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**Congestion Pricing in an Internet Market**

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# Congestion Pricing in an Internet Market.

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## Abstract

This paper analyzes a unique dataset of art auctions on eBay. We study the behavior of buyers and sellers, demand and supply, by means of a novel structural estimation approach. Our empirical framework considers the process of arrival of new bidders as well as the distribution of bidder valuations of artworks being auctioned. We use this empirical framework to quantify the effect of market congestion, and congestion pricing strategies implemented by the market intermediary. Because we explicitly model the process of arrival of new bidders, we can estimate the effect of congestion pricing on the number of bidders, the distribution of bidders' valuations, and the final selling price. Using the structural model we can also measure the impact of congestion pricing on the revenues of the artists and the market intermediary, as well as its effect on consumer surplus. Our results indicate that the congestion pricing policy acts as a coordination mechanism that facilitates the match between buyers and sellers.

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KEYWORDS: English Auctions, Internet markets, Structural estimation.

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# 1 Introduction

The Internet is an important source of current and future economic growth. A significant proportion of this growth will be generated from trade in online markets: business to business, business to customers or among individuals. Online markets are significantly different from traditional markets and we have only begun to explore their potential. For this reason, it is important to understand how Internet markets work, what type of market environment promotes Internet transactions and makes them more rewarding for buyers and sellers, as well as what type of market imperfections can prevent Internet markets from realizing their full potential. One problem that can hinder the development of a successful marketplace is that of market congestion. Congestion in an Internet market occurs when the cost of trade increases with the presence of additional agents, new sellers most likely.

Internet markets are usually under the command of a market intermediary. The market intermediary acts as a necessary catalyst for the transaction to occur, and for this reason it can charge fees for its services. It is likely that the intermediary will want to use market imperfections, like congestion, to its advantage in order to extract additional profit. Thus, when studying congestion in this environment one needs to take into account the actions of the intermediary. In this paper we analyze empirically the issue of congestion in a specific Internet market and the use of congestion-pricing strategies on the part of the market intermediary and its effect on buyers, sellers, trade, profit and welfare.<sup>2</sup>

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<sup>2</sup> Congestion pricing has been most often studied within the framework of transport networks. In the case of transport networks congestion pricing is usually employed with the goal of maximizing social welfare. In the case of Internet markets the goal instead is profit maximization on the part of the intermediary.

The existence of congestion in internet markets is not an unlikely event. Consider for example eBay, currently the most successful marketplace on the Internet, even for the case of homogeneous goods there are features like the seller's reputation, or the shipping costs, that can affect a buyer's decisions and that are not observed unless he/she incurs the cost of examining the auction listing. Thus, a seller with a bad reputation imposes a negative externality on buyers that are not willing to buy from this type of seller. It also imposes a negative externality on other sellers who are competing to get the attention of potential buyers. This effect is multiplied when the goods are intrinsically heterogeneous and difficult to categorize, like in the case of original works of art. A buyer looking for that special painting may have to browse through hundreds or even thousands of listed items. Even the most powerful search engine is not very helpful in such case. Anecdotal evidence obtained from my conversations with artists selling their work on eBay indicates that the increasing presence of imitations of original art, that is paintings that are created using industrial methods but that resemble original works of art, are making it very difficult for artist to compete on eBay. The artists seem mostly concerned by the fact that the dramatic increase in the number of listings of fake originals within the category of original art makes true originals very difficult, or costly, to find by potential buyers.

In this paper we study a specific example of congestion pricing in an Internet market. For several years now eBay has been using a rudimentary form of congestion pricing, or as they call it "Featured Plus!" This type of congestion pricing works by giving sellers the option to incur an extra fee at the time of posting their items for sale and in return having

these items listed first when buyers search for specific features, or categories. The basic idea is that potential buyers incur a lower cost of search when looking at “featured items.” In addition, the data suggests that “Featured Plus!” is also used by sellers as a “signal” of quality.<sup>3</sup>

We analyze data from art auctions by “self representing” artists. This group is composed of artists who sell their own artwork through eBay. We use this data along with a novel structural empirical framework to quantify the economic impact of the simple congestion-pricing scheme implemented by eBay. We explicitly model the process of arrival of new bidders, and as a result we can estimate the effect of congestion pricing on both the number of bidders, the distribution of bidders’ valuations and the selling price. Finally, using the structural model we are able to measure of the effect of congestion pricing in this particular market.

The paper proceeds as follows. In Section two we describe the data to be used in the analysis. Section 3 sets the stage for our empirical analysis by presenting a sketch of a theoretical model and the description of the econometric methodology. Section 4 reports estimation and policy simulation results and section 5 draws conclusions.

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<sup>3</sup> However, the “signaling” effect is not the only reason why sellers use feature price since it is also often used with homogeneous goods (like computer processors).

## 2 The Market and the Data

In July 2001 we began collecting auction data from a group of “self representing” artists on eBay, self-denominated EBSQ (an abbreviation for “e-basquiat” after the artist Jean-Michel Basquiat). This group is composed of artists who sell their own artwork through eBay. Although this group did not necessarily include all self-representing artists, to our knowledge this was the only group of its kind at the time. In most cases the item for sale was an original work of art, most often a painting but other types of artwork like collages, ceramic tiles, or sculptures were also collected. In our analysis we use data on original paintings only.

Over the period of data collection some of the artists originally on EBSQ created other groups (EBSQ+, ESR@, OW@), that were also included in our dataset. The data collected includes all the auctions associated with any one of these groups from the third week of July, 2001, to the second week of November 2001. This includes approximately eight thousand auctions of artworks by self-representing artists on eBay. The selling price of the artwork auctioned ranges from less than one dollar to hundreds, or even thousands of dollars in a small number of auctions. Between August and December, 2004, a second round of data collection was conducted on a subgroup from the original group of artists consisting of those artists who posted auctions regularly during the original data collection period. This includes about three thousand auctions.<sup>4</sup> The sample considered in this study includes auctions from

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<sup>4</sup> Whenever feasible, we collected data using computer programs. In cases where this was not possible, a specific database was used to aid in the collection of data and to minimize errors.

this subgroup of artists for the years 2001 and 2004.

Sellers offer objects for auction by posting a description of the item being auctioned, including pictures. Sellers can choose the auction length (three, five, seven or ten days) and other auction characteristics. Buyers can browse through thousands of auctions posted every day. Auctions are organized by categories, and subcategories, which simplifies the buyer's search. Buyers can also use a powerful search engine and are also able to request information from the seller about an auction anonymously via email.

Bidders participating in an auction can submit a "proxy" bid at any time before the end of the auction. As long as a bidder's proxy bid is higher than the second highest active proxy bid she will remain the highest bidder, at a price equal to the second highest bid plus a small increment. Proxy bids can be revised (increased) at any time prior to the end of the auction. Bidders can also use Snipping programs which allow them to place a predetermined bid amount at a chosen time before the end of the auction. Only bids above the existing second highest bid, plus an increment, are acceptable. The highest bidder at the end of the auction wins the item at a price equal to the second highest bid, plus a small increment.

For each auction, we collected four different types of information: item characteristics, auction characteristics, bidding history, and artist reputation. Characteristics specific to the object being auctioned include information on the height, width, style (abstract, pop, whimsical, etc.), medium (acrylic, oil, etc.) and ground (stretch canvas, paper, wood, etc.). Auction characteristics include the length of the auction, the opening bid, the shipping and handling fees, the eBay category in which the object is being listed, and whether the auction

had a reserve price. The bidding history includes all bids except for the highest bid which is not reported. At the end of each transaction the buyer and the seller have a chance to rate their level of satisfaction with the transaction (positive, neutral, negative). Thus, we collect information on the type of feedback received by the seller in previous transactions.

Several characteristics of this data make it unique. First, the data collected comprises all eBay market activity for a specific group of sellers for a long period of time, while the data collected by other researchers usually represents only a narrow snapshot of market activity. Second, by nature the intrinsic value of artwork is uncertain, especially in the case of less well-known artists. In contrast, much of the data collected by other researchers refers to items for which their market value can be determined with accuracy, like coins, stamps or computers, which lessens the value of auctions as a selling, price-finding, mechanism. Third, the data can also be used to analyze other issues of interest to economists, like artist's labor supply.

Table 2 provides descriptive statistics for the overall sample and the selected subsample. Table 3 provides descriptive statistics according to the auctions' featured/not-featured status. Featured items sell with 90% probability while non-featured items sell with 61% probability in 2001 and with 48% probability in 2004. The average selling price is about \$200.00 for featured items while it is about \$50.00 for non-featured items, in real 2004 dollars. In average, "Featured Plus!" items attract about five bidders while non-featured items attract less than two. The data suggests that "Featured Plus!" is also used by sellers as a signal of quality. In particular, featured paintings are about twice the size of non-featured



items when measured in square feet when compared with non-featured paintings. Table 4 provides frequency tables for specific characteristics of paintings (style, medium and ground). Figure 1 presents the distribution of the average probability of successful sale across artists. For the most part this distribution follows closely the 45% degree line. Thus, about 50% of artist have successful auctions less than 50% of the time, and about 20% of artists have successful auctions at least 80% of the time.

### 3 Theoretical Model and Empirical Methodology.

The empirical methodology described in this section combines a model of choice on the part of the artist as to whether to feature or not to feature a painting with a model of demand in an auction market environment. First, we present a simple example of an auction that illustrates all the technical problems that we need to address. After that, we present a theoretical model of an auction market and the econometric methodology which originates from the theory.

#### *A. A Simple Example of eBay Auctions.*

In order to aid our discussion consider an stylized example of eBay auctions presented in table 1. Differences across auctions are the result of differences in the timing (order) of arrival of bids, which results in different sequences of auction prices, denoted  $s_t$ , even though the players are the same in all auctions. There are five potential bidders with valuations,  $v_i$ , equal to \$10, \$25, \$50, \$75 and \$100, and the starting price is set by the seller at \$10. For simplicity we assume that each bidder bids only one time and her bid equals her valuation. In

particular, in auction one all potential bidders have a chance to bid. In contrast, in auction four only the bidders with the two highest valuations are able to bid, while the remaining potential bidders are unable to bid because the auction price is higher than their valuation at their time of arrival. Thus, when studying auctions on eBay we need to be aware of the following facts: the number of potential bidders may be different from the number of actual bidders; the time of arrival of bids and their magnitudes matters; and some potential bids will not be realized. Finally, although in the presented examples the number of potential bidders is predetermined in actuality this number is stochastic, a function of the auction characteristics and unobserved in many cases. All these issues will be addressed in our empirical methodology

### *B. Description of the Environment.*

We model eBay auctions as IPV, ascending-bid, second-price auctions, subject to some specific rules: The seller of the object being auctioned sets the duration of the auction,  $T$ , and the starting value,  $s_0$ ; Bids are submitted at any time within the  $[0, T]$  time interval; At each point during the auction the value of the current second highest bid, say  $s_t$ , and the number of current, or active, bidders are public information; New bids arrive sequentially at any point during the  $[0, T]$  time interval; Any new bid has to surpass  $s_t$  by a minimum increment in order to be recorded. To keep the notation simple, we will avoid mention of this minimum increment, except when absolutely necessary.

**The distribution of bidders' valuations:** Consider a particular auction ending with  $M$  bids,  $\{b_k\}_{k=1}^M$ , from  $K$  different bidders ( $K \leq M$ ) with valuations  $\{v_k^*\}_{k=1}^K$ . Define the

potential number of bidders as the number of bidders that would have chosen to bid in an otherwise identical sealed bid auction in which all bids are acceptable, and denote it by  $N$  ( $\geq K$ ). Each potential bidder,  $j$ , assigns a value,  $v_j$ , to the object being auctioned, with each value,  $v_j$ , being a random realization from a distribution  $F(v)$ . Bidders know and care only about their own valuation. By definition  $\{v_k^*\}_{k=1}^K \subset \{v_n\}_{n=1}^N$ .

**The bidders arrival process:** We consider only the time of arrival of the first bid by any new bidder. The process we have in mind is one in which at the time of her first bid the bidder incurs a certain cost necessary in order to learn about the product by reading the product description, looking at the available pictures, and perhaps even emailing the seller with any questions that the buyer may have. After the first bid, we assume that the bidder can continue bidding as many times as she wants at no additional cost. Also, the timing of any additional bid from an active bidder conveys no new information to active or potential bidders, other than its effect on the minimum acceptable bid.

We model the arrival of new bidders to an auction in a way similar to the arrival of job offers in a structural job search model (Flinn and Heckman, 1982), and also similar to how previous authors have modeled this arrival process in an auction environment (Wang, 1993). We assume that the arrival of new potential bidders at any time  $t \in [0, T]$  follows a *Poisson*( $\lambda$ ) process, but only those with high enough valuation of the object being auctioned end up bidding. Thus, the instantaneous probability of arrival of new bidders is the product of the arrival rate of potential bidders,  $\lambda$ , and the probability that the existing second highest bid, say  $s(t)$ , is below the new bidder's valuation  $v$ , or equal to  $\bar{F}(s(t)) = 1 - F(s(t))$ . Thus,

the hazard function for the arrival of new bidders is  $\alpha(t) = \lambda \bar{F}(s(t))$ . This also implies that the hazard function for the arrival of potential, unrealized, bids is  $\lambda F(s(t))$ .

### *C. Demand Estimation.*

The econometric approach to estimation of second-price ascending-bid auctions that we consider is based on that described in Canals-Cerda and Percy (2005). An advantage of this estimation approach when compared with others in the literature is that it explicitly models the process of arrival of new bidders, and as a result allow us to estimate the effect of congestion pricing on the number of bidders, the distribution of bidders' valuations and the auction final price.<sup>5</sup>

**The distribution of bidders valuations:** Bidders behavior is modeled in terms of the underlying distribution of bidders valuations and the following two behavioral assumptions:

*Assumption 1 :* Bidders do not bid more than they are willing to pay.

*Assumption 2 :* Close to the end of the auction those bidders with valuation of the item above its current minimum acceptable bid choose to bid their maximum willingness to pay, if they have not done that already.

The first assumption is identical to that in Haile and Tamer (2003). The second assumption is somewhat stronger, it guarantees that the winning bid will be equal to the second highest willingness to pay in auctions with two or more active bidders. The second assump-

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<sup>5</sup> Other papers that use data from ebay are: Hasker, Gonzalez and Sickles (2001) that assume the number of bidders to be the same across auctions, and estimate this number, and Bajari and Hortacsu (2003) that assume this number to be a function of auction characteristics. Neither paper uses data on the time of arrival of bids to aid their estimation.

tion in Haile and Tamer states that “Bidders do not allow an opponent to win at a price they are willing to beat.” This assumption would be equivalent to our second assumption under the traditional English auction framework employed in most empirical work, as long as bidders play according to their dominant strategy at some point before the end of the auction. Our assumptions are consistent with many different types of bidding behaviors. In fact, in the econometric specification we will only require that the bidder with the second highest valuation bids her true valuation before the end of the auction and that the bidder with the highest valuation bids at least as much as the second highest bidder.

Consider a particular auction that ends after time  $T$  with  $M$  bids being placed,  $\{b_k\}_{k=1}^M$ , from  $K$  different bidders ( $K \leq M$ ). Assume also that associated to this auction are  $N$  ( $\geq K$ ) potential bidders with valuations  $\{v_n\}_{n=1}^N$ . Focusing our attention on the highest bid from each bidder, say  $\{b_k^*\}_{k=1}^K$ , and organizing this set in ascending order, we obtain

$$\bar{b}_1 < \bar{b}_2 < \dots < \bar{b}_{K-1} < \max_k b_k^* = \bar{b}_K = \bar{b}_{K-1} + \Delta,$$

with  $\bar{b}_k$  representing the  $k$ -th order statistic from the set  $\{b_k^*\}_{k=1}^K$ ,  $\bar{b}_{K-1} + \Delta$  representing the winning bid, and  $\Delta$  representing the minimum increment allowed by eBay. Define also  $\bar{v}_n$  as the  $n$ -th order statistic from the set  $\{v_n\}_{n=1}^N$ . Given assumption 1, Lemma 1 in Haile and Tamer guarantees that  $\bar{b}_k \leq \bar{v}_{N-(K-k)}$ , for  $k = 1, \dots, K$ . In particular, for  $k = K$  and  $K - 1$ , we have that  $\bar{b}_K \leq \bar{v}_N$ , and  $\bar{b}_{K-1} \leq \bar{v}_{N-1}$ . Furthermore, assumption 2 implies that  $\bar{b}_{K-1} = \bar{v}_{N-1}$ . Within this framework, the event  $\left\{ (b_k)_{k=1}^M \mid N \right\}$  has the same probability

as the event  $\left\{ \left( \bar{b}_k \leq \bar{v}_{N-(K-k)} \right)_{k=1}^K, \bar{b}_{K-1} = \bar{v}_{N-1} | N \right\}$ , because assumptions 1 and 2 impose restrictions on the vector of maximum bids only.

Haile and Tamer construct their bounds based on two assumptions. Their first assumption is identical to ours. Their second assumption basically implies that if the winning bid is  $\bar{b}_K$  the bidder with the second highest valuation would not be willing to beat this price, or  $v_{N-1} < \bar{b}_K$ . This, along with assumption 1, implies that  $\bar{b}_{K-1} < v_{N-1} < \bar{b}_K = \bar{b}_{K-1} + \Delta$ , or  $|\bar{b}_{K-1} - v_{N-1}| < \Delta$ . Thus, taking into account that  $\Delta$  takes a small value, it seems that not much is lost by implementing instead our assumption 2, which implies that  $\bar{b}_{K-1} = v_{N-1}$ . In addition, our model can be manipulated more easily when conducting structural policy analysis.

#### *D. Empirical Specification.*

Denote by  $X_i$  the vector of artist specific characteristics and by  $X_j$  the vector of characteristics specific to the object being auctioned. The empirical specification models three different features of the data: The bidders' arrival process, the bidders' valuation of the object being auctioned and the buyer's feature choice.

**The Bidders' arrival process:** The theoretical model describes the arrival of new potential bidders as a *Poisson*( $\lambda$ ) process. While this is a convenient theoretical assumption it is inadequate for empirical work. Two important limitations of the Poisson model are the excess zeros problem and the overdispersion problem (Cameron and Trivedi, 1998). There are several generalizations of the Poisson model that do not suffer from these shortcomings. In particular, We model the bidder's arrival process as a finite-mixture negative binomial

model. Because this is a well known approach we will not develop the model here. With this model at hand we can define  $P(N = n|\lambda_{ij}, T)$  as the probability of  $n$  potential bidders by time  $T$ , with  $\lambda_{ij}(t)$  representing an artist-auction specific hazard. The likelihood of bidders arrival is not necessarily uniform over the duration of the auction  $[0, T]$  or across items for sale. For example, impatient bidders may choose to concentrate their attention on auctions close to their ending time. Also, the available search options on eBay make it easier for a potential buyer to find out about auctions just after they have been posted or close to their ending. Thus, we consider a flexible specification of the form  $\lambda_{ij}(t) = \lambda(t, X_i, X_j)$ , with  $(X_i, X_j)$  defined above, and allow the baseline hazard to take on different values over the duration of the auction.

The process just described refers to the arrival of potential bidders. However, the arrival process that we observe in the data includes only actual bidders, that is, the potential bidders that were able to bid because their valuation of the auctioned item was above the auction price at the time they were ready to bid. We can model the arrival of new bidders at a particular time  $t$  according to the following parameterization,  $\lambda_{ij}^* = \lambda_{ij}\bar{F}(s_{ij}(t))$ , with  $\lambda_{ij}$  defined above and  $s_{ij}(t)$  representing the minimum acceptable bid at time  $t$ .

**The Bidders' Valuation:** Denote by  $v_{ijk}$  bidder's  $k$  valuation of object  $j$  being auctioned by artist  $i$ . We impose the following structure on  $v_{ijk}$  :  $\log v_{ijk} = \Psi(X_i, X_j) + w_{ijk}$ , where  $w_{ijk} = \log v_{ijk} - \Psi(X_i, X_j)$  represents the residual, bidder specific, (log-)valuation of the object being auctioned. In particular, we consider  $\phi(X_i, X_j) = \alpha X_i + \beta X_j$  as a convenient empirical specification. Finally, we assume that the distribution of unobserved valuations

$w_{ijk}$  is distributed normal.<sup>6</sup>

**The “Featured Plus!” Choice:** As our data shows, featuring an auction increases the number of potential buyers and thus increases also the probability of sale and the expected revenues. However, these are not necessarily the only reasons for featuring a painting, there are also non-pecuniary returns from featuring a painting which may influence an artist decision to feature. For instance, artists may extract utility from the sale of their art irrespectively of the selling price. Artists may also extract utility from the fact that their painting is being viewed, and appreciated, by many potential buyers. Also, a featured auction may act as advertisement and increase the artist’s customer base. For this reason, we choose not to model the feature choice as a simple profit maximizing decision. Instead we model the feature decision on the part of the artist as a discrete choice that is a function of the artist’s characteristics as well as the characteristics of the painting being auctioned. Formally, denote by

$$F_{ij} = I(F_{ij}^* > 0), \text{ with } F_{ij}^* = F^*(X_i, X_j) + \varepsilon_{ij}.$$

with  $\{F_{ij} = 1\}$  denoting the “Featured Plus!” choice,  $F_{ij}^*$  representing the unobservable utility from featuring the artwork, and  $\varepsilon_{ij}$  representing other idiosyncratic factors affecting this choice. In this framework, the probability of “Featured Plus!” choice can be defined as  $P(F_{ij}^* > 0)$ .<sup>7</sup>

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<sup>6</sup> We plan to generalize this framework to account for sources of unobserved heterogeneity.

<sup>7</sup> Of course, the implementation of this probability in an econometric model will require some additional functional form assumptions as well as assumptions about the distribution of  $\varepsilon_{ij}$ .



**The Likelihood:** Denote the time of the first bid by bidder  $k$  as  $t_k^1$ , for  $k = 1, \dots, K$ .

The likelihood of a particular observation  $\left( (b_k, t_k)_{k=0}^M, F_{ij} | b_0 \right)$  can be computed as

$$P \left( (t_k)_{k=1}^M | (b_k)_{k=0}^M, F_{ij} \right) P \left( (b_k)_{k=1}^M | b_0, F_{ij} \right) P(F_{ij}),$$

with  $P \left( (t_k)_{k=1}^M | (b_k)_{k=0}^M, F_{ij} \right)$  equal to  $P \left( (t_k^1)_{k=1}^K | (s_k(t_k))_{k=0}^M, F_{ij} \right)$  and  $P \left( (b_k)_{k=1}^M | b_0, F_{ij} \right)$  equal to  $P \left( (\bar{b}_k)_{k=1}^K | b_0, F_{ij} \right)$ ,  $t_k^1$  denoting the time of the first bid by bidder  $k \in \{1, \dots, K\}$ ,  $s(t_k)$  representing the minimum acceptable bid an instant before a new bid arrives at time  $t_k$  and  $b_0$  representing the starting value.

The probability  $P \left( (\bar{b}_k)_{k=1}^K \right)$  is a function of the number potential of bidders, which is unknown. However, assuming that the number of potential bidders is  $N (\geq K)$  we can compute  $P \left( (\bar{b}_k)_{k=1}^K | N \right)$ . We can also determine the probability of  $N (\geq K)$  potential bidders,  $P(N | N \geq K, \lambda)$ , conditional on the arrival process  $P(\lambda)$ . As a result we can compute,

$$P \left( (\bar{b}_k)_{k=1}^K \right) = E_N \left( P \left( (\bar{b}_k)_{k=1}^K | N \right) \right) = \sum_{N=K}^{\infty} P \left( (\bar{b}_k)_{k=1}^K | N \right) P(N | N \geq K, \lambda).$$

We can easily compute  $P(N | N \geq K, \lambda)$ . In order to compute  $P \left( (\bar{b}_k)_{k=1}^K | N \right)$  it is useful to first represent this probability in terms of the underlying density of bidder's valuations  $f(v)$ .

From the relationship between order statistics and the originating random variable we have that  $g(\bar{v}_1, \dots, \bar{v}_N) = N! f(\bar{v}_1) \dots f(\bar{v}_N)$ ,  $\bar{v}_1 < \bar{v}_2 < \dots < \bar{v}_N$ , with  $\{\bar{v}_n\}_{n=1}^N$  representing the set of order statistics from a random sample of  $N$  i.i.d realizations from a random variable with

associated density  $f(\cdot)$  (Hogg and Craig, 1978). The probability  $P\left(\left(\bar{b}_k\right)_{k=1}^K | N\right)$  is equal to  $P\left(\left(\bar{b}_k \leq \bar{v}_{N-(K-k)}\right)_{k=1}^K, \bar{b}_{K-1} = \bar{v}_{N-1} | N\right)$  and can be represented as a complex integral of  $g(\bar{v}_1, \dots, \bar{v}_N)$ . Interestingly this integral has an analytic solution in the form of a recursive polynomial function of  $\{F(\bar{b}_k)\}_{k=1}^{K-1}$ . The proof of this result relies on simple integration techniques and is included in appendix 1. With these results at hand, the computation of  $P\left(\left(\bar{b}_k\right)_{k=1}^K | N\right)$  is a straightforward numerical exercise that can be handled by any modern desktop computer.

Two special cases are  $K = 0$  and 1. In particular,  $K = 0$  indicates that none of the potential bidders that had a chance to bid valued the object being auctioned above its starting bid,  $P(K = 0 | b_0, N) = P(\text{Max}_{i=1, \dots, N} \{v_i\} \leq b_0) = F^N(b_0)$ . On the other hand,  $K = 1$  indicates that only a single active bidder was willing to bid above  $b_0$ . In this case,  $P(K = 1 | N) = P(\bar{v}_{N-1} \leq b_0, b_0 \leq \bar{v}_N | N)$ . In the case of a unique potential bidder we have that  $P(K = 1 | N = 1) = P(b_0 \leq \bar{v}_1 | 1) = 1 - F(b_0)$ . In general,  $P(K = 1 | N) = P(\bar{v}_{N-1} \leq b_0 + \Delta, b_0 \leq \bar{v}_N | N)$  indicating that the highest valuation should be above  $b_0$  and all others should be below  $b_0 + \Delta$ . The general case with  $N > 1$  is developed in appendix 1.

## 4 Empirical Specification and Results.

When considering a reasonable empirical specification one needs to take into account the specific characteristics of the data at hand. In particular, looking at tables 2 and 3 we observe first that the average number of bidders is much higher for featured items than for not-featured items. Also, the average selling price is significantly larger for featured items.

Finally, the percentage of featured items is much higher for 2004 than for 2001. Based on these observations we postulate an empirical specification in which both the arrival of bidders and the bidders' valuation of objects being auctioned are modeled as two separate processes, one for featured and one for non-featured items. In addition, the "Featured Plus!" choice on the part of the artist is modeled separately for 2001 and another for 2004, to reflect important differences in the artists' propensity to feature paintings in both years. Thus, the empirical specification can be represented by six behavioral equations.

#### *A. Interpretation of Results.*

We estimate two different versions of the empirical model. In the first version differences across auctions are captured by a vector of instruments. These instruments include artist's specific characteristics, two measures of artists' reputation, as well as characteristics of the object being auctioned describing dimensions, styles, medium and ground, dummies for year, month and the auction ending day. Also included is a dummy to control for the potential impact of the terrorists attacks of September 11, 2001, on this market.<sup>8</sup> EBay allows buyers to rate their shopping experience by offering a positive (or 1) feedback, neutral feedback (or 0), or negative feedback (-1) to the seller. This rating is used to create a reputation index representing the percentage of positive feedbacks received from buyers, and we use this index as a measure of artists' reputation. Another measure of artists' reputation considered is the total number of unique buyers. In addition, the process of a rival of new bidders includes a baseline hazard that controls for differences in the rate of arrival of new bidders at different

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<sup>8</sup> The effect of the terrorist attacks of September 11 are addressed more directly in Canals and Percy (2005).

times over the duration of the auction. This baseline hazard includes a constant and four additional baseline parameters. The first baseline parameter accounts for differences in the arrival of bidders between the second day and the day before the end of the auction, the second, third and fourth baselines account for differences in the rate of arrival the last day, the last hour and the last ten minutes of the auction, respectively. Estimation results are presented in tables 5 and 6.

If there are differences in quality across artists, in addition to those captured by the vector of instruments, the model parameters could potentially be biased. For this reason, the second model estimated adds artists' specific fixed effects to the set of instruments. Unfortunately, data limitations prevent us from including fixed effects separately in each one of the six model equations. As a second best strategy we include fixed effects separately in the processes of arrival, valuation and the feature choice. In addition, because each one of these processes is modeled by two equations we assume that the artists' fixed effects are proportional in both equations.<sup>9</sup>

To preserve the simplicity and parsimony of the models we exclude from the final specification variables that are highly insignificant (t-values close to zero) in those instances in which this does not affect the overall interpretation of results. The estimation results for the first model are presented in table 5, with coefficients significant at the usual significance level of 0.95 presented in bold. For the process of bid arrivals we observe that new bidders

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<sup>9</sup> More precisely, if the coefficient associated to a particular artist dummy is  $\delta_a$  in equation one, in our model specification the coefficient associated to the same artist dummy in equation two is of the form  $\beta \cdot \delta_a$ , with  $\beta$  fixed across artists.

arrive at a higher rate close to the end of the auction. Some authors have interpreted this as a sign of complex strategic behavior on the part of the bidders (Roth and Ockenfels, 2002). Without disregarding that possibility, we would like to point out that more simple explanations also exist. For example, this type of behavior is consistent with a model in which bidders face a cost of commitment. In particular, if a bidder is interested in more than one painting in auctions that end at a similar time the bidder may choose to place a bid close to the end of the auction because bidding implies a commitment to a particular item and this commitment may be costly. Alternatively, bidding close to the end of the auction may be just the result of a simple coordination strategy when decisions about how much to bid for an specific object are made jointly by more than one person. Interestingly, the overall effect of the eBay feedback score, the percentage of positive feedback from buyers, is insignificant. This result contrasts with other papers in the literature that find the effect of this variable to be significant and important in magnitude (Melnik and Alm, 2002). It is possible to reconcile these apparently contradictory results. Observe that in our dataset all artists have feedback rates above 98% and a large number of artists have a perfect score. Thus, it is reasonable to think that buyers may find sellers in the range of 98 to 100 equally trustworthy. In contrast, the number of feedbacks from unique buyers has a significant positive effect on the arrival of new buyers. One possibility is that artists that have a large number of customers also enjoy a larger number of repeated buyers. Interestingly, the size of the painting measured in square feet has a significant positive effect on the arrival of bidders, the value of the item and the artist decision to feature. The coefficient assigned to the dummy variable associated

to year 2001 indicates a significantly higher rate of arrivals of new bidders in that year as compared with year 2004. The September 11 dummy also has a significant positive effect on the arrival of bidders and the artist decision to feature, this topic is addressed at length in Canals and Percy (2005) and we would like to direct the interested reader to that paper. There are other significant parameters in the model associated to style, medium and ground which we do not discuss. Finally, some of the equations in the model also include controls for the month and the day of the week when the auction ends. These and other parameters are only included in an equation when their effect is significant.

The estimation results for the second model are presented in table 6, with those coefficients significant at the usual level represented in bold. For the most part the estimated parameters in this model accept a similar interpretation to those in the previous model. Unfortunately, the inclusion of artists' specific fixed effects render many of the parameters in the previous model insignificant. The reason for this is simple, most artists specialize on a particular style, medium or ground. Thus, the inclusion of artists specific fixed effects absorbs most of the useful identifying variation in these variables.

### *C. Policy Analysis.*

What is the true impact of the featured status on the selling price, after controlling for differences between featured and non-featured items? How do sellers benefit from the availability of the featured alternative? What is the benefit to the market intermediary? What are the welfare benefits associated to this simple form of congestion pricing? In an attempt to answer these questions in table 7 we present results from a policy experiment

conducted using the estimation results from the model specification with artists' specific effects. This table provides information on revenues and welfare generated as a result of the transactions observed in our data. Some of the values in this table can be computed directly from the data while others, those in bold, are obtained by simulation of the estimated model.

The upper section of table 7 contains information under the status quo in which artists' can choose whether to feature or not to feature their artwork. This information includes eBay revenues, sellers' revenues, net of posting fees paid to eBay, and buyers' surplus computed by simulation as the difference between the price paid and the value of the item for the winner of the auction, under the assumption that the winner of the auction is the buyer with the highest valuation. Fees to eBay consist of three components, insertion fees, which are computed based on the minimum acceptable bid, final value fees which represent a percentage of the final selling price, and listing upgrade fees which include the fees associated with featuring the painting. The largest fee by far is usually the \$19.99 "Featured Plus!" fee. Thus, it is not surprising that the average fee paid to eBay for a featured item, \$28.21, is about ten times the average fee paid for a non-featured item. In fact, although featured items represent only 11% of our sample they also represent approximately 60% of the revenues collected by eBay from the transactions taking place in our data. Featured items also provide close to 50% of all artists' revenues, and close to 50% of the buyers' surplus, according to the models' predictions.

Thus, given the significance of the feature choice as a source of revenue for eBay, and for artists, as well as a source of welfare for buyers, it is interesting to ask what would be

the effect of disposing of this alternative. Clearly, our model can simulate this scenario only under some additional assumptions. In particular, the main assumption imposed in our simulations is that if the feature choice did not exist buyers would have posted exactly the same items on eBay and that buyers would have continued to behave in the way predicted by the model for non-featured auctions. Of course, this assumption is incorrect. One can consider reasonable scenarios in which some artists would end up posting more items while others would end up posting less, and buyers would choose to buy more or less on eBay. Finally, it is unlikely that eBay would maintain the same fee structure if the feature option was not available. However, we view this exercise as a tool for understanding the importance of the “Featured Plus!” option in this market.

The lower section of table 7 provides results from this simulation exercise. Simulations are conducted based on 100 draws from the estimated asymptotic distribution of the model parameters. For each one of these draws and for each observation in the sample we generate 100 realizations of the potential outcome resulting from posting an item with the characteristics of the corresponding data point, and consider the average outcome. Focusing our attention on those items that are featured in the status quo, the model predicts that average fees paid to eBay would be reduced substantially from \$28.21 to \$6.35. Average revenues for sellers would also decrease substantially from \$166 to \$119, and average Buyer surplus will be reduced from \$208 to \$170. These effects are due to a decrease in the number of potential buyers in these auctions. Overall, the dollar value of revenues and consumer surplus would be \$139291 instead of the original value of \$192118 from the sale of the same items as featured



under the status quo. This scenario represent a reduction of \$52827, or 27%, of the value generated by featured items, or 14% of the overall value generated in this market. Finally, observe that even in a market without the feature choice, objects from the “Featured Plus!” subsample command a much higher price and have a much higher probability of being sold than other objects in our sample. This is clear evidence of selection of the best artist and the best artwork in the “Featured Plus!” category. Thus, this seems to indicate that the “Featured Plus!” option acts as a coordination mechanism that facilitates the match of the best artists and the best artwork with buyers looking for the best art.

## 5 Conclusions.

This paper investigates the effect of a specific type of congestion price policy in the market for art by self representing artist on eBay. This type of congestion pricing policy works by giving sellers the option to incur an extra fee at the time of posting their items for sale and in return having these items listed first when buyers search for specific categories. The size of the artwork seems to be an important determinant of price after controlling for artists’ fixed effects. Also, artists’ that have been in the market for a long period of time, and that have a large customer base, benefit from the existence of repeated buyers. Ebay allows buyers to rate their shopping experience, this rating is used to create a reputation index and we use this index as a measure of artists’ reputation. Interestingly, this measure of reputation has no significant effect on the selling price. After observing that all artist have reputation ratings above 98% we conclude that this can only mean that artists place a high value on

their reputation and work very hard to maintain a satisfied customer base.

In the last part of the paper we conduct simulations based on the estimated model with artists' fixed effects. The results indicate that the "Featured Plus!" policy has an important positive effect on revenues for eBay and for sellers as well as an important positive effect on buyers' consumer surplus. Our results also suggest that the "Featured Plus!" option acts as a coordination mechanism that facilitates the match of the best artists and the best artwork with buyers looking for the best art. These results underscore the relevance of pricing policies and other kinds of market mechanisms that promote the best match between buyers and sellers.

**Table 1: Four Stylized Examples of eBay Auctions.**

Timing	Auction 1		Auction 2		Auction 3		Auction 4	
	$S_t$	$V_i$	$S_t$	$V_i$	$S_t$	$V_i$	$S_t$	$V_i$
1	\$10	\$10	\$10	\$10	\$10	\$10	\$10	\$100
2	\$10	\$25	\$10	\$50	\$10	\$75	\$75	\$75
3	\$25	\$50	\$25	\$25	\$50	\$50	\$75*	\$10*
4	\$50	\$75	\$50	\$75	\$50*	\$25*	\$75*	\$25*
5	\$75	\$100	\$75	\$100	\$75	\$100	\$75*	\$25*

We denote the true valuation of the bidder as  $V_i$  and the auction price at time  $t$  as  $S_t$ . In all cases; bidders valuations are \$10, \$25, \$50, \$75 and \$100. The auctions differ in the bidders' time of arrival. The (\*) indicates bids that will not be recorded because they are below the auction's minimum acceptable bid at time  $t$ .

**Table 2: Descriptive Statistics for the overall sample and the selected sub-sample.**

	Overall Sample								
	Featured				Not Featured				
	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
Sold	0.901	0	1	0.548	0	1			
Selling Price	194.7	10.5	1299	30.94	0.01	2435			
Num. Bids	12.01	0	52	2.203	0	48			
Num. Bidders	5.203	1	17	2.354	1	13			
Square Feet	5.371	0.292	26	1.834	0.158	30			
	<b>Auction Duration</b>								
Days	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>	
Frequency	4	15	118	426	265	776	6196	2397	
Percentage	0.71	2.66	20.96	75.67	2.75	8.05	64.31	24.88	
Number of Obs.	574				10039				
	Selected Sub-sample								
	Featured				Not Featured				
	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
Sold	0.875	0	1	0.559	0	1			
Selling Price	202.2	10.5	1299	47.45	0.01	595.0			
Num. Bids	12.46	0	52	2.383	0	38			
Num. Bidders	4.867	1	17	1.391	0	13			
Square Feet	5.389	0.292	26	2.009	0.158	24			
	<b>Auction Duration</b>								
Days	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>	
Frequency	3	14	112	405	103	312	2591	983	
Percentage	0.56	2.62	20.97	75.84	2.58	7.82	64.95	24.64	
Number of Obs.	534				3989				

**Table 3: Descriptive Statistics for featured/not featured auctions by year.**

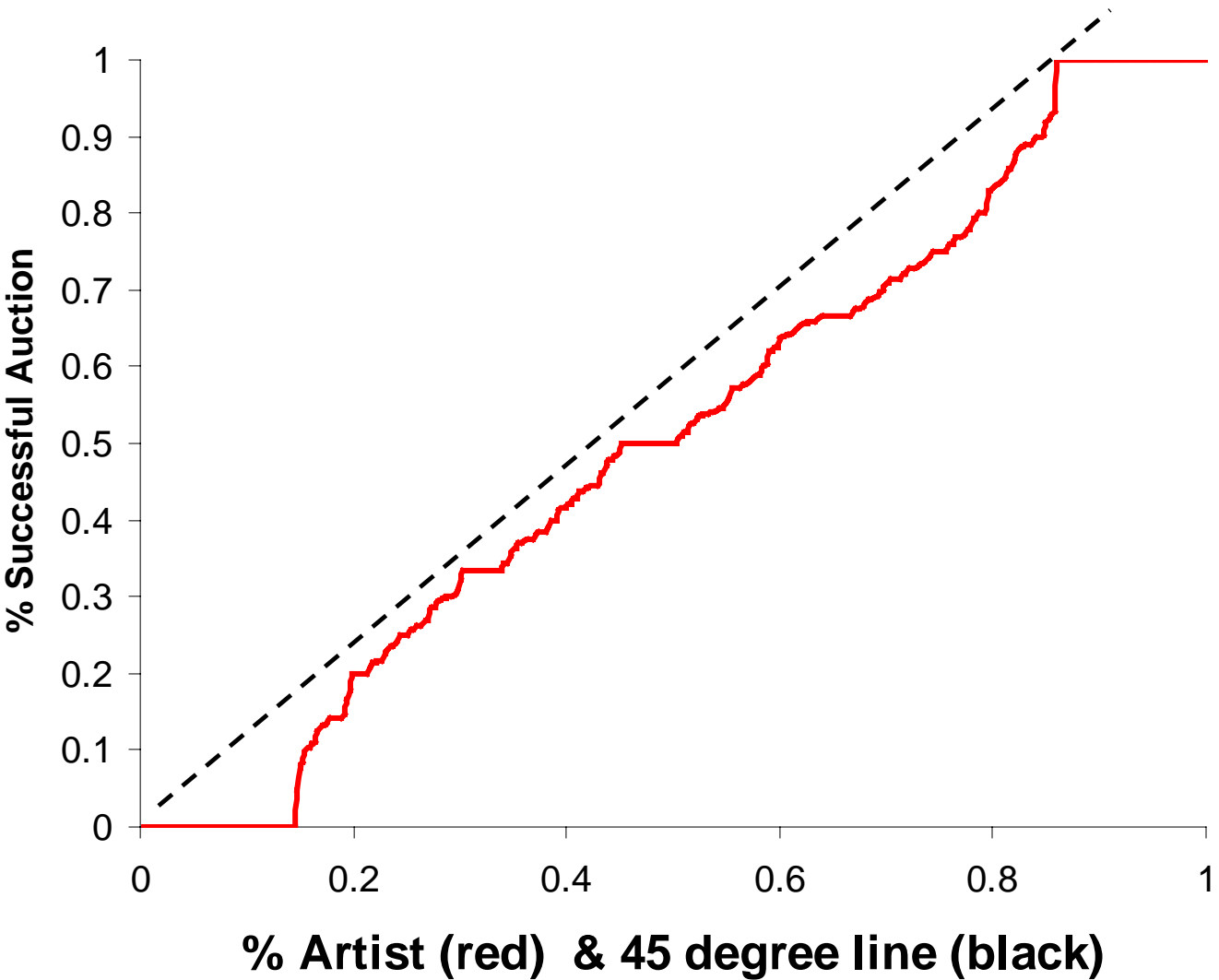
<b>Year 2001</b>								
	<b>Featured</b>			<b>Not Featured</b>				
	<b>Mean</b>	<b>Min.</b>	<b>Max.</b>	<b>Mean</b>	<b>Min.</b>	<b>Max.</b>		
Sold	0.909	0	1	0.619	0	1		
Selling Price	208.7	40.08	697.62	46.03	2.23	426.02		
Num. Bids	13.05	0	52	2.71	0	38		
Num. Bidders	5.32	0	14	1.59	0	13		
Square Feet	5.057	0.29	12.00	2.05	0.01	20.25		
<b>Auction Duration</b>								
Days	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>
Frequency	0	0	6	16	33	85	1581	531
Percentage	0	0	27.27	72.73	1148	3.81	70.90	23.81
Number of Obs.	22			2230				
<b>Measures of Artists' Reputation</b>								
			<b>Mean</b>	<b>Std.</b>	<b>Min.</b>	<b>Max.</b>		
Number of Unique Buyers ( in 100's)			1.08	1.01	0.03	5.25		
% of Positive Feedback from Buyers			99.84	0.44	98.05	100.00		
<b>Year 2004</b>								
	<b>Featured</b>			<b>Not Featured</b>				
	<b>Mean</b>	<b>Min.</b>	<b>Max.</b>	<b>Mean</b>	<b>Min.</b>	<b>Max.</b>		
Sold	0.873	0	1	0.484	0	1		
Selling Price	222.84	21.1	1441.41	49.77	3.00	595.5		
Num. Bids	12.44	0	49	1.959	0	29		
Num. Bidders	4.847	0	17	1.14	0	10		
Square Feet	5.404	0.444	24.00	1.950	0.021	24.00		
<b>Auction Duration</b>								
Days	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>
Frequency	3	14	106	389	70	227	1010	452
Percentage	0.59	2.73	20.7	75.98	3.98	12.91	57.42	25.70
Number of Obs.	512			1759				
<b>Measures of Artists' Reputation</b>								
			<b>Mean</b>	<b>Std.</b>	<b>Min.</b>	<b>Max.</b>		
Number of Unique Buyers ( in 100's)			3.91	3.10	0.39	15.73		
% of Positive Feedback from Buyers			99.80	0.44	97.65	100.00		

Selling price is in real dollars, 2004, and includes shipping and handling fees.

**Table 4: Frequency distributions and average selling price by Style, Medium and Ground.**

STYLE	FEATURED			NON FEATURED		
	Av. Price	Frequency	% Frequency	Av. Price	Frequency	% Frequency
<b>Abstract</b>	247.38	343	64.23	54.31	1475	36.98
<b>Contemporary</b>	187.38	85	15.92	38.22	1234	30.94
<b>Cubist</b>	211.46	74	13.86	78.99	390	9.78
<b>Folk</b>	101.01	45	8.43	38.94	797	19.98
<b>Modern</b>	253.54	326	61.05	77.22	260	6.52
<b>Urban</b>	110.24	7	1.31	34.06	1034	25.92
<b>Pop</b>	175.96	162	30.34	39.53	1466	36.75
<b>Other</b>	171.48	119	22.28	47.49	949	23.79
<b>Null</b>	201.76	27	5.06	42.50	2211	55.43
<b>MEDIUM</b>						
<b>Acrylic</b>	230.04	305	57.12	49.02	2238	56.10
<b>Pen and Ink</b>	0.00	0	0	33.21	109	2.73
<b>Colored Pencil</b>	0.00	0	0	17.85	17	0.43
<b>Collage</b>	236.90	66	12.36	31.44	668	16.75
<b>Oil</b>	207.40	158	29.59	87.01	573	14.36
<b>Watercolor</b>	36.90	4	0.75	32.01	195	4.89
<b>Other</b>	49.89	1	0.19	35.97	189	4.74
<b>GROUND</b>						
<b>Stretch Canvas</b>	228.79	505	94.57	79.18	1164	29.18
<b>Canvas Panel</b>	97.24	5	0.94	37.15	1300	32.59
<b>Cardboard</b>	142.84	3	0.56	23.56	198	4.96
<b>Paper</b>	65.84	8	1.50	27.48	1007	25.24
<b>Other</b>	128.06	13	2.43	54.86	320	8.02

**Figure 1: Average Artist-Specific Probability of Successful Auction**



**Table 5.- Estimation results for the model without artists' specific fixed effects.**

	Featured Choice		Bid Arrival		Item Valuation	
	2001	2004	Featured	N-Featured	Featured	N-Featured
<b>Intercept</b>	<b>-3.820</b>	<b>-4.369</b>	<b>-13.01</b>	<b>-13.90</b>	<b>3.757</b>	<b>2.651</b>
<b>Baseline1</b>			<b>-1.535</b>	<b>-0.892</b>		
<b>Baseline2</b>			<b>-0.492</b>	<b>0.136</b>		
<b>Baseline3</b>			<b>1.298</b>	<b>1.740</b>		
<b>Baseline4</b>			<b>3.956</b>	<b>4.267</b>		
<b>FEEDBACK</b>						
<b>eBay Feedback</b>						
<b># feedbacks</b>	-1.719	-0.047	<b>0.488</b>	<b>0.142</b>	0.070	-0.008
<b># feedbacks<sup>2</sup></b>	0.125	-0.006	<b>-0.034</b>	<b>-0.005</b>	-0.003	<b>0.003</b>
<b>DIMENSION</b>						
<b>Square-Foot</b>	<b>1.071</b>	<b>0.799</b>	<b>0.038</b>	<b>0.072</b>	<b>0.118</b>	<b>0.243</b>
<b>Square-Foot<sup>2</sup></b>	-0.041	<b>-0.035</b>	-0.001	<b>-0.003</b>	<b>-0.004</b>	<b>-0.008</b>
<b>STYLE</b>						
<b>Abstract</b>		0.129	0.237	<b>-0.199</b>	-0.211	<b>-0.218</b>
<b>Contemporary</b>		0.422	-0.016	<b>-0.163</b>	0.064	<b>-0.243</b>
<b>Cubist</b>		0.953	-0.224	<b>0.707</b>	-0.174	-0.047
<b>Folk</b>		-0.560	<b>0.278</b>	<b>0.396</b>	<b>-0.468</b>	<b>-0.113</b>
<b>Modern</b>		<b>1.656</b>	0.158	<b>0.298</b>	0.159	<b>0.232</b>
<b>Urban</b>		<b>-1.376</b>		-0.021	-0.183	<b>-0.107</b>
<b>Pop</b>		<b>0.871</b>	-0.056	<b>-0.125</b>	-0.033	-0.065
<b>Other</b>		0.280	0.084	<b>0.096</b>	0.063	<b>-0.148</b>
<b>Null</b>		<b>-2.485</b>	0.268	<b>0.163</b>	-0.211	<b>-0.284</b>
<b>MEDIUM</b>						
<b>Acrylic</b>			<b>-0.233</b>	<b>0.099</b>	-0.013	0.049
<b>Pen and Ink</b>				0.077		<b>0.342</b>
<b>Colored Pencil</b>				-1.139		-0.585
<b>Collage</b>			<b>-0.296</b>	-0.006	0.055	0.021
<b>Oil</b>			control	<b>-0.393</b>	control	0.002
<b>GROUND</b>						
<b>Stretch Canvas</b>			0.094	<b>0.420</b>	<b>0.347</b>	<b>0.632</b>
<b>Canvas Panel</b>				<b>0.353</b>		<b>0.263</b>
<b>Cardboard</b>				<b>0.394</b>		0.022
<b>Paper</b>				<b>0.345</b>		-0.003
<b>OTHER</b>						
<b>Year 2001</b>			<b>0.840</b>	<b>0.513</b>		<b>0.224</b>
<b>Sept. 11</b>	<b>2.002</b>			<b>0.269</b>	-0.172	0.077
<b>Month Dummy</b>	No	No	No	Yes	No	Yes
<b>Day Dummy</b>	No	Yes	Yes	Yes	No	Yes
	<b>N. Obs.</b>	4523		<b>LLF</b>	-117405.80	

Parameters significant at the usual 0.95 level are in bold.

**Table 6.- Estimation results for the model with artists' specific fixed effects.**

	Featured Choice		Bid Arrival		Item Valuation	
	2001	2004	Featured	N-Featured	Featured	N-Featured
<b>Intercept</b>	<b>-4.851</b>	<b>-4.021</b>	<b>-12.245</b>	<b>-12.269</b>	<b>3.450</b>	<b>2.303</b>
<b>Baseline1</b>			<b>-1.539</b>	<b>-0.917</b>		
<b>Baseline2</b>			<b>-0.492</b>	<b>0.136</b>		
<b>Baseline3</b>			<b>1.298</b>	<b>1.740</b>		
<b>Baseline4</b>			<b>3.956</b>	<b>4.267</b>		
<b>FEEDBACK</b>						
<b>eBay Feedback</b>						
<b># feedbacks</b>			<b>0.210</b>	<b>-0.088</b>	<b>0.187</b>	0.050
<b># feedbacks<sup>2</sup></b>			<b>-0.015</b>	0.001	-0.012	-0.002
<b>DIMENSION</b>						
<b>Square-Foot</b>	1.453	<b>1.339</b>	0.011	<b>0.066</b>	<b>0.098</b>	<b>0.273</b>
<b>Square-Foot<sup>2</sup></b>	-0.066	<b>-0.047</b>	0.001	<b>-0.004</b>	-0.002	<b>-0.009</b>
<b>STYLE</b>						
<b>Abstract</b>			0.348	<b>-0.184</b>		<b>-0.175</b>
<b>Contemporary</b>			0.176	<b>0.233</b>		-0.090
<b>Cubist</b>			-0.173	<b>0.355</b>		-0.011
<b>Folk</b>			<b>0.375</b>	<b>0.187</b>		<b>-0.152</b>
<b>Modern</b>			0.169	<b>0.293</b>		0.101
<b>Urban</b>			<b>0.350</b>	<b>0.343</b>		-0.064
<b>Pop</b>			0.042	<b>-0.090</b>		-0.059
<b>Other</b>			0.170	<b>0.202</b>		-0.099
<b>Null</b>			0.348	<b>0.121</b>		<b>-0.268</b>
<b>MEDIUM</b>						
<b>Acrylic</b>						<b>0.172</b>
<b>Pen and Ink</b>						0.131
<b>Colored Pencil</b>						-0.056
<b>Collage</b>						<b>0.156</b>
<b>Oil</b>						-0.006
<b>GROUND</b>						
<b>Stretch Canvas</b>			0.062	<b>0.158</b>	<b>0.342</b>	<b>0.475</b>
<b>Canvas Panel</b>				<b>0.371</b>		<b>0.382</b>
<b>Cardboard</b>				0.155		-0.025
<b>Paper</b>				0.047		0.010
<b>OTHER</b>						
<b>Year 2001</b>			<b>0.759</b>			0.214
<b>Sept. 11</b>	1.917			<b>0.112</b>	<b>0.476</b>	<b>0.090</b>
<b>Month Dummy</b>	No	No	No	No	No	No
<b>Day Dummy</b>	No	Yes	No	Yes	No	Yes
	<b>N. Obs.</b>	4523		<b>LLF</b>	-116176.76	

Parameters significant at the usual level are in bold.



Table7: Simulation of eBay Revenues, Artists' Revenues and Consumer Surplus based on the model of Table 6.

	AVERAGE			TOTAL		
	Featured	N-Featured	Overall	Featured	N-Featured	Overall
	Status Quo					
<b>Ebay Revenues</b>	28.21	2.56	5.59	15063.78	10219.11	25282.89
<b>Seller Revenues</b>	166.14	23.98	40.77	88720.83	95661.24	184382.10
<b>Buyer Surplus</b>	<b>208.03 (71.43)</b>	<b>53.05 (6.06)</b>		<b>88333.93</b>	<b>90743.93</b>	<b>179077.90</b>
			<b>TOTAL</b>	<b>192118.50</b>	<b>196624.30</b>	<b>388742.80</b>
	Not "Featured Option" Allowed					
<b>Ebay Revenues</b>	<b>6.35 (0.88)</b>			<b>3390.72</b>		
<b>Seller Revenues</b>	<b>119.20 (30.50)</b>			<b>63651.61</b>		
<b>Buyer Surplus</b>	<b>170.15 (27.61)</b>			<b>72248.82</b>		
<b>% Sales</b>	<b>0.80 (0.15)</b>		<b>TOTAL</b>	<b>139291.20</b>		

Simulated values, in bold, are generated using parametric bootstrap based on 100 draws from the estimated asymptotic distribution of parameters. Standard errors in parenthesis are only provided for simulated values, not-simulated values represent exact amounts computed from the current data. Sellers' revenues are net of fees paid to eBay.

## References

- Ashenfelter, O. and K. Graddy (2002): "Art Auctions: A Survey of Empirical Studies," CEPR Discussion Paper 3387.
- Ashenfelter, O. and K. Graddy (2003): "Auctions and the Price of Art," *Journal of Economic Literature*, September.
- Ashenfelter, O. and K. Graddy "Art Auctions," *Handbook of the Economics of Art and Culture*, edited by Victor Ginsburgh and David Throsby (Oxford UK: Elsevier) forthcoming.
- Bajari, P. and A. Hortacsu (2003): "The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions," *The Rand Journal of Economics*, Vol. 34, No. 2, pp. 329-355.
- Bajari, P. and A. Hortacsu (2004): "Economic Insights from Internet Auctions," *Journal of Economic Literature*, Vol. XLII, 457-486.
- Baldwin, L. H.; Marshall, R. C.; and J. F. Richard (1997): "Bidder Collusion and Forest Service Timber Sales," *Journal of Political Economy* 105(4), 657-99.
- Berry, Steven, James Levinsohn and Ariel Pakes (1995). "Automobile Prices in Market Equilibrium." *Econometrica*, Vol. 63, No. 4.: 841-890.
- Brian, D., D. Lucking-Reiley, N. Prasad, and D. Reeve (2000): "Pennies from eBay: the Determinants of Price in Online Auctions." Working Paper, University of Arizona.
- Bryant, W. D. A. and D Throsby (2005). "The economics of creativity in the arts", in Victor Ginsburgh and David Throsby (eds.), *Handbook of the Economics of Art and Culture*, (Amsterdam: North-Holland, forthcoming).
- Camerer, C., L. Babcock, G. Loewenstein and R. Thaler (1997). "Labor Supply of New York City Cabdrivers: One Day at a Time," *The Quarterly Journal of Economics*, Vol. 112, No. 2, 407-441.
- Cameron, A. C. and P. Trivedi (1998). "Analysis of Count Data." *Econometric Society Monograph No. 30*, Cambridge University Press, 1998.
- Canals-Cerda, J. and J. Percy (2005). "eBay 9/11." Working paper, University of Colorado at Boulder.
- Caserta, M. and T. Cuccia (2001). "The Supply of Arts Labour: Towards a Dynamic Approach." *Journal of Cultural Economics* 25, 185-201, 2001.
- Cohen, A. (2002). "The Perfect Store," Little, Brown and Company.

- Cowen, T. and A. Tabarrok (2000). "An Economic Theory of Avant-Garde and Popular Art, or High and Low Culture," *Southern Economic Journal*, Vol. 67, No. 2, pp. 232-253.
- Donald, S. G., and Paarsch, H. J. (1996). "Identification, Estimation and Testing in Parametric Empirical Models of Auctions within the Independent Private Values Paradigm." *Econometric Theory* 12, 517-67.
- Farber, S. H. (2003). "Is Tomorrow Another Day? The Labor Supply of New York City Cab Drivers," Princeton University, Industrial Relations Section, Working Paper #473.
- Flinn, C. and J. J. Heckman (1982). "New Methods for Analyzing Structural Models of Labor Force Dynamics." *Journal of Econometrics*. 18(1): 115-168.
- Gapinski, J. H. (1980). "The Production of Culture." *The Review of Economics and Statistics*, Vol. 62, No. 4, 578-586.
- Gapinski, J. H. (1984). "The economics of performing Shakespeare", *American Economic Review*, 74 (3): 458-466.
- Goldberg, Pinelopi K. (1995). "Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry." *Econometrica*, Vol. 63, no. 4: 891-951.
- Haile, P. A. (2001): "Auctions with Resale Markets: An Application to U.S. Forest Service Timber Sales," *American Economic Review* 91(3), 399-427.
- Haile, Philip A. and Elie T. Tamer (2003). "Inference with an Incomplete Model of English Auctions." *Journal of Political Economy*, vol. 111, no. 1: 1-51.
- Haile, Philip A. and Elie T. Tamer (2004). "Inference from English Auctions With Asymmetric Affiliated Private Values." Working paper, Yale University.
- Hasker, K.; Gonzalez, R. and R. C. Sickles (2001): "An Analysis of Strategic Behavior and Consumer Surplus in eBay Auctions," Working Paper, Rice University.
- Hendricks, K. and H. J. Paarsch (1995): "A Survey of Recent Empirical Work Concerning Auctions." *Canadian Journal of Economics*, 28 (May), 403-26.
- Hendricks, K. and R. H. Porter (2000) "Lectures on Auctions: An Empirical Perspective," mimeo, Northwestern University.
- Hogg, R. V. and A. T. Craig (1978). "Introduction to Mathematical Statistics." Macmillan ed. Fourth Edition.

- Hong, H. and M. Shum (2003): "Econometric Models of Asymmetric Ascending Auctions," *Journal of Econometrics*, 112: 327-358.
- Klemperer, P. (1999). "Auction Theory: A Guide to the Literature." *Journal of Economics Surveys* 13, 227-286.
- Klemperer, P. (2004): "Auctions: Theory and Practice," Princeton University Press.
- Krishna, V. (2002): "Auction Theory," San Diego: Academic Press.
- Laffont, Jean-Jacques, Herve Ossard, Quang Vuong (1995). "Econometrics of First-Price Auctions." *Econometrica*, Vol. 63, No. 4: 953-980.
- Levinsohn J., "The Empirics of Taxes on Differentiated Products" in R.E. Baldwin (ed.), *Trade Policy Issues and Empirical Analysis* (Chicago: University of Chicago Press), 1988.
- Lucking-Reiley, D. (2000), "Auctions on the Internet: What's Being Auctioned, and How?" *Journal of Industrial Economics*, 48, 227-252.
- McAfee, R.P. and J. McMillan (1987): "Auctions and Bidding," *Journal of Economic Literature*, Vol. XXV, 699-738.
- Melnik, M. and J. Alm (2002): "Does a Seller's eCommerce Reputation Matter? Evidence from eBay Auctions," *Journal of Industrial Economics*, Vol. 50 (3), 337-349.
- Milgrom, P. R. and R. J. Weber (1982). "A Theory of Auctions and Competitive Bidding." *Econometrica*, 50, 1089-1122.
- Mroz, T. A. (1999). "Discrete factor approximations in simultaneous equation models: Estimating the impact of a dummy endogenous variable on a continuous outcome." *Journal of Econometrics* 92, 233-274.
- Oettinger, G. S. (1999). "An Empirical Analysis of the Daily Labor Supply of Stadium Vendors," *The Journal of Political Economy*, Vol. 107, No. 2, 360-392.
- Paarsch, H. J. (1992): "Empirical Models of Auctions and an Application to British Columbian Timber Sales," Research Report n. 9212. London: Univ. Western Ontario.
- Paarsch, H. J. (1997): "Deriving an estimate of the optimal reserve price: An application to British Columbian timber sales," *Journal of Econometrics* 78, 333-357.
- Perrigne I. and Q. Vuong (1999): "Structural Econometrics of First-Price Auctions: A Survey," *Canadian Journal of Agricultural Economics*, 47, 203-223.

- Resnick, Paul and Richard Zeckhauser (2002), "Trust Among Strangers in Internet Transactions: Empirical Analysis of eBay's Reputation System," *The Economics of the Internet and E-Commerce*, Michael R. Baye, Editor, *Advances in Applied Microeconomics*, Vol. 11.
- Roth, A. E. and A. Ockenfels (2002): "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet," *American Economic Review*, 92 (4), 1093-1103.
- Roth, A. E. and A. Ockenfels (2002): "The Timing of Bids in Internet Auctions: Market Design, Bidder Behavior, and Artificial Agents." *AI Magazine*, Fall, 79-88.
- Song, Unjy (2004). "Nonparametric Estimation of an eBay Auction Model with an Unknown Number of Bidders," Working Paper, University of Wisconsin at Madison.
- Shubik, M. (1983): "Auctions, Bidding, and Markets: An Historical Sketch," *Auctions, Bidding, and Contracting*, In R. Engelbrecht-Wiggans, M. Shubik, and J. Stark eds., pp. 33-52.
- Throsby, D. (1994a): "The Production and Consumption of the Arts: A View of Cultural Economics." *Journal of Economic Literature*, Vol. 32(1), pp. 1-29.
- Throsby, D. (1994b): "A Work-Preference Model of Artist Behavior." in A. Peacock and I. Rizzo (eds.), *Cultural Economics and Cultural Policies*. Kluwer Academic Publishers, Dordrecht.
- Throsby, D. (2004): "An artistic Production Function: Theory and an Application to Australian Visual Artists." Working Paper, Macquarie University, Sydney, Australia.
- Varian Hal R. (1982). "The Nonparametric Approach to Demand Analysis." *Econometrica*, Vol. 50, no. 4: 945-974.
- Varian Hal R. (1984). "The Nonparametric Approach to Production Analysis." *Econometrica*, Vol. 52, no. 3: 579-598.
- Wang, R (1993), "Auctions versus Posted-Price Selling," *The American Economic Review*, Vol. 83, No. 4: 838-851.

# 1 Appendix 1:

**PART ONE:** Next we show how to define an explicit recursive method for computing analytically the following integral

$$\int_{\bar{b}_{K-2}}^{\bar{b}_{K-1}} f(z_{N-2}) \int_{\bar{b}_{K-3}}^{z_{N-2}} f(z_{N-3}) \dots \int_{\bar{b}_1}^{z_{N-K+2}} f(z_{N-K+1}) I(z_{N-K+1}) dz_{N-K+1} \dots dz_{N-2}, \quad (1)$$

denoted as  $I[(\bar{b}_k)_{k=1}^{K-1} | N-2]$ .

Consider first  $I(z_{N-K+1})$ , equal to

$$\int_{-\infty}^{z_{N-K+1}} f(z_{N-K}) \int_{-\infty}^{z_{N-K}} f(z_{N-K-1}) \dots \int_{-\infty}^{z_2} f(z_1) dz_1 \dots dz_{N-K-2} dz_{N-K}.$$

Observe that the probability  $P(\bar{v}_{N-K} \leq z_{N-K+1} | N-K)$  is equal to  $(N-K)! I(z_{N-K+1})$  and  $P(\bar{v}_{N-K} \leq z_{N-K+1} | N-K)$  is equal to  $F(z_{N-K+1})^{N-K}$ . Thus,

$$I(z_{N-K+1}) = (N-K)!^{-1} F(z_{N-K+1})^{N-K}.$$

Consider now the following change of variables

$$F(z_j) = F_j \rightarrow f(z_j) dz_j = dF_j, \quad j = 1, 2 \text{ and denote } B_j = F(\bar{b}_j).$$

We have that  $I(z_{N-K+1}) = (N-K)!^{-1} F_{N-K+1}^{N-K}$ .

We consider now the analytic solution of  $I[(\bar{b}_k)_{k=1}^{K-1} | N-2]$ :

A.- We begin with an illustrative example ( $K - 1 = 5, N - 2 = 6, N - K + 1 = 3$ ):

$$\begin{aligned}
& \int_{\bar{b}_4}^{\bar{b}_5} f(z_6) \int_{\bar{b}_3}^{z_6} f(z_5) \int_{\bar{b}_2}^{z_5} f(z_4) \int_{\bar{b}_1}^{z_4} f(z_3) I(z_3) dz_3 dz_4 dz_5 dz_6 \\
&= \int_{B_4}^{B_5} \int_{B_3}^{F_6} \int_{B_3}^{F_5} \int_{B_2}^{F_4} 2!^{-1} F_3^2 dF_3 dF_4 dF_5 dF_6 \\
&= \int_{B_4}^{B_5} \int_{B_3}^{F_6} \int_{B_2}^{F_5} [3!^{-1} x^3]_{B_1}^{F_4} dF_4 dF_5 dF_6 = \int_{B_4}^{B_5} \int_{B_3}^{F_6} \int_{B_2}^{F_5} 3!^{-1} [F_4^3 - B_1^3] dF_4 dF_5 dF_6 \\
&= \int_{B_4}^{B_5} \int_{B_3}^{F_6} [p_{4,3}(F_5) - p_{4,3}(B_2)] dF_5 dF_6 \\
&\quad \{\text{with } p_{4,3}(x) = 4!^{-1} x^4 - 3!^{-1} B_1^3 x\} \\
& \\
&= \int_{B_4}^{B_5} [p_{5,3}(F_6) - p_{5,3}(B_3)] dF_6 \\
&\quad \{\text{with } p_{5,3}(x) = 5!^{-1} x^5 - 3!^{-1} 2^{-1} B_1^3 x^2 - p_{4,3}(B_2) x\} \\
&= [p_{6,3}(x)]_{B_4}^{F_5} = p_{6,3}(B_5) - p_{6,3}(B_4) \\
&\quad \{\text{with } p_{6,3}(x) = 6!^{-1} x^6 - 3!^{-1} 3!^{-1} B_1^3 x^3 - 2^{-1} p_{4,3}(B_2) x^2 - p_{5,3}(B_3) x\}
\end{aligned}$$

B.- In general,

$$\begin{aligned}
& \int_{\bar{b}_{K-2}}^{\bar{b}_{K-1}} f(z_{N-2}) \int_{\bar{b}_{K-3}}^{z_{N-2}} f(z_{N-3}) \dots \int_{\bar{b}_1}^{z_{N-K+2}} f(z_{N-K+1}) I(z_{N-K+1}) dz_{N-K+1} \dots dz_{N-2} \\
&= \int_{B_{K-2}}^{B_{K-1}} \int_{B_{K-3}}^{F_{N-2}} \int_{B_1}^{F_{N-K+2}} (N-K)!^{-1} F_{N-K+1}^{N-K} dF_{N-K+1} \dots dF_{N-2} \\
&= \int_{B_{K-2}}^{B_{K-1}} \int_{B_{K-3}}^{F_{N-2}} \int_{B_2}^{F_{N-K+3}} (N-K+1)!^{-1} [x^{N-K+1}]_{B_1}^{F_{N-K+2}} dF_{N-K+2} \dots dF_{N-2} \\
&= \int_{B_{K-2}}^{B_{K-1}} \int_{B_{K-3}}^{F_{N-2}} \int_{B_3}^{F_{N-K+4}} [p_{N-K+2, N-K+1}(x)]_{B_2}^{F_{N-K+3}} dF_{N-K+3} \dots dF_{N-2} \\
&\quad \{\text{with } p_{N-K+2, N-K+1}(x) = (N-K+2)!^{-1} x^{N-K+2} - (N-K+1)!^{-1} B_1^{N-K+1} x\} \\
&= \int_{B_{K-2}}^{B_{K-1}} \int_{B_{K-3}}^{F_{N-2}} \int_{B_4}^{F_{N-K+5}} [p_{N-K+3, N-K+1}(x)]_{B_3}^{F_{N-K+4}} dF_{N-K+4} \dots dF_{N-2} \\
&\quad \{\text{with } p_{N-K+3, N-K+1}(x) = (N-K+3)!^{-1} x^{N-K+3} - (N-K+1)!^{-1} 2^{-1} B_1^{N-K+1} x^2 \\
&\quad \quad \quad - p_{N-K+2, N-K+1}(B_2) x\} \\
&= \int_{B_{K-2}}^{B_{K-1}} \int_{B_{K-3}}^{F_{N-2}} \int_{B_5}^{F_{N-K+6}} [p_{N-K+4, N-K+1}(x)]_{B_4}^{F_{N-K+5}} dF_{N-K+5} \dots dF_{N-2} \\
&\quad \{\text{with } p_{N-K+4, N-K+1}(x) = (N-K+4)!^{-1} x^{N-K+4} - (N-K+1)!^{-1} 3!^{-1} B_1^{N-K+1} x^3 \\
&\quad \quad \quad - 2^{-1} p_{N-K+2, N-K+1}(B_2) x^2 - p_{N-K+3, N-K+1}(B_3) x\}
\end{aligned}$$



$$\begin{aligned}
&= \dots = \int_{B_{K-2}}^{B_{K-1}} [p_{N-3, N-K+1}(x)]_{B_{K-3}}^{F_{N-2}} dF_{N-2} \\
&\{ \text{with } p_{N-3, N-K+1}(x) = (N-3)!^{-1} x^{N-3} - (N-K+1)!^{-1} (K-4)!^{-1} B_1^{N-K+1} x^{K-4} \\
&\quad - (K-5)!^{-1} p_{N-K+2, N-K+1}(B_2) x^{K-5} - (K-6)!^{-1} p_{N-K+3, N-K+1}(B_3) x^{K-6} \\
&\quad \dots - 2^{-1} p_{N-5, N-K+1}(B_{K-5}) x^2 - p_{N-4, N-K+1}(B_{K-4}) x \} \\
&= [p_{N-2, N-K+1}(x)]_{B_{K-2}}^{B_{K-1}} \\
&\{ \text{with } p_{N-2, N-K+1}(x) = (N-2)!^{-1} x^{N-2} - (N-K+1)!^{-1} (K-3)!^{-1} B_1^{N-K+1} x^{K-3} \\
&\quad - (K-4)!^{-1} p_{N-K+2, N-K+1}(B_2) x^{K-4} - (K-5)!^{-1} p_{N-K+3, N-K+1}(B_3) x^{K-5} \\
&\quad \dots - 3!^{-1} p_{N-5, N-K+1}(B_{K-5}) x^3 - 2^{-1} p_{N-4, N-K+1}(B_{K-4}) x^2 - p_{N-3, N-K+1}(B_{K-3}) x \}
\end{aligned}$$

Thus,  $I \left[ (\bar{b}_k)_{k=1}^{K-1} | N-2 \right]$  can be computed analytically as a polynomial function of  $\{F(\bar{b}_k)\}_{k=1}^{K-1}$ .

**PART TWO:** Next we show how to compute  $P(\bar{v}_{N-1} \leq b_0 + \Delta, b_0 \leq \bar{v}_N | N)$ . We could do that by following the same approach as in part one of this appendix. A more direct approach makes use of the expression of the joint distribution of any two order statistics (Hogg and Craig, 1978, page 160), in particular for  $(N, N-1)$ :

$$g_{N, N-1}(v_N, v_{N-1}) = N(N-1) F(v_{N-1})^{N-2} f(v_{N-1}) f(v_N), v_{N-1} \leq v_N$$

and  $P(\bar{v}_{N-1} \leq b_0 + \Delta, b_0 \leq \bar{v}_N | N)$

$$\begin{aligned}
&= \int_{b_0}^{+\infty} \int_{-\infty}^{b_0+\Delta} g_{N,N-1}(z_N, z_{N-1}) dz_{N-1} dz_N \\
&= \int_{b_0}^{+\infty} \left\{ \int_{b_0}^{\min\{b_0+\Delta, z_N\}} g_{N,N-1}(z_N, z_{N-1}) dz_{N-1} + \int_{-\infty}^{b_0} g_{N,N-1}(z_N, z_{N-1}) dz_{N-1} \right\} dz_N \\
&= \int_{b_0}^{+\infty} \int_{-\infty}^{b_0} g_{N,N-1}(z_N, z_{N-1}) dz_{N-1} dz_N + \\
&\int_{b_0}^{b_0+\Delta} \int_{b_0}^{z_N} g_{N,N-1}(z_N, z_{N-1}) dz_{N-1} dz_N + \int_{b_0+\Delta}^{+\infty} \int_{b_0}^{b_0+\Delta} g_{N,N-1}(z_N, z_{N-1}) dz_{N-1} dz_N \\
&= N(N-1) \left\{ \int_{b_0}^{+\infty} \int_{-\infty}^{b_0} F(z_{N-1})^{N-2} f(z_{N-1}) dz_{N-1} f(z_N) dz_N + \right. \\
&\quad \int_{b_0}^{b_0+\Delta} \int_{b_0}^{z_N} F(z_{N-1})^{N-2} f(z_{N-1}) dz_{N-1} f(z_N) dz_N + \\
&\quad \left. \int_{b_0+\Delta}^{+\infty} \int_{b_0}^{b_0+\Delta} F(z_{N-1})^{N-2} f(z_{N-1}) dz_{N-1} f(z_N) dz_N \right\} \\
&= N(N-1) \{I_1 + I_2 + I_3\}
\end{aligned}$$

with

$$\begin{aligned}
I_1 &= \int_{b_0}^{+\infty} \int_0^{F(b_0)} F^{N-2} dF f(z_N) dz_N = (N-1)^{-1} F(b_0)^{N-1} \int_{b_0}^{+\infty} f(z_N) dz_N \\
&= (N-1)^{-1} F(b_0)^{N-1} [1 - F(b_0)]
\end{aligned}$$

$$\begin{aligned}
I_2 &= \int_{b_0}^{b_0+\Delta} \int_{F(b_0)}^{F(z_N)} F^{N-2} dF f(z_N) dz_N \\
&= (N-1)^{-1} \int_{b_0}^{b_0+\Delta} [F^{N-1}(z_N) - F^{N-1}(b_0)] f(z_N) dz_N \\
&= (N-1)^{-1} \left\{ \int_{F(b_0)}^{F(b_0+\Delta)} F^{N-1} dF - F^{N-1}(b_0) [F(b_0+\Delta) - F(b_0)] \right\} \\
&= (N-1)^{-1} \{ N^{-1} [F^N(b_0+\Delta) - F^N(b_0)] - F^{N-1}(b_0) [F(b_0+\Delta) - F(b_0)] \}
\end{aligned}$$

$$I_3 = (N-1)^{-1} [F(b_0+\Delta)^{N-1} - F(b_0)^{N-1}] [1 - F(b_0+\Delta)]$$