

COORDINATION AND POLICY TRAPS*

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Abstract

This paper examines the ability of a policy maker to control equilibrium outcomes in an environment where market participants play a coordination game with information heterogeneity. We consider defense policies against speculative currency attacks. In contrast to Morris and Shin (1998), we find that policy endogeneity leads to multiple equilibria even when the “fundamentals” are observed with noise. The policy maker is willing to take a costly policy action only for moderate fundamentals. Market participants can use this information to coordinate on different responses to the same policy action, thus resulting in *policy traps*, where the devaluation outcome and the shape of the optimal policy are dictated by self fulfilling market expectations. Despite equilibrium indeterminacy, robust policy predictions can be made. The probability of devaluation is monotonic, the policy maker is “anxious to prove herself” by raising the interest rate only for a small region of moderate fundamentals, and the “anxiety region” vanishes as the information in the market becomes precise.

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As Mr. Greenspan prepares to give a critical new assessment of the monetary policy outlook in testimony to Congress on Wednesday, the central bank faces a difficult choice in grappling with the economic slowdown. If it heeds the clamour from much of Wall Street and cuts rates now ... it risks being seen as panicky, jeopardizing its reputation for policy-making competence. But if it waits until March 20, it risks allowing the economy to develop even more powerful downward momentum in what could prove a crucial three weeks. (Financial Times, February 27, 2001)

1 Introduction

Economic news anxiously concentrate on the information that different policy choices convey about the intentions of the policy maker and the underlying economic fundamentals, how markets may interpret and react to different policy measures, and whether government intervention can calm down animal spirits and ease markets to coordinate on desirable courses of actions. This paper investigates the ability of a policy maker to influence market expectations and control equilibrium outcomes in environments where market participants play a coordination game with information heterogeneity.

A large number of economic interactions are characterized by strategic complementarities, can be modeled as coordination games, and often exhibit multiple equilibria sustained by self-fulfilling expectations. Prominent examples include self-fulfilling bank runs (Diamond and Dybvig, 1983), debt crises (Calvo, 1988; Cole and Kehoe, 1996), financial crashes (Freixas and Rochet, 1997; Chari and Kehoe, 2003a,b), and currency crises (Flood and Garber, 1984; Obstfeld, 1986, 1996).¹ Building on the *global-games* work of Carlsson and Van Damme (1993), Morris and Shin (1998, 2000) have recently argued that equilibrium multiplicity in such coordination environments is merely “the unintended consequence” of assuming common knowledge of the fundamentals of the game, which implies that agents can perfectly forecast each others’ beliefs and actions in equilibrium. When instead different agents have different private information about the fundamentals, players are uncertain about others’ beliefs and actions, and thus perfect coordination is no longer possible.

¹Strategic complementarities arise also in economies with restricted market participation (Azariadis, 1981), production externalities (Benhabib and Farmer, 1994; Bryant, 1983), demand spillovers (Murphy, Shleifer and Vishny, 1989), thick-market externalities (Diamond, 1982), credit constraints (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997), incomplete risk sharing (Angeletos and Calvet, 2003) and imperfect product market competition (Milgrom and Roberts, 1990; Vives 1999). The reader can refer to Cooper (1999) for an overview of coordination games in macroeconomics.

Iterated deletion of strictly dominated strategies then eliminates all but one equilibrium outcome and one system of self-fulfilling beliefs.²

This argument has been elegantly illustrated by Morris and Shin (1998) in the context of currency crises. When it is common knowledge that a currency is sound but vulnerable to a large speculative attack, two equilibria coexist: One in which all speculators anticipate a crisis, attack the currency and cause a devaluation, which in turn confirms their expectations; another in which speculators refrain from attacking, hence vindicating their beliefs that the currency is not in danger. On the contrary, a unique equilibrium survives when speculators receive noisy idiosyncratic signals about the willingness and ability of the monetary authority to defend the currency against a speculative attack (the fundamentals). Devaluation then occurs if and only if the fundamentals fall below a critical state. This unique threshold in turn depends on all financial and policy variables affecting the payoffs of the speculators and the monetary authority. For example, raising domestic interest rates (i.e. increasing the opportunity cost of attacking the currency) reduces the set of fundamentals for which the crisis is inevitable; capital controls or defense borrowing also reduce the likelihood of devaluation. Morris and Shin hence conclude that, “in contrast to multiple equilibrium models, [their] model allows analysis of policy proposals directed at curtailing currency attacks.”

A flaw in this argument is that the policy choice conveys information about the fundamentals. When the fundamentals are so weak that the collapse is inevitable, there is no point in raising domestic interest rates or adopting other costly defense measures. Similarly, when the fundamentals are so strong that the size of the attack is minuscule and the peg faces no threat, there is no need to intervene. Therefore, whenever the bank raises the interest rate, the market can infer that the fundamentals are neither too weak nor too strong. This information may facilitate coordination on multiple courses of action in the market, thus interfering with the ability of the policy maker to fashion equilibrium outcomes.

In this paper, we analyze endogenous policy in a global coordination game. To settle in a particular environment, we add a first stage to the speculative currency attacks model of Morris and Shin (1998). The central bank moves first, setting the domestic interest rate. Speculators move second, taking a position in the exchange rate market on the basis of their idiosyncratic noisy information about the fundamentals, as well as their observation of the interest rate set by the central bank. Finally, the bank observes the fraction of speculators attacking the currency and decides whether to defend the currency or devalue.

²See Morris and Shin (2001) for an extensive overview of this literature. Earlier uniqueness results are in Postlewaite and Vives (1987) and Chamley (1999).

The main result of the paper (Theorem 1) establishes that the endogeneity of the policy leads to multiple equilibria, even when the fundamentals are observed with idiosyncratic noise. Let θ denote the fundamentals and r the interest rate. First, there is an *inactive policy equilibrium*, in which the central bank sets the same interest rate \underline{r} for all θ and speculators play the Morris-Shin equilibrium in the exchange-rate market. Second, there is a *continuum of active-policy equilibria*, in which the bank raises the interest rate at some level $r^* > \underline{r}$ only for $\theta \in [\theta^*, \theta^{**}]$, speculators play a lenient continuation equilibrium whenever $r \geq r^*$ and an aggressive one whenever $r < r^*$, and devaluation occurs if and only if $\theta < \theta^*$. Which equilibrium is played, how high is the rate r^* the bank needs to set in order to be spared from an attack, and what is the threshold θ^* below which devaluation occurs, are all determined by self-fulfilling market expectations. Our results thus manifest a kind of *policy traps*: In her attempt to use the policy so as to fashion the equilibrium outcome, the policy maker facilitates coordination in the market, brings back the possibility of multiple self-fulfilling equilibria, and finds herself trapped in a position where both the effectiveness of any particular policy action and the shape of the optimal policy are dictated by arbitrary market sentiments.

Different equilibria are sustained by different modes of coordination in the reaction of the market to the same policy choice, and such coordination is possible only because the endogeneity of the policy conveys information about the bank's willingness and ability to defend the currency (the fundamentals). As long as the value of maintaining the peg is increasing with θ and the size of an attack is decreasing with θ , the bank finds it optimal to pay the costs of high interest rates in order to defend the currency only for intermediate values of θ . The observation of an increase in the interest rate thus signals to the market that θ is neither too low nor too high, in which case multiple courses of action may be sustained in the market. In a *two-threshold* (active-policy) equilibrium, speculators expect the bank to defend the currency by raising the interest rate at some (arbitrary) level r^* whenever $\theta \in [\theta^*, \theta^{**}]$, they coordinate on not attacking if $r \geq r^*$, and they attack on the basis of their private information if $r < r^*$; as for $r < r^*$ the size of an attack is decreasing with θ , the bank then finds it optimal to raise the interest rate at r^* if and only if $\theta \in [\theta^*, \theta^{**}]$, thus confirming market expectations. In a *perfect-pooling* (inactive-policy) equilibrium, instead, speculators expect the policy to be inactive and coordinate on the same response to any level of the policy; the bank then never finds it optimal to raise the interest rate, once again vindicating market expectations.

The second result of the paper (Theorem 2) establishes that information heterogeneity significantly reduces the equilibrium set as compared to common knowledge and enables meaningful policy predictions despite the existence of multiple equilibria. In all *robust* equilibria of the game

with full-support noise, the probability of devaluation is monotonic in the fundamentals, the policy maker is “anxious to prove herself” by raising the interest rate only for a small region of moderate fundamentals, and the “anxiety region” vanishes as the information in the market becomes precise.

In the benchmark model, the policy maker faces no uncertainty about the aggressiveness of market expectations and the effectiveness of any particular policy choice. We relax this assumption in Section 5 by introducing sunspots on which speculators may condition their response to the policy. The same equilibrium interest rate may now lead to either no attack at all, a small attack that the bank can sustain, or a strong attack that forces devaluation. These results thus help make sense of the Financial Times quote: Once the policy maker has taken a costly policy choice, the market may be equally likely to “interpret” this action either as a signal of strength, in which case the most desirable outcome may be attained, or as a signal of panic, in which case the policy maker’s attempt to coordinate the market on the preferable course of action proves to be in vain.³

On a more theoretical ground, this paper represents a first attempt to introduce *signaling* in *global coordination games*. The receivers (speculators) use the signal (policy) as a coordination device to switch between lenient and aggressive continuation equilibria in the global coordination game, thus creating different incentives for the sender (policy maker) and resulting in different equilibria in the signaling game (policy traps). Our multiplicity result is thus different from the multiplicity result in standard signaling games. The latter is sustained by different systems of out-of-equilibrium beliefs and usually vanishes once proper refinements are imposed. In contrast, the multiplicity result in this paper is sustained by *endogenous coordination*. Moreover, all equilibria satisfy standard forward induction refinements, such as the *intuitive criterion* of Cho and Kreps (1987), and do not depend in any critical way on out-of-equilibrium beliefs.

Our multiplicity result is also different from the multiplicity that arises in standard global coordination games with exogenous public signals (e.g., Morris and Shin, 2001; Hellwig, 2002). The policy in our game conveys information about the fundamentals and thus represents a public signal. However, the informational content of this signal is endogenous, as it is itself the result of the self-fulfilling expectations of the market. Most importantly, our policy-traps result is not merely about the possibility of multiple continuation equilibria in the *coordination* game given any realization of the policy; it is rather about how endogenous coordination in the market makes the effectiveness of the policy depend on arbitrary market sentiments and leads to multiple equilibria in the *signaling* game.

³Moreover, these results reconcile the fact documented by Kraay (2001) and others that raising interest rates does not systematically prevent a currency crisis – a fact that prima facie contradicts the policy prediction of Morris and Shin (1998).

Finally, our policy traps are different from the *expectation traps* that arise in Kydland-Prescott/Barro-Gordon environments (e.g., Obstfeld, 1986, 1996; Chari, Christiano and Eichenbaum, 1998; Albanesi, Chari and Christiano, 2002). In these works, multiple equilibria originate in the government's lack of commitment and would disappear if the policy maker could commit to a (possibly state-contingent) policy rule. In our work, instead, equilibrium multiplicity originates in endogenous coordination and is orthogonal to the commitment problem. What is more, despite the risk of falling into a policy trap, the government need *not* have any incentive *ex ante* to commit to a particular interest rate.

The rest of the paper is organized as follows: Section 2 introduces the model and the equilibrium concept. Section 3 analyzes equilibria in the absence of uncertainty over the aggressiveness of market expectations. Section 4 examines to what extent one can make meaningful and robust policy predictions despite the presence of multiple equilibria. Section 5 introduces sunspots to examine equilibria in which the policy maker faces uncertainty over market reactions. Section 6 concludes.

2 The Model

2.1 Model Description

The economy is populated by a continuum of speculators of measure one indexed by i and uniformly distributed over the $[0, 1]$ interval. Each speculator is endowed with one unit of wealth denominated in domestic currency, which he may either invest in a domestic asset or convert to foreign currency and invest it in a foreign asset. In addition, there is a central banker (the policy maker), who controls the domestic interest rate and seeks to maintain a fixed peg, or some kind of managed exchange-rate system.⁴ We let $\theta \in \Theta$ parametrize the cost and benefits the bank associates with maintaining the peg, or equivalently her willingness and ability to defend the currency against a potential speculative attack. θ is private information to the bank and corresponds to what Morris and Shin (1998) refer to as “the fundamentals.” For simplicity, we let $\Theta = \mathbb{R}$ and the common prior shared by the speculators be a degenerate uniform.⁵

⁴We adopt the convention of using female pronouns for the central banker and masculine pronouns for the speculators.

⁵That the fundamentals of the economy coincide here with the type of the central bank is clearly just a simplification. Our results remain true if one reinterprets θ as the policy maker's perception of the fundamentals of the economy, and x_i as the signal speculator i receives about the information of the policy maker. Also, our results hold for any interval $\Theta \subseteq \mathbb{R}$ and any strictly positive and continuous density over Θ , provided that the game remains

The game has three stages. In stage 1, the central banker learns the value of maintaining the peg θ and fixes the domestic interest rate $r \in \mathcal{R} = [\underline{r}, \bar{r}]$. In stage 2, speculators choose their portfolios after observing the interest rate r set by the central bank and after receiving private signals $x_i = \theta + \varepsilon \xi_i$ about θ . The scalar $\varepsilon > 0$ parametrizes the precision of the speculators' private information about θ and ξ_i is noise, i.i.d. across speculators and independent of θ , with absolutely continuous c.d.f. Ψ and density ψ strictly positive over the entire real line (unbounded full support) or a closed interval $[-1, +1]$ (bounded support). Finally, in stage 3, the central bank observes the aggregate demand for the foreign currency and decides whether to maintain the peg. Note that stages 2 and 3 of our model correspond to the global speculative game of Morris and Shin (1998); our model reduces to theirs if r is exogenously fixed. On the other hand, if the fundamentals were common knowledge ($\varepsilon = 0$), stages 2 and 3 would correspond to the speculative game examined by Obstfeld (1986, 1996).

We normalize the foreign interest rate to zero and let the pay-off for a speculator be

$$u(r, a_i, D) = (1 - a_i)r + a_i D \pi,$$

where $a_i \in [0, 1]$ is the fraction of his wealth converted to foreign currency (equivalently, the probability that he attacks the peg), $\pi > 0$ is the devaluation premium, and D is the probability the peg is abandoned. That is, a speculator who does not to attack ($a_i = 0$) enjoys r with certainty, whereas a speculator who attacks the peg ($a_i = 1$) earns π if the peg is abandoned and zero otherwise.

We denote with α the aggregate demand for the foreign currency (equivalently, the mass of speculators who attack the peg) and let the payoff for the central bank be

$$U(r, \alpha, D, \theta) = (1 - D)V(\theta, \alpha) - C(r).$$

$V(\theta, \alpha)$ is the net value of defending the currency against an attack of size α and $C(r)$ the cost of raising the domestic interest rate at level r . V is increasing in θ and decreasing in α , C is strictly increasing in r , and both V and C are continuous.⁶

Let $\underline{\theta}$ and $\bar{\theta}$ be defined by $V(\underline{\theta}, 0) = V(\bar{\theta}, 1) = 0$. For $\theta < \underline{\theta}$ it is dominant for the bank to devalue, whereas for $\theta > \bar{\theta}$ it is dominant to maintain the peg. The interval $[\underline{\theta}, \bar{\theta}]$ thus represents the “critical range” of θ for which the peg is sound but vulnerable to a sufficiently large attack. Also, let $\underline{x} \equiv \inf \{x : \Pr(\theta < \underline{\theta}|x) < 1\}$ and $\bar{x} \equiv \sup \{x : \Pr(\theta < \bar{\theta}|x) > 0\}$; note that $\underline{x} = \underline{\theta} - \varepsilon$ and

“global.” We refer to Morris and Shin (2001) for a discussion of the role of degenerate uniform and general common priors in global coordination games.

⁶That the bank has no private information about the cost of raising the interest rate is not essential.

$\bar{x} = \bar{\theta} + \varepsilon$ when the noise ξ has bounded support $[-1, +1]$, whereas $\underline{x} = -\infty$ and $\bar{x} = +\infty$ when the noise has unbounded full support. Next, note that $\underline{r} > 0$ represents the efficient or cost-minimizing interest rate. We normalize $C(\underline{r}) = 0$ and, without any loss of generality, we let the domain of the interest rate be $\mathcal{R} = [\underline{r}, \bar{r}]$, where \bar{r} solves $C(\bar{r}) = \max_{\theta} \{V(\theta, 0) - V(\theta, 1)\}$; \bar{r} represents the maximal interest rate the bank is ever willing to set in order to deter an attack. To make things interesting, we assume $\bar{r} < \pi$, which ensures that it is too costly for the bank to raise the interest rate to totally offset the devaluation premium. Finally, to simplify the exposition, and without any loss of generality, we let $V(\theta, \alpha) = \theta - \alpha$. It follows that $\underline{\theta} = 0 = C(\underline{r})$ and $\bar{\theta} = 1 = C(\bar{r})$.

2.2 Equilibrium Definition

In the analysis that follows, we restrict attention to *perfect Bayesian equilibria* that satisfy the *intuitive criterion* refinement first introduced in Cho and Kreps (1987). As the global coordination game described in Section 2.1 is very different from the class of signaling games examined in the literature, in the next two definitions we formalize the equilibrium concept and the intuitive criterion test for our context. We also impose as a further refinement that the speculators' beliefs that $\theta > \bar{\theta}$ conditional on \underline{r} and x converge to 1 as $x \rightarrow \infty$. As it will become clear in Section 4, this refinement merely selects equilibria in which speculators do not totally ignore their private information when they observe \underline{r} . We refer to equilibria that satisfy this refinement as *robust equilibria*.

Definition 1 A *perfect Bayesian equilibrium* is a set of functions $r : \Theta \rightarrow \mathcal{R}$, $D : \Theta \times [0, 1] \times \mathcal{R} \rightarrow [0, 1]$, $a : \mathbb{R} \times \mathcal{R} \rightarrow [0, 1]$, $\alpha : \Theta \times \mathcal{R} \rightarrow [0, 1]$, and $\mu : \Theta \times \mathbb{R} \times \mathcal{R} \rightarrow [0, 1]$, such that:

$$r(\theta) \in \arg \max_{r \in \mathcal{R}} U(r, \alpha(\theta, r), D(\theta, \alpha(\theta, r), r), \theta); \quad (1)$$

$$a(x, r) \in \arg \max_{a \in [0, 1]} \int_{\Theta} u(r, a, D(\theta, \alpha(\theta, r), r)) d\mu(\theta|x, r) \text{ and } \alpha(\theta, r) = \int_{-\infty}^{+\infty} a(x, r) \psi\left(\frac{x-\theta}{\varepsilon}\right) dx; \quad (2)$$

$$D(\theta, \alpha, r) \in \arg \max_{D \in [0, 1]} U(r, \alpha, D, \theta); \quad (3)$$

$$\mu(\theta|x, r) = 0 \text{ for all } \theta \notin \Theta(x) \text{ and } \mu(\theta|x, r) \text{ satisfies Bayes' rule for any } r \in r(\Theta(x)), \quad (4)$$

where $\Theta(x) \equiv \{\theta : \psi\left(\frac{x-\theta}{\varepsilon}\right) > 0\}$ is the set of fundamentals θ consistent with signal x .

$r(\theta)$ is the policy of the bank and $D(\theta, \alpha, r)$ is the probability of devaluation. $\mu(\theta|x, r)$ is a speculator's posterior belief about θ conditional on private signal x and interest rate r , $a(x, r)$ is the speculator's position in the foreign-exchange market, and $\alpha(\theta, r)$ the associated aggregate

demand of foreign currency.⁷ Conditions (1) and (3) mean that the interest rate set in stage 1 and the devaluation decision in stage 3 are sequentially optimal for the bank, whereas condition (2) means that the portfolio choice in stage 2 is sequentially optimal for the speculators, given beliefs $\mu(\theta|x, r)$ about θ . Finally, condition (4) requires that beliefs do not assign positive measure to fundamentals θ that are not compatible with the private signals x , and are pinned down by Bayes' rule on the equilibrium path. To simplify notation, in what follows we will also refer to $D(\theta) = D(\theta, r(\theta), \alpha(\theta, r(\theta)))$ as the equilibrium devaluation probability for type θ .

Definition 2 Let $U(\theta)$ denote the equilibrium payoff of the central bank when the fundamentals are θ , and define $\Theta(r)$ as the set of θ for whom the choice r is dominated in equilibrium by $r(\theta)$ and $\mathcal{M}(r)$ as the set of posterior beliefs that assign zero measure to any $\theta \in \Theta(r)$ whenever $\Theta(r) \subset \Theta(x)$.⁸ A perfect Bayesian equilibrium satisfies the **intuitive criterion** if and only if, for any $\theta \in \Theta$ and for any $r \in \mathcal{R}$, $U(\theta) \geq U(r, \alpha(\theta, r), D(\theta, \alpha(\theta, r), r), \theta)$, for every $\alpha(\theta, r)$ satisfying (2) with $\mu \in \mathcal{M}(r)$.

In words, a perfect Bayesian equilibrium fails the intuitive criterion test when there is a type θ that would be better off by choosing an interest rate $r \neq r(\theta)$ should speculators' reaction not be sustained by beliefs that assign positive measure to types for whom r is dominated in equilibrium by $r(\theta)$.

Definition 3 A perfect Bayesian equilibrium is **robust** if

$$\mu(\theta > \bar{\theta}|x, \underline{r}) \rightarrow 1 \text{ as } x \rightarrow \infty. \quad (5)$$

When the noise is bounded, $\mu(\theta > \bar{\theta}|x, \underline{r}) = 1$ for any $x \geq \bar{x} = \bar{\theta} + \varepsilon$ and therefore this refinement is redundant. When instead the noise is unbounded, (5) fails if and only if $r(\theta) > \underline{r}$ for (almost) every $\theta > \bar{\theta}$. This refinement then selects equilibria that are robust to perturbations of the strategies, or the environment, for which the bank occasionally fails to raise the interest rate when the fundamentals are very strong.

2.3 Model Discussion

The particular pay-off structure and many of the institutional details of the specific coordination environment we consider in this paper are not essential for our results. For example, the cost of the

⁷That $\alpha(\theta, r) = \int_{-\infty}^{+\infty} a(x, r) \psi\left(\frac{x-\theta}{\varepsilon}\right) dx$ follows from the Law of Large Numbers when there are countable infinitely many agents; with a continuum, see Judd (1985).

⁸Formally, $\Theta(r) \equiv \{\theta \text{ such that } U(\theta) > U(r, \alpha(\theta, r), D(\theta, \alpha(\theta, r), r), \theta) \text{ for any } \alpha(\theta, r) \text{ and } \mu \text{ satisfying (2) and (4)}\}$ and $\mathcal{M}(r) \equiv \{\mu \text{ satisfying (4) and such that } \mu(\theta|x, r) = 0 \text{ for any } \theta \in \Theta(r) \text{ if } \Theta(r) \subset \Theta(x)\}$.

interest rate can be read as the cost of implementing a particular policy; this may depend also on the fundamentals of the economy, as well as the aggregate response of the market. And the value of maintaining the peg represents the value of coordinating the market on a particular action; this may depend, not only on the underlying fundamental and the reaction of the market, but also on the policy itself.

Similarly, that stage 3 is strategic is not essential. Given the payoff function of the central bank, the devaluation policy is sequentially optimal if and only if $D(\theta, r, \alpha) = 1$ whenever $V(\theta, \alpha) < 0$ and $D(\theta, r, \alpha) = 0$ whenever $V(\theta, \alpha) > 0$, for any r . All our results would hold true if the currency were exogenously devalued for any θ and α such that $V(\theta, \alpha) \leq 0$, as in the case the bank has no choice but to abandon the peg whenever the aggregate demand of foreign currency α exceeds the amount of foreign reserves θ . Indeed, although described as a three-stage game, the model essentially reduces to a two-stage game, where in the first stage the policy maker signals information about θ to the market and in the second stage speculators play a global coordination game in response to the action of the policy maker. Stage 3 serves only to introduce strategic complementarities in the actions of market participants. If instead the bank could commit never to devalue in stage 3, there would be no speculation in stage 2, and the market would not play a coordination game. Such lack of commitment is essential for the literature on *policy discretion* and *expectation traps*, but not for our results. What is essential for the kind of policy traps we identify in this paper is that the game the market plays in response to the action of the policy maker is a global coordination game; the origin of strategic complementarities is irrelevant.

In short, we suggest that our model may well fit a broader class of environments in which a stage of endogenous information transmission (signaling) is followed by a global coordination game, such as, for example, in the case of a government offering an investment subsidy in an attempt to stimulate the adoption of a new technology, or a telecommunications company undertaking an aggressive advertising campaign in attempt to persuade consumers to adopt her service.

3 Policy Traps

3.1 Exogenous versus Endogenous Policy

Suppose for a moment that the interest rate r is exogenously fixed, say $r = \underline{r}$. Our model then reduces to the model of Morris and Shin (1998). The lack of common knowledge over the fundamentals eliminates any possibility of coordination and iterated deletion of strongly dominated strategies selects a unique equilibrium profile and a unique system of equilibrium beliefs.

Proposition 1 (Morris-Shin) *In the speculative game with exogenous interest rate policy, there exists a unique perfect Bayesian equilibrium, in which a speculator attacks if and only if $x < x_{MS}$ and the bank devalues if and only if $\theta < \theta_{MS}$. The thresholds x_{MS} and θ_{MS} are defined by*

$$\theta_{MS} = \frac{\pi - \underline{r}}{\pi} = \Psi\left(\frac{x_{MS} - \theta_{MS}}{\varepsilon}\right) \quad (6)$$

and are decreasing in \underline{r} .

Proof. Uniqueness is established in Morris and Shin (1998). Here, we only characterize x_{MS} and θ_{MS} in our setting. Given that $D(\theta) = 1$ if $\theta < \theta_{MS}$ and $D(\theta) = 0$ otherwise, a speculator with signal x finds it optimal not to attack if and only if $\pi\mu(\theta < \theta_{MS}|x) \leq \underline{r}$, where $\mu(\theta < \theta_{MS}|x) = \int_{\theta < \theta_{MS}} d\mu(\theta|x)$. By Bayes rule, $\mu(\theta < \theta_{MS}|x) = 1 - \Psi((x - \theta_{MS})/\varepsilon)$. Thus, x_{MS} solves the indifference condition $1 - \Psi\left(\frac{x_{MS} - \theta_{MS}}{\varepsilon}\right) = \underline{r}/\pi$. The fraction of speculators attacking is then $\alpha(\theta, \underline{r}) = \Pr(x < x_{MS}|\theta) = \Psi\left(\frac{x_{MS} - \theta}{\varepsilon}\right)$. It follows that $V(\theta, \alpha(\theta, \underline{r})) < 0$, and hence the bank devalues, if and only if $\theta < \theta_{MS}$, where θ_{MS} solves $V(\theta_{MS}, \alpha(\theta_{MS}, \underline{r})) = 0$, i.e., $\theta_{MS} = \Psi\left(\frac{x_{MS} - \theta_{MS}}{\varepsilon}\right)$. Combining the indifference condition for the speculators with that for the central bank gives (6). It is then immediate that x_{MS} and θ_{MS} are both decreasing in \underline{r} . ■

The larger the return on the domestic asset, or the higher the cost of short-selling the domestic currency, the less attractive it is for a speculator to take the risk of attacking the currency. It follows that x_{MS} and θ_{MS} are decreasing functions of \underline{r} , which suggests that the monetary authority should be able to reduce the likelihood and severity of a currency crisis by simply raising the domestic interest rate, or more generally reducing the speculators' ability and incentives to attack.

Indeed, that would be the end of the story if the policy did not convey any information to the market. However, since raising the interest rate is costly, a high interest rate signals that the bank is willing to defend the currency and hence that the fundamentals are not too weak. On the other hand, as long as speculators do not attack when their private signal is sufficiently high, the bank faces only a small attack when the fundamentals are sufficiently strong, in which case there is no need to raise the interest rate. Therefore, any attempt to defend the peg by increasing the interest rate is interpreted by the market as a signal of intermediate fundamentals,⁹ in which case speculators may coordinate on either an aggressive or a lenient course of action. Different modes of coordination then create different incentives for the policy maker and result in different equilibria.

⁹This interpretation of the information conveyed by a high interest rate follows from Bayes' rule if the observed level of the interest rate is on the equilibrium path, and from forward induction (the intuitive criterion) otherwise.

At the end, which equilibrium is played, how high is the level of the policy the bank needs to set in order to be spared from a crisis, and what is the critical value of the fundamental below which a devaluation occurs, are all determined by self-fulfilling market expectations.

Theorem 1 (Policy Traps) *In the speculative game with endogenous policy, there exist multiple perfect Bayesian equilibria for any $\varepsilon > 0$.*

(a) *There is an **inactive-policy equilibrium**: The bank sets the cost-minimizing interest rate \underline{r} for all θ and devaluation occurs if and only if $\theta < \theta_{MS}$.*

(b) *There is a continuum of **active-policy equilibria**: Let \tilde{r} solve $C(\tilde{r}) = \frac{\pi - \tilde{r}}{\pi}$. For any $r^* \in [\underline{r}, \tilde{r}]$, there is an equilibrium in which the bank sets either \underline{r} or r^* , raises the interest rate at r^* only for $\theta \in [\theta^*, \theta^{**}]$, and devalues if and only if $\theta < \theta^*$, where*

$$\theta^* = C(r^*) \quad \text{and} \quad \theta^{**} = \theta^* + \varepsilon \left[\Psi^{-1} \left(1 - \frac{r}{\pi - \underline{r}} \theta^* \right) - \Psi^{-1}(\theta^*) \right]. \quad (7)$$

The threshold θ^ is independent of ε and can take any value in $(\underline{\theta}, \theta_{MS}]$, whereas the threshold θ^{**} is decreasing in ε and converges to θ^* as $\varepsilon \rightarrow 0$.*

All the above equilibria satisfy the intuitive criterion, are robust in the sense of (5), and can be supported by strategies for the speculators that are monotonic in both x and r .

The proof of Theorem 1 follows from Propositions 2 and 3, which we present in the next two sections. Theorem 1 states that there is a continuum of equilibria, which contrasts sharply with the uniqueness result and the policy conjecture of Morris and Shin. The central bank is subject to *policy traps*: In her attempt to use the interest rate so as to fashion the size of a currency attack, the monetary authority reveals information that the fundamentals are neither too weak nor too strong. This information facilitates coordination in the market, brings back the possibility of multiple self-fulfilling equilibria, and leads to a situation where the effectiveness of any particular policy (the eventual devaluation outcome) and the shape of the optimal policy are dictated by the arbitrary aggressiveness of market expectations. In the *inactive-policy equilibrium*, speculators expect the bank never to raise the interest rate and play the same continuation equilibrium (attacking if and only if $x < x_{MS}$) for any r . Anticipating this, the bank finds it pointless to raise the interest rate, which in turn vindicates market expectations. In an *active-policy equilibrium* instead, speculators expect the bank to raise the interest rate at r^* only for $\theta \in [\theta^*, \theta^{**}]$, coordinate on an aggressive response for any $r < r^*$ and on a lenient one for any $r \geq r^*$. Again, the bank can do no better than simply conforming to the arbitrary self-fulfilling expectations of the market.

It is worth noting that all equilibria in Theorem 1 can be supported by simple threshold strategies for the speculators that are monotonic in x . That is, given any policy choice r , a

speculator attacks the currency if and only his private signal about the fundamentals falls below a threshold which is dictated by self-fulfilling market expectations.

Finally, note that all active-policy equilibria in Theorem 1 share the following properties. First, the devaluation outcome is monotonic in the fundamentals. Second, the devaluation threshold is determined by self-fulfilling market expectations, but never exceeds the devaluation threshold that would prevail under policy inaction. Third, when the fundamentals are either very weak, or sufficiently strong, the market can easily recognize this, in which case there is no value for the policy maker to raise the interest rate; it is then only for a small range of moderate fundamentals that the market is likely to be “uncertain” or “confused” about eventual devaluation outcomes, and it is thus only for moderate fundamentals that the policy maker is “anxious to prove herself” by taking a costly policy action. Fourth, the “anxiety region” shrinks as the precision of market information increases. In Section 4, we further discuss the *robustness* of these policy predictions.

3.2 Inactive Policy Equilibrium

When the interest rate is endogenous, the Morris-Shin outcome survives as an inactive policy equilibrium, in which the interest rate remains uninformative about the probability the currency will be devalued and speculators do not condition their behavior on the interest rate.

Proposition 2 (Perfect Pooling) *There is an inactive policy equilibrium, in which the central bank sets \underline{r} for all θ , speculators attack if and only if $x < x_{MS}$, independently of the interest rate chosen by the central bank, and devaluation occurs if and only if $\theta < \theta_{MS}$.*

Proof. Since in equilibrium all θ set $r = \underline{r}$, the observation of \underline{r} conveys no information about the fundamentals θ and hence the continuation game starting after the bank sets $r = \underline{r}$ is isomorphic to the Morris-Shin game: There is a unique continuation equilibrium, in which a speculator attacks if and only if $x < x_{MS}$ and the central bank devalues if and only if $\theta < \theta_{MS}$. Equilibrium beliefs are then pinned down by Bayes’ rule and hence satisfy (5).

Next, consider out-of-equilibrium interest rates. Note that \tilde{r} solves $C(\tilde{r}) = V(\theta_{MS}, 0)$. Any $r > \tilde{r}$ is dominated in equilibrium by \underline{r} for all types: For $\theta < \theta_{MS}$, $V(\theta, 0) - C(r) < 0$; for $\theta \geq \theta_{MS}$, $C(r) > \theta_{MS} > \alpha(\theta, \underline{r})$ and thus $V(\theta, 0) - C(r) < V(\theta, \alpha(\theta, \underline{r}))$. On the other hand, any $r \in (\underline{r}, \tilde{r})$ is dominated in equilibrium by $r(\theta)$ for any θ such that $\theta < C(r)$ or $\alpha(\theta, \underline{r}) < C(r)$, where $C(r) \leq C(\tilde{r}) = \theta_{MS}$. Therefore, for any out-of-equilibrium $r \neq \tilde{r}$, one can construct out-of-equilibrium beliefs μ that satisfy (4) and the intuitive criterion, and such that $\mu(\theta < \theta_{MS} | x, r)$ is non-increasing in x and satisfies $\pi\mu(\theta < \theta_{MS} | x, r) = r$ at $x = x_{MS}$. For such beliefs, a speculator

attacks if and only if $x < x_{MS}$, forcing devaluation to occur if and only if $\theta < \theta_{MS}$. Finally, for $r = \tilde{r}$, let $\mu(\theta_{MS}|x, \tilde{r}) = 1$ for all x such that $\theta_{MS} \in \Theta(x)$, and $\mu(\theta|x, \tilde{r}) = \mu(\theta|x)$ otherwise, so that again (4) and the intuitive criterion are satisfied. Then, there is a mixed-strategy equilibrium for the continuation game following $r = \tilde{r}$, in which a speculator attacks if and only if $x < x_{MS}$ and type θ_{MS} devalues with probability \tilde{r}/π .

Given that speculators attack if and only if $x < x_{MS}$ for any r , it is optimal for the central bank to set $r(\theta) = \underline{r}$ for all θ . ■

The inactive policy equilibrium is illustrated in Figure 1. θ is on the horizontal axis, α and C on the vertical one. The devaluation threshold θ_{MS} is the point of intersection between the value of the peg θ and the size of the attack $\alpha(\theta, \underline{r})$. Figure 1 also illustrates the intuitive criterion. Note that the equilibrium payoff is $U(\theta) = 0$ for all $\theta \leq \theta_{MS}$ and $U(\theta) = \theta - \alpha(\theta, \underline{r}) > 0$ for all $\theta > \theta_{MS}$. Consider a deviation to some $r' \in (\underline{r}, \tilde{r})$ and let θ' and θ'' solve $\theta' = C(r') = \alpha(\theta'', \underline{r})$. Note that $C(r') > \theta$ if and only if $\theta < \theta'$ and $C(r') > \alpha(\theta, \underline{r})$ if and only if $\theta > \theta''$, which implies that r' is dominated in equilibrium by \underline{r} if and only if $\theta \notin [\theta', \theta'']$. Hence, if $\theta \in [\theta', \theta'']$ and the bank deviates to r' , the market “learns” that $\theta \in [\theta', \theta'']$.¹⁰ Furthermore, since $\theta_{MS} \in [\theta', \theta'']$, one can construct beliefs for which speculators continue to attack whenever $x < x_{MS}$, in which case it is pointless for the bank to raise the interest rate at r' .

Insert Figure 1 here

Clearly, any system of beliefs and strategies such that $\alpha(\theta, r) \geq \alpha(\theta, \underline{r})$ for all $r > \underline{r}$ sustains policy inaction as an equilibrium; the bank has then no choice but to set $r(\theta) = \underline{r}$ for all θ , confirming the expectations of the market. Note that a higher interest rate increases the opportunity cost of attacking the currency and, other things equal, reduces the speculators’ incentives to attack. In an inactive-policy equilibrium, however, this portfolio effect is offset by the higher probability speculators attach to a final devaluation. The particular beliefs we consider in the proof of Proposition 2 have the property that these two effects *just* offset each other, in which case speculators use the same strategy on and off the equilibrium path and condition their behavior *only* on their private information. It is then as if speculators do not pay attention to the policy of the bank.¹¹

¹⁰Throughout the paper, the expression “the market learns X by observing Y ” means that the event X becomes *common* $p = 1$ *belief* among the speculators given that Y is common knowledge; see Monderer and Samet (1989) and Kajii and Morris (1995).

¹¹Moreover, speculators do not need to “learn” how to behave off the equilibrium path; they simply continue to play the same strategy they “learned” to play in equilibrium.

3.3 Active Policy Equilibria

We now prove the existence of active-policy equilibria in which the bank raises the interest rate at r^* for every $\theta \in [\theta^*, \theta^{**}]$.

Proposition 3 (Two-Threshold Equilibria) *For any $r^* \in (\underline{r}, \tilde{r}]$, there is a two-threshold equilibrium in which the central bank sets r^* for all $\theta \in [\theta^*, \theta^{**}]$ and \underline{r} otherwise; speculators attack if and only if either $r < r^*$ and $x < x^*$, or $r \geq r^*$ and $x < \underline{x}$; finally, devaluation occurs if and only if $\theta < \theta^*$. The threshold x^* solves $\pi\mu(\theta < \theta^* | x^*, \underline{r}) = \underline{r}$, whereas θ^* and θ^{**} are given by (7). A two-threshold equilibrium exists if and only if $r^* \in (\underline{r}, \tilde{r}]$ or, equivalently, if and only if $\theta^* \in (\underline{\theta}, \theta_{MS}]$.*

Proof. Let $\hat{\theta} \in [\theta^*, \theta^{**}]$ solve $V(\hat{\theta}, \alpha(\hat{\theta}, \underline{r})) = 0$, where $\alpha(\hat{\theta}, \underline{r}) = \Psi\left(\frac{x^* - \hat{\theta}}{\varepsilon}\right)$, and note that, for any $r < r^*$, speculators trigger devaluation if and only if $\theta < \hat{\theta}$.

Consider first the behavior of the speculators. When $r = \underline{r}$, beliefs are pinned down by Bayes' rule for all x , since (7) ensures $\Theta(x) \not\subseteq [\theta^*, \theta^{**}]$ for all x .¹² Therefore,

$$\mu(\theta < \hat{\theta} | x, \underline{r}) = \mu(\theta < \theta^* | x, \underline{r}) = \frac{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right)}{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right) + \Psi\left(\frac{x - \theta^{**}}{\varepsilon}\right)},$$

which is decreasing in x . We let x^* be the unique solution to $\mu(\theta < \theta^* | x^*, \underline{r}) = \underline{r}/\pi$. For any $r \in (\underline{r}, r^*)$, we consider out-of-equilibrium beliefs μ such that $\mu(\theta < \hat{\theta} | x, r)$ is non-increasing in x and $\pi\mu(\theta < \hat{\theta} | x, r) = r$ at $x = x^*$. Furthermore, if for some $\theta \in \Theta(x)$, r is not dominated in equilibrium by $r(\theta)$, i.e., if $C(r) \leq \min\{\alpha(\theta, \underline{r}), \theta\}$, then we further restrict μ to satisfy $\mu(\theta | x, r) = 0$ for all $\theta \in \Theta(x)$ such that $\theta < C(r)$ or $\alpha(\theta, \underline{r}) < C(r)$. When $r = r^*$, Bayes' rule implies $\mu(\theta \in [\theta^*, \theta^{**}] | x, r^*) = 1$ for all x such that $\Theta(x) \cap [\theta^*, \theta^{**}] \neq \emptyset$. Finally, for any (x, r) such that either $r = r^*$ and $\Theta(x) \cap [\theta^*, \theta^{**}] = \emptyset$, or $r > r^*$, we let $\mu(\theta \geq \underline{\theta} | x, r) = 1$ for all $x \geq \underline{x}$ and $\mu(\theta < \underline{\theta} | x, r) = 0$ otherwise. Given these beliefs, (4) and (5) are satisfied, $\mu \in \mathcal{M}(r)$ for any r , and the strategy of the speculators is sequentially optimal.

Consider next the central bank. Given the strategy of the speculators, the bank prefers \underline{r} to any $r \in (\underline{r}, r^*)$ and r^* to any $r > r^*$. θ^* is indifferent between setting r^* (and not being attacked) and setting \underline{r} (and being forced to devalue) if and only if $V(\theta^*, 0) - C(r^*) = 0$, i.e. $\theta^* = C(r^*)$. Similarly, θ^{**} is indifferent between setting r^* and not be attacked and setting \underline{r} and be attacked without devaluing if only if $V(\theta^{**}, 0) - C(r^*) = V(\theta^{**}, \alpha(\theta^{**}, \underline{r}))$, i.e. $C(r^*) = \alpha(\theta^{**}, \underline{r})$, where

¹²When the noise is unbounded, this is immediate; when it is bounded, it follows from the fact that $|\theta^{**} - \theta^*| < 2\varepsilon$ and $\Theta(x) = [x - \varepsilon, x + \varepsilon]$, for any x .

$\alpha(\theta^{**}, \underline{r}) = \Psi\left(\frac{x^* - \theta^{**}}{\varepsilon}\right)$. For any $\theta < \theta^*$, \underline{r} is optimal; for any $\theta \in (\theta^*, \theta^{**})$, $\alpha(\theta, \underline{r}) > \alpha(\theta^{**}, \underline{r}) = C(r^*)$ and thus r^* is preferred to \underline{r} ; and the reverse is true for any $\theta > \theta^{**}$.

Finally, from the indifference conditions for θ^* , θ^{**} , and x^* , we obtain $x^* = \theta^{**} + \varepsilon\Psi^{-1}(\theta^*)$ and

$$\theta^{**} = \theta^* + \varepsilon \left[\Psi^{-1} \left(1 - \frac{\underline{r}}{\pi - \underline{r}} \theta^* \right) - \Psi^{-1}(\theta^*) \right].$$

It follows that $\theta^* \leq \theta^{**}$ if and only if $\theta^* \leq (\pi - \underline{r})/\pi = \theta_{MS}$. Using $\theta^* = C(r^*)$ and $\theta_{MS} = C(\tilde{r})$, we conclude that a two threshold equilibrium exists if and only if $\theta^* \in (\underline{\theta}, \theta_{MS}]$, or equivalently $r^* \in (\underline{r}, \tilde{r}]$. ■

A two-threshold equilibrium is illustrated in Figure 2. Like in the inactive policy equilibrium, the observation of any interest rate $r > \underline{r}$ is interpreted by market participants as a signal of intermediate fundamentals. However, contrary to the inactive policy equilibrium, speculators switch from playing aggressively (attacking if and only if $x < x^*$) to playing leniently (attacking if and only if $x < \underline{x}$) whenever the policy meets market expectations ($r \geq r^*$). Anticipating this reaction by market participants, the bank finds it optimal to raise the interest rate to defend the currency whenever the fundamentals are strong enough that the net value of defending the currency offsets the cost of raising the interest rate ($\theta \geq \theta^*$), but not so strong that the cost of facing a small and unsuccessful attack is lower than the cost of raising the interest rate ($\theta \leq \theta^{**}$). The thresholds θ^* and θ^{**} are determined by the indifference conditions $\theta^* = C(r^*)$ and $\alpha(\theta^{**}, \underline{r}) = C(r^*)$, as illustrated in Figure 2. Finally, note that, in any two-threshold equilibrium, the exact fundamentals θ never become common knowledge among speculators. What the market “learns” from equilibrium policy observations is only whether $\theta \in [\theta^*, \theta^{**}]$ or $\theta \notin [\theta^*, \theta^{**}]$, but this is enough to facilitate coordination.

Insert Figure 2 here

There are other out-of-equilibrium beliefs and strategies for the speculators that also sustain the same two-threshold equilibria. For example, we could have assumed speculators coordinate on the *most* aggressive continuation equilibrium (attack if and only if $x < \bar{x}$) whenever the policy falls short of market expectations ($\underline{r} < r < r^*$). Nonetheless, the beliefs and strategies we consider in the proof of Proposition 3 have the appealing property that the speculators’ strategy is the same for all $r < r^*$. It is then as if speculators can simply “ignore” any attempt of the policy maker that falls short of market expectations and continue to play exactly as if there had been no intervention.

Note that a two-threshold equilibrium exists if and only if $r^* \leq \tilde{r}$, or equivalently $\theta^* \leq \theta_{MS}$. In the next section, we will establish that this property extends to *all* robust equilibria. Here, we discuss the intuition behind this result. From the bank’s indifference conditions, $\theta^* = C(r^*) =$

$\alpha(\theta^{**}, \underline{r})$), we infer that a higher r^* raises both the devaluation threshold θ^* and the size of the attack at θ^{**} . The latter is given by $\Pr(x \leq x^* | \theta^{**})$, which is also equal to $\Pr(\theta \geq \theta^{**} | x^*)$. Therefore, $\Pr(\theta \geq \theta^{**} | x^*)$ increases with r^* . For a marginal speculator to be indifferent between attacking and non attacking conditional on $r = \underline{r}$, it must be that $\Pr(\theta \leq \theta^* | x^*)$ also increases with r^* . It follows that $\Delta\theta(r^*) \equiv \theta^{**} - \theta^*$ is decreasing in r^* . Obviously, $\Delta\theta(r^*)$ is also continuous in r^* . A two-threshold equilibrium exists if and only if $\Delta\theta(r^*) \geq 0$. By the monotonicity of $\Delta\theta(r^*)$, there exists at most one \tilde{r} such that $\Delta\theta(\tilde{r}) = 0$, and $\Delta\theta(r^*) > 0$ if and only if $r^* < \tilde{r}$. But $\Delta\theta(\tilde{r}) = 0$ if and only if $\theta^* = \theta^{**}$, in which case the continuation game following \underline{r} is (essentially) the Morris-Shin game and therefore $\theta^* = \theta^{**} = \theta_{MS}$. It follows that \tilde{r} solves $C(\tilde{r}) = V(\theta_{MS}, 0) = \frac{\pi - \underline{r}}{\pi}$ and a two-threshold equilibrium exists if and only if $r^* \in (\underline{r}, \tilde{r}]$, or equivalently, if and only if $\theta^* \in (\underline{\theta}, \theta_{MS}]$.

Finally, note that, for any r^* , the devaluation threshold θ^* is independent of ε , but θ^{**} is decreasing in ε and $\theta^{**} \rightarrow \theta^*$ as $\varepsilon \rightarrow 0$. Therefore, as private information becomes more precise, the bank raises the interest rate only for a smaller measure of fundamentals. At the limit, the interest rate policy has a spike at an arbitrary devaluation threshold $\theta^* \in (\underline{\theta}, \theta_{MS})$ dictated by market expectations. In Section 4, we will examine under what conditions this limit property extends to all robust equilibria of the game.

3.4 Discussion

We conclude this section with a few remarks about the role of coordination, signaling, and commitment in our environment.

First, consider environments where the market does *not* play a coordination game in response to the policy maker, or where there is a single speculator (receiver) playing against the bank (sender).¹³ In such environments, the policy could be non-monotonic as long as the receivers have access to outside information regarding the type of the sender (Feltovich, Harbaugh and To, 2002). Moreover, multiple equilibria could possibly be supported by different out-of-equilibrium beliefs. However, a *unique* equilibrium would typically survive the intuitive criterion or other proper refinements of out-of-equilibrium beliefs. To the contrary, the multiplicity we have identified in this paper does not depend in any critical way on the specification of out-of-equilibrium beliefs, originates merely in endogenous coordination, and would not arise in environments with a single receiver. The comparison is even sharper if we consider $\varepsilon \rightarrow 0$. The limit of all equilibria of the game with a single receiver would have the latter attacking the currency if and only if $x < \bar{\theta}$, and the bank devaluing if and only if $\theta < \bar{\theta}$. Contrast this with our findings in Theorem 1 (and in Corollary 1

¹³See, for example, Drazen (2001).

below), where any devaluation threshold $\theta^* \in (\underline{\theta}, \theta_{MS}]$ can be sustained as $\varepsilon \rightarrow 0$. We conclude that the policy traps (multiple signaling equilibria) identified in this paper are absent when market participants (receivers) do not play a coordination game against the policy maker (sender).

Second, consider environments where there is no signaling, such as standard global coordination games. As shown in Morris and Shin (2001) and Hellwig (2002), these games may exhibit multiple equilibria if market participants observe sufficiently informative *public* signals about the underlying fundamentals. The multiplicity of equilibria documented in Theorems 1 and 2, however, is substantially different from the kind of multiplicity in that literature. The policy in our model does generate a public signal about the fundamentals. Yet, the informational content of this signal is endogenous, as it depends on the particular equilibrium on which the market coordinates. In addition, endogenous policy introduces a very specific kind of public signal: The observation of active policy necessarily reveals that the fundamentals are neither too weak nor too strong, and it is this particular kind of information that is essential for restoring the ability of the market to coordinate on different courses of action. Finally, Theorem 1 is not about the possibility of multiple (continuation) equilibria in the coordination game that follows a given realization of the public signal; it is rather about how endogenous coordination in the market makes the effectiveness of the policy depend on arbitrary market sentiments and leads to multiple equilibria in the signaling game.

Lastly, consider the role of commitment. Note that policy traps arise in our environment *because* the policy maker moves first, thus revealing valuable information about the fundamentals that market participants use to coordinate their response to the policy choice. This raises the question of whether the policy maker would be better off committing to a certain level of the policy before observing θ , thus inducing a unique continuation equilibrium in the coordination game. To see that commitment is not necessarily optimal, suppose the noise ξ has full support and consider $\varepsilon \rightarrow 0$. If the policy maker commits ex ante to some interest rate r , she incurs a cost $C(r)$ and ensures that devaluation will occur if and only if $\theta < \hat{\theta}(r) \equiv \frac{\pi-r}{\pi}$; moreover, for all $\theta > \hat{\theta}(r)$, $\alpha(\theta, r) \rightarrow 0$ as $\varepsilon \rightarrow 0$.¹⁴ Hence, the ex ante payoff from committing to r is $U(r) = \Pr(\theta \geq \hat{\theta}(r))\mathbb{E}[\theta | \theta \geq \hat{\theta}(r)] - C(r)$. Let $U_c = \max_r U(r)$ and $\theta_c = \min\{\hat{\theta}(r_c) | r_c \in \arg \max_r U(r)\}$. If instead the policy maker retains the option to fashion the policy contingent on θ , the ex ante payoff depends on the particular equilibrium $(r^*, \theta^*, \theta^{**})$ the policy maker expects to be played. As $\varepsilon \rightarrow 0$, $\theta^{**} \rightarrow \theta^*$ and $\alpha(\theta, \underline{r}) \rightarrow 0$ for all $\theta \geq \theta^{**}$, meaning the bank pays $C(r^*)$ only for a negligible measure of θ . It follows that the ex ante value of discretion is $U_d = \Pr(\theta \geq \theta^*)\mathbb{E}[\theta | \theta \geq \theta^*]$. But note that $\theta_c > \underline{\theta}$, and therefore any

¹⁴These results follow from Proposition 1, replacing \underline{r} with an arbitrary r .

$\theta^* \in (\underline{\theta}, \theta_c)$ necessarily leads to $U_d > U_c$. A similar argument holds for arbitrary ε . We conclude that, even when perfect commitment is possible, the government will prefer discretion as long as she is not too pessimistic about future market sentiments.

4 Robust Policy Predictions

Propositions 2 and 3 left open the possibility that there also exist other equilibria outside the two classes of Theorem 1, which would only strengthen our argument that policy endogeneity facilitates coordination and leads to policy traps. Nonetheless, we are also interested in understanding whether and under what conditions these are the only equilibria of the policy game, in which case information heterogeneity significantly refines the equilibrium set as compared to common knowledge and permits us to make robust policy predictions.

We first note that, when the noise has unbounded full support, there is also a continuum of *one-threshold equilibria*: For any $r^* \in (\underline{r}, \bar{r}]$, there exists a perfect Bayesian equilibrium in which speculators threaten to ignore their private information and unconditionally attack the currency whenever they observe any $r < r^*$, thus forcing the bank to raise the interest rate at r^* for all θ above the devaluation threshold $\theta^* \in [\underline{\theta}, \bar{\theta}]$, where $V(\theta^*, 0) = C(r^*)$.¹⁵ These equilibria, however, are very fragile. They require beliefs such that $\mu(\theta > \bar{\theta} | x, r) = 0$ for all x whenever $r < r^*$, meaning that speculators are perfectly confident that (almost) no $\theta > \bar{\theta}$ ever sets any $r < r^*$, for it is only then that the speculators may ignore their private information. These equilibria are therefore not robust to perturbations that force speculators to use their private signal, such as when the bank fails to raise the interest rate for good fundamentals even with arbitrarily small probability.¹⁶

It is these considerations that motivated the equilibrium refinement in Definition 3. Any one-threshold equilibrium necessarily violates (5) and hence is not robust. On the other hand, any perfect Bayesian equilibrium in which a positive measure of $\theta > \bar{\theta}$ sets \underline{r} automatically satisfies (5). Therefore, (5) merely selects equilibria that do not rely on speculators interpreting \underline{r} as a *perfect* signal of devaluation and *totally* ignoring their private information when they attack. The following then provides a partial converse to Theorem 1.

Theorem 2 (Robust Equilibria) *Suppose the noise ξ has unbounded full support. Then, every robust perfect Bayesian equilibrium belongs to one of the two classes of Theorem 1. If, in addition,*

¹⁵The proof is available upon request.

¹⁶Moreover, these equilibria can not exist when the idiosyncratic noise is bounded, for then $\mu(\theta > \bar{\theta} | x, r) = 1$ for all $x > \bar{x} = \bar{\theta} + \varepsilon$ and it is dominant for each speculator not to attack whenever $x > \bar{x}$, in which case $\alpha(\theta, r) = 0$ for all $\theta > \bar{x} + \varepsilon$ and thus no $\theta > \bar{x} + \varepsilon$ ever raises the interest rate.

the distribution of the noise satisfies the monotone likelihood ratio property, then any active-policy equilibrium is a two-threshold equilibrium as in Proposition 3.

Recall that in a pooling equilibrium r is dominated by \underline{r} for every θ if and only if $r > \tilde{r}$. In other words, \tilde{r} represents the maximal interest rate that the bank would ever be tempted to deviate to in the inactive-policy equilibrium. Following Theorems 1 and 2, the policy in *any* robust equilibrium is bounded above by \tilde{r} . Equivalently, the devaluation threshold is bounded above by the inactive-policy threshold θ_{MS} . Similarly, the ex ante probability of devaluation and the ex post size of an attack are lower in any robust active-policy equilibrium than in the inactive-policy equilibrium.

We sketch the intuition for Theorem 2 here and present the formal proof in the Appendix. Consider any perfect Bayesian equilibrium of the game with unbounded full support noise. Since there is no aggregate uncertainty, the central bank can perfectly anticipate whether she will devalue when she fixes the interest rate. Hence, the bank will be willing to pay the cost of an interest rate above \underline{r} only if she anticipates not to devalue. Given this, and given that the noise has full support, the observation of an equilibrium interest rate above \underline{r} necessarily signals that there will be no devaluation. Speculators therefore never attack when they observe any equilibrium interest rate above \underline{r} . Since $C(r)$ is strictly increasing, at most one interest rate above \underline{r} may be played in any equilibrium. Let r^* denote this interest rate and define θ^* and θ^{**} as in Theorem 1. Next, let

$$\theta' = \inf\{\theta : r(\theta) = r^*\} \quad \text{and} \quad \theta'' = \sup\{\theta : r(\theta) = r^*\}$$

be, respectively, the lowest and highest type who raise the interest rate in any given equilibrium in which r^* is played. Obviously, it never pays to raise the interest rate for any $\theta \leq \theta^*$. Therefore, $\theta'' \geq \theta' \geq \theta^*$. If $\theta'' < \infty$, then (5) is automatically satisfied. Conversely, if (5) holds, it is dominant for speculators not to attack for sufficiently high x , implying that $\alpha(\theta, \underline{r}) \rightarrow 0$ as $\theta \rightarrow \infty$ and therefore the bank prefers \underline{r} to r^* for sufficiently high θ ; that is, $\theta'' < \infty$. Compare now the strategy of the speculators in any such equilibrium with the strategy in the corresponding two-threshold equilibrium. Any $\theta \geq \theta^*$ necessarily does not devalue as it can guarantee herself a positive payoff by setting $r = r^*$. If there also exist types $\theta < \theta^*$ who do not devalue, then the incentives to attack when observing \underline{r} are lower than when *all* $\theta < \theta^*$ devalue. Similarly, if there exist types $\theta \in [\theta^*, \theta'']$ who do not raise the interest rate at r^* , then the observation of $r = \underline{r}$ is less informative of devaluation than in the case where *all* $\theta \in [\theta^*, \theta'']$ set r^* . Hence, speculators are most aggressive at \underline{r} when $D(\theta) = 1$ for all $\theta < \theta^*$ and $r(\theta) = r^*$ for all $[\theta^*, \theta'']$, which is possible if and only if $\theta'' = \theta^{**}$. Equivalently, the size of the attack in the two-threshold equilibrium corresponding to r^* represents an upper bound on the size of the attack in any active-policy equilibrium in which r^* is

played. It follows that $\theta'' \leq \theta^{**}$. Since $\theta^{**} < \theta^*$ whenever $r^* > \tilde{r}$, this immediately rules out the possibility of active-policy equilibria in which $r^* > \tilde{r}$. On the other hand, for any $r^* \leq \tilde{r}$, we have

$$\theta^* \leq \theta' \leq \theta'' \leq \theta^{**}.$$

In words, the anxiety region of any robust equilibrium is contained by the anxiety region of the corresponding two-threshold equilibrium. By iterated deletion of strictly dominated strategies, one can also show that all $\theta < \theta^*$ necessarily devalue and that $\theta' = \theta^*$, which proves the first part of Theorem 2.

Observe next that the monotonicity of the devaluation policy implies monotonicity of the speculators' strategy as long as the posterior probability of devaluation is monotonic in the speculators' private signals. The latter is necessarily true when the noise distribution satisfies the monotone likelihood ratio property (MLRP), that is, when $\psi'(z)/\psi(z)$ is decreasing in z . In this case, the size of the attack $\alpha(\theta, \underline{r})$ is decreasing in θ , and therefore all $\theta \in [\theta^*, \theta'']$ raise the interest rate at r^* . But then, by definition of θ^{**} , it follows that $\theta'' = \theta^{**}$, which completes the second part of the theorem. Finally, when the speculators' posteriors fail to be monotonic in x , we can not exclude the possibility there exist active-policy equilibria different from the two-threshold equilibria; nevertheless, in *any* such equilibrium, devaluation necessarily occurs if and only if $\theta < \theta^*$ and the policy is active only for $\theta \in [\theta^*, \theta^{**}]$.

We earlier noted that the perfect-pooling and two-threshold equilibria can be supported by strategies for the speculators that are monotonic in the private information x . If we restrict attention to such simple x -monotonic strategies, we can dispense the MLRP condition in the second part of Theorem 2. We thus conclude that the perfect-pooling and the two-threshold equilibria of Propositions 2 and 3 represent the most plausible outcomes of our policy game.

An immediate corollary of Theorem 2 is that as private information becomes more precise, the anxiety region in any robust equilibrium shrinks. In the limit, the policy converges to a spike around a single point θ^* , namely the devaluation threshold, which is dictated merely by market sentiments.

Corollary 1 (Limit) *Suppose the noise ξ has unbounded full support. The limit of any robust equilibrium as $\varepsilon \rightarrow 0$ is either perfect pooling or the following: For arbitrary $\theta^* \in (\underline{\theta}, \theta_{MS}]$, the policy is $r(\theta) = \underline{r}$ for all $\theta \neq \theta^*$ and $r(\theta) = r^*$ for $\theta = \theta^*$, where $C(r^*) = V(\theta^*, 0)$; speculators attack if and only if $r < r^*$ and $x < x^*$, with $x^* = \theta^*$; and devaluation occurs if and only if $\theta < \theta^*$.*

At this point, it is interesting to compare the above results with the set of equilibrium policies that would arise if fundamentals were common knowledge.

Proposition 4 (Common Knowledge) *Suppose $\varepsilon = 0$. An interest-rate policy $r : \Theta \rightarrow \mathcal{R}$ can be part of a subgame perfect equilibrium if and only if $C(r(\theta)) \leq V(\theta, 0)$ for $\theta \in [\underline{\theta}, \bar{\theta}]$ and $r(\theta) = \underline{r}$ for $\theta \notin [\underline{\theta}, \bar{\theta}]$.*

Proof. For $\theta < \underline{\theta}$, it is dominant for the bank to set \underline{r} and devalue and for speculators to attack. Similarly, for $\theta > \bar{\theta}$, the bank never devalues, speculators do not to attack, and there is no need to raise the interest rate. Finally, take any $\theta \in [\underline{\theta}, \bar{\theta}]$. The continuation game following any interest rate r is a coordination game with two (extreme) continuation equilibria, no attack and full attack. Let $r(\theta)$ be the minimal r for which speculators coordinate on the no-attack continuation equilibrium. Clearly, it is optimal for the bank to set $r(\theta) > \underline{r}$ if and only if $V(\theta, 0) - C(r(\theta)) \geq 0$, or equivalently $C(r(\theta)) \leq \theta$. ■

That is, if the fundamentals were common knowledge, the equilibrium policy $r(\theta)$ could take essentially any shape in the critical range $[\underline{\theta}, \bar{\theta}]$. For example, it could have multiple discontinuities and multiple non-monotonicities. Similarly, the devaluation outcome $D(\theta)$ could also take any shape in $[\underline{\theta}, \bar{\theta}]$ and need not be monotonic. These results contrast sharply with our results in Theorem 2 and Corollary 1. When the information about fundamentals is very precise (i.e. ε is small but positive), the policy is active only for a small range of intermediate fundamentals $[\theta^*, \theta^{**}]$, this range vanishes as $\varepsilon \rightarrow 0$, and the devaluation outcome is necessarily monotonic in θ . We conclude that introducing small idiosyncratic noise in the observation of the fundamentals *does* reduce significantly the equilibrium set, as compared to the common knowledge case. The global-game methodology thus maintains a strong selection power even in our multiple-equilibria environment.

Another interesting implication of Corollary 1 is that, in the limit, all active-policy equilibria are observationally equivalent to the inactive-policy equilibrium in terms of the interest rate policy, although they are very different in terms of the devaluation outcome. An econometrician may then fail to predict the probability of devaluation on the basis of information on the fundamentals and the policy of the monetary authority: For any fundamentals $\theta \in (\underline{\theta}, \theta_{MS})$, the same interest rate \underline{r} may result in two different outcomes – either a large attack triggering a devaluation, if market expectations are aggressive (in the sense that $\theta^* > \theta$), or a small attack failing to cause devaluation, if market expectations are sufficiently lenient ($\theta^{**} < \theta$). An even sharper dependence of observable outcomes on unobservable market sentiments arises if one introduces sunspots, as we discuss next.

5 Uncertainty over the Aggressiveness of Market Expectations

The analysis so far assumed the monetary authority was able to anticipate perfectly the aggressiveness of market expectations, which we identify with the threshold r^* at which speculators switch from an aggressive to a lenient response to the policy. In reality, however, market expectations are hard to predict, even when the underlying economic fundamentals are perfectly known to the policy maker. Indeed, as suggested in the Financial Times quote, the reaction of the market to any given policy choice may be influenced by random sentiments, or animal spirits.

To capture this possibility, we introduce payoff-irrelevant sunspots, on which speculators may condition their responses to the actions of the policy maker. Instead of modelling explicitly the sunspots, we assume directly that r^* is a random variable with c.d.f. Φ over a compact support $\mathcal{R}^* \subseteq \mathcal{R}$. The realization of the random variable r^* is common knowledge among the speculators, but is unknown to the bank when she sets the policy. Uncertainty over the aggressiveness of market expectations then generates random variation in the effectiveness of any given policy choice and leads to random variation in the devaluation outcome. Different sunspot equilibria are associated with different distributions (Φ, \mathcal{R}^*) and result in different equilibrium policies $r(\theta)$.

Proposition 5 *Take any random variable r^* with support $\mathcal{R}^* \subseteq (\underline{r}, \bar{r})$ and distribution Φ . For $\varepsilon > 0$ sufficiently small,¹⁷ there exist thresholds $\theta^* \in (\underline{\theta}, \theta_{MS})$ and $\theta^{**} \in (\theta^*, \bar{\theta})$ and a robust equilibrium such that: The central bank follows a non-monotonic policy with $r(\theta) = \underline{r}$ for $\theta \notin [\theta^*, \theta^{**}]$, and $r(\theta) \in \mathcal{R}^*$ with $r(\theta)$ non-decreasing in θ for $\theta \in [\theta^*, \theta^{**}]$; devaluation occurs with certainty for $\theta < \theta^*$, with positive and decreasing probability for $\theta \in [\theta^*, \theta^{**}]$, and never occurs for $\theta > \theta^{**}$. Finally, θ^* is independent of ε , whereas $\theta^{**} \rightarrow \theta^*$ as $\varepsilon \rightarrow 0$.*

When $\theta < \theta^*$, raising the policy to any level in \mathcal{R}^* is too costly compared to the expected value of defending the peg, in which case the bank finds it optimal to set \underline{r} and devalue with certainty. When instead $\theta \in [\theta^*, \theta^{**}]$, it pays to raise the interest rate at some level in \mathcal{R}^* so as to lower the probability of a speculative attack. Since the value from defending the currency is increasing in θ , so is the optimal policy in the range $[\theta^*, \theta^{**}]$. Finally, for $\theta \geq \theta^{**}$, the size of the attack at \underline{r} is so small that the bank prefers the cost of such an attack to the cost of a high interest rate. The thresholds θ^* and θ^{**} are again given by the relevant indifference conditions for the bank, but differ from the ones we derived in the absence of sunspots. The definition of the thresholds and the complete proof of the above proposition are provided in the Appendix.

¹⁷The assumption that ε is sufficiently small is not essential; it ensures $\theta^{**} < \bar{\theta}$, which we use only to simplify the construction of these equilibria (see the Appendix).

The equilibria of Proposition 5 are qualitatively similar to the two-threshold equilibria of Proposition 3. The policy maker is anxious to prove herself only for a small range of moderate fundamentals, and this range vanishes as $\varepsilon \rightarrow 0$. The interval \mathcal{R}^* represents the set of random thresholds r^* such that speculators coordinate on an aggressive response whenever $r < r^*$ and on a lenient one whenever $r \geq r^*$. If one takes a sequence of sunspot equilibria such that \mathcal{R}^* converges to a single point r^* , the thresholds θ^* and θ^{**} in Proposition 5 converge to the ones given in (7). That is, the two-threshold equilibria of Proposition 3 are the limit of sunspot equilibria.¹⁸

With random variation in the aggressiveness of market expectations, policy traps take an even stronger form. Not only the policy maker has to adopt a policy that simply confirms market expectations, but also the equilibrium outcome of any given policy action is determined by animal spirits and market sentiments. Once the bank has raised interest rates in an attempt to defend the currency, the market may be equally likely to “interpret” the bank’s action either as a signal of strength, in which case the currency is spared from an attack, or as a signal of panic, in which case the bank’s attempt to defend the currency proves to be in vain. In this sense, the results of this section help understand and formalize the kind of arguments that commonly appear in financial news, like the one we quoted from Financial Times in the beginning of the paper.

Finally, a fact documented in the currency crises literature (e.g., Kraay, 2001) is that, in a large number of historical episodes of currency attacks, raising interest rates does not systematically succeed in deterring a large speculative attack and/or preventing an exchange-rate collapse. This fact contradicts Morris and Shin’s (1998) prediction that policy and fundamentals uniquely determine final outcomes, but is consistent with the sunspot equilibria of Proposition 5. The same interest rate may lead in equilibrium either to no attack at all, or an attack small enough for the bank to sustain, or an attack large enough to force devaluation. Hence, in the eyes of an econometrician, raising the interest rate may appear equally successful and unsuccessful as a defense strategy.

6 Concluding Remarks

In this paper we investigated the ability of a policy maker to influence market expectations and control equilibrium outcomes in economies where agents play a global coordination game. We found that policy endogeneity facilitates coordination in the market and results in multiple self-fulfilling equilibria, even when the fundamentals of the economy are observed with idiosyncratic noise. The multiple equilibria take the form of policy traps, where the policy maker is forced to

¹⁸Proposition 5 does not exhaust all possible sunspot equilibria. We conjecture, however, that an analog of Theorem 2 and Corollary 1 can be established for sunspot equilibria as well.

conform to the arbitrary expectations of the market instead of being able to fashion the equilibrium outcome. There is an inactive-policy equilibrium in which market participants coordinate on totally “ignoring” any attempt of the policy maker to affect market behavior, as well as a continuum of active-policy equilibria in which market participants coordinate on the level of the policy beyond which they stop playing aggressively against the policy maker. These results suggest also the possibility of another form of policy traps. Suppose the policy maker has more than one policy instrument (potential signals) at her disposal. In currency crises, for example, the government may impose capital controls or borrow reserves from abroad in addition to raising interest rates and restricting domestic credit to speculators. Which of these instruments the policy maker will use, and in what combination, is again to be determined largely by market expectations.

Although this paper focused on the particular example of self-fulfilling currency attacks, our approach may extend to other environments where government policy can serve as a coordination device among market participants. Monetary policy in economies with staggered pricing, fiscal and growth policies in economies with investment complementarities, and stabilization policies or regulatory intervention during financial or debt crises, are only a few examples where our results might be relevant. On a more theoretical ground, the main contribution of this paper is in showing that endogenous information transmission (signaling) in global coordination games facilitates coordination and leads to multiple equilibria in the signaling game. An interesting extension in this direction is to consider *noisy* signaling in global games. It can be shown that all equilibria of Theorem 1 are robust to the introduction of small bounded noise in the speculators’ observation of the policy, whether the noise is aggregate or idiosyncratic.¹⁹

In conclusion, the methodological message of this paper is two-fold. First, just as multiplicity in simple common-knowledge coordination games like Obstfeld (1986, 1996) is largely the unintended consequence of ignoring agents’ uncertainty about others’ beliefs and actions, uniqueness in global coordination games like Morris and Shin (1998, 2001) may be the unintended consequence of ignoring the role of endogenous information revelation. Second, even if it does eliminate multiplicity, information heterogeneity significantly reduces the equilibrium set as compared to the case of common knowledge and enables robust predictions.

¹⁹The proof of this claim is available upon request.

Appendix

Proof of Theorem 2. We prove the result in a sequence of five Lemmas. First, note that

Lemma 1. When x has unbounded full support, there is at most one interest rate $r^* > \underline{r}$ played in any equilibrium.

□ Any interest rate $r > \underline{r}$ is played in equilibrium only if it leads to no devaluation, i.e. only if $D(\theta) = 0$; all types who devalue set $r = \underline{r}$. Speculators attack if and only if the expected devaluation premium is higher than the interest rate differential, i.e. $\pi \int_{\Theta} D(\theta) d\mu(\theta|x, r) \geq r$. Since on the equilibrium path any interest rate $r > \underline{r}$ results in a posterior $\mu(\Theta_0|x, r) = 1$, where $\Theta_0 \equiv \{\theta : D(\theta) = 0\}$ is the set of fundamentals for which devaluation does not occur, no speculator ever attacks when he observes an equilibrium $r > \underline{r}$, and thus $\alpha(\theta, r) = 0$ for any equilibrium $r > \underline{r}$. Since $C(r)$ is strictly increasing in r , this also implies that, in any equilibrium, at most one interest rate $r^* \neq \underline{r}$ will be chosen by the central bank. ■

If no interest rate other than \underline{r} is played in equilibrium, this leads to the pooling equilibrium in (a) of Theorem 1. Hence, in what follows, we consider equilibria in which $r^* \in (\underline{r}, \bar{r}]$ is played by some θ . Define the thresholds

$$\theta^* = C(r^*) \quad \text{and} \quad \theta^{**} = \theta^* + \varepsilon \left[\Psi^{-1} \left(1 - \frac{r}{\pi - \underline{r}} \theta^* \right) - \Psi^{-1}(\theta^*) \right].$$

For any $r^* \in (\underline{r}, \tilde{r}]$, $\theta^* \leq \theta^{**}$, whereas for $r^* \in (\tilde{r}, \bar{r}]$, $\theta^* > \theta^{**}$. Let

$$\theta' = \inf\{\theta : r(\theta) = r^*\} \quad \text{and} \quad \theta'' = \sup\{\theta : r(\theta) = r^*\}$$

By definition, $\theta' \leq \theta''$. Moreover, in any robust equilibrium, $\theta'' < \infty$. Indeed, (5) implies there exists $x^+ < \infty$ such that $\mu(D = 1|x, \underline{r}) \equiv \int_{\Theta} D(\theta) d\mu(\theta|x, \underline{r}) \leq \underline{r}/\pi$ for all $x > x^+$ and therefore $\alpha(\theta, \underline{r}) \leq \Psi\left(\frac{x^+ - \theta}{\varepsilon}\right)$ for all θ . Let then $\theta^+ \equiv x^+ - \varepsilon \Psi^{-1}(\theta^*) < \infty$ and note that, for any $\theta \geq \theta^+$, $\alpha(\theta, \underline{r}) \leq \Psi\left(\frac{x^+ - \theta}{\varepsilon}\right) \leq \Psi\left(\frac{x^+ - \theta^+}{\varepsilon}\right) = \theta^* = C(r^*)$, which implies that all $\theta > \theta^+$ necessarily set \underline{r} . Hence, $\theta'' \leq \theta^+ < \infty$. We next show

Lemma 2. For any $r^* \in (\underline{r}, \bar{r}]$, $\theta' \geq \theta^*$ and $\theta'' \leq \theta^{**}$.

□ Since for any $\theta < \theta^*$, r^* is strictly dominated by \underline{r} , it immediately follows that $\theta' \geq \theta^*$. On the other hand, any $\theta > \theta^*$ can always set r^* , face no attack, and ensure a payoff $V(\theta, 0) - C(r^*) > 0$. Therefore, necessarily $D(\theta) = 0$ for all $\theta > \theta^*$. However, there may exist types $\theta < \theta^*$ that also do not devalue in equilibrium. Define $\delta(x)$ as the probability, conditional on x , that $\theta < \theta^*$ and

$D = 0$. Further, define $p(x)$ as the probability, conditional on x , that $\theta \in [\theta^*, \theta'']$ and $r(\theta) = \underline{r}$. Then, the probability of devaluation conditional on x and \underline{r} is given by

$$\mu(D = 1|x, \underline{r}) = \frac{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right) - \delta(x)}{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right) + p(x) + \Psi\left(\frac{x - \theta''}{\varepsilon}\right)}.$$

Clearly, a speculator never attacks whenever $\mu(D = 1|x, \underline{r}) < \underline{r}/\pi$. Define

$$F(x; \theta^*, \theta'') \equiv \frac{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right)}{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right) + \Psi\left(\frac{x - \theta''}{\varepsilon}\right)} = \left[1 + \frac{\Psi\left(\frac{x - \theta''}{\varepsilon}\right)}{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right)}\right]^{-1}.$$

Note that $F(x; \theta^*, \theta'')$ is strictly decreasing in x , and let $\hat{x}(\theta^*, \theta'')$ solve

$$F(\hat{x}; \theta^*, \theta'') = \underline{r}/\pi. \quad (8)$$

Since $\delta(x) \geq 0$ and $p(x) \geq 0$, we have $\mu(D = 1|x, \underline{r}) \leq F(x; \theta^*, \theta'')$ for all x . It follows that, whenever $x > \hat{x}(\theta^*, \theta'')$,

$$\mu(D = 1|x, \underline{r}) \leq F(x; \theta^*, \theta'') < F(\hat{x}; \theta^*, \theta'') = \underline{r}/\pi$$

and therefore any speculator with $x > \hat{x}(\theta^*, \theta'')$ does not attack in equilibrium. That is, in any equilibrium in which r^* is played, speculators are necessarily at most as aggressive as they would be if it were the case that $D(\theta) = 1$ for all $\theta < \theta^*$ and $r(\theta) = r^*$ for all $\theta \in [\theta^*, \theta'']$. This is intuitive for (i) a positive probability that $D(\theta) = 0$ for some $\theta < \theta^*$ reduces the incentives to attack for every x , and (ii) a positive probability that $r(\theta) = \underline{r}$ for some $\theta \in [\theta^*, \theta'']$ reduces the probability that the observation of \underline{r} signals devaluation and therefore also reduces the incentives to attack conditional on \underline{r} and x . It follows that for any θ

$$\alpha(\theta, \underline{r}) \leq \Psi\left(\frac{\hat{x}(\theta^*, \theta'') - \theta}{\varepsilon}\right).$$

From the indifference condition for θ'' , we have that $\alpha(\theta'', \underline{r}) = C(r^*)$. Since $C(r^*) = \theta^*$, it follows that $\theta^* \leq \Psi\left(\frac{\hat{x}(\theta^*, \theta'') - \theta''}{\varepsilon}\right)$. From the indifference conditions of Proposition (3), and using $\hat{x}(\theta^*, \theta^{**}) = x^*$, we also have that $\theta^* = \Psi\left(\frac{\hat{x}(\theta^*, \theta^{**}) - \theta^{**}}{\varepsilon}\right)$. Hence, in any equilibrium in which r^* is played, θ'' must satisfy

$$\hat{x}(\theta^*, \theta^{**}) - \theta^{**} \leq \hat{x}(\theta^*, \theta'') - \theta''.$$

From (8), $\partial \hat{x}(\theta^*, \theta'') / \partial \theta'' < 1$ and therefore $\hat{x}(\theta^*, \theta'') - \theta''$ is strictly decreasing in θ'' . We conclude that $\theta'' \leq \theta^{**}$. ■

Recall that $\theta^* \leq \theta^{**}$ if and only if $r^* \leq \tilde{r} \equiv C^{-1}(\theta_{MS})$. For any $r^* > \tilde{r}$, Lemma 2 implies $\theta'' \leq \theta^{**} < \theta^* \leq \theta'$, which is a contradiction, since by definition $\theta' \leq \theta''$. Therefore, there exists no robust equilibrium in which $r^* \in (\tilde{r}, \bar{r}]$ is played. On the other hand, in any robust equilibrium where $r^* \in [\underline{r}, \tilde{r}]$ is played, necessarily $\theta^* \leq \theta' \leq \theta'' \leq \theta^{**}$.

We next show, by iterated deletion of strictly dominated strategies, that in any robust equilibrium in which r^* is played, devaluation occurs if and only if $\theta < \theta^*$, which in turn implies that $\theta' = \theta^*$.

Lemma 3. $D(\theta) = 1$ for all $\theta < \theta^*$, $D(\theta) = 0$ for all $\theta > \theta^*$, and $\theta' = \theta^*$.

□ Given an arbitrary equilibrium policy $r(\theta)$, we consider the continuation game that follows $r = \underline{r}$ and construct the iterated deletion mapping $\mathcal{T} : [\underline{\theta}, \theta'] \rightarrow [\underline{\theta}, \theta']$ as follows. Take any $\tilde{\theta} \in [\underline{\theta}, \theta']$ and let

$$G(x; \tilde{\theta}) \equiv \frac{1 - \Psi\left(\frac{x - \tilde{\theta}}{\varepsilon}\right)}{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right) + p(x) + \Psi\left(\frac{x - \theta''}{\varepsilon}\right)}.$$

$G(x; \tilde{\theta})$ thus represents the probability of $\theta < \tilde{\theta}$, conditional on x and \underline{r} . Note that $\lim_{x \rightarrow -\infty} p(x) = \lim_{x \rightarrow +\infty} p(x) = 0$ and therefore $\lim_{x \rightarrow -\infty} G(x; \tilde{\theta}) = 1$ and $\lim_{x \rightarrow +\infty} G(x; \tilde{\theta}) = 0$. It follows that, for every $\tilde{\theta}$, there is at least one solution to the equation $G(x; \tilde{\theta}) = \underline{r}/\pi$. Define then $\tilde{x} = \tilde{x}(\tilde{\theta})$ as the lowest solution, $\tilde{x}(\tilde{\theta}) \equiv \min\{x \mid G(x; \tilde{\theta}) = \underline{r}/\pi\}$, and let $\tilde{\theta} = \tilde{\theta}(\tilde{\theta})$ be the unique solution to $\tilde{\theta} = \Psi\left(\frac{\tilde{x} - \tilde{\theta}}{\varepsilon}\right)$. Note that $G(x; \tilde{\theta}) > \underline{r}/\pi$ for every $x < \tilde{x}$ and $\Psi\left(\frac{\tilde{x} - \theta}{\varepsilon}\right) > \theta$ for every $\theta < \tilde{\theta}$. That is, if all $\theta < \tilde{\theta}$ are expected to devalue, all $x < \tilde{x}$ necessarily find it optimal to attack, which in turn implies that any $\theta < \tilde{\theta}$ necessarily devalues, unless θ is playing r^* rather than \underline{r} , which happens in equilibrium only if $\theta \geq \theta'$. The iterated deletion operator \mathcal{T} is thus defined by

$$\mathcal{T}(\tilde{\theta}) \equiv \min\left\{\theta', \tilde{\theta}\right\}$$

Observe that G is strictly increasing in $\tilde{\theta}$, implying that \tilde{x} and therefore $\tilde{\theta}$ are also strictly increasing in $\tilde{\theta}$.²⁰ We conclude that the mapping \mathcal{T} is weakly increasing for all $\tilde{\theta} \in [\underline{\theta}, \theta']$. Obviously, \mathcal{T} is also bounded above by θ' . Finally, note that $\theta^* \leq \theta' \leq \theta^{**} < \infty$ and $\theta^* \leq \theta_{MS}$, but so far we have ruled out neither $\theta' \leq \theta_{MS}$, nor $\theta' > \theta_{MS}$.

Next, we compare \mathcal{T} with the iterated deletion operator of the Morris-Shin game without signaling (or, equivalently, of the continuation game at \underline{r} when the pooling equilibrium is played).

²⁰In general, \tilde{x} and therefore $\tilde{\theta}$ need not be continuous in $\tilde{\theta}$. Continuity is ensured when Ψ satisfies the MLRP, in which case G is strictly decreasing in x , implying that $G(x, \tilde{\theta}) = \underline{r}/\pi$ has a unique solution and this solution is continuously increasing in $\tilde{\theta}$. All we need, however, is monotonicity of the lowest solution, which is true for any Ψ .

This operator, $\mathcal{F} : [\underline{\theta}, \bar{\theta}] \rightarrow [\underline{\theta}, \bar{\theta}]$, is defined by $\mathcal{F}(\hat{\theta}) = \hat{\hat{\theta}}$, where $\hat{x} = \hat{x}(\hat{\theta})$ and $\hat{\hat{\theta}} = \hat{\hat{\theta}}(\hat{\theta})$ are the unique solution to $1 - \Psi\left(\frac{\hat{x} - \hat{\theta}}{\varepsilon}\right) = \underline{r}/\pi$ and $\hat{\hat{\theta}} = \Psi\left(\frac{\hat{x} - \hat{\theta}}{\varepsilon}\right)$. \mathcal{F} has a unique fixed point at $\hat{\theta} = \hat{\hat{\theta}} = \theta_{MS}$, and satisfies $\hat{\theta} < \mathcal{F}(\hat{\theta}) < \theta_{MS}$ whenever $\hat{\theta} \in [\underline{\theta}, \theta_{MS})$ and $\hat{\theta} > \mathcal{F}(\hat{\theta}) > \theta_{MS}$ whenever $\hat{\theta} \in (\theta_{MS}, \bar{\theta}]$. Moreover, since $G(x, \theta) > 1 - \Psi\left(\frac{x - \theta}{\varepsilon}\right)$ for all x and θ , we have $\tilde{x}(\theta) > \hat{x}(\theta)$ and therefore $\tilde{\theta}(\theta) > \hat{\theta}(\theta)$. We conclude that, for any $\theta \in [\underline{\theta}, \theta']$, either $\tilde{\theta}(\theta) \geq \theta'$, in which case $\mathcal{T}(\theta) = \theta'$, or $\tilde{\theta}(\theta) < \theta'$, in which case $\mathcal{T}(\theta) > \mathcal{F}(\theta)$.

Finally, we consider the sequence $\{\theta^k\}_{k=1}^{\infty}$, where $\theta^1 = \underline{\theta}$ and $\theta^{k+1} = \mathcal{T}(\theta^k)$ for all $k \geq 1$. This sequence represents iterated deletion of dominated strategies starting from $\underline{\theta}$. Since this sequence is monotone and bounded above by θ' , it necessarily converges to some limit $\theta^{\infty} \leq \theta'$. This limit must be a fixed point of \mathcal{T} , that is, $\theta^{\infty} = \mathcal{T}(\theta^{\infty})$. We first prove that either $\theta^{\infty} = \theta'$, or $\theta^{\infty} > \theta_{MS}$. Suppose $\theta^{\infty} < \theta'$. This can be true only if $\tilde{\theta}(\theta^{\infty}) < \theta'$. If it were the case that $\theta^{\infty} \leq \theta_{MS}$, we would then have $\mathcal{T}(\theta^{\infty}) > \mathcal{F}(\theta^{\infty}) \geq \theta^{\infty}$, which would be a contradiction. Therefore, $\theta^{\infty} = \theta'$ whenever $\theta' \leq \theta_{MS}$, whereas $\theta^{\infty} > \theta_{MS}$ whenever $\theta' > \theta_{MS}$. We next prove that $\theta^{\infty} = \theta' = \theta^*$. Suppose that $\theta' > \theta^*$. If $\theta' \leq \theta_{MS}$, then $\theta^{\infty} = \theta' > \theta^*$ and all $\theta \in (\theta^*, \theta')$ would devalue in equilibrium. If instead $\theta' > \theta_{MS}$, then $\theta^{\infty} > \theta_{MS} \geq \theta^*$ and again all $\theta \in (\theta^*, \theta^{\infty})$ would devalue in equilibrium. But either case is impossible as any $\theta > \theta^*$ can ensure no devaluation and a positive payoff by setting r^* . Therefore, it is necessarily the case that $\theta' = \theta^*$. This also implies that $\theta' \leq \theta_{MS}$ and therefore $\theta^{\infty} = \theta' = \theta^*$, which completes the proof of this lemma. ■

We next show that, if the posterior beliefs given \underline{r} are monotonic in x , then speculators follow a threshold strategy and therefore the size of an attack is decreasing in the fundamentals θ . This in turns implies that the bank sets r^* for all $\theta \in [\theta^*, \theta^{**}]$.

Lemma 4. If in equilibrium $\mu(\theta \leq \theta^* | x, \underline{r})$ is monotonic in x , then necessarily $\alpha(\theta, \underline{r})$ is strictly decreasing in θ , in which case $\theta'' = \theta^{**}$, and $r(\theta) = r^*$ for all $\theta \in [\theta^*, \theta^{**}]$.

□ Following Lemma 3, the probability of devaluation conditional on x and \underline{r} is given by

$$\mu(D = 1 | x, \underline{r}) = \mu(\theta \leq \theta^* | x, \underline{r}) = G(x; \theta^*) \equiv \frac{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right)}{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right) + p(x) + \Psi\left(\frac{x - \theta''}{\varepsilon}\right)},$$

with $\lim_{x \rightarrow -\infty} G(x; \theta^*) = 1$ and $\lim_{x \rightarrow +\infty} G(x; \theta^*) = 0$. When $G(x, \theta^*)$ is monotonic in x , there is a unique x' such that $G(x'; \theta^*) = \underline{r}/\pi$ and thus speculators follow a threshold strategy with $a(x, \underline{r}) = 1$ if $x < x'$ and $a(x, \underline{r}) = 0$ if $x > x'$. It follows that $\alpha(\theta, \underline{r})$ is strictly decreasing in θ and, since θ'' solves $\alpha(\theta'', \underline{r}) = C(r^*)$, all $\theta \in [\theta^*, \theta'']$ necessarily set r^* . But then, $p(x) = 0$ for any x , and $x' = \hat{x}(\theta^*, \theta'')$, implying that $\alpha(\theta, \underline{r}) = \Psi\left(\frac{\hat{x}(\theta^*, \theta'') - \theta}{\varepsilon}\right)$. Thus, θ'' solves $\Psi\left(\frac{\hat{x}(\theta^*, \theta'') - \theta''}{\varepsilon}\right) = C(r^*)$,

and from Proposition (3), $\widehat{x}(\theta^*, \theta^{**}) - \theta^{**} = \widehat{x}(\theta^*, \theta'') - \theta''$. From the monotonicity of $\widehat{x}(\theta^*, \theta) - \theta$ established in Lemma 2, it follows that $\theta'' = \theta^{**}$. ■

The above result presumed monotonicity of the posterior $\mu(\theta \leq \theta^* | x, \underline{r})$ in x . Since all $\theta \leq \theta^*$ set \underline{r} , this is likely to be the case for a wide range of noise structures. In the next, we prove this is necessarily the case at least when the noise ξ follows a distribution which satisfies the MLRP.

Lemma 5. If Ψ satisfies the MLRP, then $\mu(\theta \leq \theta^* | x, \underline{r})$ is monotonic in x .

□ Let $I(\theta)$ be the probability that θ sets \underline{r} . Using $p(x) + \Psi\left(\frac{x-\theta''}{\varepsilon}\right) = \int_{\theta^*}^{\infty} \frac{1}{\varepsilon} \psi\left(\frac{x-\theta}{\varepsilon}\right) I(\theta) d\theta$, we have that

$$\frac{G(x; \theta^*)}{1 - G(x; \theta^*)} = \frac{1 - \Psi\left(\frac{x-\theta^*}{\varepsilon}\right)}{\int_{\theta^*}^{\infty} \frac{1}{\varepsilon} \psi\left(\frac{x-\theta}{\varepsilon}\right) I(\theta) d\theta},$$

and therefore

$$\frac{d\left(\frac{G(x; \theta^*)}{1 - G(x; \theta^*)}\right)}{dx} = \frac{-\frac{1}{\varepsilon} \psi\left(\frac{x-\theta^*}{\varepsilon}\right)}{\int_{\theta^*}^{\infty} \frac{1}{\varepsilon} \psi\left(\frac{x-\theta}{\varepsilon}\right) I(\theta) d\theta} - \frac{\left[1 - \Psi\left(\frac{x-\theta^*}{\varepsilon}\right)\right] \frac{d}{dx} \left(\int_{\theta^*}^{\infty} \frac{1}{\varepsilon} \psi\left(\frac{x-\theta}{\varepsilon}\right) I(\theta) d\theta\right)}{\left[\int_{\theta^*}^{\infty} \frac{1}{\varepsilon} \psi\left(\frac{x-\theta}{\varepsilon}\right) I(\theta) d\theta\right]^2}.$$

It follows that $dG(x; \theta^*)/dx \leq 0$ if and only if

$$\frac{\int_{-\infty}^{\theta^*} \frac{1}{\varepsilon^2} \psi'\left(\frac{x-\theta}{\varepsilon}\right) d\theta}{\int_{-\infty}^{\theta^*} \frac{1}{\varepsilon} \psi\left(\frac{x-\theta}{\varepsilon}\right) d\theta} - \frac{\int_{\theta^*}^{\infty} \frac{1}{\varepsilon^2} \psi'\left(\frac{x-\theta}{\varepsilon}\right) I(\theta) d\theta}{\int_{\theta^*}^{\infty} \frac{1}{\varepsilon} \psi\left(\frac{x-\theta}{\varepsilon}\right) I(\theta) d\theta} \leq 0.$$

Using the fact that $I(\theta) = 1$ for all $\theta \leq \theta^*$, the above is equivalent to

$$\mathbb{E}_{\theta} \left[\frac{\frac{1}{\varepsilon} \psi'\left(\frac{x-\theta}{\varepsilon}\right)}{\psi\left(\frac{x-\theta}{\varepsilon}\right)} \middle| \theta \leq \theta^*, x, \underline{r} \right] - \mathbb{E}_{\theta} \left[\frac{\frac{1}{\varepsilon} \psi'\left(\frac{x-\theta}{\varepsilon}\right)}{\psi\left(\frac{x-\theta}{\varepsilon}\right)} \middle| \theta > \theta^*, x, \underline{r} \right] \leq 0,$$

which holds true necessarily if ψ'/ψ is monotone decreasing. ■

Combining Lemmas 1, 2, and 3, we conclude that any robust equilibrium belongs necessarily to either (a) or (b) in Theorem 1. If, in addition, Ψ satisfies the MLRP, Lemmas 4 and 5 imply that any equilibrium in (b) is a two-threshold equilibrium, which completes the proof of the theorem.

Proof of Proposition 5. Take any random variable r^* with support $\mathcal{R}^* = [\underline{r}^*, \bar{r}^*] \subseteq (\underline{r}, \bar{r})$ and distribution Φ . We want to show that for ε sufficiently small, there exist thresholds $x^*, \theta^* \in (\underline{\theta}, \theta_{MS})$, and $\theta^{**} \in (\theta^*, \bar{\theta})$, a system of beliefs μ , and a robust equilibrium such that: for any $r < \underline{r}^*$, speculators attack if and only if $x < x^*$; for $\underline{r}^* \leq r < r^*$, if and only if $x < \bar{x}$, and for $r \geq r^*$ if and only if $x < \underline{x}$; the bank follows a non-monotonic policy with $r(\theta) = \underline{r}$ for $\theta \notin [\theta^*, \theta^{**}]$, and $r(\theta) \in \mathcal{R}^*$

with $r(\theta)$ non-decreasing in θ for $\theta \in [\theta^*, \theta^{**}]$; devaluation occurs with certainty for $\theta < \theta^*$, with positive and decreasing probability for $\theta \in [\theta^*, \theta^{**}]$, and never occurs for $\theta > \theta^{**}$.

The proof is in four steps: Steps 1 and 2 characterize the thresholds θ^* , θ^{**} , and x^* ; Step 3 examines the behavior of the bank; and Step 4 examines the beliefs and the strategy of the speculators.

Step 1. Let $\mathcal{R}^* = [\underline{r}^*, \bar{r}^*]$, with $\underline{r} < \underline{r}^* < \bar{r}^* < \tilde{r}$. Define

$$\widehat{U}(\theta) \equiv \begin{cases} \max_{r \in \mathcal{R}^*} \{-C(r)\} & \text{if } \theta < \underline{\theta}, \\ \max_{r \in \mathcal{R}^*} \{\theta \Phi(r) - C(r)\} & \text{if } \theta \in [\underline{\theta}, \bar{\theta}], \\ \max_{r \in \mathcal{R}^*} \{\theta - (1 - \Phi(r))\Psi\left(\frac{\bar{x} - \theta}{\varepsilon}\right) - C(r)\} & \text{if } \theta > \bar{\theta}, \end{cases}$$

and $\widehat{r}(\theta)$ as the corresponding arg max. Note that $\widehat{U}(\theta)$ is strictly increasing in θ whenever $\widehat{r}(\theta) > \underline{r}^*$ and $\widehat{U}(\theta) = -C(\underline{r}^*)$ otherwise. Moreover, $\widehat{U}(\underline{\theta}) = -C(\underline{r}^*) < 0$ and $\widehat{U}(\theta_{MS}) \geq \theta_{MS} - C(\bar{r}^*) > \theta_{MS} - C(\tilde{r}) = 0$. Therefore, there exists a unique $\theta^* \in (\underline{\theta}, \theta_{MS})$ such that $\widehat{U}(\theta^*) = 0$, $\widehat{U}(\theta) < 0$ for all $\theta < \theta^*$, and $\widehat{U}(\theta) > 0$ for all $\theta > \theta^*$.

Step 2. For any $\theta \geq \theta^*$, define the functions $v(\theta; \theta^*)$ and $\widehat{x}(\theta; \theta^*)$ as follows:

$$v(\theta; \theta^*) = \theta - \Psi\left(\frac{\widehat{x}(\theta; \theta^*) - \theta}{\varepsilon}\right) \quad \text{and} \quad \frac{1 - \Psi\left(\frac{\widehat{x} - \theta^*}{\varepsilon}\right)}{1 - \Psi\left(\frac{\widehat{x} - \theta^*}{\varepsilon}\right) + \Psi\left(\frac{\widehat{x} - \theta}{\varepsilon}\right)} = \frac{\underline{r}}{\pi}$$

Note that $\partial \widehat{x}(\theta; \theta^*) / \partial \theta \in (0, 1)$ and therefore $\partial v(\theta; \theta^*) / \partial \theta > 1$. Note also that, for any $\theta > \theta^*$, $\Psi\left(\frac{\widehat{x}(\theta; \theta^*) - \theta}{\varepsilon}\right) \rightarrow 0$ as $\varepsilon \rightarrow 0$. To see this, suppose instead that $\lim_{\varepsilon \rightarrow 0} \Psi\left(\frac{\widehat{x}(\theta; \theta^*) - \theta}{\varepsilon}\right) = \omega$ for some $\omega > 0$. This can be true only if $\lim_{\varepsilon \rightarrow 0} \widehat{x}(\theta; \theta^*) \geq \theta$, in which case $\theta > \theta^*$ implies $\lim_{\varepsilon \rightarrow 0} \widehat{x}(\theta; \theta^*) > \theta^*$ and therefore $\lim_{\varepsilon \rightarrow 0} \Psi\left(\frac{\widehat{x}(\theta; \theta^*) - \theta^*}{\varepsilon}\right) = 1$. But then

$$\lim_{\varepsilon \rightarrow 0} \left\{ \frac{1 - \Psi\left(\frac{\widehat{x}(\theta; \theta^*) - \theta^*}{\varepsilon}\right)}{1 - \Psi\left(\frac{\widehat{x}(\theta; \theta^*) - \theta^*}{\varepsilon}\right) + \Psi\left(\frac{\widehat{x}(\theta; \theta^*) - \theta}{\varepsilon}\right)} \right\} = \frac{0}{0 + \omega} = 0 < \frac{\underline{r}}{\pi},$$

which is a contradiction. Consider now the function $g(\theta) = v(\theta; \theta^*) - \widehat{U}(\theta)$. From the envelope theorem, $\partial \widehat{U}(\theta) / \partial \theta$ is bounded above by 1, and hence $g(\theta)$ is strictly increasing in θ . Moreover, by definition of $\widehat{x}(\theta; \theta^*)$, we have $\Psi\left(\frac{\widehat{x}(\theta^*; \theta^*) - \theta^*}{\varepsilon}\right) = \frac{\pi - \underline{r}}{\pi} = \theta_{MS}$ and therefore $v(\theta^*; \theta^*) = \theta^* - \theta_{MS}$. Since $\theta^* < \theta_{MS}$ and $\widehat{U}(\theta^*) = 0$, we conclude $g(\theta^*) < 0$. Next, note that θ^* and $\widehat{r}(\cdot)$ are independent of ε , whereas $\Psi\left(\frac{\widehat{x}(\theta; \theta^*) - \theta}{\varepsilon}\right) \rightarrow 0$ as $\varepsilon \rightarrow 0$ for every $\theta > \theta^*$. (See the argument above). Since $\bar{\theta} > \theta^*$ and $C(\widehat{r}(\bar{\theta})) > 0$, it follows that there exists $\bar{\varepsilon} > 0$ such that $\Psi\left(\frac{\widehat{x}(\bar{\theta}; \theta^*) - \bar{\theta}}{\varepsilon}\right) < C(\widehat{r}(\bar{\theta}))$ whenever

$\varepsilon < \bar{\varepsilon}$. But then $v(\bar{\theta}; \theta^*) > \bar{\theta} - C(\hat{r}(\bar{\theta})) \geq \hat{U}(\bar{\theta})$ and therefore $g(\bar{\theta}) > 0$. We conclude that, for $\varepsilon < \bar{\varepsilon}$, there exists a unique $\theta^{**} \in (\theta^*, \bar{\theta})$ such that $g(\theta^{**}) = 0$, $g(\theta) < 0$ for $\theta < \theta^{**}$, and $g(\theta) > 0$ for $\theta > \theta^{**}$. Moreover, note that, as $\varepsilon \rightarrow 0$, $v(\theta; \theta^*) \rightarrow \theta > \hat{U}(\theta)$ for every $\theta > \theta^*$; it follows that $\theta^{**} \rightarrow \theta^*$ as $\varepsilon \rightarrow 0$. Next, let

$$x^* \equiv \hat{x}(\theta^{**}; \theta^*) \quad \text{and} \quad \tilde{U}(\theta; \theta^*) \equiv \theta - \Psi\left(\frac{x^* - \theta}{\varepsilon}\right).$$

Compare now $\tilde{U}(\theta; \theta^*)$ with $v(\theta; \theta^*)$. That $\hat{x}(\theta^{**}; \theta^*) = x^*$ implies $\tilde{U}(\theta^{**}; \theta^*) = v(\theta^{**}; \theta^*)$, while the fact that $\hat{x}(\theta; \theta^*)$ is increasing in θ implies $\tilde{U}(\theta; \theta^*) < v(\theta; \theta^*)$ for all $\theta < \theta^{**}$ and $\tilde{U}(\theta; \theta^*) > v(\theta; \theta^*)$ for all $\theta > \theta^{**}$. Combining this result with the properties of the function $g(\theta)$, we conclude that $\tilde{U}(\theta; \theta^*) < v(\theta; \theta^*) < \hat{U}(\theta)$ for all $\theta < \theta^{**}$, $\tilde{U}(\theta; \theta^*) = v(\theta; \theta^*) = \hat{U}(\theta)$ at $\theta = \theta^{**}$, and $\tilde{U}(\theta; \theta^*) > v(\theta; \theta^*) > \hat{U}(\theta)$ for all $\theta > \theta^{**}$. Finally, let $\hat{\theta}$ be the unique solution to $\tilde{U}(\hat{\theta}; \theta^*) = 0$ and note that $\hat{\theta} \in (\theta^*, \theta^{**})$, since $\tilde{U}(\theta^{**}; \theta^*) > 0 > \tilde{U}(\theta^*; \theta^*)$.

Step 3. Consider now the behavior of the bank. Given the strategy of the speculators, the bank prefers \underline{r} to any $r < \underline{r}^*$, and \bar{r}^* to any $r > \bar{r}^*$. Furthermore, since $\hat{r}(\theta)$ dominates any $r \in \mathcal{R}^*$, the bank prefers $\hat{r}(\theta)$ to any $r \geq \underline{r}^*$. We thus need to compare only the payoff from playing $\hat{r}(\theta)$ with that from playing \underline{r} . Playing $\hat{r}(\theta)$ yields $\hat{U}(\theta)$, while playing \underline{r} yields $\underline{U}(\theta) = \max\{0, \tilde{U}(\theta; \theta^*)\}$. Note that $\underline{U}(\theta) = 0$ if $\theta \leq \hat{\theta}$ and $\underline{U}(\theta) = \tilde{U}(\theta) > 0$ if $\theta > \hat{\theta} \in (\theta^*, \theta^{**})$. It follows that $\hat{U}(\theta) < 0 = \underline{U}(\theta)$ for all $\theta < \theta^*$, $\hat{U}(\theta) > 0 = \underline{U}(\theta)$ for all $\theta \in (\theta^*, \hat{\theta}]$, $\hat{U}(\theta) > \tilde{U}(\theta) = \underline{U}(\theta) > 0$ for all $\theta \in (\hat{\theta}, \theta^{**})$, and $\underline{U}(\theta) = \tilde{U}(\theta) > \hat{U}(\theta)$ for all $\theta > \theta^{**}$. Therefore, it is indeed optimal for the bank to play $\hat{r}(\theta)$ whenever $\theta \in [\theta^*, \theta^{**}]$ and \underline{r} otherwise.

Step 4. Finally, consider the behavior of the speculators. For any $r < \underline{r}^*$, devaluation occurs if and only if $\theta < \hat{\theta}$. In equilibrium, at $r = \underline{r}$, the probability of devaluation is given by

$$\mu(\theta < \hat{\theta} | x, \underline{r}) = \mu(\theta < \theta^* | x, \underline{r}) = \frac{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right)}{1 - \Psi\left(\frac{x - \theta^*}{\varepsilon}\right) + \Psi\left(\frac{x - \theta^{**}}{\varepsilon}\right)}.$$

By construction, x^* solves $\pi\mu(\theta < \hat{\theta} | x^*, \underline{r}) = \underline{r}$, and since $\mu(\theta < \hat{\theta} | x, \underline{r})$ is decreasing in x , attacking the currency if and only if $x < x^*$ is indeed optimal. For any out-of-equilibrium $r < \underline{r}^*$, we consider beliefs μ such that $\mu(\theta < \hat{\theta} | x, r)$ is non-increasing in x and $\pi\mu(\theta < \hat{\theta} | x, r) = r$ at $x = x^*$. For any $r \geq \underline{r}^*$, let $\mu(\theta \in (\underline{\theta}, \bar{\theta}) | x, r) = 1$ for all $x \in [\underline{x}, \bar{x}]$, $\mu(\theta < \underline{\theta} | x, r) = 1$ for $x < \underline{x}$, and $\mu(\theta > \bar{\theta} | x, r) = 1$ for $x > \bar{x}$. In particular, for any $r \in r(\Theta)$, with $r \geq \underline{r}^*$, $\mu(\theta \in \theta^{-1}(r) | x, r) = 1$ for all x such that $\Theta(x) \cap \theta^{-1}(r) \neq \emptyset$, where $\theta^{-1}(r) = \{\theta : r(\theta) = r\}$. In addition, for any out-of-equilibrium r , if r is not dominated in equilibrium by $r(\theta)$ for some $\theta \in \Theta(x)$, then we further restrict μ to satisfy

$\mu(\theta|x, r) = 0$ for all $\theta \in \Theta(x)$ such that $\theta - C(r) < U(\theta)$. These beliefs satisfy (4), as well as $\mu \in \mathcal{M}(r)$; and given these beliefs, the strategy of the speculators is sequentially optimal for any r and x . Finally, it is immediate that $\mu(\theta > \bar{\theta}|x, \underline{r}) \rightarrow 1$ as $x \rightarrow \infty$, so that the equilibrium is robust in the sense of (5).

References

- [1] Albanesi, Stefania, V.V. Chari, and Lawrence Christiano, (2002), “Expectation Traps and Monetary Policy,” NBER working paper 8912.
- [2] Angeletos, George-Marios, and Laurent Calvet (2003), “Idiosyncratic Production Risk, Growth, and the Business Cycle,” NBER working paper.
- [3] Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan (2003), “Information, Coordination, and the Dynamics of Currency Crises,” work in progress.
- [4] Azariadis, Costas (1981), “Self-fulfilling Prophecies,” *Journal of Economic Theory* 25, 380-96.
- [5] Benhabib, Jess, and Roger Farmer (1994), “Indeterminacy and Increasing Returns,” *Journal of Economic Theory* 63, 19-41.
- [6] Bernanke, Ben, and Mark Gertler (1989), “Agency Costs, Net Worth, and Business Fluctuations,” *American Economic Review* 79, 14-31.
- [7] Bryant, John (1983), “A Simple Rational Expectations Keynes-Type Model,” *Quarterly Journal of Economics* 98, 3, 525-28.
- [8] Calvo, G. (1988), “Servicing the Public Debt: the Role of Expectations,” *American Economic Review*, 78(4), 647-661.
- [9] Carlsson, Hans, and Eric van Damme (1993), “Global Games and Equilibrium Selection,” *Econometrica* 61, 5, 989-1018.
- [10] Chamley, Christophe (1999), “Coordinating Regime Switches,” *Quarterly Journal of Economics* 114, 3, 869-905.

- [11] Chari, V.V., Lawrence Christiano, and Martin Eichenbaum (1998), "Expectation Traps and Discretion," *Journal of Economic Theory* 81, 2, 462-92.
- [12] Chari, V.V., and Patrick Kehoe (2003a), "Hot Money," Federal Reserve Bank of Minneapolis staff report 228.
- [13] Chari, V.V., and Patrick Kehoe (2003b), "Financial Crises as Herds," Federal Reserve Bank of Minneapolis working paper 600.
- [14] Cho, In-Koo, and David Kreps (1987), "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics* 102, 2, 179-221.
- [15] Cole, Harold, and Timothy Kehoe (1996), "Self-fulfilling Debt Crises," Federal Reserve Bank of Minneapolis staff report 211.
- [16] Cooper, Russell (1998), *Coordination Games, Complementarities and Macroeconomics*, Cambridge University Press.
- [17] Diamond, Douglas, and Philip Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy* 91, 401-19.
- [18] Diamond, Peter (1982), "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy* 90, 5, 881-94.
- [19] Drazen, Alan (2001), "Interest Rate Defense Against Speculative Attack as a Signal: A Primer," University of Maryland mimeo.
- [20] Feltovich, Nick, Rick Harbaugh, and Ted To (2002), "Too Cool for School? Signaling and Countersignaling," *Rand Journal of Economics* 33, 630-649.
- [21] Flood, Robert and Peter Garber (1984), "Gold Monetization and Gold Discipline," *Journal of Political Economy* 92, 1, 90-107.
- [22] Freixas, Xavier, and Jean-Charles Rochet (1997), *Microeconomics of Banking*, Cambridge and London: MIT Press.
- [23] Hellwig, Christian (2002), "Public Information, Private Information and the Multiplicity of Equilibria in Coordination Games," *Journal of Economic Theory*, 107, 191-222.
- [24] Judd, Kenneth, (1985) "The Law of Large Numbers with a Continuum of IID Random Variables," *Journal of Economic Theory* 35, 19-25.

- [25] Kajii A. and S. Morris (1997), "Common p -beliefs: The General Case," *Games and Economic Behavior*, 18, 73-82.
- [26] Kiyotaki, Nobuhiro, and John Moore (1997), "Credit Cycles," *Journal of Political Economy* 105, 211-248.
- [27] Kraay, Aart (2001), "Do High Interest Rates Defend Currencies During Speculative Attack?", World Bank mimeo.
- [28] Milgrom, Paul, and John Roberts (1990), "Rationalizability, Learning and Equilibrium in Games with Strategic Complementarities," *Econometrica*, 58, 1255-1277.
- [29] Monderer, D. and D. Samet (1989), "Approximating Common Knowledge with Common Beliefs," *Games and Economic Behavior*, 1, 170-190.
- [30] Morris, Stephen and Hyun Song Shin (1998), "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks", *American Economic Review*, 88, 3, 587-597.
- [31] Morris, Stephen and Hyun Song Shin (2000), "Rethinking Multiple Equilibria in Macroeconomics," *NBER Macro Annual*.
- [32] Morris, Stephen and Hyun Song Shin (2001), "Global Games - Theory and Applications," in *Advances in Economics and Econometrics*, 8th World Congress of the Econometric Society (M. Dewatripont, L. Hansen, and S. Turnovsky, eds.), Cambridge University Press, Cambridge, UK.
- [33] Murphy, Kevin M., Andrei Shleifer, and Robert Vishny (1989), "Industrialization and the Big Push," *Journal of Political Economy* 97, 5, 1003-26.
- [34] Obstfeld, Maurice (1986), "Rational and Self-Fulfilling Balance-of-Payments Crises", *American Economic Review* 76, 1, 72-81.
- [35] Obstfeld, Maurice (1996), "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review* 40, 3-5, 1037-47.
- [36] Postlewaite, Andrew, and Xavier Vives (1987), "Bank Runs as an Equilibrium Phenomenon," *Journal of Political Economy* 95, 3, 485-91.
- [37] Vives, Xavier (1999), *Oligopoly Pricing: Old Ideas and New Tools*, MIT Press.

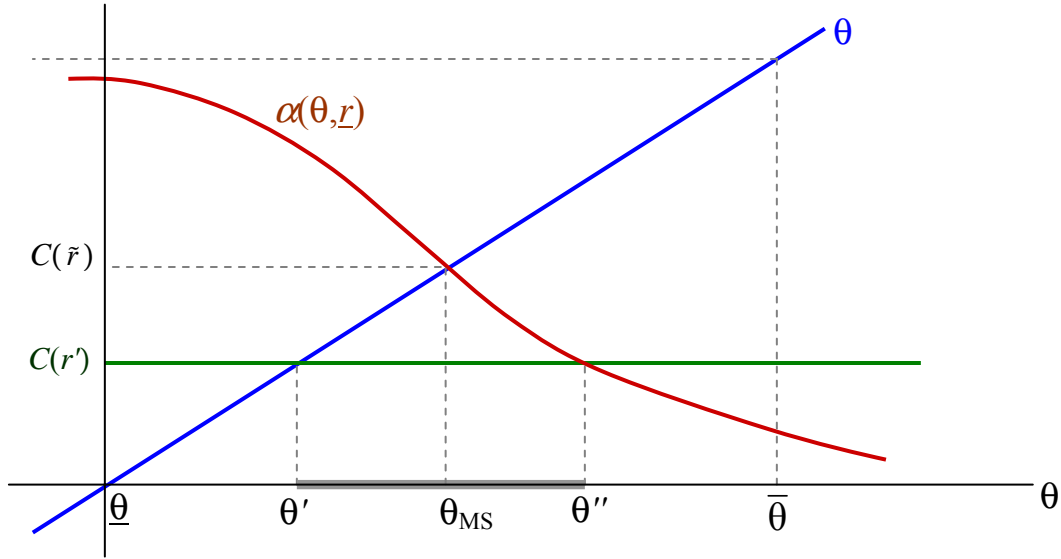


Figure 1

There exists an *inactive policy equilibrium*, in which the central bank sets \underline{r} for all θ , the size of the attack is $\alpha(\theta, \underline{r})$, and devaluation occurs if and only if $\theta < \theta_{MS}$. Any $r' \in (\underline{r}, \tilde{r}]$ is dominated in equilibrium by \underline{r} if and only if $C(r') > \theta$ or $C(r') > \alpha(\theta, \underline{r})$, that is, if and only if $\theta \notin [\theta', \theta'']$. It follows that, for any $\theta \in [\theta', \theta'']$, if the central bank deviates from \underline{r} to r' , the market “learns” that $\theta \in [\theta', \theta'']$. Speculators then coordinate on the same behavior as when $r = \underline{r}$, thus eliminating any incentive for the bank to raise the interest rate.

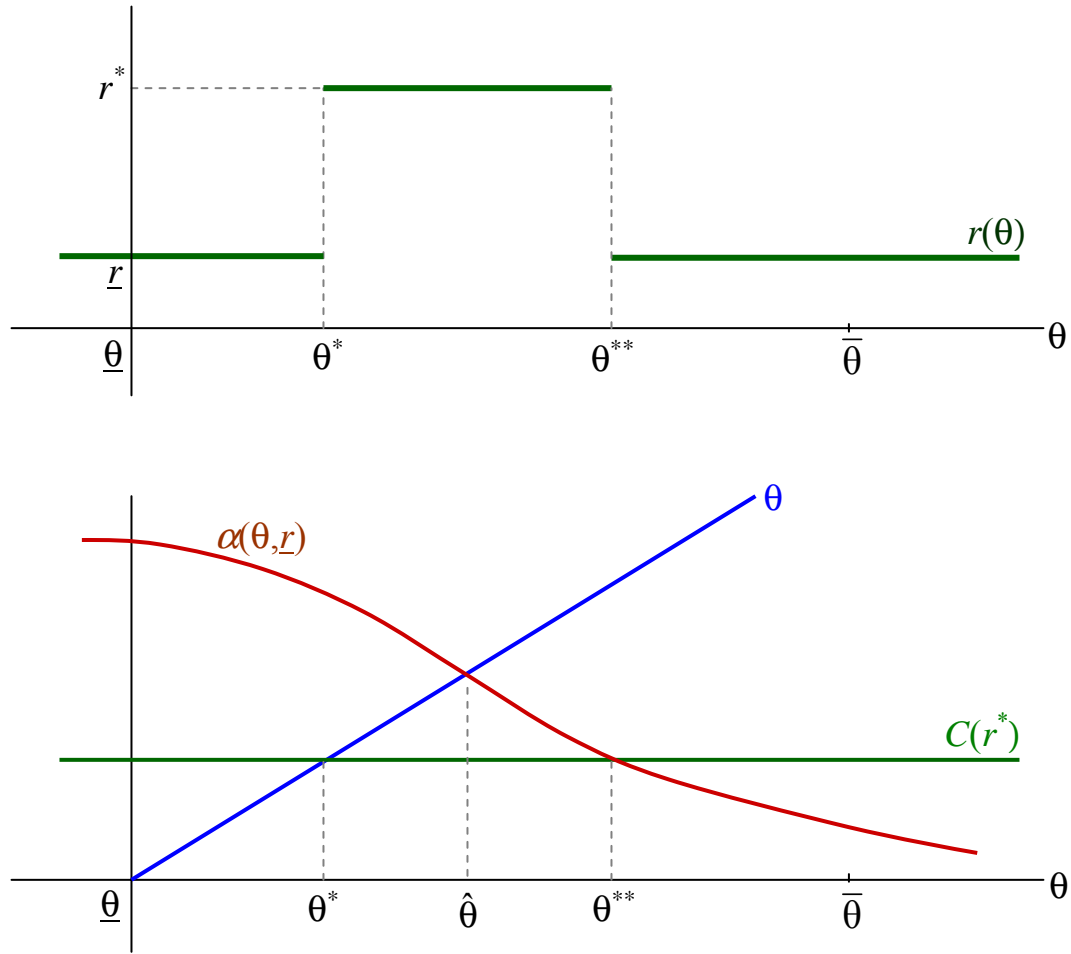


Figure 2

For each $r^* \in (\underline{r}, \tilde{r}]$, there is a *two-threshold equilibrium* in which the interest rate is $r(\theta) = r^*$ if $\theta \in [\theta^*, \theta^{**}]$ and $r(\theta) = \underline{r}$ otherwise, and in which devaluation occurs if and only if $\theta < \theta^*$. When the central bank raises the interest rate at r^* , the market “learns” that $\theta \in [\theta^*, \theta^{**}]$ and coordinates on no attack. When instead the bank sets \underline{r} , speculators attack if and only if their signal is sufficiently low, in which case the size of the attack is decreasing in θ . It follows that it is optimal for the central bank to raise the interest rate at r^* if and only if $C(r^*) \leq \bar{\theta}$ and $C(r^*) \leq \alpha(\theta, \underline{r})$, that is, if and only if $\theta \in [\theta^*, \theta^{**}]$.