

Selecting the Best? Spillover and Shadows in Elimination Tournaments*

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Abstract

Innovation competitions between firms and personnel tournaments within firms are both examples of contests where competitors are eliminated over successive stages of play. In this paper, we consider how match outcomes in elimination tournaments are shaped by past, current, and future competition. We present a two-stage model that yields the following results: (a) a shadow effect of future competition—the weaker the expected competitor in the next stage, the greater the probability that the stronger player wins in the current stage; (b) an effort spillover effect—with negative (positive) spillover, more effort in earlier rounds leads to a lower (higher) probability of winning in later stages; (c) noise around effort increases the probability that the weaker player wins; (d) under certain conditions, the weaker player is more likely to win in the final stage, relative to earlier stages; and (e) a steeper prize structure improves the stronger player's probability of success in all stages. We test our theory predictions using data from professional tennis matches and betting markets. We find evidence that negative spillover and the shadow of a tough future opponent increases the probability that a weak competitor wins.

Keywords: *Elimination tournament, dynamic contest, contest design, effort choice, betting markets.*

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1 Introduction

Competition for employment and education, innovation funding, and design opportunities can all be framed as multi-stage elimination tournaments in which players are knocked out over successive stages of the event. These contests are often designed to increase player effort—indeed, much of the theoretical and empirical literature focuses on contests as incentive mechanisms. Yet, tournaments may also serve as selection mechanisms, identifying the “best” candidates as overall winners. In labor tournaments where employees’ latent talents are not directly observable, firms may organize contests to reveal workers’ relative abilities. For example, in searching for a CEO, a firm may use a tournament to identify and promote the highest-ability candidate, not simply the one who puts forth the most effort.

In this paper, we study how the strategies of heterogeneous players in match-pair elimination tournaments are shaped by past, current, and future competition. More specifically, we examine how these intertemporal effects influence a tournament’s ability to reveal the strongest player as the winner. Negative spillover from past stages may make current effort more costly and depress performance, while the shadow of tough future competition decreases a player’s expected future payoffs and also may lead to lower current effort. The differential impact of past and future competition across players in a given match changes the effectiveness of tournaments as a selection mechanism—both negative spillover and tough future competition increase the probability that a weak candidate wins overall. Our results have practical implications; whether the contest aims to encourage effort, select a strong winner, or both, we find evidence suggesting that firms, educators, and other contest designers may need to consider the role of past and future competition in structuring incentives.

In personnel tournaments, workers risk elimination as they advance through corporate management levels. Employees who perform relatively well in each stage are promoted, awarded progressively larger compensation, and have the opportunity to advance even higher in the firm. Employees who perform relatively poorly are passed over in promotions or, in some cases, demoted or fired. In most contexts, retention of the highest quality worker is most desirable. For example, GE’s former CEO, Jack Welch, designed an explicit elimination tournament to select his successor (Konrad 2009).

Competition between firms may also take on an elimination tournament structure. For example, GE announced in 2010 that it would award \$200 million to the winning firm in a three-stage elimination contest, the Ecomagination Challenge, to develop smart grid technologies. More commonly, architectural firms may compete for large contracts; a university might invite initial ideas for a new classroom building and then select a small fraction of those applicants to submit more detailed designs to determine the final winner. Competition

among investment banks for clients may involve similar stages of proposals and commitments.

For students, the stages are levels of education—only students who perform well in high school are admitted to well-ranked colleges; students who top their collegiate classes advance to prestigious graduate programs; and only the best Ph.D. students from those elite universities are rewarded with selective academic positions. While not the rule, individuals who have been eliminated in early stages are unlikely to rejoin the competition at more advanced stages.

Some sporting events are also structured as elimination tournaments. Professional tennis is likely the most familiar and is the setting for the empirical tests in this paper. Combative sports and competitive bridge and poker contests are also often multi-stage knock-out events. Political races also may involve elimination stages—a candidate must win his party’s primary election to compete in the general election to hold office.

In each of these examples, effort is clearly important; firms want to hire designers, bankers and innovators who will invest heavily in the activity at hand, voters want their representatives to work hard on their behalf, and spectators enjoy high action games. However, selection may also be a prime objective of the contest organizer—a client may desire the most creative design firm, voters may value the most skilled politician, and a board may want the smartest executive to lead the company.

We explore elimination tournaments formally with a two-stage match-pair model. Our analysis yields five main results: First, we identify a shadow effect of future competition—the weaker the expected competitor in the next stage, the greater the probability that the stronger player wins in the current match. Second, we find an effort spillover effect—with negative (positive) spillover, more effort in earlier stages leads to a lower (higher) probability of winning in later stages, holding the opponent’s effort fixed. When both players experience equal degrees of negative spillover, the weaker player has an increased probability of winning. Third, we show that noise around effort increases the probability that the weaker player wins. Fourth, we identify an underdog advantage, showing the conditions under which the weaker player is more likely to win in the final stage, relative to earlier stages. Finally, we show a prize spread effect, where a steeper structure improves the stronger player’s probability of success in all stages.

We test our theoretical predictions using data from professional tennis matches and betting markets. Examining the effect of changes in the skill of the expected competitor in the next round, we find evidence of a shadow effect in all but the last rounds of play. Spillover in tennis tournaments appears to have a negative impact—more effort exertion in the previous rounds is associated with less success in the current round. Comparing best-of-three and best-of-five events, we find evidence supporting the prediction that increased noise around

effort positively impacts the probability that the weaker competitor wins. We do not find support for an underdog advantage in the final tournament stage, suggesting that the prize conditions for this hypothesized phenomenon may not be satisfied in the data. Examining tennis betting markets, we find evidence that bookmakers' prices reflect both spillover from past competition and the shadow of future opponents.

In early work on knock-out tournaments, Rosen (1986) models a multi-stage contest where forward-looking players have Tullock-style contest success functions. Similar to the model presented below, players' strategies in his model depend on the anticipated behavior of current and future opponents. Rosen's main result explains the skewed compensation distributions found in many firms—extra rewards are required in late stages of these elimination tournaments to maintain equal levels of effort across stages. Searls (1963) studies single- and double-elimination contests and finds that best-of-three single-elimination events are most likely to select the highest ability player as the winner. Groh *et al.* (2008) describe the optimal seeding of heterogeneous players according to the contest designer's objective. Modeling contests as all-pay auctions, they find that common seeding rules that match weakest to strongest players in the semifinals maximize the probability that the strongest player wins overall.

Ryvkin (2009) considers the elasticities of a player's equilibrium effort with respect to his own ability and the abilities of his opponents across several tournament formats. In elimination tournaments with weakly heterogeneous players, he finds that the abilities of opponents in the more distant future have a lower impact on a player's equilibrium effort than does the ability of the current opponent. Ryvkin also shows that, when players' relative abilities are uniformly distributed, a “balanced” seeding can eliminate the dependence of a player's equilibrium effort on his opponents' abilities.

Sunde (2009) tests the incentive effect of player heterogeneity using data from selected professional tennis tournaments. He finds that heterogeneity impacts the effort choice of the stronger player more than it changes the effort of the weaker player in a match. In his analysis, this means that the weaker player wins fewer games per set and the stronger player wins more games per set as heterogeneity increases. However, these effects are not symmetric: for an equal change in rank disparity, the increase in the number of games won by the stronger player is smaller than the decrease in the number of games lost by the weaker player. In contrast to Sunde's work, we study the role of skill heterogeneity across multiple stages of an event—that is, we examine the incentive impact of ability differences with past, current, and (expected) future opponents. The effects of player heterogeneity on effort in one-shot tournaments has been studied both theoretically (e.g. Baik, 1994; Moldovanu and Sela, 2001; Szymanski and Valletti, 2005; Minor, 2011) and empirically (e.g. Knoeber and

Thurman, 1994; Brown, 2011).

Data from professional tennis has been used in other research: Walker and Wooders (2001) used video footage and data from the finals of 10 Grand Slam events to identify mixed strategies. Malueg and Yates (2010) study best-of-three contests using four years of data from professional tennis matches with evenly-skilled opponents. They find that the winner of the first set of a match tends to exert more effort in the second set than does the loser and, in the event of a third set, players exert equal effort. Forrest and McHale (2007) use professional tennis tour and bookmaking data and find a modest long-shot bias. Gonzalez-Diaz *et al.* (2010) use data from US Open tournaments to assess individual players’ abilities to adjust their performance depending on the importance of the competitive situation—that is, trade off better performance in high-stakes situations for worse performances in low-stakes situations. They identify heterogeneity in players’ ability to make this trade-off and suggest that this heterogeneity drives differences in players’ long-term success. Using detailed data from the men’s and women’s professional tennis circuits, Gilsdorf and Sukhatme (2008a and 2008b) find that larger marginal prizes increase the probability that the stronger player wins.

The paper is organized as follows: Section 2 presents a two-stage model of an elimination tournament. We derive several propositions and outline the testable hypotheses. In Section 3, we describe our data and empirical strategy for testing the empirical predictions of the model. Section 4 describe the results. In Section 5, we discuss spillover and shadows in the context of betting markets. We conclude in Section 6 and discuss the implications of our findings for contest designers.

2 Theory

Rosen (1986) uses a probabilistic Tullock-style contest model to study elimination tournaments, while Lazear and Rosen (1981) use an additive noise structure in their foundational work on one-shot labor tournaments. Distinctly, we use Lazear and Rosen’s original additive noise model to focus on the dynamics of a multi-stage elimination tournament. This specification gives us some of the familiar results from Rosen (1986) and also allows us to identify conditions that reverse these predictions. In addition, we draw further comparative static results related to the strategic effort exertion of players across multiple stages based on the future *and* the past.

We study a new theoretical version of knockout tournaments that we describe as “sequentially-resolved elimination tournaments.” In this setting, matches occur sequentially in a given stage. That is, in each stage of competition, matches between pairs of players are staggered across time. As a consequence, players in later matches of the same stage learn the identity

of their potential future opponent when their future opponent's match is over. This structure is in contrast with other models of elimination tournaments where all matches in a given stage occur simultaneously (for example, see Stracke (2011)). The sequential play is often found in practice; for example, in firm-level tournaments, it is rare for multiple promotions to division vice-president to occur simultaneously. Instead, the identity of the new appointee is known to other workers still competing for a parallel executive spot—the hopeful workers now know their future opponent for advancement beyond vice-president. To our knowledge, we are the first to consider such a format both theoretically and empirically.

2.1 Model Set-Up

Consider a two-stage elimination tournament with four players, where the players who win in the first stage advance to the final stage. The overall tournament winner receives a prize of V_W , while the second-place competitor receives a prize V_L . Let $V_W > V_L > 0$ and define the prize spread $\Delta V = V_W - V_L$. Let player i 's total cost be a function of his effort x_i and his cost type c_i . We denote player i 's costs as $c_i\gamma(x_i)$, where $\gamma'(x_i) > 0$, $\gamma'(0) = 0$ and $\gamma''(x_i) > 0$.

Matches in the first-stage are sequential. Assume that players 3 and 4 compete first. Then, player 1 faces player 2 knowing the outcome of the previous match. Without loss of generality, we assume that player 3 won his match against player 4. We also assume cost type, c_i , varies across all players.

2.1.1 Final Stage

Assume that player 1 won his first-stage match. To find the equilibrium of the multi-stage game, we begin by analyzing the strategies of player 1 and his opponent player 3 in the *final* stage. Define player 1's expected payoff function as

$$\pi_{1,final} = P_1(x_1, x_3) \Delta V - c_1\gamma(x_1) + V_L \quad (1)$$

where his probability of winning takes the following form:

$$P_1(x_1, x_3) = \begin{cases} 1 & \text{if } x_1 + \varepsilon_1 > x_3 + \varepsilon_3 \\ \frac{1}{2} & \text{if } x_1 + \varepsilon_1 = x_3 + \varepsilon_3 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where $x_i + \varepsilon_i$ is player i 's level of output. Note that output is a function of both effort x_i and a random noise term ε_i . In definition (2), the probability that player 1 wins is increasing in

his own effort and decreasing in the effort of his opponent.

Define $\varepsilon = \varepsilon_3 - \varepsilon_1$ and let ε be distributed according to some distribution G such that probability (2) can be written as

$$P_1(x_1, x_3) = P_1(x_1 - x_3 > \varepsilon) = G(x_1 - x_3) \quad (3)$$

Now, player 1's payoff function (1) can be written as

$$\pi_{1,final} = G(x_1 - x_3) \Delta V - c_1 \gamma(x_1) + V_L \quad (4)$$

and his first order condition is

$$\frac{\partial \pi_{1,final}}{\partial x_1} = G'(x_1 - x_3) \Delta V - c_1 \gamma'(x_1) = 0 \quad (5)$$

Following Konrad (2009) and Ederer (2010), we assume that G is distributed uniformly with the following support¹

$$G \sim U \left[-\frac{1}{2}a, \frac{1}{2}a \right]$$

and, therefore,

$$G' = \frac{1}{a}$$

The assumption that G is uniformly distributed removes the strategic interdependence of players' current period effort choices (Konrad, 2009). This allows us to isolate the consequences of past effort choices and potential future competition on current-stage effort.

Rewriting the first order condition (5) yields:

$$\frac{\partial \pi_{1,final}}{\partial x_1} = \frac{\Delta V}{a} - c_1 \gamma'(x_1) = 0$$

which we can rearrange as the following expression:

$$\gamma'(x_i) = \frac{\Delta V}{ac_i} \text{ for } i = 1, 3 \quad (6)$$

We can think of $\gamma'(x_i)$ as player i 's marginal cost and $\frac{\Delta V}{ac_i}$ as his marginal benefit. Assume for the remainder of the analysis that player 1 is the stronger player ($c_1 < c_3$). Then, expression (6) implies player 1 exerts more equilibrium effort in the final stage ($x_1^* > x_3^*$).

In the final round, since both players are guaranteed at least second prize V_L , increasing the first prize amounts to increasing the stakes of the contest. As expected, higher stakes

¹See the Appendix for conditions on the primitives of the model that assure that G is well-defined.

leads to more effort from both players, though the stronger player increases his effort more than the weaker player. Also, increasing the noise around effort reduces equilibrium effort, particularly for the stronger player. Finally, as expected, effort choices are increasing in ability.

2.1.2 First Stage

Define z_1 and z_2 as the efforts of players 1 and 2 in the first stage. Player 1's expected payoff function in the first stage is

$$\pi_{1,first} = P_1(z_1, z_2) \tilde{V}_1 - c_1 \gamma(z_1) \quad (7)$$

where \tilde{V}_1 is the continuation value of player 1. Define player 1's continuation value as the equilibrium payoff in the final stage:

$$\tilde{V}_1 = \pi_1(x_1^*, x_3^*(c_3)).$$

Equation (7) yields the first order condition

$$\frac{\partial \pi_{1,first}}{\partial z_1} = \frac{\tilde{V}_1}{a} - c_1 \gamma'(z_1) = 0$$

which we can rearrange, for either player, as the following expression

$$\gamma'(z_i) = \frac{\tilde{V}_i}{ac_i} \text{ for } i = 1, 2 \quad (8)$$

As in the final stage, equilibrium effort is increasing in ability and the continuation value. Increasing noise has an adverse effect on first stage effort.

Recall that, at the start of their match, players 1 and 2 already know the outcome of the other first-stage match between players 3 and 4. Of course, this means that players 3 and 4 did not know exactly the identity of their future opponent. Instead, we assume that they formed an expectation of their continuation value as follows:

$$\mathbb{E}[\tilde{V}_i] = p_{1|i} \tilde{V}_i(x_i^*, x_1^*) + (1 - p_{1|i}) \tilde{V}_i(x_i^*, x_2^*) \text{ for } i = 3, 4$$

where $p_{1|i}$ is the equilibrium probability that player 1 wins knowing that he will face player i in the final stage.² Note that player i cannot influence this probability $p_{1|i}$ because it is a

²When player 1 is stronger than player 2, $\tilde{V}_i(x_i^*, x_1^*) < \mathbb{E}[\tilde{V}_i] < \tilde{V}_i(x_i^*, x_2^*)$ for $i = 3, 4$.

function of the *realized* outcome of the completed, first-stage match between players 3 and 4. This simplifies our analysis because player i 's first-stage effort z_i does not change this probability $p_{1|i}$. Thus, for players 3 and 4, we can restate the expression of their equilibrium effort (8) as

$$\gamma'(z_i) = \frac{E[\tilde{V}_i]}{ac_i} \text{ for } i = 3, 4$$

and the analysis described above for players 1 and 2 applies similarly.

Now, we can compare the difference in effort choices between the stronger and weaker player across stages to determine differences in their probabilities of winning, holding fixed the skill differential between players. For example, we can compare the outcome of a final-stage match between players of cost types 1 and 3 with the outcome of a first-stage match where players of cost types 1 and 3 compete.

Assume that a player of cost type 1 is stronger than a player of cost type 3 ($c_1 < c_3$). As long as the final-stage losing prize V_L is large enough relative to ΔV , then effort disparity across stages is ordered: $x_1^* - x_3^* < z_1^* - z_3^*$. Since effort disparity is greater in the first stage than in the final stage, expression (3) necessarily means that the stronger player has a greater probability of winning in the first stage relative to the final stage. That is, when certain prize conditions are satisfied, the weaker player has a lesser probability of losing against a stronger player in the final stage relative to his probability of losing against that same opponent in the first stage of an otherwise identical tournament. The intuition is as follows: As we increase the losing prize V_L while holding prize spread ΔV constant, the difference in first-stage efforts ($z_1^* - z_3^*$) increases because the stronger player is more sensitive to changes in the continuation value. In contrast, the difference in final stage efforts ($x_1^* - x_3^*$) remains the same, since the losing prize does not enter the first order condition for the final stage. This result is in contrast to Rosen (1986) who finds that there is always an “underdog advantage” in his Tullock-style contest.

This leads to our first proposition:

Proposition 1 $P_{1,final}(x_1^*, x_3^*(c_3)) < P_{1,first}(z_1^*, z_2^*(c_2))$ when $c_2 = c_3$ and $V_L > \Delta V + c_2\gamma\left(h\left(\frac{\Delta V}{ac_2}\right)\right)$. With a sufficiently large second place prize relative to the first place prize, the probability that the weaker player wins in the final stage is greater than the probability that he wins in the first stage, holding opponent skill constant.

Proof. See Appendix 7.2. ■

2.2 Shadow of Future Competition

The model can also be used to understand the impact of known (or expected) future competition on current effort decisions. Consider a decrease in the skill of the future opponent, player 3 (i.e. c_3 increases). This change has the effect of increasing the continuation value for both players 1 and 2 in the *first* stage. Since player 1 has a lower cost of effort than player 2, player 1 will increase his first-stage effort more than player 2 since his return to a change in the continuation value is greater than for player 2.

From the final-stage first order condition, equation (6), it follows that x_3^* decreases as c_3 increases.

To understand the effect of decreasing x_3^* on the final stage payoff $\pi_{1,final}$, we take the derivative of equation (4)

$$\frac{\partial \pi_{1,final}^*}{\partial x_3^*} = G'(x_1^* - x_3^*) \Delta V = -\frac{\Delta V}{a}$$

That is, as the final stage opponent's effort decreases, the player 1's final stage equilibrium payoff increases. A change in the final round opponent's skill will have an equal effect on player 2: $\frac{\partial \pi_{1,final}^*}{\partial x_3^*} = \frac{\partial \pi_{2,final}^*}{\partial x_3^*}$.

Since the change in the continuation value is the same for players 1 and 2, the stronger player will increase his effort (z_1^*) more than player 2 will increase his effort (z_2^*). We can add a term, W , to represent the (equal) change in the continuation value to equation (8)

$$\gamma'(z_i) = \frac{\tilde{V}_i + W}{ac_i}$$

Note that the impact of the increase in the continuation value is larger for player 1's effort relative to the effect on the effort of player 2 $\left(\frac{W}{ac_1} > \frac{W}{ac_2}\right)$. Thus, if $\gamma'(z_i)$ is not too convex (e.g., quadratic costs), the effort disparity is increased, and this further improves the stronger player's probability of winning in the first stage.³ This analysis gives us the following proposition:

Proposition 2 *Assuming $\gamma''' \leq 0$ (e.g. quadratic costs), as the skill of the future competitor in the final stage declines, the stronger player becomes even more likely to win in the first stage (and the weaker player becomes even less likely to win in the first stage).*

³Marginal benefit increases more for the stronger player than for the weaker player because $c_1 < c_3$. However, if the marginal cost of effort is too convex, the stronger player may not increase his effort as much as the weaker player. Conditions on the third derivative, namely $\gamma''' \leq 0$, ensure that the stronger player increases his effort more than the weaker player for a given change in the strength of the future competitor.

2.3 Prize Effect

We can explore how changes in the prize structure affects equilibrium effort choice. In particular, we study the effect of increasing the spread, ΔV , between first and second prize. To simplify our analysis, we assume that participants' costs are quadratic and hold constant the skill level of players in each stage.

2.3.1 Prize Effect - Final Stage

From equation (6), we know that final period effort is

$$x_i^* = \frac{\Delta V}{2ac_i}$$

Since $c_1 < c_2$, increasing ΔV increases the effort of the stronger player more than for the weaker player. That is, the effort disparity between players increases as the relative stakes increase.

2.3.2 Prize Effect - First stage

From our analysis above, we can write out player i 's continuation value when he faces quadratic costs and player j :

$$\tilde{V}_i = \left(\frac{\frac{\Delta V}{2ac_i} - \frac{\Delta V}{2ac_j} + \frac{a}{2}}{a} \right) \Delta V - c_i \left(\frac{\Delta V}{2ac_i} \right)^2 + V_L$$

Using equation (8) and our expression for \tilde{V}_i , we can solve explicitly for first-stage equilibrium effort:

$$z_i^* = \frac{\left(\frac{\frac{\Delta V}{2ac_i} - \frac{\Delta V}{2ac_j} + \frac{a}{2}}{a} \right) \Delta V - c_i \left(\frac{\Delta V}{2ac_i} \right)^2 + V_L}{2c_i a}$$

The following expression describes the change in equilibrium effort resulting from a change in ΔV :

$$\begin{aligned} \frac{\partial z_i^*}{\partial \Delta V} &: \frac{\frac{\Delta V}{a^2} \left(\frac{1}{c_i} - \frac{1}{c_j} \right) + \frac{1}{2} - \frac{\Delta V}{2a^2 c_i}}{2c_i a} \\ &= \frac{\frac{\Delta V}{a^2} \left(\frac{1}{c_i} - \frac{1}{c_j} \right) + \frac{1}{2}}{2c_i a} \end{aligned}$$

This leads to the following proposition:

Proposition 3 *With quadratic costs, for a given prize spread increase, the stronger player is even more likely to win in either stage (and the weaker player is even less likely to win in either stage).*

Proof. See Appendix 7.2. ■

This proposition suggests that tournaments with steeper prize structures will experience fewer upsets.

2.4 Effort Spillover

We can also examine effort spillover between stages of the tournament.⁴ Spillover can take either a positive or negative form. Positive spillover might reflect learning (by doing), skill building or momentum within a firm. For example, an innovation team whose proposal advances to a second stage of funding might benefit from its first-stage experience, both technical and relational. With positive spillover, second-stage effort is less costly than first stage effort. In contrast, negative spillover might reflect fatigue or reduced resources in later stages. For example, architects competing in design competitions might exhaust their creative resources in early stages and have only limited energy for second-stage proposals. In this case, second-stage effort is more costly than first-stage effort.

Consider a scenario where effort expended by a player in the first stage influences his effort in the final stage. We can rewrite player 1's final-stage payoff as

$$\pi_{1,final} = G(x_1 - x_3) \Delta V - c_1 \gamma(x_1, z_1) + V_L$$

where the cost function reflects current and past effort.

First, consider the case where a player's marginal cost of effort in the final stage is unaffected by previous stage effort: $\frac{\partial \gamma(x_1, z_1)}{\partial x_1 \partial z_1} = 0$. For example, if previous effort appears only as a fixed cost in the final stage, we would expect no change in final-stage effort. For example, a design team that submits an innovative proposal in the first stage might require specialized equipment to complete the building phase in the second stage.

To study a negative spillover effect, we let a player's marginal cost of effort in the final stage be increasing in first-stage effort: $\frac{\partial \gamma(x_1, z_1)}{\partial x_1 \partial z_1} > 0$. Consider again expression (6). We see that final stage equilibrium effort is strictly decreasing in first stage effort because the marginal cost of final-stage effort is increasing in first-stage effort. With positive spillover, a player's marginal cost of effort in the final stage is decreasing in first-stage effort: $\frac{\partial \gamma(x_1, z_1)}{\partial x_1 \partial z_1} < 0$.

⁴Different notions of spillover have been explored in the literature in settings where players with exogenous, fixed resources make effort allocation decisions over multiple periods of play. For recent examples, see Sela and Erez (2011) and Harbaugh and Klumpp (2005).

Now, from expression (6), final-stage equilibrium effort is strictly increasing in first-stage effort.

We can re-write player 1's marginal cost of effort with spillover as $\gamma'(x_1, z_1) = \gamma'(x_1, 0) k$ where $k > 1$ for net negative spillover and $k < 1$ for net positive spillover. Revisiting expression (6), we can rewrite marginal cost as

$$\gamma'(x_1) k = \frac{\Delta V}{ac_1} \quad \text{or} \quad \gamma'(x_1) = \frac{\Delta V}{a\tilde{c}_1}$$

where $\tilde{c}_1 = c_1 k$. Straightforward calculations show that $\frac{\partial \pi_{1,final}}{\partial c_1} < 0$.⁵ Therefore, when $\tilde{c}_1 > c_1$, final period profit \tilde{V}_1 is less than when there was no effort spillover. Similarly, when $\tilde{c}_1 < c_1$, final period profit \tilde{V}_1 is greater than when there was no spillover.

The presence of the negative (positive) spillover also reduces (increases) *first-stage* effort; in expression (8), lower (higher) \tilde{V}_1 leads to lower (higher) effort.

Since negative spillover decreases final-stage equilibrium effort, an increase in first-stage effort implies a lower probability of success in the final stage, holding the opponent's effort and skill constant. Of course, the opposite is true for positive spillover. The direction and impact of spillover depends on the context and, thus, is an empirical question.

Proposition 1 considers the underdog advantage across rounds within an event, but we can also consider it across events. In particular, the degree of noise in determining the winner and the amount of spillover between stages within an event changes the weaker player's probability of success.

Proposition 4 $G(z_1^* - z_2^*) \rightarrow 0.5$ and $G(x_1^* - x_3^*) \rightarrow 0.5$, when (i) $a \rightarrow \infty$ or (ii) $k \rightarrow \infty$ where player i 's effective cost type is kc_i for all i . In any stage, an increase in noise or a common proportional increase in effective cost type increases the probability that the weaker player wins.

Proof. For Proposition 4i, from expressions (6) and (8), we can see that as the support of G goes to infinity, equilibrium effort converges to zero. That is, as the noise around effort increases, the marginal return to effort declines. In the limit, effort has no impact

⁵We can see this from the following:

$$\begin{aligned} \frac{\partial}{\partial c_i} \tilde{V}_i &= \frac{\partial}{\partial c_i} \left(\left(\frac{\frac{\Delta V}{2ac_i} - \frac{\Delta V}{2ac_j} + \frac{a}{2}}{a} \right) \Delta V - c_i \left(\frac{\Delta V}{2ac_i} \right)^2 + V_L \right) \\ &= -\frac{(\Delta V)^2}{2a^2 c_i^2} + \frac{(\Delta V)^2}{4a^2 c_i^2} \\ &= -\left(\frac{\Delta V}{2ac_i} \right)^2 < 0 \end{aligned}$$

on a player's probability of success and, therefore, no effort is exerted. For Proposition 4ii, a similar logic applies—as cost types converge to infinity, effort becomes so costly that no effort is exerted. Thus, the weaker player should fare better in events that are noisier or proportionally more costly for all players in every round. ■

This proposition suggests that weaker players might support costlier competitive conditions—for example, a weaker player might advocate for more stringent standards or more difficult tasks. Of course, the two effects outlined in Propositions 4 might yield an ambiguous prediction if we compared a very noisy event with limited spillover to an event with less noise but very costly spillover.

2.5 Combined Shadow and Spillover Effects

The previous analysis has considered separately the effects of effort spillover and the shadow of future competition. Next, we present an analysis when both effects are at play. Combining the effects does not change the general predictions of the previous analysis—spillover continues to even the playing field, while weaker future competition does the opposite.

2.5.1 Spillover and Shadow - Final Stage

We begin with the final stage and fix players' abilities across the stages (i.e. the pairs of opponents in both stages have the same set of cost types). For illustration purposes and computational ease, we again assume quadratic participant costs. Our first order condition for the final stage yields equilibrium effort choice

$$x_i^* = \frac{\Delta V}{2ac_i k_i(z_i)}$$

where $k_i(\cdot)$ reflects the degree of spillover from the previous stage and is an increasing function of first stage effort z_i . As expected, greater first-stage effort results in lower equilibrium effort in the final stage. Further, this effect is amplified for the stronger type since $c_1 < c_2$. The final stage spillover effect is therefore

$$\frac{\partial x_i^*}{\partial k_i(z_i)} = \frac{-\Delta V}{2ac_i k_i(z_i)^2} < 0$$

Therefore, a given level of spillover ($k_1 = k_2 = k$) reduces the disparity between participants' efforts in the final stage, since $\frac{\partial x_1^*}{\partial k(z_1)} < \frac{\partial x_2^*}{\partial k(z_2)} < 0$.

2.5.2 Spillover and Shadow - First Stage

Next, we consider effort decisions in the first stage and write player i 's payoff as⁶

$$\pi_i = G_{first}(\cdot) \left(\left(\frac{\frac{\Delta V}{2ac_i k_i} - \frac{\Delta V}{2ac_j k_j} + \frac{a}{2}}{a} \right) \Delta V - c_i k_i \left(\frac{\Delta V}{2ac_i k_i} \right)^2 + V_L \right) - c_i z_i^2$$

The first order condition for the first stage is

$$\begin{aligned} \frac{\partial \pi_i}{\partial z_i} = & \frac{\left(\frac{\frac{\Delta V}{2ac_i k_i} - \frac{\Delta V}{2ac_j k_j} + \frac{a}{2}}{a} \right) \Delta V - c_i k_i \left(\frac{\Delta V}{2ac_i k_i} \right)^2 + V_L}{a} \\ & + G_{first}(\cdot) \left(\frac{-\Delta V^2}{2a^2 c_i k_i^2} + \left(\frac{\Delta V^2}{4a^2 c_i k_i^2} \right) \right) \frac{\partial k_i}{\partial z_i} - 2c_i z_i = 0 \end{aligned}$$

which then gives us the following expression for first-stage equilibrium effort:

$$z_i^* = \underbrace{\frac{\left(\frac{\frac{\Delta V}{2ac_i k_i} - \frac{\Delta V}{2ac_j k_j} + \frac{a}{2}}{a} \right) \Delta V - c_i k_i \left(\frac{\Delta V}{2ac_i k_i} \right)^2 + V_L}{2c_i a}}_{\text{shadow effect}} + \underbrace{\frac{G_{first}(\cdot) \left(\frac{-\Delta V^2}{2a^2 c_i k_i^2} + \left(\frac{\Delta V^2}{4a^2 c_i k_i^2} \right) \right) \frac{\partial k_i}{\partial z_i}}{2c_i}}_{\text{spillover effect}} \quad (9)$$

With no spillover ($k = 1$), the left term is precisely the shadow effect we described in Section (2.2). The right term reflects spillover. When $k = 1$, this spillover term is greater for the stronger player and, thus, the stronger player reduces his effort more than the lesser player (since $G_{first}(\cdot) \geq \frac{1}{2}$ and $c_1 < c_2$). Since the stronger player exerts more equilibrium effort in the first stage, he will necessarily suffer more spillover in the final stage (assuming players face a common $k(z_i)$ function). Thus, spillover has the effect of evening the playing field in both stages. That is, *ceteris parabus*, spillover increases the chance of an upset.

2.6 Model Predictions

The theory model outlined above provides the following predictions:

1. **Shadow of Future Competitors:** The worse-ranked the expected competitor in the next stage, the greater the probability that the stronger player wins in the current stage.
2. **Effort Spillover between Stages:** In contests exhibiting negative spillover, more effort exertion decreases the probability that a player wins in the next stage, holding

⁶We write $k_i(z_i)$ as k_i to simplify the notation in this section.

his opponent's effort fixed. In equilibrium, increased negative spillover decreases the probability that the stronger player wins in the final stage. With positive spillover, more first stage effort increases the probability that the stronger player wins in the final stage.

3. **Noise in Effort:** The noisier the effort-to-output relationship, the more likely the weaker player wins in either stage.
4. **Underdog Advantage in Final Stage:** Fixing the competitors and given a sufficiently large second-place prize, the probability of winning is greater for the weaker player in the final stage, relative to his probability of winning in the first stage.
5. **Prize Spread:** A steeper prize structure improves the stronger player's probability of success in all stages.

A strength of this particular model is that, while these hypotheses emerge from differences in the abilities and efforts of players, the testable implications can be framed in terms of outcomes. That is, while effort is notoriously difficult to assess in field data, we can test the predictions of the model by observing players' wins and losses under different scenarios. In the following sections, we describe our data and empirical analysis.

3 Data

Professional tennis offers an ideal environment in which to test the empirical implications of the theory. Tennis events are single-elimination tournaments—only winning players advance to successive stages until two players meet in the final stage to determine the overall winner. Prizes increase across stages with the largest prize going to the overall winner. The distribution of prizes is known in advance of all tournaments. The financial stakes are substantial and vary across events—for example, the total purse for the 2009 US Open singles competition was \$16 million with a \$1.7 million prize for first place, while the total purse for the 2009 SAP Open was \$531,000 and the winner received \$90,925.

The structure of tennis tournaments is particularly conducive to studying the shadow of future competition—both players (and the econometrician) know the competitors in the parallel match. In some cases, players can know exactly who they would face in the next round; in other cases, they can make reasonable predictions about upcoming opponents. Moreover, player ability is also observable to players and researchers—past performance, as well as world rankings statistics, are widely available. For example, in the 2007 Swiss Indoors tournament in Basel, players in the first round, Del Potro and Russell, knew that

their next opponent would be either Roger Federer or Michael Berrer. Of course, given the ability difference between these possible future opponents, Del Potro and Russell were likely predicting that their second-round opponent would be Federer.

Two distinct formats are used in professional tennis events—the winner of most matches is determined through the best of three sets, while select events are best-of-five.⁷ This variation allows us to consider the role of noise in elimination contests—the best-of-three format is expected to produce noisier outcomes than best-of-five events. Intuitively, more sets is analogous to more draws from a distribution, leading to a more “precise” overall effort-to-output return.

3.1 Professional Tennis Match Data

To test the predictions outlined in the theory, we examine the behavior of professional tennis players in 615 international tournaments on the ATP World Tour between January 2001 and June 2010. The data include game-level scores and player attributes for men’s singles matches (available at <http://www.tennis-data.co.uk>). The four “Grand Slam” events—the Australian, French, and US Opens, and Wimbledon—are included in the data. All of the tournaments are multi-round, single-elimination events played over several days.

Tournament draws are organized by seeds; in general, seeding is determined by players’ official world rankings in the week before the event. Draws include 28, 32, 48, 56, 96 or 128 players. Of the 615 events in the data, 433 tournaments consist of five rounds of play—rounds 1 and 2, quarterfinals, semifinals, and the final. Six rounds are played in 128 events. Fifty-four tournaments, including the Grand Slam events, consist of seven rounds of play—rounds 1 to 4, quarterfinals, semifinals, and the final. Most ATP events are best-of-three sets, while the Grand Slam events are best-of-five sets.

Figure 1 is a typical draw for a five-round, 32-player tournament. The placement of seeded players is determined by ATP rules—in general, in the first round, the highest-ranked players face the lowest-ranked players. Depending on the number of competitors and the tournament draw structure, first-round byes may be awarded to the top-ranked players.⁸

World rankings (officially called the South African Airways ATP Rankings) are based on points that players accumulate over the previous 12 months. The ATP points directly reflect the pyramid structure of tournaments. More points are awarded to players who advance in top tournaments; for example, a Grand Slam winners earns the maximum points awarded

⁷To win a set, a player must win at least six games and at least two games more than his opponent. A game is won by the player who wins at least four points and at least two more than his opponent. Set tie-break rules vary by tournament.

⁸Byes automatically advance a player to the next round.

for a single event.⁹ ATP rankings are simply a rank-order of all players by their accumulated points. In our analysis, we use the ATP rankings to account for players' skill levels.¹⁰

Table 1 presents summary statistics from over 28,000 men's professional tennis matches. On average, matches are decided after 23 games; however, players play more games on average in the final round than in the first or semifinal rounds ($p\text{-value} < 0.01$). Match winners are significantly more skilled than losers ($p\text{-value} < 0.01$). Tournament winners typically rank 30th in the world, while second-place finishers are 45th in the rankings. Tournaments' seeding formats generally pair the weakest players against the strongest players in the first round. Consequently, the disparity in rankings decreases as players advance. To consider the competitive balance of matches across rounds, we also report the rankings ratio (worse rank divided by better rank). While mean rankings ratios remain relatively stable across rounds, the variance appears to decline. The skill-related summary statistics suggest that while high-skill players do not *always* win their matches, on average, opponents become closer in ability as tournaments progress.

4 Results

In this section, we use a series of empirical tests to examine the predictions of the theory model in Section 2.6. For each hypothesis, the discussion of the results is organized as follows: First, we outline the prediction to be tested. Then, we describe the econometric specification that allows us to test this hypothesis. We estimate all equations using OLS with a robust variance estimator; results are quantitatively very similar for a probit specification and are not reported. Finally, we describe and interpret the findings.

4.1 The Shadow of Future Competition

Proposition 2 states that weaker future competition will increase the stronger competitor's probability of success in the current stage. This prediction follows from the observation that, while weaker future competition will cause both players to increase their effort in the current period, the current effort of the better-ranked player increases even more than the current effort of his worse-ranked opponent.

⁹For details of the world ranking system, see the 2011 ATP World Tour Media Guide, available online at www.atpworldtour.com.

¹⁰Klaassen and Magnus (2003) suggest a transformation of rankings to account for differences in ability between high- and low-skilled players. They calculate a player's ability as $R = K + 1 - \log_2(\text{ranking})$, where K is the total number of rounds in the tournament and *ranking* is the player's tournament seed. All of our analyses are robust to this alternative measure of skill heterogeneity—results with the Klaassen-Magnus transformation are qualitatively very similar to the results using ATP rankings and are not reported.

The following specification tests this hypothesis:

$$strongwins_{mt} = \beta_0 + \beta_1 Future_{mt} + \beta_2 Current_{mt} + \gamma X_t + \varepsilon_{mt} \quad (10)$$

where $strongwins_{mt}$ is a binary indicator of whether the better-ranked player of match m won in a stated round of tournament t , $Future_{mt}$ represents the expected ability of the opponent in the next round, $Current_{mt}$ represents the heterogeneity of players' skills in the current match, X_t is a matrix of tournament-level controls, and ε_{mt} is the error term.

In the reported regression, $Current_{mt}$ is the ratio of the rank of the worse player and the better player. $Future_{mt}$ is the rank associated with the stronger player in the parallel match. For example, for the 2007 Swiss Indoors tournament (see Figure 1), the expected future opponent for the match between Del Potro and Russell would be Roger Federer. For the Del Potro-Russell match, $Current_{mt} = \frac{71}{49}$ and $Future_{mt} = 1$. Note that this construction of $Future_{mt}$ is a conservative one—we are assuming that the future competitor will always be the better of the two potential opponents in the next round.¹¹ This means that, on average, we are understating the continuation value for players in the current round. Consequently, our coefficient estimates on $Future_{mt}$ will understate the actual shadow effect. Results are qualitatively similar if we instead use an average of future opponents' rankings.

Tournament-specific fixed effects capture average event-level characteristics and control for differences between tournaments (e.g. media attention). Total purse size for any given event has not varied substantially across time. For example, the purse for the US Open has grown by an average of 3% each year from 1997 to 2011. Over the same period, the inflation rate was roughly the same. Therefore, real purse size was relatively stable and the purse effect is captured by tournament dummies. Additionally, because some tournaments have changed venues over time, we include additional controls for surface and court type.

Results: Table 2

Table 2 reports the estimated coefficients for regression (10) by tournament round using rankings to measure player skill. In all rounds before the quarterfinals, the coefficient on the shadow effect ($Future_{mt}$) is positive and statistically significant ($p\text{-value} < 0.01$). That is, the weaker the future opponent (i.e. a larger rank value), the greater the probability that the stronger player wins in the current round, controlling for current players' skills. The estimated effects for the quarterfinals and semifinals are also positive, although not statistically significant at conventional levels—this may reflect both small sample sizes and limited variation in opponents' skills at advanced stages of these tournaments.

The magnitude of the coefficients may also be interpreted—we include mean and standard

¹¹Unfortunately, our data do not report match start and end times. That is, we cannot determine the exact sequence of matches in a round.

deviation values of future opponent rank in the table. For a one standard-deviation increase in future opponent’s rank (decrease in ability), we estimate that the probability that the stronger player wins in the current round increases by 2.3 to 5.5 percentage points. Given that the average probability that the stronger player wins is approximately 65%, on average, the shadow effect represents a 5% increase in the probability of winning.

As expected, the coefficient on skill disparity in the current match is also positive and statistically significant ($p\text{-value} < 0.01$), indicating that increased heterogeneity between the players increases the probability that the stronger player wins.

We estimate, but do not report, results for regression (10) using ATP points as a measure of player skill. Estimates are qualitatively similar to results using ATP rankings.

4.2 Effort Spillover Across Rounds

Proposition 4 considers the role of spillover in effort choice. The direction of the spillover effect is often an empirical question; however, one might expect negative spillover in events that require intense effort exertion over a short period of time. In professional tennis, players may face a higher cost of effort if their total exertion in previous matches induced lasting fatigue. To identify an empirical spillover effect, we estimate the following equation:

$$wins_{it} = \beta_0 + \beta_1 PastGames_{it} + \beta_2 RelativeStrength_{it} + \gamma X_t + \varepsilon_{it} \quad (11)$$

where $wins_{it}$ is a binary indicator of whether player i won the match in a stated round of tournament t , $PastGames_{it}$ is the number of games played in previous round(s) of the tournament by player i (either all previous rounds or just the most recent previous round), $RelativeStrength_{it}$ reflects player i ’s relative skill compared to his current opponent, X_t is a matrix of tournament-level controls similar to those in regression (10), and ε_{it} is the error term.

In the reported regressions, $RelativeStrength_{it}$ is the ratio of player i ’s current opponent’s rank and player i ’s own rank. Therefore, $RelativeStrength_{it} < 1$ when player i is weaker than his opponent and $RelativeStrength_{it} > 1$ when player i is the stronger player.

Results: Tables 3 and 4

Table 3 reports regression results for specification (11) by tournament round using rankings and accounting for the number of games played in all previous rounds. Here, we identify a cumulative spillover effect. Coefficient estimates for $PastGames_{it}$ are negative and statistically significant (at $p\text{-values}$ from < 0.01 to < 0.1), suggesting a negative spillover. The magnitudes of the coefficients suggest that, on average, a one standard-deviation increase in the number of previous games (over all previous rounds) is associated with a 7 percentage

point decline in the probability of winning in the current match.¹² For the stronger player in an average match, this represents a 11% decline in the probability of winning; for weaker players, this reflects a 20% decline in success.

We estimate, but do not report, a similar regression using ATP points to control for players' relative skills. Again, coefficient estimates for $PastGames_{it}$ are negative and statistically significant (at p -values from < 0.01 to < 0.05). In fact, the magnitudes of the coefficients are remarkably similar and lead to the same interpretation as in Table 3.

Coefficient estimates for $RelativeStrength_{it}$ are statistically significant at conventional levels in almost all rounds in Tables 3; the positive estimates suggest that a player's probability of winning increases with improvements in his relative skill.

Tables 4 reports results when $PastGames_{it}$ reflects only the number of games played in the most recent round. While the coefficient on $PastGames_{it}$ is negative and statistically significant in early rounds, we find little period-to-period effort spillover between quarterfinal, semifinal and final rounds. This (non) result is not surprising given that tournament schedules often give competitors days of rest between the last few rounds. As expected, for early rounds, the period-to-period spillover is smaller than the cumulative spillover estimates in Tables 3. Here, a one standard-deviation increase in the number of games in a player's previous match is associated with a 3 percentage point decline in the probability of winning in the current match.

The estimates for $RelativeStrength_{it}$ in the period-by-period analysis take on the same sign and significance as in the cumulative spillover regressions. Using rankings and ATP points (not reported), the coefficients indicate that a player is more likely to win as he improves relative to his opponent.

4.3 Combined Spillover and Shadow Effects

Section 2.5 studies the knock-out tournament model with both spillover from past effort and the shadow of a future competitor. Overall, our analysis reveals that the combined effects produce the same predictions that we identified when we explored shadow and spillover independently. In theory, the weaker the expected future competitor, the greater the probability that the stronger player wins in the current round. In contrast, effort spillover from the previous round actually levels the playing field by reducing the probability that the stronger player wins in the current round.

The following specification allows us to study the effects of shadow and spillover simul-

¹²This is a conservative estimate, calculated using only summary statistics for best-of-3 matches.

taneously:

$$\begin{aligned} strongwins_{mt} = & \beta_0 + \beta_1 Future_{mt} + \beta_2 Current_{mt} \\ & + \beta_3 SPastGames_{it} + \beta_4 WPastGames_{it} + \gamma X_t + \varepsilon_{mt} \end{aligned} \quad (12)$$

where $strongwins_{mt}$ is a binary indicator of whether the better-ranked player in match m won in a stated round of tournament t , $Future_{mt}$ represents the expected ability of the opponent in the next round, $Current_{mt}$ represents the heterogeneity of players' skills in the current match, $SPastGames_{it}$ is the number of games played in all previous rounds of the tournament by the better-ranked player, $WPastGames_{it}$ is the number of games played in all previous rounds of the tournament by the worse-ranked player, X_t is a matrix of tournament-level controls, and ε_{mt} is the error term.

Results: Table 5

Overall, results in Table 5 provide further support for the empirical existence of spillover and shadow effects. Note that the first and last columns of the tables omit estimates for spillover and shadows, respectively—there is no spillover for players in the first round of a tournament and players face no shadow in the final round.

As in Table 2, coefficient estimates for the shadow of the future competitor are positive and statistically significant in all rounds before the quarterfinals. Indeed, the coefficients on $Future_{mt}$ are very similar in magnitude in Tables 2 and 5. For a one standard-deviation increase in the next-round opponent's rank, the probability that the stronger player wins in the current round increases by approximately 4 percentage points.

Coefficient estimates for the two spillover variables take on predicted signs—more previous games for the stronger player decreases the probability he wins in the current match, while in general more previous games for the weaker player increases the chance that the stronger player wins.

The history of the stronger player appears to drive his current success more than the history of his opponent—from expression (9) of our model, we expect the stronger player to be more adversely affected than the weaker player for a given increase in spillover. Indeed, in the data, the effect of previous games played by the stronger player is often larger than the effect of the weaker player's previous games. T-tests comparing the magnitude of the spillover estimates ($H_o : \beta_3 = -\beta_4$) reject equality in the 2nd round, 4th round, quarter- and semi-finals ($p\text{-values} < 0.05$). We cannot reject the null of equal effects in the final round, perhaps because players tend to be relatively well-matched in terms of ability.

Comparing across rounds, the effects of spillover from both stronger and weaker players' histories are smaller in the final periods of tournaments relative to early rounds. This may

be because many events provides additional rest periods for players between the later rounds of play, while early-round schedules often have players competing on consecutive days.

4.4 Noise around Effort

Proposition (4) also states that an increase in the noise around effort will benefit the weaker player. Intuitively, an increase in the noisiness of effort pushes the contest towards a lottery where each player has equal probability of success. Clearly, this improves the odds of the weaker playing winning. To test this hypothesis, we compare best-of-three and best-of-five matches, where we expect best-of-three matches to be noisier than matches with more sets.

We use the following specification:

$$strongwins_{mt} = \lambda_0 + \lambda_1 Current_{mt} + \lambda_2 BestOfFive_t + \eta Y_t + \varepsilon_{mt}$$

where $Current_{mt}$ reflect the heterogeneity of players (as above), $BestOfFive_t$ is a binary indicator that equals 1 if the tournament has a best-of-five format and 0 otherwise, and Y_t is a matrix of controls for surface and court type. In this regression, we do not include tournament-specific fixed effects since the “best-of” format does not vary across years for a given tournament. Also, because the best-of-five format is only used in tournaments with four rounds of play before the quarterfinals, we limit the analysis to tournaments with seven rounds total.

Results: Table 6

Coefficient estimates in Table 6 suggest that stronger players are more likely to win in best-of-five events ($p\text{-value} < 0.01$)—in the reported specification, the stronger player has nearly 6 percentage points greater probability of winning a best-of-five match.

Since best-of-five events have substantially higher prizes and larger prize spreads than best-of-three tournaments, one might worry that the results in Table 6 reflect the prize spread effect in Proposition 3. To test this hypothesis, we examine only best-of-3 tournaments and compared Master Series events to other ATP tournaments. The typical spread between first and second place prizes in the Masters Series is nearly four times the prize gap of other events. While Gilsdorf and Sukhatme (2008b) find that larger marginal prizes increase the probability that the stronger player wins in women’s tennis (where all matches are best-of-three), we find little evidence of this prize effect in men’s events. In other work by Gilsdorf and Sukhatme (2008a) examining men’s tennis, they find again that larger marginal prizes increase the likelihood of a win by the better player. However, our model would suggest that this effect could be a result of both prize spread and noise effects, since high-stakes tournaments are also less noisy—Grand Slam events are both high prize and best-of-five.

4.5 Underdog Advantage

Rosen (1986) predicts that the weaker competitor’s likelihood of success should always be higher in the final round relative to the probability that he wins earlier in an elimination tournament. Our proposition 1 provides a similar prediction, but requires that the second-place prize be sufficiently large relative to the gap between first and second-place prizes.

In the ATP match data, the weaker competitor (underdog) wins 34.1% of the matches in final rounds and 32.6% of matches in earlier rounds. However, this difference is not statistically significant at conventional levels. For robustness, we also ran a comparison while controlling for player skill and tournament-level heterogeneities. Again, we failed to find statistically-significant differences between the weaker players’ probability of winning in final and non-final rounds. Although not conclusive, our analysis suggests that the losing prizes may not be sufficiently large (relative to the winning prizes) to induce an underdog advantage in the final round.

5 Betting Markets, Spillover and Shadows

The efficiency of prediction and betting markets has been studied extensively in the literature; for examples, see the survey by Vaughn Williams (1999). Prediction markets are founded on the argument that by aggregating information, competitive markets should result in prices that reflect all available information (Fama 1970). Therefore, driven by aggregated information and expectations, prediction market prices may offer good forecasts of actual outcomes (Spann and Skeira 2003).

Similarly, betting odds reflect bookmakers’ predictions of future outcomes. Betting odds may change as new public or private information becomes available to the bookmaker and with changes in the volume of bets that may be driven by individual bettors’ private information. As with formal prediction markets, we might expect betting odds to provide good forecasts.

Spann and Skeira (2008) compare forecasts from prediction markets and betting odds using data for German premier soccer league matches. They find that prediction markets and betting odds provide equally accurate forecasts. This result seems reasonable, since betting companies with inaccurate and inefficient odds should not survive.

Although we cannot account for all information available to the market, our findings are consistent with the claim that the tennis betting market exhibits a form of information efficiency. In the discussion below, we provide statistical evidence that prices in the tennis betting market reflect information about spillover from players’ past exertion and the shadow

of future competition.

5.1 Shadow and Spillover in Betting Odds

To examine whether betting markets incorporate information about the effects of shadow and spillover, we estimate a regression similar to equation (12). Now, instead of a binary indicator of the actual outcome, the dependent variable is the probability that the stronger player wins the match as implied by betting markets.

Our data include closing odds from professional bookmakers for pre-match betting.¹³ Woodland and Woodland (1999) note that bookmakers adjust odds based on the volume of bets, making the odds available as the betting market closes particularly rich in information. In our analysis, we use the median of the available odds data since the data from no single firm covered all matches.¹⁴ Overall, there was little variation between odds posted by different bookmakers for the same match, perhaps because participants in tennis betting markets tend to be specialists and there is little casual betting (Forrest and McHale 2007).¹⁵

The accuracy of odds market predictions suggests that information beyond simple rankings are being priced in the market. Between 2001 and 2010, predictions from the market are correct for 69% of the 25,633 matches for which betting data are available. Given that the stronger player actually wins in 65% of matches, one might not be surprised by this accuracy if the market always predicted that the stronger player wins. However, in 18% of the matches, the betting odds imply that the weaker player is expected to win. Interestingly, these market predictions are accurate nearly 63% of the time. That is, these betting markets do almost as well predicting an upset as they do predicting a win by the stronger player. This is particularly notable since a naive assessment of the ATP rankings in these matches might suggest that the odds are still solidly against the weaker player—on average, the weaker player’s rank is 2.1 times higher (worse) than his opponent.

Results: Table 7

Table 7 reports results for round-level regressions where the dependent variable is the probability that the stronger player wins as implied by the betting market. Overall, coeffi-

¹³Data from 11 betting firms (Bet365, Bet&Win, Centrebet, Expekt, Ladbrokes, Gamebookers, Interwetten, Pinnacles, Sportingbet, Stan James, and Unibet) are included in our main dataset obtained from www.tennis-data.co.uk. Several betting firms also offer in-play betting, but we focus our analysis on pre-match bets only.

¹⁴We calculate the probability odds from the decimal odds in the original data. Probability odds are $1/(\text{decimal odds}-1)$.

¹⁵A positive long-shot bias—where the market undervalues the true favorite and overvalues the long-shot—has been documented in tennis odds by Forrest and McHale (2007). However, the authors find this small bias is consistent over a broad range of match-ups. In contrast to some markets, they do not find any range with a negative long-shot bias.

cient estimates suggest that the betting predictions incorporate information about players’ past and expected future competition.

Coefficient estimates for the effect of a stronger future opponent are positive and statistically significant for all rounds except for the shadow of the final round on the semifinals (p -values range from < 0.01 to 0.1). It is not surprising that we do not identify the effect of the shadow of the final on the semifinal, given the compressed distribution of skill at the end of the tournament.

Since the betting market closes only at the start of the match (and after the end of earlier rounds), players’ past exertion information is readily available. Indeed, coefficient estimates for the stronger and weaker players’ previous number of games are statistically significant (p -values < 0.01) and take on the expected signs. More previous games played by the stronger player is associated with a decrease in expectations of his success, while more previous games played by the weaker player is associated with an increase in expectations that the stronger player wins. As in Section 4.3 and as predicted by theory, the magnitude of these coefficients suggests that stronger players are more adversely affected by a given level of spillover relative to weaker players.

Greater heterogeneity in players’ abilities may increase the market’s expectation that the stronger player wins—the coefficient on rank ratio is positive and statistically significant in all rounds (p -values < 0.01).

Overall, we find strong evidence that prices in tennis betting markets reflect both the shadow and spillover effects predicted by our model. Interestingly, we again find no evidence of an underdog advantage—higher predicted odds for the weaker player in the final relative to earlier rounds—in the betting data.

6 Conclusion

In this paper, we explore a class of contests we call “sequentially-resolved elimination tournaments.” We present a two-stage, match-pair tournament model that provides several sharp results: (a) a shadow effect of future competition—the weaker the expected competitor in the final stage, the greater the probability that the stronger player wins in the first match; (b) an effort spillover effect—with negative (positive) spillover, more effort in the first stage leads to a lower (higher) probability of winning in final stage; (c) a noise effect—noise around effort increases the probability that the weaker player wins in either stage; (d) an underdog advantage—we describe the conditions under which the weaker player is more likely to win in the final stage, relative to the probability he wins in the early stage; and (e) a prize spread effect—increasing stakes improves the stronger player’s probability of success in both stages.

We test our theoretical hypotheses using data from professional tennis matches and betting markets. We find evidence of a substantial shadow effect, where a weaker future competitor increases the probability that the stronger player wins the current match. We also identify negative spillover in tennis tournaments—more effort exertion in the previous rounds is associated with significantly less success in the current round. Comparing best-of-three and best-of-five events, we find evidence supporting the prediction that the noise around effort impacts the probability that the weaker competitor wins. We do not find support for an underdog advantage in the final tournament stage, suggesting that the prize conditions for this hypothesized phenomenon may not be satisfied in the data. Moreover, we are not able to identify an empirical prize-spread effect. In a supplemental analysis, we use probability odds data from bookmakers to show that betting markets also recognize and price in the spillover and shadow effects.

6.1 Discussion

Our findings have implications in terms of the structure of elimination tournaments. Tournaments are often designed to identify high-ability candidates in environments where the contest organizer cannot readily observe innate talent. Our results suggest ways by which a contest designer can improve the likelihood that the strongest candidate succeeds. Limiting negative spillover by allowing competitors opportunities to refresh their resources between stages increases the probability that the stronger type wins. Firms may also want to encourage positive spillover through learning. For example, in an innovation contest, firms should be given adequate time between stages to raise additional funds and pursue more advanced technology improvements. Similarly, a firm may wish to institute “work-life balance” program that promotes employee wellness, discourages career-related burnout, and improves the probability that the firm’s labor tournament promotes the strongest workers.

Contest organizers may also wish to avoid “noisy” competition where the effort-to-output technology is less precise; for example, a larger panel of decision-makers in innovation, design or personnel tournaments may yield more discriminating selection.

Higher powered prizes also enhance selection across stages—large prize spreads, as well as small loser prizes, will reduce the chance of selecting the lower-ability candidate.

In contrast, if a contest designer is concerned with the unevenness of competition, it can design a more balanced contest with more negative spillover, noisier effort-to-output activities, and a flatter prize structure. Some contest designers may want to maximize individual or total effort across some or all stages. The study of these objectives is beyond the scope of our current paper and is left to future research.

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7 Appendix

7.1 Conditions for $G(\cdot)$

To ensure that probabilities are well-defined, we require two conditions on the primitives:

$$0 < \frac{h\left(\frac{\Delta V}{ac_1}\right) - h\left(\frac{\Delta V}{ac_3}\right) + \frac{a}{2}}{a} < 1 \quad \text{and} \quad 0 < \frac{h\left(\frac{\tilde{V}_1}{ac_1}\right) - h\left(\frac{\tilde{V}_2}{ac_2}\right) + \frac{a}{2}}{a} < 1$$

where $h \equiv [\gamma'(\cdot)]^{-1}$. These conditions ensure that $G(\cdot) \in (0, 1)$ for both stages in equilibrium. Depending on the model parameters, one condition will determine the upper bound of $G(\cdot)$ and the other condition will determine the lower bound.

7.2 Proofs

Proposition 1: $P_{1,final}(x_1^*, x_3^*(c_3)) < P_{1,first}(z_1^*, z_2^*(c_2))$ when $c_2 = c_3$ and $V_L > \Delta V + c_2\gamma\left(h\left(\frac{\Delta V}{ac_2}\right)\right)$. With a sufficiently large second place prize relative to the first place prize, the probability that the weaker player wins in the final stage is greater than the probability that he wins in the first stage, holding opponent skill constant.

Proof. Proposition 1 identifies an underdog advantage in the final stage—that is, the weaker player has a greater probability of winning in the final stage over the first stage. This occurs when $G_{semi} > G_{final}$ where

$$\begin{aligned} G_{final} &= \frac{h\left(\frac{\Delta V}{ac_1}\right) - h\left(\frac{\Delta V}{ac_3}\right) + \frac{a}{2}}{a} \\ G_{semi} &= \frac{h\left(\frac{\tilde{V}_1}{ac_1}\right) - h\left(\frac{\tilde{V}_2}{ac_2}\right) + \frac{a}{2}}{a} \end{aligned}$$

in other words,

$$h\left(\frac{\tilde{V}_1}{ac_1}\right) - h\left(\frac{\tilde{V}_2}{ac_2}\right) > h\left(\frac{\Delta V}{ac_1}\right) - h\left(\frac{\Delta V}{ac_2}\right) \quad (13)$$

Assume that the skill level of opponents in the first and final stages are equal, $c_2 = c_3$.

We must show that $\tilde{V}_2 > \Delta V$. This will prove the inequality in expression (13) since the difference in the pairs of h functions is increasing in that stage's prize and we know that

$$\tilde{V}_1 > \tilde{V}_2.$$

$$\begin{aligned}\tilde{V}_2 &> \Delta V \\ (1 - G_{final}(\cdot)) \Delta V - c_2 \gamma(x_2) + V_L &> \Delta V \\ V_L &> G_{final}(\cdot) \Delta V + c_2 \gamma(x_2) \\ V_L &> G_{final}(\cdot) \Delta V + c_2 \gamma\left(h\left(\frac{\Delta V}{ac_2}\right)\right)\end{aligned}$$

The final inequality provides a sufficient condition for an underdog advantage. Note that when we fix ΔV the value of the RHS of the inequality is also fixed. Then, holding ΔV fixed, we can find some $V_L > 0$ that satisfies the inequality. However, recall that G_{semi} must be less than 1; yet, G_{semi} is increasing in V_L . To see that some values satisfy $G_{semi} \leq 1$, we allow the prize levels to converge: $V_L \rightarrow V_W$. In this case, as $\Delta V \rightarrow 0$, the RHS of the inequality approaches 0 and the inequality is then easily satisfied since $V_L > 0$. ■

For a simple example, set: $c_1 = 0.7$; $c_2 = 1$; $V_L = 0.1$; $\Delta V = 1$; $a = 1.5$. With quadratic costs, our inequality fails—i.e., the weaker player is *even less* likely to win in the final stage. However, with $V_L = 0.5$, the weaker player is *more* likely to win in the final stage. Thus, given a large enough second-place prize relative to the first-place prize, the weaker player is more likely to win against the same opponent in the final stage relative to earlier stages.

Proposition 3: *With quadratic costs, for a given prize spread increase, the stronger player is even more likely to win in either stage (and the weaker player is even less likely to win in either stage).*

Proof. Let player 1 be the stronger player and player 2 be the weaker player so that $c_1 < c_2$. The result for the final stage effort follows immediately from equation (6). To compare the effect of changing the prize spread ΔV on players' first stage equilibrium efforts, we consider the following expression:

$$\begin{aligned}\frac{\partial z_1^*}{\partial \Delta V} - \frac{\partial z_2^*}{\partial \Delta V} &= \frac{\frac{\Delta V}{a^2} \left(\frac{1}{c_1} - \frac{1}{c_2} \right) + \frac{1}{2}}{2c_1 a} - \frac{\frac{\Delta V}{a^2} \left(\frac{1}{c_2} - \frac{1}{c_1} \right) + \frac{1}{2}}{2c_2 a} \\ &= \frac{\Delta V}{2a^3} \left(\frac{1}{c_1^2} - \frac{1}{c_1 c_2} \right) + \frac{1}{4c_1 a} - \frac{\Delta V}{2a^3} \left(\frac{1}{c_2^2} - \frac{1}{c_1 c_2} \right) - \frac{1}{4c_2 a} \\ &> \frac{\Delta V}{2a^3} \left(\frac{1}{c_1^2} - \frac{1}{c_2^2} \right) > 0\end{aligned}$$

The final inequality is always met, since $c_1 < c_2$ by assumption. Therefore, the stronger player increases his effort more than the weaker player for a given increase in ΔV . Conse-

quently, this increased effort disparity increases the probability that the stronger player wins in the either stage. ■

Figure 1 - Example Draw from 2007 Davidoff Swiss Indoors in Basel

1	1	FEDERER, Roger	SUI	R. FEDERER [1]	
	2	BERRER, Michael	GER	61 36 63	R. FEDERER [1]
	3	DEL POTRO, Juan Martin	ARG	J. DEL POTRO	61 64
	4	RUSSELL, Michael	USA	52 Ret'd	
S	5	KIEFER, Nicolas	GER	N. KIEFER	R. FEDERER [1]
	6	BERLOCQ, Carlos	ARG	61 61	63 62
	7	KOUBEK, Stefan	AUT	S. KOUBEK	
7	8	CHELA, Juan Ignacio	ARG	63 64	
4 WC	9	BLAKE, James	USA	J. BLAKE [4]	R. FEDERER [1]
	10	HERNANDEZ, Oscar	ESP	63 64	76(6) 76(5)
	11	KARLOVIC, Ivo	CRO	I. KARLOVIC	
WC	12	CHIUDINELLI, Marco	SUI	I. KARLOVIC	46 76(4) 64
	13	NALBANDIAN, David	ARG	63 62	I. KARLOVIC
	14	WAWRINKA, Stanislas	SUI	S. WAWRINKA	67(5) 76(2) 76(13)
	15	LOPEZ, Feliciano	ESP	76(5) 62	
	16	BERDYCH, Tomas	CZE	T. BERDYCH [5]	75 64
5	17	MATHIEU, Paul-Henri	FRA	T. BERDYCH [5]	
8 WC	18	DANCEVIC, Frank	CAN	76(4) 36 63	
	19	DELIC, Amer	USA	P. MATHIEU [8]	
Q	20	GOLUBEV, Andrey	RUS	P. MATHIEU [8]	67(7) 64 61
	21	MONTANES, Albert	ESP	A. GOLUBEV	
	22	BAGHDATIS, Marcos	CYP	64 63	M. BAGHDATIS
	23	CALLERI, Agustin	ARG	M. BAGHDATIS	62 76(5)
3	24	FERRER, David	ESP	M. BAGHDATIS	67(1) 63 63
6	25	CANAS, Guillermo	ARG	A. CALLERI	
	26	ACASUSO, Jose	ARG	64 76(3)	
	27	GINEPRI, Robby	USA	G. CANAS [6]	
	28	NIEMINEN, Jarkko	FIN	64 63	J. NIEMINEN
Q	29	REISTER, Julian	GER	J. NIEMINEN	64 36 75
Q	30	KARANUSIC, Roko	CRO	J. NIEMINEN	
Q	31	HAEHNEL, Jerome	FRA	J. NIEMINEN	63 75
2	32	GONZALEZ, Fernando	CHI	R. KARANUSIC	
				F. GONZALEZ [2]	
				64 76(5)	
				F. GONZALEZ [2]	
				76(5) 64	

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source: ATPworldtour.com

Table 1 - Summary Statistics for ATP World Tour Events January 2001 to May 2010

	All Rounds	1st Round	2nd Round	3rd Round	4th Round	Quarterfinals	Semifinals	The Final
# of Matches Played	28370	14497	7237	1897	433	2461	1230	615
Average # of games played	23.1 (10.4)	21.6 (11.4)	24.6 (8.6)	26.7 (10.1)	31.7 (11.5)	23.3 (7.7)	23.5 (7.6)	25.3 (8.7)
Average Rank of Winner	58.6 (73.2)	71.7 (83.4)	52.5 (61.1)	32.5 (55.0)	21.5 (57.3)	43.8 (53.4)	37.0 (49.6)	29.7 (39.2)
Average Rank of Loser	95.5 (121.0)	119.7 (147.7)	90.9 (97.0)	54.9 (64.4)	39.9 (58.1)	62.0 (65.3)	50.5 (56.3)	43.9 (55.2)
Average Rank Ratio (Worse / Better by Rank)	6.8 (20.9)	5.7 (21.5)	8.2 (20.6)	8.9 (18.9)	10.4 (20.1)	6.7 (24.3)	6.6 (12.8)	6.7 (13.4)

Note: Data contain player and performance information for 615 tournaments. Values in parentheses are standard deviations.

Table 2 - Shadow of (Potential) Future Opponent on Probability of Winning

Dependent Variable:

Stronger Player Wins in Current Period (0 or 1)

	2nd on 1st Round	3rd on 2nd Round	Qfinals on 2nd Round	4th on 3rd Round	Qfinals on 3rd Round	Qfinals on 4th	Sfinals on Qfinals	Final on Sfinals
Expected Future Opponent Rank	0.08% *** (0.0001)	0.10% *** (0.0003)	0.07% *** (0.0002)	0.15% *** (0.0008)	0.16% *** (0.0006)	0.41% *** (0.0016)	0.06% (0.0004)	0.07% (0.0007)
Current Rank Ratio (Worse / Better Rank)	0.16% *** (0.0002)	0.30% *** (0.0004)	0.21% *** (0.0003)	0.39% *** (0.0008)	0.31% *** (0.0006)	0.36% *** (0.0011)	0.10% ** (0.0004)	0.60% *** (0.0011)
# of observations	12575	3759	4891	858	1461	432	2450	1220
Mean Future Opponent Rank	47.1	25.2	36.0	17.6	18.5	11.8	28.5	23.5
Std. Dev. Future Opponent Rank	42.4	23.4	32.6	18.8	20.9	13.5	28.3	25.9

Notes: "Expected Future Opponent Rank" is the rank of the stronger player in the parallel event. That is, it is the rank of the stronger of the potential opponents in the next round. Values in parentheses are robust standard errors. ***, ** and * denote statistical significance at *p-values* of 1%, 5% and 10%, respectively. Regressions include tournament-level fixed effects.

Table 3 - Effort Spillover from All Previous Tournament Periods

Dependent Variable:

Winning in Current Period (0 or 1)

	2nd Round	3rd Round	4th Round	Quarterfinals (2 previous)	Quarterfinals (3 previous)	Quarterfinals (4 previous)	Semifinals	The Final
All Previous Games	-0.74% *** (0.0005)	-0.58% *** (0.0006)	-0.38% *** (0.0010)	-0.36% *** (0.0009)	-0.64% *** (0.0011)	-0.23% * (0.0012)	-0.14% *** (0.0003)	-0.16% *** (0.0004)
Rank Ratio (Opponent / Player Rank)	0.50% *** (0.0007)	0.63% *** (0.0011)	0.75% *** (0.0017)	0.95% *** (0.0017)	0.10% * (0.0006)	1.35% *** (0.0044)	0.91% *** (0.0017)	0.91% *** (0.0031)
# of observations	14462	3792	866	3454	1032	432	2460	1230
Mean # of Previous Games	18.9	34.9	49.9	43.8	56.2	68.5	130.4	149.5
Std. Dev. of Previous Games	9.8	13.7	15.9	9.7	15.5	16.4	50.8	48.7

Notes: Values in parentheses are robust standard errors. ***, ** and * denote statistical significance at *p-values* of 1%, 5% and 10%, respectively.

Regressions include tournament-level fixed effects. Summary statistics from previous games include only best-of-three matches.

Table 4 - Effort Spillover from Previous Tournament Period Only

Dependent Variable:

Winning in Current Period (0 or 1)

	1st to 2nd Round	2nd to 3rd Round	3rd to 4th Round	2nd to Qfinals	3rd to Qfinals	4th to Qfinals	Qfinals to Sfinals	Sfinals to Final
Games in Previous Period	-0.74% *** (0.0005)	-0.44% *** (0.0010)	-0.20% (0.0017)	-0.32% ** (0.0013)	-0.47% ** (0.0024)	-0.15% (0.0025)	-0.18% (0.0015)	-0.22% (0.0022)
Rank Ratio (Opponent / Player Rank)	0.50% *** (0.0007)	0.71% *** (0.0013)	0.80% *** (0.0018)	0.97% *** (0.0017)	0.12% * (0.0007)	1.43% *** (0.0044)	0.98% *** (0.0017)	0.95% *** (0.0032)
# of observations	14462	3792	866	3454	1032	432	2460	1230
Mean # of Previous Games	18.9	22.6	22.9	22.4	22.4	22.1	22.5	22.6
Std. Dev. of Previous Games	9.8	6.4	6.7	6.6	6.7	7.2	6.6	6.6

Notes: Values in parentheses are robust standard errors. ***, ** and * denote statistical significance at *p-values* of 1%, 5% and 10%, respectively.

Regressions include tournament-level fixed effects. Summary statistics from previous games include only best-of-three matches.

Table 5 - Combined Spillover and Shadow Effects

Dependent Variable: **Stronger Player Wins in Current Period (0 or 1)**

	2nd on 1st Round	3rd on 2nd Round	Qfinals on 2nd Round	4th on 3rd Round	Qfinals on 3rd Round	Qfinals on 4th	Sfinals on Qfinals	Final on Sfinals	The Final (no shadow)
Expected Future Opponent Rank	0.079% *** (0.0001)	0.084% *** (0.0003)	0.071% *** (0.0002)	0.165% ** (0.0008)	0.150% *** (0.0006)	0.421% *** (0.0015)	0.057% (0.0004)	0.077% (0.0006)	
Stronger Player's Previous Games		-0.350% *** (0.0009)	-0.435% *** (0.0008)	-0.442% *** (0.0013)	-0.513% *** (0.0011)	-0.581% * (0.0031)	-0.310% *** (0.0008)	-0.158% *** (0.0004)	-0.132% ** (0.0007)
Weaker Player's Previous Games		0.219% ** (0.0009)	0.175% * (0.0009)	0.110% (0.0011)	0.094% (0.0010)	0.163% (0.0021)	0.139% * (0.0008)	0.062% * (0.0004)	0.087% * (0.0005)
Current Rank Ratio (Worse / Better Rank)	0.161% *** (0.0005)	0.281% *** (0.0004)	0.194% *** (0.0003)	0.340% *** (0.0006)	0.265% *** (0.0005)	0.338% *** (0.0008)	0.089% ** (0.0004)	0.543% *** (0.0012)	0.363% (0.0032)
# of observations	12575	3759	4891	858	1461	432	2450	1220	615

Notes: "Expected Future Opponent Rank" is the rank of the stronger player in the parallel event. That is, it is the rank of the stronger of the potential opponents in the next round. Values in parentheses are robust standard errors. ***, ** and * denote statistical significance at *p-values* of 1%, 5% and 10%, respectively. Regressions include tournament-level fixed effects.

Table 6 - Noise around Effort

<i>Dependent Variable:</i>	Stronger Player Wins (0 or 1)
	<hr/> Rankings <hr/>
Current Skill Ratio	0.22% *** (0.0004)
Best-of-5 Dummy	5.90% *** (0.0078)
# of observations	<hr/> 26545 <hr/>

Notes Values in parentheses are robust standard errors. ***, ** and * denote statistical significance at *p-values* of 1%, 5% and 10%, respectively. Regressions include tournament-level fixed effects.

Table 7 - Predicting Betting Probability Odds by Spillovers and Shadows (Rankings)

Dependent Variable: **Implied Probability the Stronger Player Wins (%)**

	2nd on 1st Round	3rd on 2nd Round	Qfinals on 2nd Round	4th on 3rd Round	Qfinals on 3rd Round	Qfinals on 4th	Sfinals on Qfinals	Final on Sfinals	The Final (no shadow)
Expected Future Opponent Rank	0.083% *** (0.0001)	0.095% *** (0.0001)	0.041% *** (0.0001)	0.075% *** (0.0003)	0.075% *** (0.0002)	0.054% (0.0006)	0.021% * (0.0001)	0.020% (0.0002)	
Stronger Player's Previous Games		-0.312% *** (0.0003)	-0.340% *** (0.0003)	-0.363% *** (0.0004)	-0.369% *** (0.0003)	-0.508% *** (0.0011)	-0.291% *** (0.0003)	-0.102% *** (0.0001)	-0.094% *** (0.0002)
Weaker Player's Previous Games		0.116% *** (0.0003)	0.114% *** (0.0003)	0.084% *** (0.0004)	0.091% *** (0.0003)	0.116% (0.0008)	0.069% *** (0.0002)	0.036% *** (0.0001)	0.053% *** (0.0002)
Current Rank Ratio (Worse / Better Rank)	0.147% *** (0.0005)	0.249% *** (0.0004)	0.178% *** (0.0002)	0.318% *** (0.0006)	0.248% *** (0.0005)	0.329% *** (0.0007)	0.117% (0.0007)	0.415% *** (0.0009)	0.338% *** (0.0009)
# of observations	11983	3617	4586	834	1414	425	2339	1169	591

Notes: "Expected Future Opponent Rank" is the rank of the stronger player in the parallel event. That is, it is the rank of the stronger of the potential opponents in the next round. Values in parentheses are robust standard errors. ***, ** and * denote statistical significance at *p-values* of 1%, 5% and 10%, respectively. Regressions include tournament-level fixed effects.