variable length, of approximately one month.

[6]The announcement of the weekly auction takes place, as a rule, on Monday at 15:00, together with the publication of the Eurosystem's forecast of the average daily liquidity needs of the banking system until the next open market operation, stemming from the so-called autonomous factors. At the same time, the ECB also publishes the benchmark allotment which corresponds, in general, to the amount of reserves that, based on past fulfilment and the projected autonomous factors, would bring the average reserve holdings one week ahead in line with the reserve requirement plus a technical, small amount for excess reserves.

[7]Bids may be submitted until 9:30 a.m. on Tuesday. The result of the auction is published by the ECB on the wire services Reuters and Bloomberg at 11:20 a.m. on the auction day. The announcement gives the total allotment amount, total bid amount, number of bidders, minimum and maximum bid rates, weighted average allotment rate, marginal rate and percentage of allotment at the marginal rate. Within a short time-window after the publication of the results one does not observe, in general, any movement in very-short term money market interest rates.

the reversed winner's curse found in the empirical literature on ECB tenders by showing that it may be related to an identification problem. The remainder of the paper is organized as follows. Section 2 sets out the theoretical model of optimal bidding and Section 3 explains the econometric methodology. The data used in the study is described in Section 4 and the results are presented in Section 5. The main conclusions are presented at the end.

2. Theoretical framework

As mentioned in Section 1, the Eurosystem conducts weekly tenders whereby refinancing is provided to the banking system. The liquidity is allotted via standard tender procedures, "pay-as-you-bid " and pro-rata allotment at the cut-off price (marginal rate).[8] Formally these auctions are multi-unit auctions (or share auctions) with discriminatory pricing and a reserve price. Existing empirical work on the ECB main refinancing operations has relied on panel data analysis where the main theoretical predictions are derived in analogy to single-unit auctions (see Nyborg et al., 2002 and Scalia et al., 2005). Both papers conclude that bid shading by participants to ECB tenders decreases with interest rate uncertainty, which is against the prediction of standard single-unit auction theory (with common values and discriminatory pricing), the winner's curse.

The theory tested in this paper builds on the seminal paper by Wilson (1979) on auctions of shares. More specifically, the theoretical model is designed to capture the essential features of central bank auctions like those of the ECB. For complete references and details on the derivations, as well as for the discussion of uniform vs. discriminatory pricing, and homogeneous vs. heterogenous bidders, the reader is referred to Ewerhart et al.(2006). Here, the discussion will focus exclusively on the discriminatory pricing, homogeneous agents model.

[8] The Eurosystem has the option of conducting either fixed rate (volume) or variable rate (interest) tenders. The main refinancing operations have been conducted as variable rate tenders, with a minimum bid rate, since June 2001. In the variable rate tenders banks may submit bids for up to ten different pairs of interest rate/quantity levels. The interest rates bid must be expressed as multiples of 0.01 percentage point. The minimum bid amout is EUR 1,000,000 and bids exceeding this amount must be expressed as multiples of EUR 100,000. Counterparties are expected to cover the amounts alloted to them (not their bids) by a sufficient amout of eligible underlying assets. For further details on the tender procedures see "The implementation of monetary policy in the euro area: general documentation on Eurosystem monetary policy instruments and procedures", ECB, February 2005, downloadable from www.ecb.int.

Model A central bank puts up for sale a random quantity, the total allotment $\widetilde{Q} \geq 0$, of liquidity (i.e. a perfectly divisible good). There are two alternative interpretations for uncertainty about aggregate allotment. First, the central bank may possess a superior knowledge about the aggregate liquidity shortage facing the banking system. Second, there may be a fraction of non-strategic bidders. In practice, both effects contribute to the uncertainty about the residual supply perceived by the individual bidder (bank). For reasons of tractability we assume that $\widetilde{Q}$ is uniformly distributed in $\left[0, \overline{Q}\right]$. There are $i = 1, 2, ..., n$ bidders which do not observe the total allotment prior to the submission of bids. The central bank does not exploit its information about the incoming bid schedules to affect the distribution of $\widetilde{Q}$. Marginal valuations are assumed to be linearly decreasing from a maximum valuation $\overline{v} > 0$ that is common to all bidders. Thus, bidder i's marginal valuations for quantities $q_i \geq 0$ are formally given by $v_i(q_i) = \overline{v} - B_i^{-1} q_i$, for an exogenous parameter $B_i > 0$. We consider a symmetric set-up where $B_1 = B_2 = ... = B_n$. The tender mechanism asks each bidder to submit a bid schedule that specifies, for any price $p \geq 0$, the amount $x_i(p_i) \geq 0$ that bidder i is willing to buy at p. A schedule $x_i(p_i)$ is admissible if it is non-increasing, left-continuous, and if $x_i(p_i) = 0$ for a sufficiently high p. It is assumed that only admissible bid schedules are accepted by the auctioneer. Let $x(p) = \sum_{i=1}^{n} x_i(p)$ denote total demand at price p, and $P^*(\widetilde{Q}) = \left\{p \geq 0 | x(p) \leq \widetilde{Q}\right\}$ the set of prices at which total demand can be satisfied. The stop-out price is defined as the infimum $p^*(\widetilde{Q}) = \inf P^*(\widetilde{Q})$ of such prices.

Individual allotments are determined by satisfying all bids strictly above the stop-out price, and by applying rationing at the margin, if necessary. Define $x_i^+(p^*) = \lim_{p \to p^*, p > p^*} x_i(p)$ as bidder's i demand at a price just above p^*, and let $x^+(p^*) = \sum_{i=1}^{n} x_i(p^*)$, denote the corresponding aggregate. Bidder i obtains an allotment

$$q_i^*(\widetilde{Q}) = x_i^+(p^*(\widetilde{Q})) + \frac{x_i(p^*(\widetilde{Q})) - x_i^+(p^*(\widetilde{Q}))}{x(p^*(\widetilde{Q})) - x^+(p^*(\widetilde{Q}))} \left\{\widetilde{Q} - x^+(p^*(\widetilde{Q}))\right\}, \qquad (2.1)$$

in state $\widetilde{Q}$. Thus, when demand exceeds supply, the allotment is composed of a complete allocation of the part of the bid schedule that lies above the stop-out price, and a pro-rata allocation of any flat segment of the bid schedule that lies at the stop-out price. The tuple $(p^*, q_1^*, q_2^*, ..., q_n^*)$ consisting of the stop-out price and the individual allotments will be referred to as the outcome of the tender.

Bidders are risk-neutral, assumed to maximize expected profits. Define the inverse bid schedule as $b_i(q_i) = \inf\{p \geq 0 | x_i(p) \leq q_i\}$. Under discriminatory pricing, the bidder i pays his own bid $b_i(q_i)$ for any marginal unit, so that the resulting profit from an outcome $(p^*, q_1^*, q_2^*, ..., q_n^*)$ amounts to

$$\Pi_i = \int_0^{q_i^*} \{v_i(q_i) - b_i(q_i)\}\, dq_i. \tag{2.2}$$

Equilibrium An equilibrium can be found for $n \geq 2$, when bidders $i = 1, 2, ..., n$, have identical marginal valuations $v_i(q_i) = \overline{v} - B^{-1}q_i$. Assume also that $\overline{Q} < n\overline{v}B$. Bidder i submits the piecewise linear bid schedule

$$x_i(p) = \left\{ \begin{array}{cc} 0 & for\ p > \overline{v}^d \\ B^d(\overline{v}^d - p) & for\ p_{\min} < p \leq \overline{v}^d \\ \overline{Q}/n & for\ p \leq p_{\min} \end{array} \right\} \tag{2.3}$$

for $i = 1, 2, ..., n$, where

$$\overline{v}^d = \overline{v} - \frac{\overline{Q}}{(2n-1)B} \tag{2.4}$$

$$B^d = \frac{2n-1}{n-1}B \tag{2.5}$$

$$p_{\min} = \overline{v} - \frac{\overline{Q}}{nB}. \tag{2.6}$$

are the maximum price bid, the slope of the inverse bid schedule, and the minimum stop-out price, respectively.

The equilibrium marginal rate in the model is stochastic as it depends on the allotment. The expected marginal rate is equal to the rate that obtains when the central bank allots half of the maximum quantity and it is given by

$$E(p^{\text{mar}}) = \overline{v} - \frac{3(n-1)\overline{Q}}{2n(2n-1)B} \tag{2.7}$$

and when $n \to \infty$ the quantity allotted is $\overline{Q}(n) = n\,\overline{Q}$. Then, the maximum price at which a bid is placed will converge to

$$\lim_{n\to\infty} \overline{v}^d = \overline{v} - \frac{\overline{Q}}{2B}, \tag{2.8}$$

and the expected marginal rate will converge to

$$\lim_{n\to\infty} E(p^{\text{mar}}) = \overline{v} - \frac{3\overline{Q}}{4B}. \tag{2.9}$$

Strategic behavior does not disappear in the limit. Moreover, in the limit the aggregate bid schedule will converge to

$$x(p) = \underbrace{(2B\overline{v} - \overline{Q})}_{intercept} - \underbrace{2B}_{slope}p, \tag{2.10}$$

showing that the aggregate bid schedule will move in a parallel manner, upwards if $\overline{v}$ increases (e.g. increase in the level of the market interest rate) and downwards if $\overline{Q}$ increases (e.g. increase in liquidity volatility).

Illustration of the model The solution of the model is illustrated in Figure 1. The true linear demand curve (dotted line) is represented above a piecewise linear bid schedule ($x_i(p)$) which was drawn for $\overline{Q}(n) = 300$ (EUR billion). The other parameters are set as follows: $\overline{v} = 2.06$ (the one-week EONIA swap rate level[9]), $B = 20$, $n = 300$, i.e. calibrated to match euro area data (see Section 3 for details). Auction prices correspond to interest rates in percent. Full allotment is at $\overline{Q}(n)/n = 1$, to which it corresponds a stop-out price at 2.01.

Equilibrium is determined at the interception of the individual bid schedule with the residual supply curve, i.e. the supply diminished by the allotments made to the other bidders at a given price. An equilibrium is depicted such that the allotment ratio is 50%; the stop-out price (marginal tender rate) is at 2.0225, and the corresponding repo rate (equal to the true marginal valuation) is at 2.035 with a bid-shading component of 1.25 basis points. Suppose the central bank set a minimum bid rate at 2.0. Thus, in this particular case, the spread between the swap rate and the minimum bid rate would be 6 basis points; the spread between the repo rate[10] and the minimum bid rate would be 3.5 basis points, and the

[9]EONIA (euro overnight index rate) is a weighted average of the interest rates on unsecured overnight lending transactions denominated in euro, as reported by a panel of contributing banks. The one-week EONIA swap rate is the main reference for banks when they prepare their bids, given that this segment of the swap market is very liquid and the Eurosystem's refinancing operations have one-week maturity. Given that the underlying EONIA refers to unsecured loans, bids submitted to ECB repo operations should be below that rate.

[10]The theoretical repo rate does not correspond to the private market repo rate (the so-called GC rate). The former should lie somewhat above the latter because it is collateralized with less liquid paper.

spread between the marginal tender rate and the minimum bid rate would be 2.25 basis points. These values are close to those often observed in the euro money market with an ECB's minimum bid rate at 2.0.

Consider next an allotment with $\overline{Q}(n) = 400$ with the remaining parameters unchanged. This is equivalent to an increase in liquidity uncertainty (see Figure 2). In this case full allotment is at $\overline{Q}(n)/n = 1.33$ and stop-out price just below 2.0, which is not feasible if the central bank's minimum bid rate is at 2.0. The expected equilibrium is depicted such that an allotment ratio of 50% prevails. The (expected) stop-out price is at 2.015, while the repo rate remains at 2.035 with a bid-shading component of 2 basis points. Thus, in this particular case, the spread between the swap rate and the minimum bid rate would be 6 basis points; the spread between the repo rate and the minimum bid rate, 3.5 basis points, and the spread between the marginal tender rate and the minimum bid rate would be 1.5 basis points. This exercise illustrates why using a measure of market price volatility (zero in this case) would not allow estimating the impact of liquidity uncertainty on bid shading (increase in bid shading). This is an example of an identification problem that might have affected the empirical literature.

Finally consider again an allotment with $\overline{Q}(n) = 300$, however with a higher swap rate ($\overline{v} = 2.08$). An ex-post equilibrium is depicted (which is not equal to the expected one) such that an allotment ratio of 60% prevails, which would be obtained with an unchanged residual supply (Figure 3). In this case volatility in market interest rates would be associated with a decline in bid shading (which would be wrongly interpreted as a reversal of the winner's curse). As this example shows, the residual supply facing an individual bidder not changing much when very short-term market rates increase is a sufficient condition for bid-shading to decrease with price uncertainty.[11]

Empirical predictions The theoretical model suggests five testable predictions about individual bidding behavior and interest rate spreads:

1. The strategic inverse bid schedule is flatter than the true demand. The bid schedule is steeper than the true marginal valuation curve.

2. Bid-shading decreases with the number of bidders; however, it does not disappear even when n becomes very large.

[11] This is a sufficient condition if an increase in the level of the overnight interest rate is associated with an increase in its volatility, which is expected via a risk premium effect.

3. Bid shading increases with supply uncertainty.

4. The impact of price uncertainty on bid shading is ambiguous.

5. In equilibrium there is a positive spread between the repo rate and the marginal (stop-out) price.

The theoretical model suggests three testable predictions at an aggregate level:

1. An increase in liquidity uncertainty should have a negative impact on the intercept of the aggregate bid schedule (downward shift of the curve).

2. An increase in the short-term market interest rate should have a positive impact on the intercept of the aggregate bid schedule (upward shift of the curve).

3. The slope of the aggregate bid schedule should not be affected either by liquidity uncertainty or the level of market interest rates. However, changes in the number of market participants should affect both the slope and the intercept of the curve (flattening of the curve).

3. Econometric methodology

3.1. Estimation of a structural model

The linear model of the auction with discriminatory pricing, which has been solved explicitly by Ewerhart et al. (2006) , can be tested following a general econometric approach as suggested by Hortaçsu (2002a). We apply the structural empirical approach to auctions, which is an expanding field recently surveyed by Athey and Haile (2004). We proceed in three steps:

1. Estimation of the equilibrium price for each auction.

2. Estimation of the bid-shading components.

3. Tests on individual bidding behavior.

4. Tests on the aggregate bid schedule.

Denote bidder i's marginal utility from winning q units of the good, $v_i(q)$. The utility maximization problem can be written as

$$\max_{y_i(.)} \int_0^{\infty} \left\{ \int_0^{q_i} [v(q) - b_i(q)]\, dq \right\} dH(p^c, x_i(p)), \tag{3.1}$$

where $q_i = x_i(p)$, and $H(p^c, x_i(p))$ and $dH(p^c, x_i(p))$ are the cumulative distribution and density functions of the market clearing price (p^c), conditional on submitting a demand function $x_i(p)$, respectively, i.e. p^c is such that $x_i(p^c) = Q - \sum_{j \neq i} x_j(p^c)$ and $H(p^c, x_i(p)) = \Pr\{p^c \leq p^t | x_i(p)\}$.

The Euler necessary condition for the maximization of the objective function is then

$$v(x_i(p)) = p + \frac{H(p^c, x_i(p))}{H_p(p^c, x_i(p))}, \tag{3.2}$$

where $H_p(p^c, x_i(p)) = \dfrac{\partial H(p^c, x_i(p))}{\partial p^c}$; $v(x_i(p))$ is the true marginal valuation given to quantity q by bidder i. It is equal to the price bid p plus the bid-shading component, measured by the inverse hazard ratio. The above optimality condition allows to nonparametrically identify the marginal valuations of the bidders using observed bids.

3.2. Estimation of the auction's equilibrium price

Since in the case of ECB tenders most banks submit just one bid at each auction, only average individual bidding functions have been estimated. Estimation has been performed by considering jointly the data for all the auctions for each bank, aggregating the bids over all the auctions, and averaging the bids to obtain the final data. This has required the exclusion from the sample of all banks that bid at the same price in all auctions.

The OLS estimator has been employed, considering both linear and log-log specifications. The equilibrium price for each auction has been computed by equating the aggregate bidding function, obtained by horizontal summing of the inverse individual bidding functions, and total supply, and solving for the equilibrium price (interest rate). Hence, by denoting the estimated inverse aggregate bid function as $p = \hat{\alpha} - \hat{\beta} Q^d$, the equilibrium price has been computed from the market equilibrium condition, $Q^d = Q^s$ as $p^c = \hat{\alpha} - \hat{\beta} Q^s$.

3.3. Nonparametric estimation of the bid-shading components

There are T auctions in the sample and N_t bidders participate at auction t, $t = 1, ..., T$. The procedure to estimate the bid-shading components works as follows:

i) select auction t and bidder i;

ii) from the sample of $\mathrm{N}_t - 1$ vectors draws a random sample of $\mathrm{N}_t - 1$ individual intercept and slope vectors with replacement;

iii) use the random sample to compute the residual supply function and intersect with bidder i's bidding function to determine the market clearing price (p^c);

iv) repeat for B times the previous steps to determine the empirical cumulative conditional distribution of the market clearing price $\hat{H}(p^c, x_i(p))$, taking into account the truncation implied by the minimum bid rate;

v) then, with reference to the estimated equilibrium price for the auction $p^{c,t}$, compute the probability $\Pr\{p^c \leq p^{c,t} | x_i(p)\} = \hat{H}(p^{c,t}, x_i(p))$ and the value of the density function at $p^{c,t}$, i.e. $\frac{\partial \hat{H}(p^{c,t}, x_i(p))}{\partial p}$, as $\frac{\hat{H}(p^{c,t}, x_i(p)) - \hat{H}(p^0, x_i(p))}{p^{c,t} - p^0}$, where p^0 is the ordered price statistic before the equilibrium price $(... < p^0 < p^{c,t} < ...)$. The bid-shading component can then be computed;

vi) repeat the previous steps for each of the bidders participating to auction t.

vii) repeat the previous steps for each auction.

Kernel estimation has been employed at point *v*). Given that the price distribution is truncated to the left, i.e. the bid rate cannot fall below the minimum bid rate, a Gaussian truncated kernel has been employed for the estimation of the equilibrium price density function.[12] Finally, standard errors for the bid-shading components have been obtained by bootstrapping the empirical distribution of the bid-shading components for each auction.[13]

3.4. Tests of individual bidding behavior

On the basis of the estimated slopes and intercepts, heterogeneity across bidder can be assessed and measured by standard statistical tools. Tests on bidding behavior can be carried out.

[12] See Pagan and Ullah (1999).

[13] For a similar approach see Hortaçsu (2002b).

3.4.1. Test 1: Is more successful bidding associated with more aggressive bidding?

The first test of bidding behavior is a general one, not directly related to the theoretical model but nonetheless interesting on its own. The following cross sectional regressions were performed

$$\begin{aligned} \bar{s}_i &= \theta_{\alpha_0} + \theta_{\alpha_1}\hat{\alpha}_i + \varepsilon_{\alpha i}, \\ \bar{s}_i &= \theta_{\beta_0} + \theta_{\beta_1}|\hat{\beta}_i| + \varepsilon_{\beta i}, \end{aligned} \tag{3.3}$$

where $\bar{s}_i$ is the average shortfall over the auctions in which bidder i has participated, and $\hat{\alpha}_i$ and $|\hat{\beta}_i|$ are the estimated intercept and (absolute) slope parameters of the individual (inverse) bidding functions. The shortfall in a given auction has been measured as the ratio of the quantity demanded by the bidder and the quantity actually allocated to the bidder. Thus, an increase in this measure means less success at the auction. It is expected that $\theta_{\alpha_1} < 0$ and $\theta_{\beta_1} < 0$, denoting that an increase in shortfall is associated with less aggressive behavior measured by lower $\hat{\alpha}_i$ and lower $|\hat{\beta}_i|$. This test can be interpreted as a test on whether bidding strategically pays-off. This is important given the potential existence of non-strategic bidders when their number is very large.

3.4.2. Test 2: Is the strategic inverse bid schedule flatter than the true demand?

The test can be implemented by running the cross sectional regression

$$|\hat{\beta}_i| = \theta_{q0} + \theta_q\bar{q}_i + \varepsilon_{\beta i}, \tag{3.4}$$

where $\bar{q}_i$ is the average quantity bid by bidder i over all the auctions in which it has participated. It is expected $\theta_q < 0$, which can be interpreted as bid-shading decreasing in the quantity bid. This is consistent with the idea of a true valuation schedule steeper than the observed bid schedule.

3.4.3. Test 3: The sources of bid-shading

To assess whether a relationship between the amount of bid-shading and the uncertainty in the value of the good auctioned, supply uncertainty, and the number of bidders can be found, the following cross sectional regression has been estimated

$$bs_i = \gamma_{0_1} + \gamma_{1_1}\hat{\sigma}_i + \gamma_{2_1}\hat{\sigma}_{zi} + \gamma_{3_1}N_i + \varepsilon_{bsi},$$

where bs_i is the average of the estimated bid shading components, considering all the bidders participating at auction i, obtained using the above described approaches, $\hat{\sigma}_i$ is price value uncertainty, measured by the conditional standard deviation of the one-week Eonia swap rate for the week preceding auction i[14] or by the price intercept dispersion; N_i is the number of participants to auction i, and $\hat{\sigma}_{zi}$ is a proxy for liquidity supply uncertainty for auction i, measured by the conditional standard deviation of the cumulated liquidity forecast error for auction i.[15] It is expected that $\gamma_{2_1} > 0$ and $\gamma_{3_1} < 0$. The sign of the parameter γ_{1_1} is open to different interpretations and predictions about its sign. We interpret it as capturing the price level effect of an increase in the volatility of the one-week EONIA swap rate. In fact, if an increase in price uncertainty is associated with an increase in the spread between the one-week EONIA swap rate and the ECB minimum bid rate (e.g. a risk premium effect), this is equivalent to an increase in the parameter $\overline{v}$ of the theoretical model. Then, a sufficient condition for bid-shading to decline when the interest rate (and its volatility) increases is that the residual supply facing bidder i should not change in equilibrium.

3.4.4. Test 4: The aggregate bid schedule

To test the implications of the model for the aggregate bid schedule we look at the time series variation in the (inverse) aggregate bid schedule. The following regressions were performed

$$\begin{aligned} \hat{\alpha}_t &= \delta_{0_1} + \delta_{1_1}r_t + \delta_{2_1}\hat{\sigma}_{zt} + \delta_{3_1}sh_{t-1} + \delta_{4_1}N_t + \varepsilon_{\alpha t}, \\ |\hat{\beta}_t| &= \delta_{0_2} + \delta_{1_2}r_t + \delta_{2_2}\hat{\sigma}_{zt} + \delta_{3_2}sh_{t-1} + \delta_{4_2}N_t + \varepsilon_{\beta t}, \end{aligned} \quad (3.5)$$

where $\hat{\alpha}_t$ and $|\hat{\beta}_t|$ are the estimated aggregate intercept and (absolute) slope parameters of the inverse demand function, $p(Q)$, for auction t; r_t is the one-week Eonia swap rate level at the time of the auction (observed by market participants); $\hat{\sigma}_{zt}$

[14] The daily volatility of the one-week Eonia swap rate has been computed by means of a GARCH(1,1) model. The weekly volatility has been computed by summing the daily volatility over the five working days of the week.

[15] It refers to the conditional standard deviation of the cumulated liquidity forecast error made by the Eurosystem, over eigth days, on the allotment day, computed by means of a GARCH (1,1) model.

is the proxy for supply uncertainty; sh_{t-1} is the (inverse) shortfall (i.e. is the bid-to-cover ratio) for auction $t-1$; and N_t is the number of bidders at the auction.

It is expected that $\delta_{1_1} > 0, \delta_{2_1} < 0; \delta_{1_2} = \delta_{2_2} = 0; \delta_{4_1}, \delta_{4_2} < 0$. No clear predictions can be drawn from the theory about the signs of $\delta_{3_1}, \delta_{3_2}$.

4. The data

The data set includes all bids submitted to the 31 weekly main refinancing operations conducted by the ECB between 16 March 2004 and 11 October 2004. During the period under analysis the maturity of the ECB repo operations was also weekly. The average number of bidders was 359 with an average of 515 bids, thus giving 1.44 bids per bidder. This illustrates the fact that most bidders bid for all the quantity at a single price. The average bid amount was EUR 300 billion, with a maximum of EUR 344 billion and a minimum of EUR 224 billion. Thus, in the calibration exercise presented in Section 2, we set as benchmark values $Q(n) = 300$ and $n = 300$. The average allotted amount was EUR 239 billion, with a maximum of EUR 263 billion and a minimum of EUR 206 billion, suggesting a relatively stable supply environment. The bid-to-cover ratio moved around an average value of 1.26, which suggests relatively successful bidding. In the sample period the marginal MRO rate was 2.007 on average, the average repo rate 2.011, the weighted average MRO rate was 2.0148 on average, and the average EONIA swap rate 2.0306. Thus, the spread between the repo and the marginal rate was 0.4 basis points, indicative of a small, though positive bid shading component. In the calibration exercise we used somewhat higher market rates for the sake of clarity in the illustration.

5. Empirical results

Not all the data are usable for the empirical analysis. In fact, the implementation of the (averaged) parametric disaggregated approach requires the availability of at least two different bids placed during the 31 auctions in the sample, not necessarily at the same auction. After having excluded from the sample the banks which placed only a single bid over the 31 auctions or always bided at the same price, 525 banks and 15753 bids (representing a value of EUR 9297.607 billion) are left, against a total of 593 banks and 15973 bids (for a total value of EUR 9327.326 billion). Although the number of excluded banks relative to the number of banks in the sample is not negligible (12%; 68 banks), the number of excluded bids is negligible both in terms of total number (0.25%; 220 bids) and total value (0.3%; EUR 29.719 billion). Hence, the analysis carried out by means of the disaggregated parametric approach should not be affected by sample trimming, albeit subject to the caveat that the estimated bidding functions are only representative of the average behavior of each agent. Yet, in the light of the short sample employed (March 2004 - October 2004) and the relatively smooth liquidity supply and bidding environment that characterized the euro area over the period investigated, the results drawn from the average analysis are expected to be reliable. Moreover, the period under analysis was marked by absence of short-term expectations of key ECB interest rate changes.

5.1. Bidders' heterogeneity

A first evaluation of the presence of heterogeneity across bidders can be carried out through the analysis of the estimated bidding functions for each single bidder. As discussed in the methodological section, bidding functions for each agent and auction have been estimated by means of OLS regressions using both a linear and log-log specification. Given the characteristics of the data analyzed, only average bidding functions could be estimated for each agent. Summary statistics are reported in Table 1, where figures have been normalized relatively to the average allotment value, while in Figure 4 the estimated empirical distributions, after log transformation, are plotted. Only results obtained for the linear model have been reported, since the latter specification appeared to be superior to the log-log model in terms of fit (the average R^2 is equal to 0.98 for the linear model and 0.95 for the log-log model). As can be noted from Table 1 and Figure 4, there is evidence of heterogeneity across bidders, with 70% of the slopes and intercepts falling in the range (-0.002, -0.16) and (0.008, 0.85), respectively (the estimated standard deviations are equal to 0.93 and 1.89, with mean values equal to -0.20 and 0.41, for slopes and intercepts respectively).

Evidence of heterogeneity is also provided by the estimated price (interest rate) elasticities, ranging between -203 and -33 (estimated mean and standard deviations are -94 and 34). Despite the variability found, in all cases the evidence points to highly elastic (inverse) bidding functions. Computing the price elasticities using the log-log model, rather than using the average bids values, does not modify this conclusion, with quantiles also numerically very similar to the ones obtained from the linear model.

As shown by the QQ-plots reported in Figure 4, the distribution of the estimated slopes and intercepts is very close to a lognormal one, while for the elasticities the evidence is less compelling, due to the left tail being heavier than predicted. An important open question thus is whether the presence of heterogeneity is sufficient to empirically reject the theoretical results implied by an homogeneous agents framework.

5.2. Aggregate bid schedule

The aggregate bidding function for each auction has been obtained from the aggregation of the individual bidding functions, as previously discussed in the methodological section. As done for the single agent bidding function, both a linear model and a log-log model have been estimated. Since the log-log model provided a superior fit for the aggregated bidding function (the average R^2 is equal to 0.71 for the linear model and 0.89 for the log-log model)[16], in Table 2 only the quantiles for the estimated coefficients from the log-log model are reported, also including the price elasticity computed from the linear model for comparison[17], while in Figure 5 the estimated empirical distributions are plotted. As is shown in Table 2, the estimated elasticities show some variation, ranging between -476 and -266, with mean and standard deviation values equal to -355 and 69, respectively. Moreover, the probability of an elasticity lower than -400 is close to 0.4, above -400 and below -300 about 0.3, and above -300 about 0.3 as well. Over the sample investigated, the estimated elasticity has been subject to swings of up 30% from one auction to the other, with little evidence of serial correlation in the estimated parameter. Yet, the elasticities are always very large, pointing to a very strong reactivity of liquidity demand in all the cases analyzed, and therefore to little heterogeneity across auctions, at least from this point of view. Similar findings hold for the estimated intercepts, while the elasticities estimated from the linear model show lower heterogeneity, ranging between -158 and -67, still showing very strong price reactivity of the aggregate bidding functions. Interestingly, as shown by the QQ-plots, also at the aggregate level there is evidence that the log normal distribution is appropriate to describe the distribution of intercepts and slopes (elasticities).

[16] Interestingly, the linear model seems however to provide a better fit to the data than the log-log model since auction 25.

[17] The latter has been computed in correspondence of the average aggregate bid.

5.3. Bid-shading analysis

In Figure 6 the empirical distribution for the estimated bid-shading components (multiplied by 100), are plotted, while in Table 2 quantiles for the values of the components and the test for significance of the estimated components are reported. Two findings seem to be of particular interest. First, the estimated bid-shading components tend to be small, ranging between 0.2 b.p. and 0.8 b.p., with average value of 0.5 b.p. and a standard deviation equal to 0.12 b.p. Second, the estimated bid-shading components tend to be statistically significant. The null of zero bid-shading component, against the alternative of positive bid-shading component, can in fact be rejected at the 1% significance level 90% of the times. Given the large number of bidders participating at each auction and across auctions, finding positive, statistically significant bid-shading components provides evidence that bid-shading does not disappear even when n becomes very large.

5.4. Tests on individual bidding behavior

In Table 3 the results of the tests on bidding behavior described in the methodological section are reported. The regressions have been estimated by OLS and heteroskedasticity consistent standard errors have been computed. Moreover, in order to control for the different magnitude of the variables employed the dependent and independent variables have been standardized.

Test 1: Is more successful bidding associated with more aggressive bidding? The key parameters to answer this question are θ_{α_1} and θ_{β_1}, which are expected to be both negative as an increase in $\bar{s}_i$ (less success) should be correlated with less aggressive bidding behavior.

From Table 3 it is possible to note that both estimated parameters are negative and statistically significant. However, the R^2 of the slope regression is virtually zero (0.01), while that of the intercept regression is non negligibly larger (0.12).

Test 2: Is the strategic inverse bid schedule flatter than the true demand? The key parameter for this test is θ_q, which is expected to be negative in the case large bid volumes are accompanied by less bid-shading. From Table 3 it is possible to note that this hypothesis is weakly supported by the data. In fact, the linkage between slopes and the bid quantities is negative, but significant only at the 10% level. However, the R^2 of the regression is virtually zero (0.01), suggesting that bidders' heterogeneity is little explained by this size variable.

Test 3: Sources of bid shading The key parameters for this test are γ_{1_1}, γ_{2_1}, γ_{3_1}. Theoretical results suggest that $\gamma_{2_1} > 0$; and $\gamma_{3_1} < 0$; No clear cut prediction can be made about γ_{1_1}. From the results it is possible to note that there is evidence that bid-shading tends to fall as value uncertainty and the number of bidders increase ($\hat{\gamma}_{1_1}$, $\hat{\gamma}_{1_3} < 0$) and to increase as supply uncertainty increases ($\hat{\gamma}_{3_2} > 0$). The linkage of bid-shading with value uncertainty is significant only when the one-week Eonia rate volatility is employed as a measure of value uncertainty. As a general result, using the standard deviation of the estimated intercepts to proxy value uncertainty leads to less significant estimates, both in terms of estimated coefficients and R^2 of the regressions, than when the volatility of the one-week Eonia rate is employed.[18]

Overall, the evidence is in line with the theoretical predictions. The finding that the parameter γ_{1_1}has a negative sign is interesting. Assuming that it is capturing the risk premium effect of an increase in the volatility of the one-week EONIA swap rate, a sufficient condition for a decrease in bid shading is that the residual supply facing individual bidder should not have changed much. The fact that the bid-to-cover ratio in the sample has increased somewhat when market and tender rates increased suggests that this might have been the case.

5.5. Test 4: Aggregate bid curve

In Table 4 the results of the tests on the aggregate bid curve described in the methodological section are reported. The regressions have been estimated by OLS and, in order to control for the different magnitudes of the variables, the dependent and the independent variables have been standardized. Tests on the residuals, for both equations, revealed neither autocorrelation nor heteroskedasticity. The $\overline{R}^2$ are high for both equations (90% for the slope regression and 70% for the intercept regression).

Overall the results are coherent with the predictions. First, the impact of the level of the one-week EONIA swap rate is positive, but statistically significant only for the intercept, not for the slope of the (inverse) aggregate bid curve ($\delta_{1_1} > 0$ and $\delta_{1_2} = 0$). Second, supply uncertainty has a negative, statistically significant impact on the intercept ($\delta_{2_1} < 0$); however, and against the prediction of the theory, supply uncertainty seems has a positive impact on the slope of the (inverse) aggregate bid schedule ($\delta_{2_2} \neq 0$). This effect may be due to the heterogeneity of

[18] The highest, average and lowest R^2 for the parametric disaggregated approach are 0.73, 0.66 and 0.58, respectively.

bidders. Third, a higher bid-cover ratio is associated with an upward shift in the bid curve at the following auction ($\delta_{3_1} > 0$). Fourth, as predicted, an increase in the number of bidders leads to a decline in the level and a flattening of the bid curve ($\delta_{4_1}, \delta_{4_2} < 0$).

6. Conclusions

Overall the results suggest that strategic and optimal behavior is prevalent in ECB tenders. Despite the documented heterogeneity across bidders, bidding behavior in ECB tenders seems consistent with optimal behavior in a multi-object discriminatory pricing auction. There is evidence of a statistically significant bid shading component, even though the number of bidders is very large. We argue that the economic analysis of the winner's curse in the context of the open market operations performed by the Eurosystem may have been impaired by an identification problem. Bid-shading increases with liquidity uncertainty and decreases with the number of participants and with price uncertainty. The latter suggests that when the EONIA swap rate increases the residual supply facing individual bidders does not change much.

References

[1] Athey, Susan and P. A. Haile, 2004, Nonparametric Approaches to Auctions, (forthcoming Handbook of Econometrics).

[2] Ewerhart, C., N. Cassola, N. Valla, 2006, Declining Valuations and Equilibrium Bidding in Central Bank Refinancing Operations, (forthcoming ECB Working Paper).

[3] Haile, P.A., H. Hong and M. Shum, 2003, Nonparametric Tests for Common Values in First-Price Bid-Sealed Auctions, mimeo, University of Yale.

[4] Hortaçsu, A., 2002a, Mechanism Choice and Strategic Bidding in Divisible Good Auctions: An Empirical Analysis of the Turkish Treasury Auction Market, mimeo, University of Chicago.

[5] Hortaçsu, A., 2002b, Bidding Behavior in Divisible Good Auctions: Theory and Evidence from the Turkish Treasury Auction Market, mimeo, University of Chicago.

[6] Nyborg, K., U. Bindseil, and I. Strebulaev, 2002, Bidding and performance in repo auctions: evidence from the ECB open market operations, ECB Working Paper No. 157, July 2002.

[7] Pagan, A. and Ullah, A., 1999, *Nonparametric Econometrics*, Cambridge University Press, Cambridge, United Kingdom.

[8] Scalia, A., G. Bruno, and M. Ordine, 2005, Bank's participation in the Eurosystem auctions and money market integration, paper presented to the ECB Conference on Monetary Policy Implementation, 20-21 January 2005.

[9] Wilson, R., 1979, Auctions of Shares, *Quarterly Journal of Economics,* 93, 675-698.

Table 1: Estimated parameters, quantiles.

Q	β_L^d	α_L^d	ε_L^d	β_{LL}^d	β_{LL}^a	α_{LL}^a	ε_L^a
0.01	−2.6004	0.0007	−203.12	−240.28	–	–	–
0.05	−0.9268	0.0016	−154.64	−178.69	−476.39	189.16	−158.72
0.10	−0.4288	0.0033	−135.00	−162.78	−465.51	196.95	−156.33
0.20	−0.1649	0.0075	−118.74	−138.98	−419.89	203.59	−143.55
0.30	−0.0659	0.0136	−106.39	−122.35	−401.76	221.13	−123.79
0.40	−0.0361	0.0232	−98.64	−112.58	−384.85	235.93	−112.13
0.50	−0.0207	0.0408	−89.46	−102.88	−351.29	256.31	−107.75
0.60	−0.0114	0.0729	−80.40	−92.36	−334.67	279.51	−103.05
0.70	−0.0067	0.1303	−73.98	−84.95	−306.43	288.01	−89.07
0.80	−0.0037	0.3322	−67.06	−75.97	−286.83	297.92	−81.75
0.90	−0.0016	0.8484	−58.83	−67.13	−267.05	325.05	−68.15
0.95	−0.0008	1.6879	−51.46	−58.35	−266.32	337.81	−67.21
0.99	−0.0004	4.6742	−33.18	−39.07	–	–	–
mean	−0.1996	0.4067	−94.12	−108.53	−355.30	259.72	−108.21
std.dev.	0.9288	1.8951	33.68	39.62	68.86	48.82	30.46

The table reports the quantiles for the estimated slopes (β), intercepts (α) and price (bid rate) elasticities (ε) obtained from the disaggregated (*d;* single bidder) and aggregated (a; single auction) models. The linear model is denoted by L, while the log-log model by LL. Note that the slope parameter in the log-log model measures the price (bid rate) elasticity.

Table 2: Estimated bid-shading components and significance tests, quantiles.

Q	P_{d_k}	$p\text{-}val_{d_k}$
0.01	0.2161	0.0000
0.05	0.2806	3E-6
0.10	0.3186	1E-5
0.20	0.3683	4E-5
0.30	0.4017	8E-5
0.40	0.4312	0.0002
0.50	0.4586	0.0003
0.60	0.4878	0.0005
0.70	0.5202	0.0010
0.80	0.5571	0.0019
0.90	0.6144	0.0048
0.95	0.6634	0.0102
0.99	0.7914	0.0329
mean	0.4653	0.0022
std.dev.	0.1185	0.0065

The table reports the quantiles for the estimated bid-shading components and for the p-values of the one-sided test for statistical signficance of the estimated bid-shading components. The bid-shading components have been computed with kernel estimation (P_{d_k}). Figures have been multiplied by 100.

Table 3: Tests on bidding behaviour

	P_{d_k}	$P_{d_{k_2}}$
θ_{α_1}	-0.342^{**}	
	(0.081)	
θ_{β_1}	-0.117^{**}	
	(0.026)	
θ_q	-0.093	
	(0.055)	
γ_{1_1}	-0.352^{*}	-0.144
	(0.169)	(0.175)
γ_{2_1}	0.306^{**}	0.455^{**}
	(0.133)	(0.136)
γ_{3_1}	-0.676^{**}	-0.494^{**}
	(0.085)	(0.089)

The table reports the estimated parameters for the ausiliary test regressions. Heteroschedastic standard errors are reported in brackets. * denotes significance at the 5% level, ** denotes signficance at the 1% level. P_{d_k} denotes the results obtained by the disaggregated parametric approach with kernel estimation, using the conditional standard deviation of the one-week Eonia rate as proxy for value uncertainty; $P_{d_{k_2}}$ denotes the results obtained by the disaggregated parametric approach with kernel estimation, using the standard deviation of the estimated intercepts as proxy for value uncertainty.

Table 4: Tests on aggregate bidding behaviour

i	$\lvert\beta\rvert$	α
δ_{1_i}	0.03	0.29^{*}
	(0.07)	(0.14)
δ_{2_i}	0.12^{*}	-0.33^{**}
	(0.06)	(0.11)
δ_{3_i}	0.09	0.45^{**}
	(0.07)	(0.13)
δ_{4_i}	-0.95^{**}	-0.74^{**}
	(0.066)	(0.061)

The table reports the estimated parameters for the auxiliary test regressions. Standard errors are reported in brackets. * denotes significance at the 5% level; ** denotes signficance at the 1% level; $i = int, slp$, refer to the test for the intercept and the absolute slope respectively.

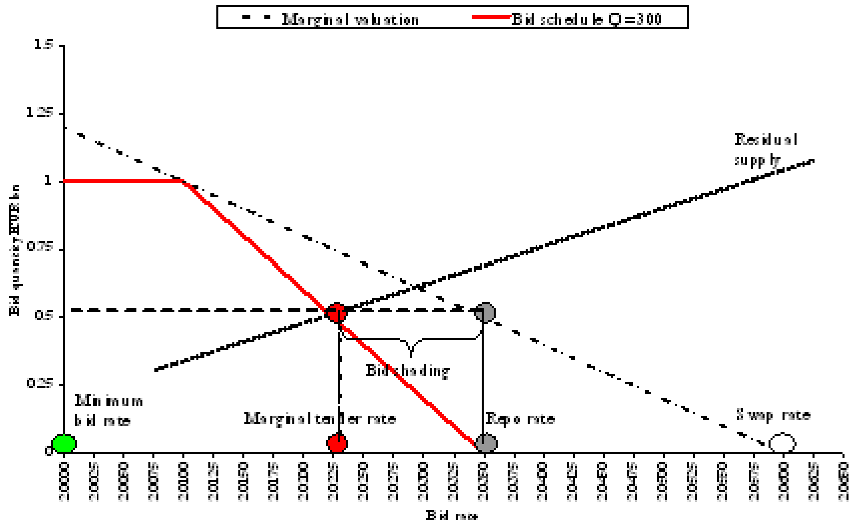

Figure 1. Equilibrium bidding

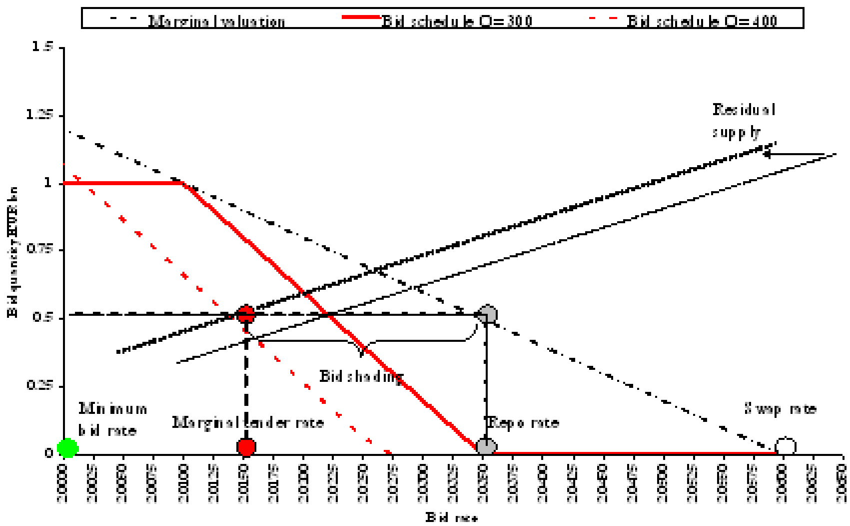

Figure 2. Increase in bid shading

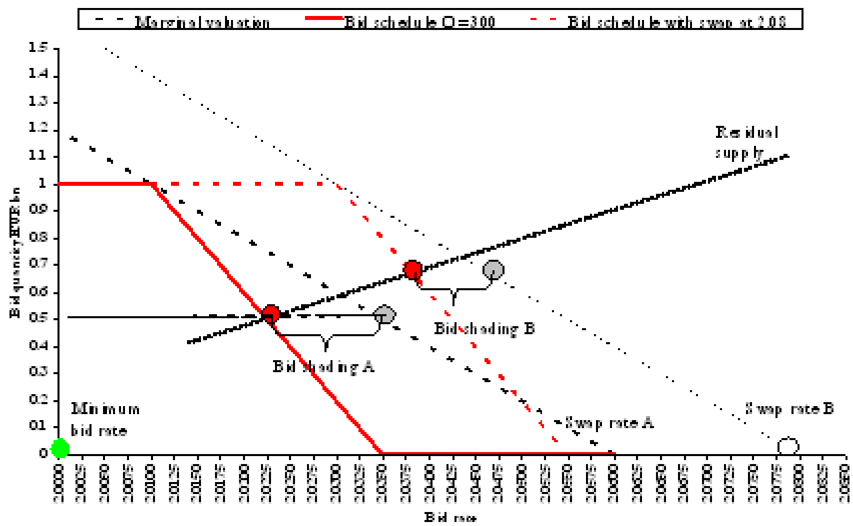

Figure 3. Decrease in bid shading

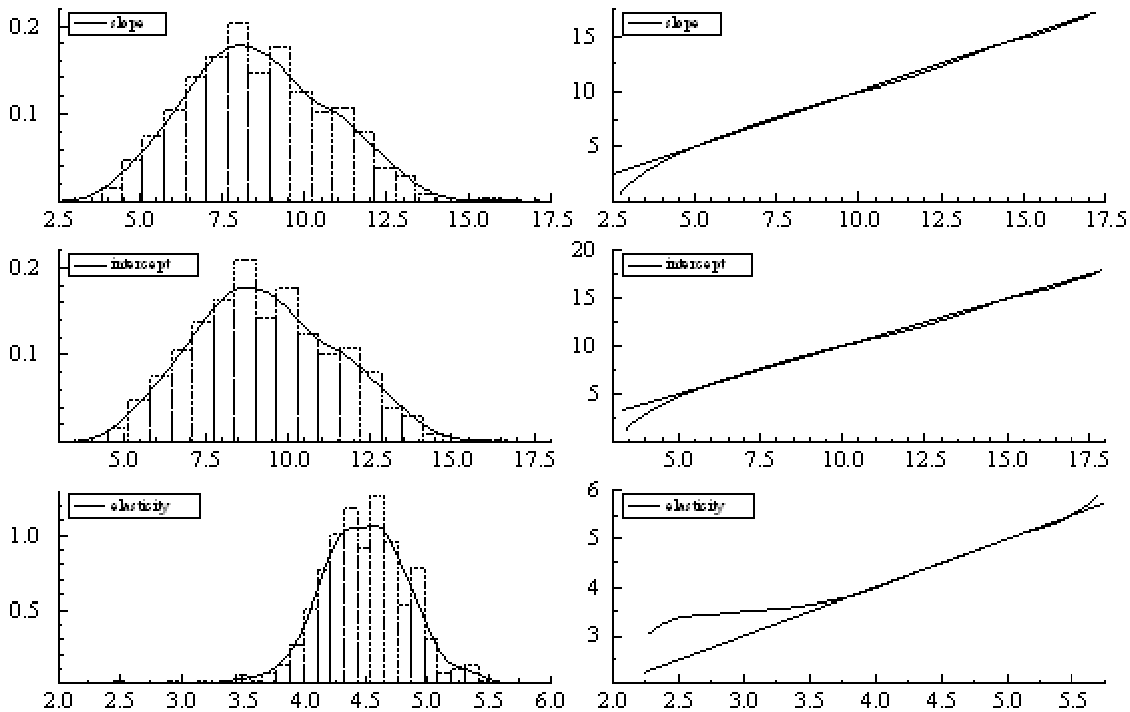

Figure 4: Empirical distributions and QQ-plots. Estimated log intercepts, log absolute slopes and log absolute elasticities.

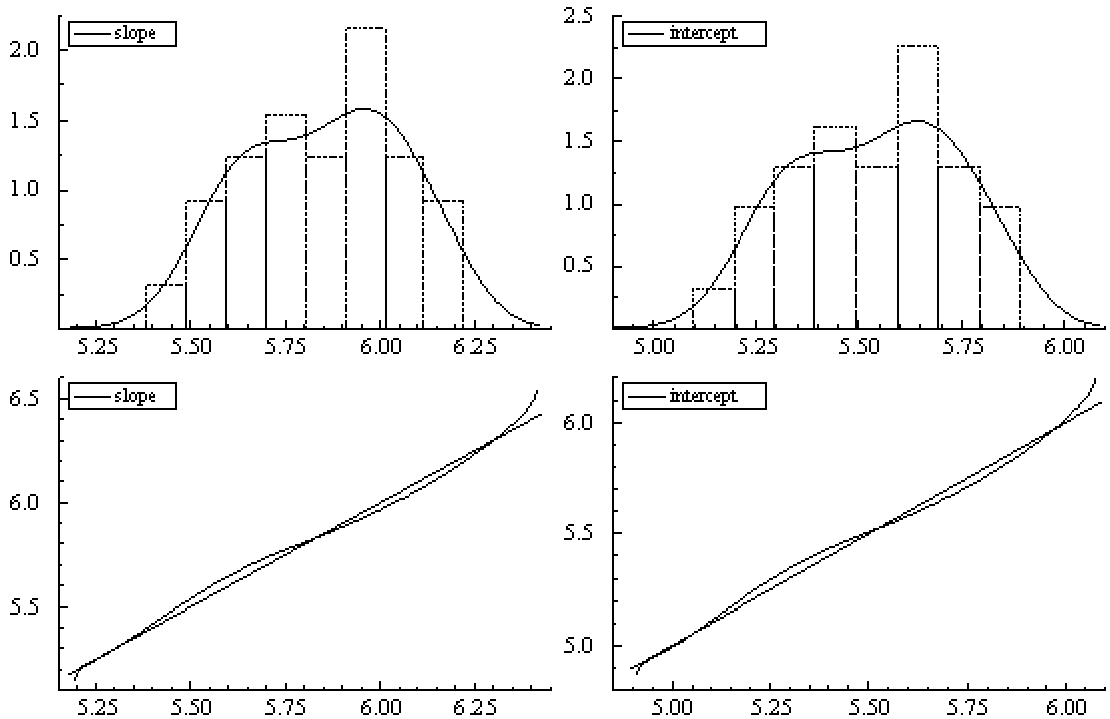

Figure 5: Empirical distributions and QQ-plots. Estimated log absolute slopes and log intercepts, aggregate log-log models.

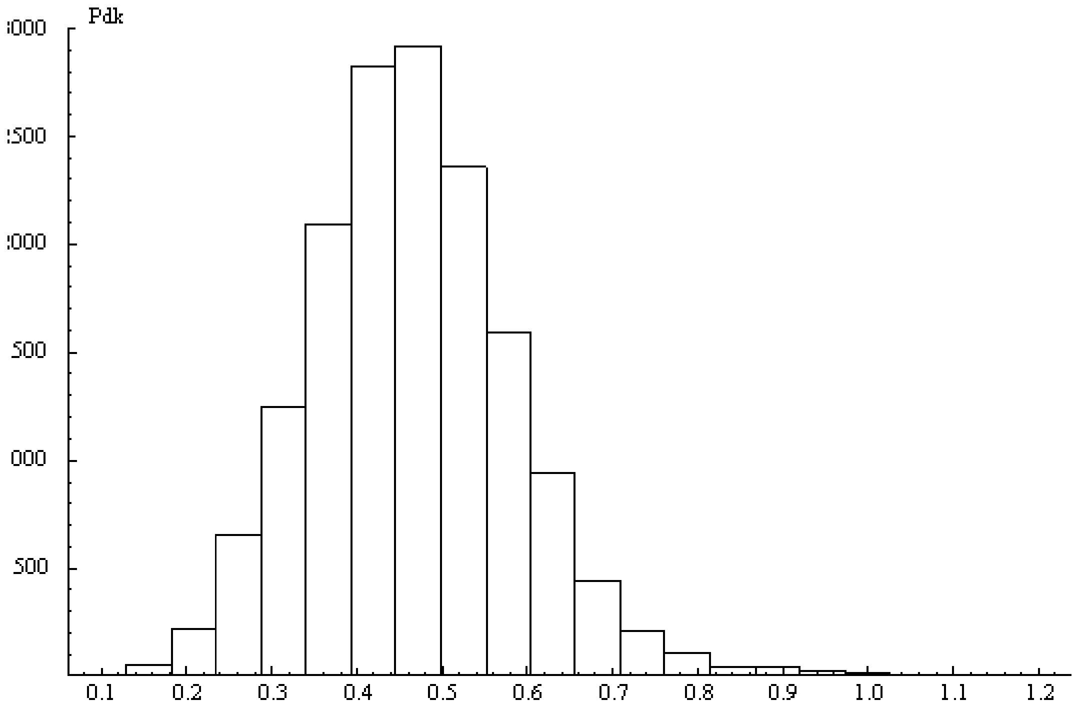

Figure 6: Bid-shading components, empirical distributions, disaggregated parametric approach with kernel estimation. Figures have been multiplied by 100.