The Role of Exclusive Contracts in Facilitating Market Transactions

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Abstract

Vertical contracts between a buyer and a seller often contain exclusivity provisions that restrict their ability to transact with third parties in the future. In this paper we develop a matching model in which matched agents first decide whether or not to adopt exclusivity provisions and then bargain over the price of a good in the presence of private information. We show that by making disagreement more costly exclusivity provisions reduce the risk of inefficient bargaining breakdowns and that agents find it optimal to adopt exclusive contracts in thin markets but not in thick markets. For intermediate levels of market thickness all agents, matched and unmatched, would be better off if courts did not enforce the exclusivity provisions.

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1 Introduction

Vertical contracts between a buyer and a seller often contain exclusivity provisions that restrict the ability of either of them to transact with third parties. The economics literature has focused on two broad explanations for the use of exclusive contracts. One strand of the literature focuses on the anti-competitive effects of such contracts and analyzes to what extent they can restrict entry by potentially more efficient firms (see, for instance, Aghion and Bolton (1987), Rasmusen, Ramseyer, and Wiley (1991), Spier and Whinston (1995), Bernheim and Whinston (1998)). The other strand of the literature has analyzed the conditions under which exclusive contracts mitigate potential hold up problems and can thus be efficiency enhancing (see, for instance, Segal and Whinston (2000)). In this paper we analyze a third potential reason for the use of exclusive contracts. In particular, we argue that agents who commit to such contracts ex ante are able to reduce the risk of inefficient bargaining breakdowns ex post. In other words, we argue that exclusive contracts can mitigate ex post inefficiencies that arise when a buyer and a seller haggle over the sharing of quasi-rents and thus waste time and resources before reaching an agreement.

It has long been argued (see, in particular, Williamson (1985)) that the mitigation of ex post inefficiencies is a key purpose of vertical contracts. To understand how vertical contracts in general, and exclusive contracts in particular, affect ex post inefficiencies it is of course necessary to be specific about what causes the ex post inefficiencies in the first place. In this paper we assume that the ex post inefficiencies are due to the presence of private information. It is well known (see Myerson and Satterthwaite (1983)) that when two agents bargain with each other over the sharing of quasi-rents in the presence of private information they may fail to realize all gains from trade. We argue that by committing to exclusive contracts ex ante, agents may be able to reduce such ex post bargaining inefficiencies.

In particular, we argue that by restricting trade with third parties exclusive contracts reduce the agents' disagreement payoffs which induces them to adopt less aggressive bargaining strategies and thus increases the probability that they reach an agreement. Countervailing this positive effect of exclusive contracts are the lower disagreement payoffs that the agents realize if they do fail to reach an agreement with each other. We show that the positive effect dominates, and the agents therefore adopt exclusive contracts, if the agents' quasi-rents are large and that the negative effect dominates if the quasi-rents are small.

An argument that is very similar to ours has been made less formally in the legal

literature. In particular, Walker (1999) discusses the use of right of first refusal provisions in many commercial transactions. He makes two key points for which he also provides some anecdotal evidence. First, he argues that by granting a right of first refusal to a particular buyer, a seller may drastically reduce the probability that other potential buyers show an interest in, and be willing to make a bid for, the good that is to be transacted. Second, he argues that keeping away third parties may be in the interest of the buyer and the seller since it makes efficient agreements between them more likely.[1] The motivation for the use of rights of first refusal that he presents is very similar to our motivation for the use of exclusive contracts. Indeed he acknowledges that third parties could be kept away, and bilateral bargaining thus be facilitated, through other contracts, for instance some type of exclusive contract. He argues that many agents prefer to use rights of first refusal because "the right of first refusal does an effective job of restraining transferability of interests" and because it "has become a legally acceptable means of discouraging sales, and courts traditionally hostile to restraints on alienability have tended to invalidate more obvious restrictions."

Whether or not courts should enforce exclusive contracts is a controversial question and the answer depends crucially on why agents agree to such contracts in the first place. We therefore analyze whether courts should enforce exclusive contracts if agents adopt them for the reason we focus on in this paper, namely to reduce bargaining inefficiencies. The answer to this question is not immediately obvious, since the commitment to an exclusive contract by one pair of agents, while potentially efficiency enhancing for them, reduces the trading opportunities of other agents. In other words, the adoption of exclusive contracts gives rise to search externalities which might introduce a wedge between privately and socially optimal contractual choices.[2]

To address this issue we consider a matching market in which agents who fail to

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[1]Walker (1999), p. 34: "This part asks why contracting parties would adopt a right of first refusal and forego upside realization potential on the sale of property. The hypothesized answer is that the right of first refusal provides insurance against bargaining breakdown between the contracting parties. In essence the argument runs as follows: The contracting parties do sacrifice some upside potential if outsiders who value the property highly at the time of sale are driven away, but ex ante the parties are more concerned about the possibility that the rightholder will have the highest value at sale time, and that something could go wrong in a negotiation between the seller and the rightholder that would jeopardize the high insider value."

[2]The potentially important welfare implications of different rules regulating the breach of contracts in a search market were first explored by Diamond and Maskin (1979). Although our model also generates search externalities arising form different contract arrangements, our analysis is, however, rather different from theirs. Daimond and Maskin focus on a situation in which bargaining is efficient, whereas in our paper the main purpose of contractual arrangements is to mitigate bargaining inefficiencies.

reach an agreement with their trading partner today may be matched with another agent tomorrow. The level of the quasi-rents generated by a representative buyer-seller match is then determined by the probability that each agent in the pair has of meeting alternative trading partners who are not bound by exclusive contracts. In particular, these quasi-rents are large when the market matching rate is low. We show that when the market matching rate is low agents commit to exclusive contracts to ensure that trade with their current partners take place and large quasi-rents are not dissipated, whereas when the matching rate is high agents find it optimal not to commit to exclusive contracts. Although each agent takes the matching rate as given, in equilibrium this is actually affected by the contractual choices of the agents. In particular, the market is thin and, in the presence of increasing returns to matching, the matching rate is low if a large number of agents have locked themselves in exclusive contracts. This interaction between contractual arrangements and the matching rate gives rise to strategic complementarities which can lead to multiple equilibria and market failures.

We show that if the exogenous number of agents in the market is very small or if the effectiveness of the matching technology is very poor, i.e. if the matching rate is very low for exogenous reasons, the unique market equilibrium has all buyer-seller pairs adopting exclusive contracts. Analogously, if the matching rate is very high for exogenous reasons, the only equilibrium has no agent adopting exclusive contracts. For intermediate levels of market frictions, we have multiple equilibria, in particular one stable equilibrium with all buyer-seller pairs adopting exclusive contracts and one stable equilibrium with nobody doing so. We show that in this region with multiple equilibria, the equilibrium with no exclusive contracts always dominates that with exclusive contracts. In this case, all agents could be made better off if courts did not enforce exclusive contracts.

Two empirical papers have investigated the relationship between market thickness and vertical contracts and the informal arguments they make are closely related to ours. Pirrong (1993) investigates bulk shipping markets and argues that "[...] time and space factors in shipping markets may create 'temporal specificities' that encourage costly haggling between shippers and carriers over quasi rents if they rely on spot contracts. [...] Since buyers and sellers competing for quasi rents may waste resources in costly bargaining, forward contracting should dominate spot contracting when these considerations are important." (Pirrong, 1993, pp.130-134). The contractual solutions to these ex post inefficiencies that he describes vary greatly but they all restrict the ability of the shipper and the carrier to use alternative trading partners (footnote with

contract details). He then describes how the use of such contracts in bulk shipping markets lead to 'contractual specificities': "Contractual specificities exist when a contract between a shipper and carrier expires, but all other vessels the shipper can utilize and all the other shippers the carrier can service are under contract for a considerable time thereafter. This shipper and carrier are in a bilateral monopoly situation during the interval between the end of their contract and the termination of the next expiring agreement." (Pirrong, 1993, p.140) He argues that because of the 'temporal' and 'contractual specificities' shippers and carriers rely mostly on long term contracts in thin markets and on spot trading in thick markets. He provides empirical evidence for this claim.

In the context of the trucking industry Hubbard (2000) describes the inefficiencies that can arise when shippers and carriers negotiate or renegotiate over prices once a truck has been sent to a shipper to be loaded: "[...] shippers may try to extract quasi-rents from carriers by threatening delays. For example, they may refuse to tender loads until carriers grant them rate concessions. Shippers may face similar risks. For example, drivers may refuse to allow their truck to be loaded without concessions." (Hubbard, 2000, p.372) He argues that the associated costs are much greater in thin than in thick trucking markets and finds empirical evidence that agents are more likely to use contracts rather than engage in spot transactions when markets are thin. The contracts that the agents use again vary greatly but all make it more difficult for the agents to transact with third parties (footnote: contract details).

The rest of the paper is organized as follows. In the next section we describe and analyze the main model. First, in section 2 we abstract from the details of matching process and consider the contracting choice of a representative buyer-seller pair. This allows us to clearly highlight the trade-offs that the agents face when they decide on any exclusive contract. Second, in section 3 we explicitly model the matching process to analyze the complementarities in the contracting choice between different agents. The main model that we describe in sections 2 and 3 contains two key assumptions which we relax in sections 4 and 5. The first key assumption is that exclusive contracts are non-renegotiable. This assumption allows us to analyze whether such contracts should indeed be enforced which is an important policy question. Also, we argue below that in some environments it is realistic to assume that contracts cannot be renegotiated. However, it is also interesting to analyze the implications of allowing for renegotiation of the exclusive contract and we do so in section 4. The second key assumption in the main model is that contracts are incomplete, in the sense that agents cannot contract over the bargaining game that takes place ex post. In section

5 we discuss the implications of allowing the agents to contract over the exclusivity clause and the ex post bargaining game. Section 6 concludes.

2 Bilateral Contracting and Bargaining

Before presenting the full model of trade in a matching market, in this section we introduce and analyze the bargaining and contracting problem faced by a representative buyer-seller pair. In particular, we study in which way the expected utilities of the parties to the bargaining game are affected by changes in their disagreement payoffs and thus in which way the parties might decide to manipulate their disagreement payoffs by commiting to exclusive contracts. The next section will embed the bargaining and contracting problems described in this section in a more general model of market exchange in which disagreement payoffs are determined by both contractual arrangements and the probability of locating new trading partners.

2.1 The bargaining game

Consider a buyer and a seller who have been brought together by some matching process – to be described in the next section – and have the opportunity to exchange some good. The seller can produce only one unit of the good at a cost c and the buyer needs only one unit of the good for which he has a valuation equal to v. The cost c is drawn from a uniform distribution with support $[0, 1]$ and the valuation v from a uniform distribution with support $[\alpha, \alpha + 1]$. The distributions of c and v are independent and common knowledge and the parameter $\alpha \in [0, 1]$ is a measurement of the expected gains from trade: the larger α, the larger the expected gains from trade between the buyer and the seller. The realizations of c and v are, however, private information, i.e. they are observed only by the buyer or the seller to whom they refer. Once a buyer and a seller have observed the realization of their own valuation or cost, they bargain over the price of the good according to the rules of a double auction (Chatterjee and Samuelson (1983). In particular, a seller of type c asks for a price $p_s(c)$ and, simultaneously, a buyer of type v offers a price $p_b(v)$. If $p_b(v) \geq p_s(c)$, trade takes place at a price $p(c, v) = [p_s(c) + p_b(v)]/2$. If instead $p_b(v) < p_s(c)$, trade does not take place and the agents realize their disagreement payoffs, which we denote by d_s for the seller and d_b for the buyer, and $d = d_s + d_b$ for the aggregate disagreement payoff. Note that our assumption that the bargaining game is exogenously given and takes the form of a double auction is less restrictive than it might seem. This is the

case since Myerson and Satterthwaite (1983) have shown that the linear equilibrium of the double auction, on which we will focus below, implements the most efficient trading rule if the buyer's and the seller's valuations are independently distributed on $[0, 1]$. Thus, at least for a part of our parameter space, the bargaining game that we assume is the most efficient bargaining game and the agents would agree to it if they could also contract over the bargaining game.

2.2 The contracting problem

The main purpose of these papers is to analyze how agents can use exclusive contracts in order to improve their welfare when they have to trade in the presence of private information. To do so, we assume that, after having being matched but before learning any private information, the buyer and the seller in the representative pair described above can contract over their ability to trade with third parties in the future. In particular we assume that they can contract over the probabilities, e_b for the buyer and e_s for the seller, with which they are allowed to transact with a new counterpart should they fail to trade with each other. We further denote by m the probability that any agent locates a new counterpart in the next period and by $U_{b,2}$ and $U_{s,2}$ the utilities that a buyer and a seller, respectively, expect to attain by transacting with this new counterpart. Note that the matching rate m and the continuation utilities $U_{b,2}$ and $U_{s,2}$ are taken as given by each single agent and will be derived endogenously in the two period matching model introduced in the next section. Therefore a buyer and a seller who have agreed on a contract specifying e_b and e_s bargain over the price of the good to be traded knowing that their disagreement payoffs are $d_b = e_b m U_{b,2}$ and $d_s = e_s m U_{s,2}$, respectively. For future convenience, we also define $U_2 \equiv U_{b,2} + U_{s,2}$. Since agents are risk neutral and liquidity unconstrained, at the contracting stage they choose e_b and e_s to maximize their joint utility, and, if needed, use front payments to share the surplus.

We assume for the time being that exclusive contracts are non-renegotiable.[3] There are two reasons why exclusive contracts may be non-renegotiable, which we discuss in turn. *(i) Court-enforced commitment:* An exclusive contract is obviously non-renegotiable if courts enforce it even if, ex post, the buyer and the seller would both prefer it not to be enforced. Whether or not courts should indeed enforce these contracts is an important policy issue that we will address further below in the paper, when we discuss the normative properties of the market equilibrium of our full match-

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[3]We relax this assumption in section 4.

ing model. *(ii) Technological commitment:* An exclusive contract can also be non-renegotiable to the extent that it involves an irreversible technological commitment that makes it more difficult for the agents to transact with third parties. Economic agents can achieve such commitments by, for example, locating their plants near each other or investing in machinery which is very specific to their relationship. In this case the investment can be interpreted as determining the probabilities e_s and e_b that it is possible for the seller or the buyer to reverse their technological choice and transact with third parties in the future. The results of the model that we present in this section do not depend on whether exclusive contracts are non-renegotiable because of court-enforced or technological commitment. However, the results of the more general model that we present in section 5, in which we allow agents to contract over both the exclusive contract and the bargaining game used to transact the good, depend crucially on the type of commitment.

2.3   Analysis

We now solve the contracting and bargaining game by backward induction. We first find and discuss the solution to the bargaining game for given contractual decisions e_b and e_s, and thus for given disagreement payoffs d_b and d_s, and then find the e_b and e_s contractually chosen by the parties to maximize their joint utility.

It is well known – see Chatterjee and Samuelson (1983) – that the double-auction game has a plethora of Bayesian Nash equilibria. Here we focus on linear equilibria in which the price offered or asked for by an agent is a linear function of her own type, i.e. on equilibria in which $p_s(c) = \beta_s + \rho_s c$ and $p_b(v) = \beta_b + \rho_b v$, where β_i and ρ_i, $i \in \{s, b\}$, are type-independent parameters.

An agent who knows her own valuation of the good, but not the valuation of her counterpart, will offer or ask a price that maximizes her expected utility given the distribution of all the possible prices offered or asked by her counterpart and her own outside option. Denote by $U_s(c, p_s)$ the expected utility of a seller who knows that she is of type c and asks for a price p_s. Analogously, $U_b(v, p_b)$ denotes the expected utility of a buyer who knows that she is of type v and offers a price p_b. We have that

$$U_s(c, p_s) = \pi(p_s \leq p_b) \left[\frac{p_s + \mathrm{E}\left[p_b | p_s \leq p_b\right]}{2} - c - d_s \right] + d_s, \tag{1}$$

$$U_b(v, p_b) = \pi(p_s \leq p_b) \left[v - \frac{\mathrm{E}\left[p_s | p_s \leq p_b\right] + p_b}{2} - d_b \right] + d_b. \tag{2}$$

7

where $\pi(p_s \leq p_b)$ is the probability that $p_s \leq p_b$ – i.e. that trade takes place – and $E[p_i|p_s \leq p_b]$ is the expected value of p_i conditional on trade taking place.

The (linear) equilibrium strategies of the double auction are prices $p_s^*(c) = \beta_s + \rho_s c$ and $p_b^*(v) = \beta_b + \rho_b v$, such that

$$p_s^*(c) = \arg\max_{p_s} U_s(c, p_s),$$
$$p_b^*(v) = \arg\max_{p_b} U_b(v, p_b).$$

The following proposition describes the equilibrium strategies in more detail.

Proposition 1 *The strategies*

$$p_s^*(c) = \begin{cases} \tilde{p}_s(c) & \text{if } \tilde{p}_s(c) \geq \tilde{p}_b(a), \\ \tilde{p}_b(a) & \text{otherwise,} \end{cases} \qquad p_b^*(v) = \begin{cases} \tilde{p}_b(v) & \text{if } \tilde{p}_b(v) \leq \tilde{p}_s(1), \\ \tilde{p}_s(1) & \text{otherwise,} \end{cases}$$

$$\tilde{p}_s(c) = \frac{1}{12}\left[3(1 + \alpha) + 9d_s - 3d_b + 8c\right], \tag{3}$$

$$\tilde{p}_b(v) = \frac{1}{12}\left[(1 + \alpha) + 3d_s - 9d_b + 8v\right] \tag{4}$$

constitute the unique linear Bayesian Nash equilibrium of the double auction played by a seller of type c and a buyer of type v.

Proof. In the Appendix. ∎

Note that, given the strategies outlined in Proposition 1, some very good types – i.e. low cost sellers with $c \leq \frac{1}{4}[-1+3\alpha-3d]$ and high valuation buyers with $v \geq \frac{1}{4}[5+\alpha+3d]$, where $d \equiv d_b + d_s$ is the aggregate disagreement payoff – always trade. On the contrary, some very bad types – i.e. high cost sellers with $c \geq \frac{3}{4}[(1 + \alpha) - d]$ and low valuation buyers with $v \leq \frac{1}{4}[(1 + \alpha) + 3d]$ – never trade. All other 'middle' types ask for or offer a price that implies some positive, but less than one, probability of trade. A necessary and sufficient condition for trade to take place between a seller of type c and a buyer of type v is given in the following lemma.

Lemma 2 *A seller of type c and a buyer of type v trade if and only if*

$$v - c \geq d + \frac{1}{4}\left((1 + \alpha) - d\right). \tag{5}$$

Proof. The necessary and sufficient condition in (5) can be obtained in a straightforward manner by taking into account that trade takes place if and only if $p_b^*(v) \geq p_s^*(c)$, where $p_b^*(v)$ and $p_s^*(c)$ are given in Proposition 1. ∎

Lemma 2 has some important implications for the rest of our analysis which is worth pointing out here. First, given the existence of private information, bargaining is inefficient in the sense that some trades that would be efficient – i.e. trades between agents for which $v - c \geq d$ – might not be effected in equilibrium (Chatterjee and Samuelson (1983)). Second, the probability that any pair of agents trade depends only on the sum of their disagreement payoffs, d, and not on their distribution between the buyer and the seller. Finally, the probability that any pair of agents trade is decreasing in d, as can be seen from the fact that the right hand side of (5) is increasing in d. This property of the bargaining between sellers and buyers will play a crucial role in the rest of our analysis and owes to the fact that, when disagreement is less costly, agents adopt more aggressive bargaining strategies which imply a higher probability of trade breakdowns.

Since in the following sections we will allow agents to sign contracts that influence the value of their disagreement payoffs in a market equilibrium, it is important to characterize how the expected utility of these agents varies with the value d of these disagreement payoffs. In particular, given that contracting with side payments aims at maximizing the aggregate utility of a buyer-seller match, we are particularly interested in determining the effects of changes in d on this aggregate utility. The expected aggregate utility of a representative buyer-seller pair can be written as

$$U(d) = E_{c,v} \left[U_s(c, p_s^*) + U_b(v, p_b^*) \right]. \tag{6}$$

Using the results of Lemma 2 in equations (1) and (2), taking expectations over v and c, and adding one obtains

$$U(d) = d + \mathrm{E}_{c,v} \left[(v - c - d) \, \pi \left(v - c \geq \frac{3}{4}d + \frac{1}{4}(1 + \alpha) \right) \right]. \tag{7}$$

Equation (7) shows that a marginal change in the aggregate disagreement payoff d affects the aggregate expected utility U of the agents in two opposing ways. On the one hand, for a given probability π of trade, a marginal increase in d increases U, as it allows the agents to realize a higher payoff when they do not trade. On the other hand, this marginal increase in d induces the seller and the buyer to bargain more aggressively and thus reduces the probability π that ex-post efficient trades take place,

9

thereby reducing U. This trade-off between the probability of trade and disagreement payoffs is the key trade-off considered by agents when deciding the optimal degree of contractual commitment in the next sections. The following lemma clarifies when either of these two opposing effects dominates.

Lemma 3 *U is convex in the aggregate disagreement payoff d.*

Proof.   In the Appendix.   ■

The solid line in Figure 1 represents the typical shape of U as a function of the aggregate disagreement payoffs d.[4] The intuition underlying the possibility that U is first decreasing and then increasing in the aggregate disagreement payoff can be briefly summarized as follows. When the disagreement payoffs are very low, the parties are very likely to reach an agreement when bargaining over the price of the good to be traded, and are therefore very unlikely to have to resort to their disagreement payoffs. A marginal increase in d just causes a reduction in the probability of trade without conferring large benefits to the parties. However, when d is large, the parties are very likely to disagree and thus are very likely to realize d. In this latter case a marginal increase in d is therefore beneficial. The fact that U is convex in the aggregate disagreement payoffs d has important implications for the optimal choice of contractual structure.

We now solve for the optimal degree of exclusivity that the parties decide to adopt in their contracts. Recall that, for given e_b and e_s, the aggregate disagreement payoff is given by $d = m(e_b U_{b,2} + e_s U_{s,2})$. Since the agents are risk-neutral and not liquidity constrained, they choose contracts that specify e_s and e_b in order to maximize $U(d)$ as given in (7). Since U is convex in d, each pair of matched agents acts as to either maximize or minimize the aggregate disagreement payoff d. This implies that every pair chooses to contractually set either $e_b = e_s = 1$ or $e_b = e_s = 0$. For expositional convenience we will refer to the former equilibrium outcome as to an equilibrium with no exclusive contracts (NC), and to the latter outcome as to an equilibrium with exclusive contracts (EC). The disagreement payoff if the parties agree to an exclusive contract is $d_{EC} = 0$, since in this case $e_b = e_s = 0$. If instead the parties decide to write no exclusive contract, that is to set $e_b = e_s = 1$, the disagreement payoff is entirely determined by the value of market search in the next period and is given

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[4]Note that U could also be everywhere increasing in d, in which case the discussion that follows should be adjusted accordingly.
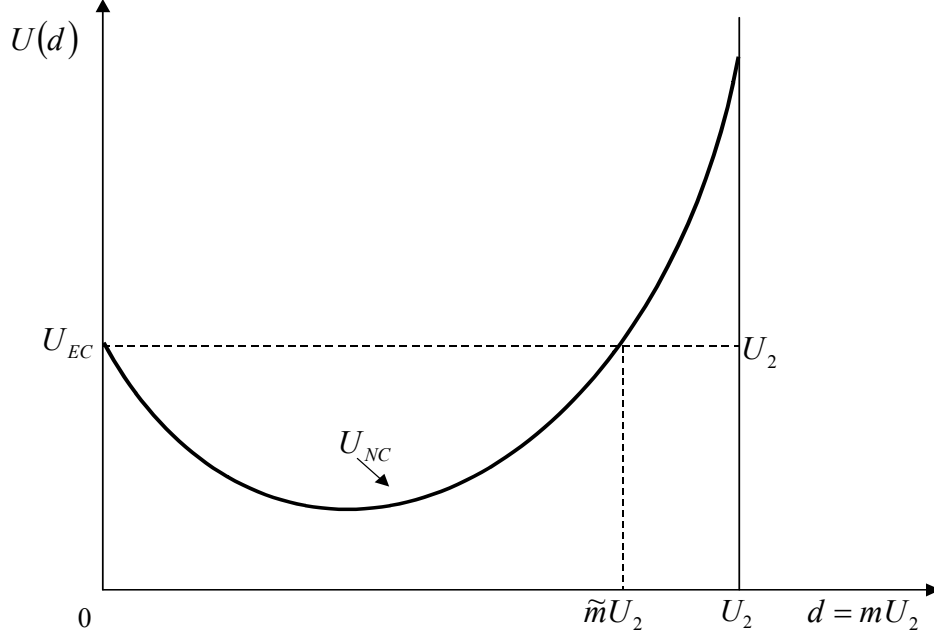
Figure 1: Expected aggregate utility under exclusive contracts and no contracts.

by $d_{NC} = m(U_{b,2} + U_{s,2}) = mU_2 > 0$. For any pair of agents, the aggregate expected utility from committing to an exclusive contract, U_{EC}, can be obtained by substituting $d_{EC} = 0$ for d in (7), and the aggregate expected utility from not committing to any contract, U_{NC}, can be obtained by substituting $d_{NC} = mU_2$ for d in the same equation. In equilibrium, agents write exclusive contracts if and only if $U_{EC} > U_{NC}$. The following lemma shows that exclusive contracts are chosen in equilibrium if and only if the matching rate, m, is sufficiently low. In order to prove the lemma we assume for the time being that $U(d = 0) = U_2$, a fact that will be always true in the two-period matching model that we introduce in the next section.

Lemma 4 *There exists $\tilde{m} = \tilde{m}(c, \alpha) \in [0, 1]$ such that all agents commit to exclusive contracts if $m \leq \tilde{m}$ and to no contract if $m \geq \tilde{m}$.*

Proof.   A formal proof is given in the Appendix. An intuitive proof relying on a simple graphical argument is however presented below. ∎

An intuitive proof of Lemma 4 can be followed by looking at Figure 1. The solid line represents the utility U_{NC} of a pair of agents who did not commit to an exclusive

11

contract as a function of their aggregate disagreement payoff mU_2. It is important to notice that the intercept of the U_{NC} curve at $d = 0$ is equal to the expected utility U_{EC} that the agents would obtain if they had signed an exclusive contracts and their joint disagreement payoff were therefore $d = 0$ for all m. The value of U_{EC} is represented by the dashed horizontal line in the figure. It is straightforward to see that for $m < \tilde{m}$ it must be that $U_{EC} > U_{NC}$ and the parties therefore adopt exclusive contracts. The opposite holds if instead $m > \tilde{m}$. Clearly, in the case, not depicted in the figure, in which U_{NC} is everywhere increasing in m, we would have $\tilde{m} = 0$.

3 Market Equilibrium

We now embed the bargaining problem introduced in the previous section in a market environment with many buyers and sellers who need to go through a costly process of search in order to locate a counterpart. The main insight provided by this section is that the effectiveness of the search process and the contractual structure adopted by the parties interact to endogenously determine the value d of the disagreement payoffs for any pair of agents. This two way interaction between the endogenous market matching rate and the equilibrium contractual structure gives rise to strategic complementarities in the choice of contracts, which in turn makes multiple equilibria and market failures possible.

3.1 The matching model

We consider a market that lasts for two periods, denoted by $t = 1, 2$. At the beginning of period 1 there is a unit mass of sellers and a unit mass of buyers who are risk-neutral, liquidity unconstrained, and do not discount the future. In this first period, each buyer is matched with a seller, and vice versa, with probability $a < 1$. After having been matched a buyer and a seller contract over their ability to trade with third parties in period 2 and then bargain over the price of the good, as discussed above. If trade takes place both agents leave the market permanently. If instead the agents fail to trade the good in period 1, they can re-enter the market in period 2 and, provided they are contractually allowed to do so, are matched with a new trading partner with probability m. This probability is obtained by assuming that the total number of matches M in period 2 is determined by the matching function $M = \phi M(N_b, N_s)$, where $N_b = N_s = N$ is the (endogenous) number of buyers and sellers searching in the market in period 2 and ϕ is a parameter measuring the effectiveness of the

matching process. We assume that this matching function is homogeneous of degree $1 + \sigma$, $\sigma \geq 0$, and we normalize $M(1, 1) = 1$. If matched in period 2, agents submit again bids according to the rules of the double auction and, since the market closes at the end of this period, if they fail to trade they realize their outside option that we normalize to zero. The agents' expected utilities from the bargaining game being played in period 2 depends therefore only on the expected gains from trade α, and, in keeping with the notation used in the previous section, we denote them by $U_{b,2}$ and $U_{s,2}$ and their sum by $U_2 = U_{b,2} + U_{s,2}$. Furthermore notice that these expected utilities in period 2 are the same as the expected utilities in period 1 when the disagreement payoffs are equal to zero, that is $U_1(d = 0) = U_2$ as we had assumed until now. Given this structure, the contracting and bargaining problem faced by the agents in period 1 is exactly the same as the one discussed in section 2. However, we now derive the value of m endogenously.

3.2   Market equilibria and strategic complementarities

We now close the model analyzed in sections 2 by solving for the endogenous value of the matching rate m and characterize the type of contracts that we observe in the general equilibrium of this market.

Since the matching function is homogeneous of degree $1 + \sigma$, the probability that each agent meets a trading partner is

$$m = \frac{M}{N} = \phi N^\sigma, \tag{8}$$

If there are constant returns to matching, i.e. if $\sigma = 0$, this probability is independent of the number of agents searching. If instead $\sigma > 0$, there are increasing returns in matching and the probability that any given agent finds a trading partner in period 2 is increasing in the number N of agents searching on each side of the market. The number of agents on side j of the market in period 2 is given by the number $(1 - a)$ of agents who have not found a match in period 1 plus the number $a(1 - \pi)e_j$ who were matched but have failed to trade the good in period 1 and have not committed to an exclusive contract. We can therefore write the matching rate in period 2 for an agent on side i of the market as

$$m_i = \phi[(1 - a) + a(1 - \pi)e_j]^\sigma. \tag{9}$$

The matching rate in period 2 if all agents have signed an exclusive contract in

period 1 is then obtained by setting $e_j = 0$, $j \in \{b, s\}$, in (9) above and is given by

$$m_{EC} = \phi(1 - a)^\sigma. \tag{10}$$

If instead no agent has committed to an exclusive contract in period 1, i.e. if $e_j = 1$, $j \in \{b, s\}$, the matching rate in period 2 is

$$m_{NC} = \phi(1 - a\pi_{NC})^\sigma, \tag{11}$$

where π_{NC} is the probability that a representative agent trades in period 1 if he has signed no exclusive contract.[5]

Making use of equation (5) this probability can be written as

$$\pi_{NC} = \begin{cases} \frac{1}{4}[1 + 3(\alpha - m_{NC}U_2)] & \text{if } m_{NC}U_2 \le \alpha - \frac{1}{3}, \\ \frac{3}{4}[1 + (\alpha - m_{NC}U_2)] & \text{otherwise.} \end{cases} \tag{12}$$

Equations (11) and (12) together determine the matching rate m_{NC} in the case in which the parties did not write any exclusive contract in period 1.[6] Note that for any given value of ϕ we have that $m_{NC} \ge m_{EC}$, since when the parties do not adopt exclusive contracts there is a larger number of agents searching in period 2, which improves the prospects of finding a trading partner in period 2 for all agents. This fact gives rise to strategic complementarities in the agents' choice of contracts. To see this consider the contracting problem faced in period 1 by a representative pair of matched agents. If this pair expects all other agents not to adopt exclusive contracts in period 1, each agent in the pair knows that there is a high probability that she will be easily matched in period 1, should she fail to trade with her current partner. In Lemma 4 we have shown that when agents expect the matching rate in period 2 to be high, in particular higher that $\tilde{m}$, they decide not to adopt an exclusive contract. However, if the agents in our representative pair expect all other agents to adopt exclusive contracts in period 1, they expect a low matching rate in period 2, and they therefore

---

[5]Note that there is a third possibility of interest, namely a situation in which a share $\gamma \in (0, 1)$ of agents chooses not to sign exclusive contracts and a share $(1 - \gamma) \in (0, 1)$ chooses to write exclusive contracts. However, as we discuss below in Proposition 5, an equilibrium with a positive number of agents adopting each of these arrangements is locally unstable and therefore less interesting than the two degenerate equilibria on which we focus.

[6]It can be shown that the matching rate m is uniquely determined for any value of ϕ and increasing in the latter. This does not depend on the particular simple bargaining game, and thus on the particular expression for π_{NC}, adopted in this section, but is always true provided that π_{NC} is decreasing in m and that $\pi_{NC} \in (0, 1)$ for all m.

choose to also adopt an exclusive contract. The presence of complementarities makes multiple equilibria possible, that is two economies with exactly the same parameters may have different equilibrium contractual arrangements. The following proposition describes precisely when this is the case.

Proposition 5 *There exist ϕ' and ϕ'', $\phi'' \geq \phi'$, such that*

1. *for $\phi < \phi'$ there exists a unique equilibrium in which all agents set $e_b = e_s = 0$ (exclusive contracts),*

2. *for $\phi > \phi''$ there exists a unique equilibrium in which all agents set $e_b = e_s = 1$ (no exclusive contracts).*

3. *for $\phi' \leq \phi \leq \phi''$ there exist*

   - *a (locally stable) equilibrium in which all agents set $e_b = e_s = 0$ (exclusive contracts),*

   - *a (locally stable) equilibrium in which all agents set $e_b = e_s = 1$ (no exclusive contracts)*

   - *a (locally unstable) equilibrium in which a fraction $\tilde{\gamma} \in (0,1)$ of agents adopts no exclusive contracts and a fraction $(1-\tilde{\gamma}) \in (0,1)$ of agents adopts exclusive contracts.*

If $\sigma = 0$, we have that $\phi' = \phi''$. If $\sigma > 0$ we have $\phi'' > \phi'$ and multiple equilibria co-exist.

Proof. We first prove the existence of the equilibria in which all agents take the same action, we then analyze the case in which strictly positive shares of agents take different actions. Define ϕ' and ϕ'' as those values of ϕ for which

$$\tilde{m} = m_{NC}(\phi') = m_{EC}(\phi''),$$

where $\tilde{m}$ is defined in Lemma 4, $m_{NC}(\cdot)$ in equations (11) and (12), and $m_{EC}(\cdot)$ in equation (10). Since, for all ϕ, $m_{NC} \geq m_{EC}$ and since both matching rates are increasing in ϕ, it must be the case that $\phi'' \geq \phi'$. Furthermore, for $\phi \leq \phi''$ we have that $m_{EC} \leq \tilde{m}$, which from Lemma 4 implies that the equilibrium is characterized by exclusive contracts. Analogously, for $\phi \geq \phi'$ we have that $m_{NC} \geq \tilde{m}$, which from Lemma 4 implies that the equilibrium is characterized by no exclusive contracts.
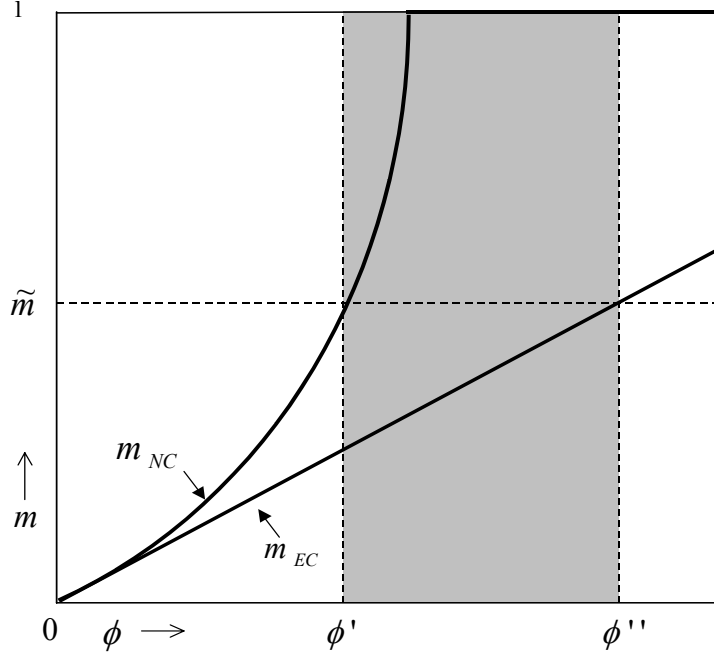
Figure 2: Endogenous matching probability and equilibrium contractual choice.

Next, consider the case in which a fraction $\gamma \in (0,1)$ of the agents does not adopt exclusive contracts, with the rest adopting exclusive contracts. For this to be an equilibrium, agents must be indifferent between the two alternatives. We know from Lemma 4 that this is the case if $m = \tilde{m}$. Since the number of agents searching in period 2 is given by $N = (1 - a) + a(1 - \pi)\gamma$, using equation (8) we have that the matching rate in this case is given by

$$m = \phi[(1 - a) + a(1 - \pi)\gamma]^{\sigma}, \tag{13}$$

where π is given in equation (12) with m in the place of m_{NC}. For $\tilde{\gamma}$ to be an equilibrium it must solve (13) above and (12) for $m = \tilde{m}$, where $\tilde{m}$ is given in Lemma (4). It can furtheermore be proved that this equilibrium is not stable to small perturbations of γ, since (as long as π does not fall too much, to be chekcked) an increase in γ rbings about an increase in m above $\tilde{m}$ which in turn induces even more agents not to sign an exclusive contract, raising γ even more. ∎

The intuition underlying the results and the proof of Proposition 5 is represented

16

in Figure 2. The two curves represent the relationship between the endogenously determined matching rate m and the parameter ϕ with and without exclusive contracts.[7] In particular, the lower curve, denoted by m_{EC}, represents the equilibrium matching rate when all agents adopt exclusive contracts, whereas the upper curve, denoted by m_{NC}, represents the equilibrium matching rate when no agent adopts exclusive contracts . The horizontal line $m = \tilde{m}$ represents the threshold value of the matching rate at which agents switch from exclusive contracts to no exclusive contracts, as derived in Lemma 4. Consider first the shaded region in which $\phi' \leq \phi \leq \phi''$. At any given ϕ in this region both an equilibrium with exclusive contracts and an equilibrium with no exclusive contracts exist. If all agents choose not to adopt exclusive contracts in period 1, the probability of trade π_1 in this period is low, a large number of agents searches the market in period 2, and the matching rate m_{NC} is high, in particular $m_{NC} > \tilde{m}$. Therefore any given pair of agents holding these expectations about the behavior of the rest of the agents in the economy also chooses not to adopt exclusive contracts, which is therefore an equilibrium outcome. An analogous reasoning applies if all agents adopt exclusive contracts, in which case $m_{EC} < \tilde{m}$ and the adoption of exclusive contracts by all agents is an equilibrium. The presence of these complementarities implies that, for intermediate levels of ϕ, ex-ante identical economies can have very different equilibrium contractual arrangements.

Also note that if $\phi < \phi'$, then the adoption of exclusive contracts is the unique equilibrium, as both m_{NC} and m_{EC} are smaller than $\tilde{m}$. In this case, notwithstanding the presence of complementarities, the level of market frictions is simply too high for an equilibrium with no exclusive contracts to be sustainable. If instead $\phi > \phi''$, then the unique equilibrium has no exclusive contracts, owing to the opposite logic of the case discussed above.

It is important to note that when multiple equilibria exist, they are Pareto-rankable. In particular, as the following proposition establishes, the equilibrium with no exclusive contracts Pareto dominates the equilibrium with exclusive contracts, since all agents have higher expected welfare at the time of the opening of the market in the former than in the latter case.

Proposition 6 *For $\phi' \leq \phi \leq \phi''$, social welfare is higher in the equilibrium with no exclusive contracts than in the equilibrium with exclusive contracts.*

---

[7]Note that, besides being a measure of the efficiency of the matching technology for given number of agents searching, when $\sigma > 0$ the parameter ϕ can also be interpreted as a measure of the exogenously given number of initial participants in this market. In other words, it is appropriate to interpret high values of ϕ as indicating a thick market.

Proof. In period 1 a representative agent can be either matched, which happens with probability a, or unmatched, which happens with probability $(1-a)$. If an agent is unmatched in period 1, he can expect to be matched in period 2 with probability m and to obtain utility U_2. As proved in Proposition 5, m is higher in the equilibrium without exclusive contracts than in that with exclusive contracts, i.e. $m_{NC} > m_{EC}$. Therefore the expected utility of those agents who are not matched in period 1 is higher in the equilibrium without exclusive coontracts than in that with exclusive contracts.

We now prove that, for $\phi \in [\phi', \phi'']$, also the expected utility of an agent who is matched at the beginning of period 1 is higher in an equilibrium without exclusive contracts than in one with exclusive contracts, or in other words that $U_{1,NC}(m_{NC}) > U_{1,EC}(m_{EC})$, where the notation makes explicit the possibility that the matching rate in the two equilibria could be different. First, note that if the parties write an exclusive contract, their disagreement payoffs in period 1 are equal to zero and their expected utility in period 1 is the same as their expected utility in period 2, i.e. $U_{1,EC}(m_{EC}) = U_2$. U_2 is therefore the expected utility obtained by every agent who is matched in period 1 in an equilibrium with exclusive contract, independently of the matching rate m prevailing in period 2. We know from Lemma 4 that in an equilibrium with no exclusive contracts it must be $U_{1,NC}(m_{NC}) > U_{1,EC}(m_{NC}) = U_2$. This establishes that for all ϕ for which both equilibria exist, it must be $U_{1,NC}(m_{NC}) > U_2 = U_{1,EC}(m_{EC})$, and therefore that every agent is better off in the equilibrium without exclusive contracts than in that with exclusive contracts.

An inutitive illustration of the proof is given in Figure 3. The solid curve represents the expected utility of a pair of agents as a function of the matching rate when they are not bound by an exclusive contract. Consider a given $\phi \in [\phi', \phi'']$. We know from Proposition 5 that for this ϕ we have $m_{EC} < \tilde{m} < m_{NC}$, as in Figure 3. The utility level $U_{1,NC}$ associated with m_{NC} is actually the expected utility level enjoyed by every agent in the equilibrium with no exclusive contracts. The expected utility of a pair of matched agents who agree to an exclusive contract when everybody else chooses an exclusive contract is instead given by the horizontal line $U_{1,EC} = U_2$. It is therefore straightforward to see that the expected utility of a pair of matched agents is higher in an equilibrium without exclusive contracts than in one with exclusive contracts. ∎
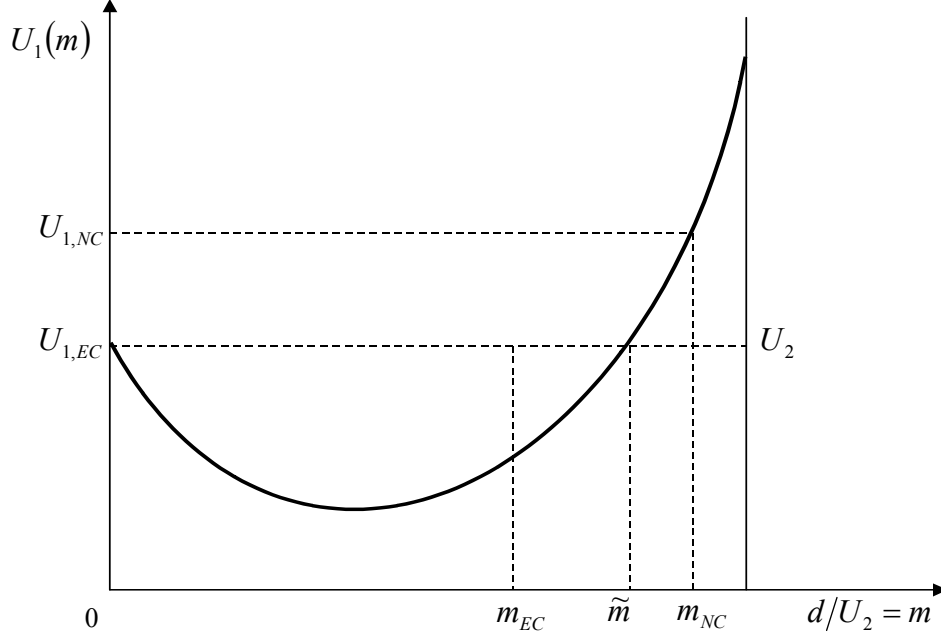
Figure 3: Welfare comparisons with multiple equilibria.

4 Renegotiable Exclusive Contracts and Break-up Fees

We have so far assumed that the agents can commit not to renegotiate exclusive contracts ex post. While this may be realistic in some environments, it clearly is not in others. In this section we therefore study the implications of allowing for ex post renegotiation of exclusive contracts.

Note that if agents do not trade in period 1 they will always find it optimal to renegotiate the exclusive contract. The expectation that any exclusive contract will not be enforced ex post makes its adoption of no use ex ante. However, in this section we show that, as long as there are some positive search costs, agents can credibly reduce their disagreement payoffs by including appropriately chosen break up fee provisions in the contracts.

In particular, we maintain the same model as that presented in the previous section, but we introduce two additional assumptions. First, we assume that the agents can include in their exclusive contract a provision that in case an agent decides to break

the contract and attempt to trade with a new trading partner in period 2, he or she has to pay a break up fee to the other agent. We denote by f_b the break up fee that the buyer has to pay to the seller and by f_s the break up fee that the seller has to pay to the buyer. We allow for ex post renegotiation of these initial break up fees and assume that agents Nash bargain over the sharing of the surplus deriving from renegotiation. We assume that the buyer and the seller have equal bargaining power and that they first Nash bargain over the buyer's break up fee and then over the seller's break up fee. Second, we assume that in order to be matched in period 2 agents must pay a search cost of c in period 1. Clearly the model is only of interest if, in the absence of any exclusive contract, the agents find it profitable to incur the search cost. To ensure that this is the case, and to facilitate comparisons with the previous sections, we take c to be very small.

As in the previous section, if $m > \tilde{m}$ agents do not want to write any contract that restricts their ability to trade with third parties, and the present section has nothing to add in this case. However, if $m < \tilde{m}$, agents find it optimal to credibly commit ex ante not to trade with third parties in the future: this section discusses how this can be achieved in the absence of a commitment not to renegotiate exclusive contracts.

Given that if a pair of agents do not trade in period 1 they always scrap the exclusivity contract forbidding them to trade in period 2, the only way for agents to credibly reduce their disagreement payoff in period 1 is to make sure that no agent in the pair pays the search cost in this period so that no agent will have the opportunity to trade in period 2. This can be done by specifying in the initial contract break up fees that are robust to ex post renegotiation and sufficiently high to make the expected value of search in period 1 negative for both the buyer and the seller. Denote by f_i^* the break up fee that would result after the agents have had the possibility of renegotiating the initial break up fee f_i. This break up fee f_i^* prevents search by agent i if and only if

$$m(U_{i,2} - f_i^*) - c < 0,$$

or, after rearranging terms,

$$f_i^* > U_{i,2} - \frac{c}{m}. \tag{14}$$

If the post-renegotiation break up fees satisfies the condition above for $i = b, s$, neither agent pays the search cost c and the disagreement payoffs of the bargaining game in

period 1 are equal to zero for both agents. In other words, the agents can replicate the outcome they could achieve if non-renegotiable exclusive contracts were available.

We still need, however, to determine which initial break up fees f_i would give rise to the post renegotiation break up fees f_i^* that satisfy the condition in (14). The relationship between initial and post-renegotiation break up fees is illustrated in the following lemma.

Lemma 7 *For a given initial break up fee f_i, $i \in \{b, s\}$, the post renegotiation break up fee f_i^* is*

$$
f_i^* = \begin{cases} f_i & \text{if } f_i \leq U_{i,2}, \\ U_{i,2}/2 & \text{otherwise.} \end{cases} \tag{15}
$$

Proof. Assume that the initial break up fee owed by agent i is $f_i > U_{i,2}$ and that agent i has paid the search cost in period 1. Assume further that agents i and j did not trade in period 1 and can renegotiate their contract at the beginning of period 2. If the agents agree to renegotiate the break up fee and agent i searches for an alternative trading partner, the expected aggregate utility accruing to agents i and j is $mU_{i,2}$. The agents' threat points are both zero since, if the break-up fee is not renegotiated, agent i will not search for another trading partner and therefore not have to pay the break-up fee. Given these threat points, Nash bargaining therefore splits the total gains from trade $mU_{i,2}$ equally among the agents, implying a post-renegotiation break up fee of $f_i^* = U_{i,2}/2$. If instead the initial break up fee is $f_i \leq U_{i,2}$, agent i searches for an alternative trading partner even if the break-up fee is not renegotiated. There are then no gains from renegotiation and thus $f_i^* = f_i$. ∎

Thus, to maximize the 'punishment' for searching for alternative trading partners, i.e. to maximize the effective break up fee f_i^*, the agents need to agree to an initial break up fee that is neither too high nor too low, in particular, they need to agree to $f_i = U_{i,2}$. Specifying a higher initial break up fee is going to lead to a lower effective break up fee. Essentially, initial break up fees that are too high will get renegotiated downwards since the agent who is meant to pay them can credibly threaten not to search. Lower break up fees do not get renegotiated since the threat not to search is not credible. Thus the relationship between f_i and f_i^* is non-monotonic, a lower f_i can lead to a higher f_i^*.

If the parties want to reduce their disagreement payoffs to zero, which is the case if $m < \tilde{m}$, they have to specify initial break up fees f_i which lead to post-renegotiation break up fees f_i^* satisfying (14). Thus, if $m < \tilde{m}$ the agents agree on any break-up fee that satisfies $f_i \in [U_{i,2} - c/m, U_{i,2}]$.

5 General Bargaining Game

We now extend the model presented section 2.1 by allowing the buyer and the seller to contract over the ex post bargaining game and also by allowing for more general distributions of the valuation v and cost c.

More specifically, the general model that we consider in this section differs from the model of section 2.1 in four ways. First, we now assume that agents can contract not only over their ability to trade with third parties, but also over the ex post bargaining game that will be used to determine the price of the good. The agents can choose any bargaining game that satisfies the balanced budget constraint, i.e. in which the payments made by the buyer equal the payments received by the seller. Risk-neutral and liquidity unconstrained agents have clearly an incentive to choose the most efficient bargaining game, i.e. the bargaining game that maximizes their joint surplus.

Second, we allow for more general distributions of the buyers' valuations v and of the sellers' costs c. In particular, we assume that v has support $[\underline{v}, \overline{v}]$, a cumulative density function $F_b(v)$, and a density function $f_b(v)$. Similarly, the distribution of the seller's cost c has support $[\underline{c}, \overline{c}]$, a cumulative density function $F_s(c)$, and a density function $f_s(c)$. The density functions $f_b(v)$ and $f_s(c)$ are continuous and strictly positive and the distributions satisfy the monotone hazard rate conditions, which is the case if $\frac{f_b(v)}{1 - F_b(v)}$ is increasing and $\frac{f_s(c)}{F_s(c)}$ is decreasing.

Third, the bounds on the disagreement payoffs must now satisfy $\overline{d} \equiv mU_2 < \overline{v} - \underline{c}$ and $\underline{d} \equiv 0 > \underline{v} - \overline{c}$. The first restriction ensures that, at least sometimes, there are gains from trade while the second restriction ensures that, at least sometimes, there are no gains from trade. These restrictions simplify the exposition and it is trivial to analyze the implications of relaxing them.

Finally, we now assume that the agents can decide whether or not to participate in the ex post bargaining game after learning the realizations of v and c at the interim stage. We do so since it is well known that in the absence of an interim participation constraint first best efficiency can be achieved in a bilateral bargaining situation with two-sided asymmetric information (see d'Aspremont-Gérard-Varet (1979)). Because of the interim participation constraint we now need to distinguish between interim

and ex post disagreement payoffs. The former refers to the payoffs that the agents realize if either of them decides not to participate in the bargaining game and the latter refers to the payoffs they realize if they both decide to participate but fail to reach an agreement. Exclusive contracts determine the disagreement payoffs but how exactly they do so depends on whether they involve *technological* or *court enforced commitment.* In particular, if the exclusive contracts involve technological commitment then the interim and the ex post disagreement payoffs are the same and are given by $d_b = me_b U_{b,2}$ and $d_s = me_s U_{s,2}$. This is the case since a technological commitment, for instance locating the buyer's plant close to the seller's, takes place ex ante and permanently changes the agents' disagreement payoffs, independently of the stage at which they are realized. In contrast, exclusive contracts that only involve court enforcement and are not backed up by some irreversible investment can affect the interim and the ex post disagreement payoffs differently. Such contracts could, for example, mandate that the agents are not allowed to trade with third parties if they do not bargain with each other after having observed their private information, but could instead allow agents to trade with third parties if they have made a reasonable attempt at bargaining but this attempt has failed. We now first analyze the case in which the exclusive contracts involve technological commitments and then discuss the case in which they involve court enforced commitment.

5.1 Technological Commitment

Instead of studying the very large set of all possible indirect bargaining games to which the agents can commit, we can restrict ourselves, without loss of generality, to Bayesian incentive compatible direct mechanisms. This is because of the well-known Revelation Principle which states that, for any Bayesian Nash equilibrium of any bargaining game, there exists a Bayesian incentive compatible direct mechanism that leads to the same outcome (see, for example, Myerson (1979, 1981)).

Suppose then that, after having learned their trade payoffs, the buyer and the seller make respective announcements $\hat{v}$ and $\hat{c}$ of their trade payoffs. A direct mechanism specifies the probability of trade $q(\hat{c}, \hat{v})$ and expected price of good $t(\hat{c}, \hat{v})$ as a function of these announcements.

Since the agents are risk neutral and liquidity unconstrained, they choose the trading rule $q^*(\cdot)$, the transfer rule $t^*(\cdot)$, and the exclusivity clauses e_b^* and e_s^* that maximize their joint utility

$$U(d, q(c, v)) = d + E_{v,c} \left[(v - c - d) q(c, v) \right],$$

where $d \equiv m(e_b U_b + e_s U_s)$ is their joint disagreement payoff, and subject to the incentive compatibility and interim individual rationality constraints. The incentive compatibility constraints ensure that each agent finds it optimal to make truthful announcements of his or her type and the interim individual rationality constraints ensure that, after learning their type, the agents prefer participating in the bargaining game to realizing the disagreement payoffs. Note that the exclusive contracts cannot be made contingent on the agents' announcements since the technological investments that make them non-renegotiable take place at the ex ante contracting stage, before the agents learn their types. The following lemma, which follows immediately from Myerson and Satterthwaite (1983), formally describes the constrained maximization problem.

Lemma 8 *The optimal trading rule $q^*(\cdot)$ and the optimal exclusivity clauses e_b^* and e_s^* solve*

$$\max_{e_b,\ e_s,\ q(c,v)} U(d,\ q(c,v)),$$

subject to

$$E_{c,v}\left[\left(v - c - d - \frac{1 - F_b(v)}{f_b(v)} - \frac{F_s(c)}{f_s(c)}\right) q(c,v)\right] \geq 0. \tag{IR}$$

Proof. In the appendix. ■

We can use the results of Lemma 8 to establish the following result.

Lemma 9 *The aggregate utility of a buyer-seller pair is quasi-convex in the aggregate disagreement payoff d.*

Proof. In the Appendix. ■

Having proved that U is quasi-convex in d, the rest of the argument is the same as that of section 2.3. In particular, quasi-convexity implies that U is maximized at the extreme values of d. Therefore the parties will either commit to exclusive contracts, by choosing $e_b = e_s = 0$ which implies $d_{EC} = 0$, or not commit to any exclusive contract, by choosing $e_b = e_s = 1$ which implies $d_{NC} = mU_2$. Just as in Lemma (4) of section 2.3, quasi-convexity also implies that there exists an $\tilde{m}$ such that if $m < \tilde{m}$ the agents

24

will commit to exclusive contracts, whereas if $m > \tilde{m}$ the parties will not commit to any contract. The analysis of the positive an normative properties of market equilibria presented in section 3 carries therefore over to the more general bargaining framework of this section.

The trade-offs that the agents face when deciding on whether or not to sign an exclusive contract are also very closely related to those discussed in section 2. To see this note that the change in their aggregate utility caused by switching from a non-exclusive contract to an exclusive contract are given by

$$U_{EC} - U_{NC} = -(d_{NC} - d_{EC})\,(1 - E[q(d_{EC})]) + E_{c,v}\left[(v - c - d_{NC})(q(d_{EC})) - q(d_{NC}))\right].$$

The first term on the right hand side captures the reduction in utility that is due to the lower payoff the agents realize in case they still disagree after committing to an exclusive contract. The second term on the right hand side captures instead the change in utility that is due to the change in the probability of trade. As in the model of section 2, a reduction in the aggregate disagreement payoff increases the probability that the agents trade ex post. The reason for this is that a lower aggregate disagreement payoff relaxes the participation constraint (see (IR)) and thus increases the set of feasible trading rules from which the agents can choose at the contracting stage. Also as in the model of section 2, the welfare effect of the increase in the probability of trade is ambiguous: while small reductions in the aggregate disagreement payoff only increase the probability that the agents realize ex ante efficient trades, large reductions in the aggregate disagreement payoff may induce them to also realize trades that are ex ante inefficient. Finally, and again just as in the model of section 2, for certain configurations of the parameters, and in particular for certain distributions of v and c, it can be the case that $\tilde{m} = 0$, i.e. that exclusive contracts are never optimal. However, the example we give below shows that for uniform distribution we indeed have $\tilde{m} > 0$ if the gains from trade are sufficiently large.

An example with uniform distributions

Suppose that c is uniformly distributed on $[0,1]$ and v is uniformly distributed on $[\alpha, 1 + \alpha]$. It follows from the analysis above that, for any $d \in [0, mU_2]$, the optimal trading rule is given by

$$q(c, v, d) = \begin{cases} 1 & \text{if } v - c - d \geq \frac{\lambda(d)}{1 + 2\lambda(d)}\,(1 - d) \\ 0 & \text{otherwise,} \end{cases}$$

where $\lambda(d) \in (0, \infty)$ solves

$$E_{c,v}\left[(2(v-c) - 1 - d)\, q(c, v, d)\right] = 0. \tag{16}$$

One can plot $U\left(d, q(c, v, d)\right)$ as a function of d for different values of α. This reveals that, for sufficiently large α, the aggregate utility U of a buyer-seller pair can indeed be decreasing in d (figure to be added). For the same reasoning underlying the intuitive proof of Lemma 4, which is explained in Figure 1, this implies that there exists an $\tilde{m} > 0$ below which exclusive contracts are optimal.

5.2   Court-enforced Commitment

Suppose now that agents can commit to exclusive contracts that are not backed up by technological commitments. It is then no longer the case that the interim and the ex post disagreement payoffs need to be the same. The agents would then always like to set the interim disagreement payoffs as low as possible, so as to relax the interim individual rationality constraint, and the ex post disagreement payoffs as high as possible, so as to maximize the payoffs they realize if they fail to reach an agreement. Thus, if courts could enforce contracts that affect the interim disagreement payoffs and the ex post disagreement payoffs differently, then the agents would commit to exclusive contracts that prevent them from trading with third parties at the interim stage but allow such trades at the ex post stage. However, if courts cannot enforce contracts that reduce the interim disagreement payoffs, then the agents will never find it optimal to use exclusive contracts.

In summary, the results of the model of section 2 and 3 continue to hold even in the presence of general bargaining if exclusive contracts involve technological commitments. If, however, exclusive contracts involve only court-enforced commitment then the agents find it optimal to commit to exclusive contracts that reduce their interim disagreement payoffs but not their ex post disagreement payoffs.

6   Conclusions

To be added.

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Appendix

Proof.   (Proposition 1)

To be added.   ∎

Proof.   (Lemma 3)

Given that c is uniformly distributed on $[0,1]$ and v is uniformly distributed on $[\alpha, 1+\alpha]$, equation (7) can be written as

$$
U = \begin{cases}
d + \int_{\alpha}^{1+\alpha} \int_{0}^{v-(\frac{1}{4}(1+\alpha)+\frac{3}{4}d)} (v - c - d)\,\mathrm{d}c\mathrm{d}v & \text{if } d \leq \alpha - 1/3, \\
d + \int_{\frac{1}{4}(1+\alpha)+\frac{3}{4}d}^{1+\alpha} \int_{0}^{1} (v - c - d)\,\mathrm{d}c\mathrm{d}v & \text{otherwise}
\end{cases}
$$

Solving the double integrals one obtains

$$
U = \begin{cases}
\frac{1}{192}\left[(1+3d)(5+3d)^2 - 3\alpha\kappa\right] & \text{if } d \leq \alpha - 1/3, \\
d + \frac{9}{64}(1+\alpha-d)^3 & \text{otherwise.}
\end{cases}
\tag{17}
$$

where $\kappa \equiv -29 - 33\alpha + 9\alpha^2 + (66 - 27\alpha)d + 27d^2$. It is straightforward to verify that U is everywhere a continuous and differentiable function of $d \in [0, 1+\alpha]$, including at $d = \alpha - 1/3$. Differentiating U twice with respect to d one obtains:

$$
\frac{\mathrm{d}^2 U}{\mathrm{d}d^2} = \begin{cases}
\frac{18}{192}\left[11 + 9(d - \alpha)\right] > 0 & \text{if } d \leq \alpha - 1/3, \\
\frac{27}{32}(1+\alpha-d) > 0 & \text{otherwise,}
\end{cases}
\tag{18}
$$

which proves that U is everywhere convex in $d \in [0, 1+\alpha]$.   ∎

Proof.   (Lemma 4)

Consider the difference $\Delta U = U_{NC} - U_{EC}$. The agents choose to write no contract if this difference is positive and to write an exclusive contract if this difference is negative.

It is important for the logic of the proof to note that if the parties write an exclusive contract their expected utility U_{EC} is equal to $U_{NC}(m = 0)$, i.e. to the utility that they would obtain if they had signed no exclusive contract but $m = 0$ and thus $d = 0$. Using equation (7), and noting that $d = mU_2$, we can therefore write

$$
\Delta U(m) = U_{NC}(m) - U_{NC}(0) = mU_2 + E(mU_2) - U_{NC} = 0.
$$

29

where $E(mU_2) = E_{c,v}\left[(v - c - mU_2)\pi\left(v - c \geq \frac{3}{4}mU_2 + \frac{1}{4}(1 + \alpha)\right)\right]$. Since $U_{NC}(m)$ is continuous and convex so is $\Delta U(m)$. Furthermore, $\Delta U(0) = 0$ and, given our assumption that $U_{NC}(0) = U_2$, $\Delta U(1) = E(U_2) > 0$. We therefore have two possibilities: either $\Delta U(m) > 0$ for all $m > 0$, in which case $\tilde{m} = 0$ and exclusive contracts are never adopted; or $\Delta U(m) < 0$ for some $m > 0$, in which case there exists a $\tilde{m} \in (0, 1)$ such that $\Delta U(\tilde{m}) = 0$. In the latter case, given that ΔU is convex, $\tilde{m}$ is unique, $\Delta U < 0$ for $m < \tilde{m}$, and $\Delta U > 0$ for $m > \tilde{m}$. $\blacksquare$

Proof. (Lemma 9)

Let the Lagrangian be given by

$$L(q(c, v, d), \lambda(d), d) = d + E_{c,v}\left[\left(v - c - d - \lambda(d)\left(v - c - d - \frac{1 - F_b(v)}{f_b(v)} - \frac{F_s(c)}{f_s(c)}\right)\right)q(c, v, d)\right]$$

where

$$q(c, \pi, d) = \begin{cases} 1 & \text{if } \pi - c - d(A) \geq \frac{\lambda(d)}{1 + \lambda(d)}\left(\frac{1 - F_b(\pi)}{f_b(\pi)} + \frac{F_s(c)}{f_s(c)}\right) \\ 0 & \text{otherwise,} \end{cases}$$

and $\lambda(d) > 0$ solves

$$E_{c,\pi}\left[\left(\pi - c - d - \frac{1 - F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)}\right)q(c, \pi, d)\right] = 0.$$

The trading rule $q(c, \pi, d)$ is the optimal trading rule for a given aggregate disagreement payoff d, as first derived in Myerson and Satterthwaite (1983). We show that $L(q(c, v, d), \lambda(d), d)$ is quasi-convex in d.

To prove quasi-convexity, we show that any $\hat{d} \in (\underline{d}, \overline{d})$ that is a local, interior extremum is not a local maximum. Using an envelope theorem-type argument it can be shown that, $\forall d \in (\underline{d}, \overline{d})$,

$$\frac{dL(q(c, v, d), \lambda(d), d)}{dd} = \frac{\partial L(q(c, v, d), \lambda(d), d)}{\partial d}.$$

If $\hat{d}$ is a local, interior extremum it must be the case that

$$\frac{dL(q(c, v, \hat{d}), \lambda(\hat{d}), \hat{d})}{dd} = \frac{\partial L(q(c, v, \hat{d}), \lambda(\hat{d}), \hat{d})}{\partial d} = 0.$$

Since

$$\frac{\partial^2 L(q(c, v, d), \lambda(d), d)}{\partial d^2} = 0$$

30

it follows that, $\forall d \in (\underline{d}, \overline{d})$,

$$L(q(c,v,\widehat{d}), \lambda(\widehat{d}), \widehat{d}) = L(q(c,v,\widehat{d}), \lambda(\widehat{d}), d). \tag{19}$$

Note next that, since, $\forall d < \widehat{d}$,

$$E_{c,v}\left[\left(v - c - d - \frac{1 - F_b(v)}{f_b(v)} - \frac{F_s(c)}{f_s(c)}\right) q(c,v,\widehat{d}), \lambda(\widehat{d}), \widehat{d}\right] > 0,$$

$$L(q(c,v,\widehat{d}), \lambda(\widehat{d}), d) < L(q(c,v,d), \lambda(d), d) \quad \forall d < \widehat{d}. \tag{20}$$

It then follows from (19) and (20) that

$$L(q(c,v,d), \lambda(d), d) > L(q(c,v,\widehat{d}), \lambda(\widehat{d}), \widehat{d}) \quad \forall d < \widehat{d}$$

so that $\widehat{d}$ cannot be a local, interior maximum.

Since $L(\cdot)$ is therefore quasi-convex, it is always maximized at the corner. If $L(\cdot)$ is monotonically increasing in $d \in [\underline{d}, \overline{d}]$, $\widetilde{m} = 0$ and if it is monotonically decreasing, $\widetilde{m} = 1$. Otherwise $\widetilde{m}$ is implicitly defined by the unique $m \in (0,1)$ that solves $L(q(c,v,d), \lambda(d), d) = L(q(c,v,\underline{d}), \lambda(\underline{d}), \underline{d})$. ∎