Varying Life Expectancy and Social Security

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Abstract

Life expectancy differs across socio economic groups and according to individual health endowments. In this paper, we shall argue that the scarcity of studies on the design of PAYG system with heterogenous life expectancy is due to simplifying assumptions. As a matter of fact, most applied papers addressing normative questions make a double assumption of additivity (additively separable individuals utility functions across time and a utilitarian social welfare function). In this case, the allocation within a given period of time is independent of the allocation in other periods of life. In this paper, the cardinal individual utility function is given by a (non linear) transformation of an additive utility function. The social optimum is obtained when individuals living longer retire later and consume less than short lived individuals. In a second best framework, we find that the first best optimum cannot be implemented. Every agents beneficiate from a lower consumption when retired. Agents with higher life expectancy are left with higher level of instantaneous post-retirement consumption. Still pre-retirement consumption is decreasing with life expectancy. Finally, in order to implement this second best optimum, the Social Security design introduces a positive tax on continued activity but only after individuals' optimal retirement age.

1 Introduction

It is well documented that life expectancy differs across socio economic groups and according to individual (random) health endowments. For example, in France, women life expectancy at age 60 is 25% larger than the one of men of the same age. Similarly, life expectancy at age 60 of a professional is 3% higher than the one of a skilled worker and 17% higher than the one of an unskilled worker. Since Social Security provides transfers contingent on survival, the combination of differential mortality and Social Security transfers may generate important redistribution, between socioeconomic groups and more generally between individuals.

Several papers measured this redistribution (Coronado et al. (2000) and Liebman (2001)). Still there are very few normative analysis that discuss how PAYG systems should adapt to differential mortality. Should long lived individuals work longer? Should they have lower pensions, etc... These questions have remained undebated issues. An exception is the contribution of Diamond (2003). However, in Diamond's framework, heterogenous life expectancy only plays a role because it is assumed that there is asymmetric information on the disutiliy of work.

In this paper, we shall argue that the scarcity of studies on the design of PAYG system with heterogenous life expectancy is due to the invisible hand that incites economists to focus on simplifying (but unappealing) assumptions. As a matter of fact, most applied papers addressing normative questions with intertemporal agents make a double assumption of additivity. First, individuals cardinal utility functions are assumed to be separable additive across time. Second the Social Welfare function is assumed to be a weighted sum of individual cardinal utility functions. This double assumption of additivity, is for example found in Brito et al. (1991), Feldstein (1987), Calvo et al. (1988), Miles et al.(2002), Golosov et al. (2003) as well as in the study of Diamond (2003) mentioned above.

The double additivity implies that the social planner preferences are separable additive with respect to individuals' instantaneous utilities. Consequently the allocation (say of consumption, labor, etc...) within a given period of time is independent of the allocation in other periods of life. In particular, the optimal allocation of labor and consumption between Tim and Tom in a given period is unaffected by the fact that Tim will live much longer than Tom. The double assumption of additivity simply kills the chicken in the egg: there is not much to say about heterogenous life expectancy.

We will see, however that when this double assumption of additivity is relaxed, accounting for heterogenous life expectancy is crucial for the design of Social Security. In this paper we use a standard utilitarian approach and therefore maintains the assumption that social welfare is the sum of individual cardinal utility functions. We also keep the assumption that individuals preferences are weakly separable. The cardinal individual utility function is then obtained by a transformation of an additive utility function. The novelty is that we no longer constrain this transformation to be linear. Actually we will provide several arguments explaining why it seems reasonable to consider concave transformations. These arguments relate to the notions of aversion towards multiperiod inequality and the risk aversion towards the length of life.

Our contribution does not escape from the criticism we addressed above. It is simple and relies on probably unrealistic assumptions on individual preferences (nobody really believes that the assumption of weak separability is plausible). Still it is sufficient to show that heterogeneity of life expectancy should play a key role in the design of Social Security. In our framework, the social optimum is obtained when individuals living longer retire later and consume less than short lived individuals.

The interest of relaxing the double additivity assumption is enhanced when looking at the problem with asymmetric information. We consider the case where agents have private information on their life expectancy. The first best optimum cannot be implemented. In the second best, there is a wedge between pre and post retirement consumptions: every agents beneficiate from a lower consumption when retired. Agents with higher life expectancy are left with higher level of instantaneous post-retirement consumption. Still the pre-retirement consumption is decreasing with the life span. In order to implement this second best optimum, the Social Security design introduces a positive tax on continuing activity but only after the individuals optimal retirement age. As in Cremer et al. (2004), we find a downward distortion on the trade-off between consumption and retirement age but only after the optimal age of retirement. Our paper amounts therefore to the literature that argues that a good Social Security does not necessarily need to be marginally fair.

The paper is structured as follows. In section 2, we provide the main assumptions on individual preferences and the planner instruments. In section 3, we give the results under the double assumption of additivity. Section 4 discusses why we think it is necessary to relax this double assumption of additivity. Section 5 then discusses how the theory extends in such a case.

2 The Model

There is no uncertainty in our framework. Consider an individual with life span equal to T. At each time, the individual consumes $c(t)$ and can supply either one or no unit of labor. The labor disutility at each time t is $r(t)$ with $r'(t) \geq 0$. The discount rate is assumed to equal zero. Denoting z the age of retirement, preferences are given by the following ordinal utility function:

$$V(C, z, T) = \int_0^T u(c(t))\, dt - \int_0^z r(t)\, dt = \int_0^T u(c(t))\, dt - R(z) \qquad (1)$$

where C is a vector of consumptions at each time and $u(.)$ is increasing and strictly concave. $R(z) = \int_0^z r(t)\, dt$ denotes the disutility for a working life of length z. Note that this specification assumes that preferences are separable between consumption and work. Throughout the paper, we assume that the individual has no access to the capital market so that he cannot save.

There is no heterogeneity in instantaneous labor income. The monetary unit is chosen so that the labor income is equal to 1. At the beginning of her working life, an agent chooses among the available social security plans. Each plan is generally characterized by an age of retirement, z and a consumption path $c(t)$.

An agent with life-span T who is retiring at age z has a lifetime income given by:

$$Y(z) = z - \Gamma(z)$$

where $\Gamma(z)$ defines the net contribution to the Social Security system. As an example, consider a constant payroll tax $\tau(z)$ and a Social Security benefit $p(z)$ depending on the age of retirement. $\Gamma(z)$ can be rewritten as

$$\Gamma(z) = z\tau(z) - (T - z)\, p(z)$$

The function $\Gamma_T(z)$ is the difference between total contributions and total benefits received from Social Security system by an individual with life expectancy T retiring at date z. A negative $\Gamma_T(z)$ can be interpreted as being a net transfer from the pension system to the individual. Differentiating $\Gamma_T(z)$ with respect to z yields a tax on "continued activity":

$$\Gamma_T'(z) = \tau(z) + z\tau'(z) + p(z) - (T - z)\, p'(z). \tag{2}$$

This marginal tax has been estimated in Gruber and Wise (1999). It represents taxes and social security contributions as well as the foregone benefits from each extra working year. It is eventually compensated by a variation of taxes or per-period benefits. A worker deciding to work ε additional year of work increases his lifetime income by $(1 - \Gamma_T'(z))\varepsilon$. A pension system typically does not introduce a marginal tax on continued activity ("marginally fair") if for any T type, the optimal age at retirement chosen by an individual satisfies $\Gamma'(z) = 0$.

Throughout the paper, we assume a stationary population composed of individuals differing in their life expectancy. The length of life is distributed over $[T_{\min}, T_{\max}]$ with a distribution function $F(T)$ and a density $f(T)$. Assuming a stationary population and a zero interest rate, the resource constraint of the economy is:

$$\int_{T_{\min}}^{T_{\max}} z_T f(T) dT = \int_{T_{\min}}^{T_{\max}} \left(\int_0^T c_T(t) \right) f(T) dT \tag{3}$$

where z_T and $c_T(t)$ denote the age of retirement and instantaneous consumptions of agents living T years.

3 Normative results with the double additivity assumption.

In this section we assume that the Planner 's objective is to maximize the following social welfare function

$$SWF = \int_{T_{\min}}^{T_{\max}} V(C_T, z_T, T) f(T) dT$$

There are two important underlying assumptions. First the planner is assumed to be utilitarian. Second, it is assumed that the function V not only describes how the individuals rank lives differing by their lengths, consumption paths, and retirement ages, but also provides a correct measure of individual happiness. As we shall argue in Section 4, such an assumption is particularly unappealing. Still it remains by far the most common assumption in the economic literature.

3.1 First best

The social planner chooses z_T and C_T (for all $T \in [T_{\min}, T_{\max}]$) in order to maximize

$$SWF = \int_{T_{\min}}^{T_{\max}} V\left(C_T, z_T, T\right) f\left(T\right) dT$$

subject to the resource constraint (3). Given the separability between consumption and labor and the fact that the interest and discount rates are equal to zero, the optimum involves a constant consumption path i.e. $c_T(t) = c_T$ for every t and $T \in [T_{\min}, T_{\max}]$.

The type T utility can thus be rewritten as $V(c_T, z_T, T) = Tu\left(c_T\right) - R\left(z_T\right)$. The first order conditions yield $r\left(z_T\right) = u'\left(c_T\right)$ and $u'\left(c_T\right)$ is independent of T. In other words, the age of retirement and the constant consumption level of an individual are independent of his life span. This pooling property results from the double additivity assumption which makes the social planner give the same social weight to *per-period* consumption and labor disutility wether it comes from an individual with high or low life span.

Proposition 1 *The first best optimum is such that for any* $T < \tilde{T}$*:*

(i) $c_T = c_{\tilde{T}}$

(ii) $z_T = z_{\tilde{T}}$

There are three direct consequences of the double additivity assumption.

First, it is clear that such a social security scheme redistributes lifetime income from the short lived to the long lived individuals. The net contribution of a type T individual to the system is equal to $z - Tc$ where, using the resource constraint, $z = c \int_{T_{\min}}^{T_{\max}} Tf(T)dT$. Therefore every individuals with a life duration T that is lower than the mean of the life duration $\overline{T} = \int_{T_{\min}}^{T_{\max}} Tf(T)dT$ are net contributors to the system while those who live longer are net recipients.

Second, when the government cannot observe the individual's life-span but only knows its distribution over the population, the first best optimum is still implementable since it is a pooling optimum.

Third, it is possible to decentralize this optimum with simple instruments that do not introduce any tax on prolonged activity. As an example, denote z_0 and p_0 the age at retirement and the pension in the first best. Let the Social Planner propose the following set of pension plans:

$$\{(z, \tau(z), p(z)) | \tau(z) = 2 - p_0 - \frac{z}{z_0} + A(z - z_0)^2 \text{ and } p(z) = p_0\}$$

where A is a (large) positive constant.

It is easy to check that for every T:

$$\begin{aligned} \Gamma_T(z_0) &= -\tau(z_0) - p\left(z_0\right) - z_0\tau'(z_0) + (T - z)\,p'\left(z_0\right) \\ &= -2 + p_0 + 1 - p_0 + 1 = 0 \end{aligned}$$

i.e. the pension plan introduces no tax on prolonged activity at the point z_0. It remains to show that z_0 is the optimal retirement age chosen by every types of individuals. Any individual with life span T chooses z in order to maximize

$$V\left(z\right) = zu(1 - \tau(z)) + (T - z)u(p(z)) + R(z)$$

The first order condition is

$$V'(z) = u(1 - \tau(z)) - zu'(1 - \tau(z))\tau'(z) - u(p\left(z\right)) - r(z)$$

In the first best $u'(1 - \tau(z_0)) = r(z_0)$ and $u(1 - \tau(z_0)) = u(p_0)$. Thus $V'(z) = 0$ for $z = z_0$ and for A large enough $V''(z_0) > 0$ so that z_0 is the optimal individual choice.

4 Cardinal measure of individual happiness

There are two assumptions underlying the double additive specification of the Social Welfare function. First it is assumed that the Social Planner is utilitarian. Second, the additively separable function V is assumed to provide a correct cardinal measure of individual happiness. Whether utilitarianism is as an acceptable theory of justice is a philosophical question that we will not discuss in the paper. Our argument will focus on the second assumption: we will explain why, within the utilitarian approach, it seems unappealing to assume that cardinal individual utility functions are additively separable.

Individual ordinal utility functions being already specified, and given by $V(c, z, T)$, the natural candidates for the cardinal utility functions are of the form $G(V(c, z, T))$ where G is an increasing transformation. While the function V allows to decide whether a life is preferred to another one, the function G provides the scale for the measurement of individual happiness. Take for example two different outcomes (c_i, z_i, T_i), $i = 1, 2$ and assume that:

$$V(c_1, z_1, T_1) > V(c_2, z_2, T_2)$$

Individuals prefer life 1 to life 2. The individuals' happiness must be greater with life 1. Therefore we must have $G(V(c_1, z_1, T_1)) > G(V(c_2, z_2, T_2))$, which is fulfilled if G is increasing. However, individuals preferences are not informative on the gap between happiness with life 1 and happiness with life 2. This gap is given by

$$G(V(c_1, z_1, T_1)) - G(V(c_2, z_2, T_2))$$

and obviously depends on the function G. Knowledge of the function G is then crucial for a Social Planner that aims at maximizing the aggregate happiness in the society.

The standard assumption is that G is linear. There is no doubt that technically speaking, it is a very convenient choice. Nonetheless, there is no reason for technicality to be a relevant criterion. Below, we develop two parallel arguments to support the choice of a concave function G. The first one is related to the notion of aversion for multi-period inequality.

The second one is related to individual preferences under uncertainty and particularly, the risk aversion towards the length of life.

4.1 Aversion for multiperiod inequality.

The concept of aversion for multiperiod inequality has been initially introduced by Atkinson and Bourguignon (1982) and more recently discussed in Gottschalk and Spolaore (2002). It can be made very intuitive by considering the simple case of a two period and two individuals setting who have preferences represented by the ordinal utility function $V(c_1, c_2) = u(c_1) + u(c_2)$. The utilitarian SWF becomes

$$SWF = G(u(c_1^1) + u(c_2^1)) + G(u(c_1^2) + u(c_2^2))$$

where c_t^i is the consumption of individual i in period t, and G, the increasing transformation that relates ordinal and cardinal utility functions. Now, consider two levels of instantaneous consumptions a and b, with $a < b$ and compare the following two social outcomes:

$$\begin{aligned} \text{Outcome A} \quad &: \quad (c_1^1, c_2^1) = (a, a) \text{ and } (c_1^2, c_2^2) = (b, b) \\ \text{Outcome B} \quad &: \quad (c_1^1, c_2^1) = (a, b) \text{ and } (c_1^2, c_2^2) = (b, a) \end{aligned}$$

With outcome A individual 1 has a low level of consumption in both periods while the second individual has a high level of consumption in both periods. With outcome B, both individuals alternate between low and high level of consumption. One should note that in both cases there are in each period one individual with consumption a and another one with consumption b. Thus the *within* period inequality is the same with outcomes A and B.

However, A and B differ in terms of *multiperiod* inequality. Indeed, when looking at the two periods together, outcome A seems much more unequal than outcome B: with outcome A individual 1 is worse off than individual 2, while with outcome B they have identical lifetime utility. It seems reasonable to argue that a well behaved social planner should prefer outcome B. It is straightforward to see that it is the case if and only if the function G is strictly concave. This provides our first argument for choosing a concave function G.

The concavity of the function G, and more precisely $(-G''/G')$, measures the planner's aversion for multiperiod inequality. When G is linear, $(-G''/G')=0$ and the Social Planner is indifferent to multiperiod inequality. The greater is $(-G''/G')$ the greater is the planner's aversion for multiperiod inequality. Two different functions G_1 and G_2 such that $-G_1''G_1'=-G_2''/G_2'$ provide the same social preferences.[1].

4.2 Intertemporal correlation aversion and risk aversion with respect to the length of life

Another way to support the choice of a concave function G is to refer to Harsanyi's (1955) axiomatization of utilitarianism. Harsanyi considers preferences over risky alternatives and focuses on the case where the social planner and individuals have preferences that can be represented by VNM utility functions. He shows that if the Social Planner preferences satisfy an ex-ante Pareto criterion then the planner's utility function must be a linear combination of individuals utility functions. Thus, if we follow Harsanyi's axiomatic constructions, the function $G(V\,(c,z,T))$ has to be a VNM utility function representing individual preferences under uncertainty. Therefore, the properties of the function G can be related to fundamental characteristics of individuals' preferences. In particular, as discussed in Bommier (2003a), one can make the link between G and the two related concepts that are intertemporal correlation aversion and risk aversion with respect to the length of life.

Intertemporal correlation aversion measures whether an individual prefers lotteries that affect instantaneous consumption at different moments in time

[1]Note that if $-G_1''/G_1'=-G_2''/G_2'$ then $G_1=kG_2+m$ for two constants k and m.

to be negatively or positively correlated. Bommier (2003b) provides a definition and interpretation of an index of intertemporal correlation. He shows that this index is proportional to $-G''/G'$ with the specification of preferences we use in the present paper.

As for risk aversion with respect to the length of life, consider preferences over a constant path of consumption c and a retirement age z given by $G(V(c, z, T)) = G(Tu(c) - R(z))$ where T is the length of life. The absolute risk aversion with respect to the length of life, defined by $-\left(\partial^2 G(V(c, z, T))/\partial T^2\right) / \left(\partial G(V(c, z, T))/\partial T\right)$, is equal to $u(c)\left(-G''/G'\right)$. When G is linear $-G''/G' = 0$ and individuals are risk neutral with respect to the length of life, which is in contradiction with empirical evidence.[2] They are risk averse when G is concave, and the index of risk aversion with respect to the length of life is proportional to $-G''/G'$.

5 Normative results relaxing the double additivity assumption

5.1 First Best Problem

In the first best problem, a social planner is choosing the consumption paths and the retirement ages in order to maximize

$$SWF = \int G(V(C_T, z_T, T))f(T)dT$$

subject to the resource constraint (3). Again the first best is obtained when $c_T(t) = c_T$ for every t and $T \in [T_{\min}, T_{\max}]$. The problem thus amounts to choose c_T and z_T in order to maximize

$$\int_{T_{\min}}^{T_{\max}} G\left(Tu\left(c_T\right) - R(z_T)\right) f\left(T\right) dT$$

subject to the resource constraint (3).

The first order conditions imply $r\left(z_T\right) = u'\left(c_T\right)$ and $u'\left(c_T\right) G'\left(U\left(c_T, z_T, T\right)\right)$ is independent of T. The difference with the double additive case, is that G

[2] See Bommier and Villeneuve (2004).

is not linear. As a consequence, the type T constant consumption path and the retirement age is not independent of T.

Proposition 2 *When the social planner's objective is generalized utilitarian, the first best optimum is characterized by: For any $T < \tilde{T}$,*

(i) $c_T > c_{\tilde{T}}$

(ii) $z_T < z_{\tilde{T}}$

At the optimum, those who live longer retire later and consume less. As opposed to the double additive approach, long lived agents may not be net recipients of the redistribution system. Whether long lived agents are net recipients or not now clearly depends upon the concavity of G. The levels of aversion towards multiperiod inequality and the risk aversion towards the length of life are thus crucial to determine the optimal level of redistribution between individuals of different life-span.

5.2 Second Best Problem

The first best solution has been derived under the assumption that individual life spans T are observable. In the following we assume that the government cannot observe T while he can observe the retirement age and the distribution of individuals over $[T_{\min}, T_{\max}]$. Moreover, we suppose that it cannot propose time limited contracts. In other words, it cannot propose any contract (c, z, D) where c is given only until age D. This assumption clearly reflects the fact that the Social Security offers an annuity with a non limited duration. We assume further that the government offers a constant annuity after the age of retirement.

It results that the first best optimum is not implementable under asymmetric information. Individuals who live longer would optimally choose to claim to be short lived so as to enjoy a higher per period consumption (or annuity) and a shorter career.

The problem of the government is thus the first best problem to which we add a global incentive constraint preventing any type of individuals to mimic the other types of individuals. Formally, the problem is:

$$\underset{c_T,\, z_T}{Max} \int_{T_{\min}}^{T_{\max}} G\left(z_T u\left(c_T^1\right) + (T - z_T)\, u\left(c_T^2\right) - R(z_T)\right) f(T)\, dT$$
$$\int_{T_{\min}}^{T_{\max}} \left(z_T - z_T c_T^1 - (T - z_T)\, c_T^2\right) f(T)\, dT \;\geq 0$$
$$z_T u\left(c_T^1\right) + (T - z_T)\, u\left(c_T^2\right) - R(z_T) \geq z_{T'} u\left(c_{T'}^1\right) + (T - z_{T'})\, u\left(c_{T'}^2\right) - R(z_{T'}) \qquad \forall T, T'$$

The complete solution of this problem is given in the appendix. Below, we just sketch the results that are crucial for our analysis. First, we determine wether the consumption paths and the retirement ages follow the same properties as in proposition 2**.** Second, we analyze the design of the tax system decentralizing the second best optimum.

Using the definition of $V\left(c_T^1, c_T^2, z_{T,}T\right)$, the incentive compatibility constraints can be rewritten as

$$V\left(c_T^1, c_T^2, z_{T,}T\right) \geq V\left(c_{T'}^1, c_{T'}^2, z_{T',}T'\right) + \left(T - T'\right) u\left(c_{T'}^2\right) \tag{4}$$

This requires that for T' approaching T, we have in the limit $\dot{V}\left(c_T, z_{T,}T\right) = u\left(c_T^2\right)$ for all T where a dot means that the variable is derived with respect to T. Moreover given that $u\left(c_T^2\right)$ is the slope of $V\left(c_T, z_{T,}T\right)$ at T, the inequality in (4) requires that $V\left(c_T, z_{T,}T\right)$ be convex. This is fulfilled if and only if $\dot{c}_T^2 \geq 0$. In other words, the optimum always involves a non decreasing post retirement consumption path with respect to the life duration.

Proposition 3 *The second best optimum is characterized by: for any* $T, \tilde{T} \in [T_{\min}, T_{\max}]$ *such that* $T < \tilde{T}$*,*

(i) $c_T^2 \leq c_{\tilde{T}}^2$

(ii) $c_T^1 \geq c_{\tilde{T}}^1$

(iii) $z_T \gtreqless z_{\tilde{T}}$

In a second best framework, long-lived individuals will obtain a higher post retirement consumption. Still, pre retirement consumption level will be lower for long-lived individuals, as it was the case in a symmetric information framework. Since the local incentive constraint just involves post retirement consumption to be increasing with T, the government can still implement a pre-retirement consumption that is decreasing with T . Note however that bunching cannot be ruled out in which case it may be desirable to offer the same allocation for individuals distributed over a certain interval of life expectancies.[3] As opposed to the first best, the retirement ages cannot be ranked without ambiguity. There are two opposite income effects at work. On the one hand, the positive relation between c^2 and T implies a negative relation between z and T. On the other hand, the negative relation between c^1 and T calls for a positive relation between z and T. The total effect is thus ambiguous.

Proposition 4 *For any individual with life expectancy* $T \in]T_{\min}, T_{\max}[$, $c_T^1 > c_T^2$.[4]

It is desirable to favor consumption before retirement in order to relax incentive problems. To understand this, write the type T marginal rate of substitution between c^1 and c^2 as follows:

$$MRS_{c^1,c^2}^T = -\frac{(T-z)\,u'\left(c^2\right)}{zu'\left(c^1\right)}$$

which decreases with T for given levels of c^1, c^2 and z. In other words, short lived individuals must be compensated less before retirement to accept a given level of reduction of their post retirement consumption. Thus distorting downward the choice of c^2 is a way to relax an otherwise binding self selection constraint.

[3] See the appendix in section A.2.

[4] For individuals with life expectancy $T_{\max}$ and $T_{\min}$, consumptions flows may be smoothed along the life cycle i.e $c_{T_{\max}}^1 = c_{T_{\max}}^2$. This is the usual no distortion at the top and the bottom results. These results hold if there is no bunching at these specific points.

Proposition 5 *For every individuals, the marginal tax on continued activity is null before retirement but positive after retirement.*[5]

Interestingly we see in the appendix that the Social Security scheme implementing the second best is such that there exists an implicit tax on continued activity but only after the (chosen) retirement age. To understand this, write the type T marginal rate of substitution between post retirement consumption and the life cycle labor income:

$$MRS_{c^2,z} = \frac{r(z) + u(c^2) - u(c^1)}{(T-z)\,u'(c^2)}$$

which decreases with T for given levels of c^1, c^2 and z. In other words short lived individuals must be compensated more after retirement to accept to work longer than a mimicking long lived individual. Consequently, a downward distortion on the retirement age is a way to relax incentive constraints. As the marginal rate of substitution between c^1 and z is independent of T, the trade off between these two variables is not distorted. In other words, the Social Security system implementing the second best optimum is such that it is (marginally) actuarially fair until the optimal age of retirement. But every additional income earned from labor after this date are taxed.

6 Conclusion

Maximization of a Social Welfare function subject to resource and incentive constraints often proves to be a difficult and technical task. In practice, economists resort to simplifying assumptions in order to maintain the difficulty at reasonable level. It is for no other reason that normative issues concerning intertemporal agents are generally addressed under the assumption that individual cardinal utility functions are separable additive.

Ethically speaking, such an assumption is hardly defendable. Each individual life is modelled as a sequence of independent incarnations and the

[5] Again, no distortion at the top and the bottom may occur in which case the marginal tax is nill for individuals with life expectancies $T_{\min}$ and $T_{\max}$.

notions of justice between individuals is replaced by a notion of justice between incarnations. It is assumed that there is no more connection between Tim in year 0 and Tim in year 1 than between Tim in year 0 and Tom in year 1. It is the nature of human existence that is simply denigrated.

Poor assumptions may lead to ignore important questions. One of them is the question of the optimal design of Social Security with heterogeneity in life expectancy. Under reasonable assumptions, a Social Planner will prefer that long lived individual work longer and have lower instantaneous consumptions. When information is perfect the first best optimum can be implemented through an appropriate design of the Pay-As-You-Go pension system. However, contributions and pensions have to be defined according to individual life expectancy. This becomes clearly impossible when life expectancy is a private information of individuals. In such a case, the planner's second best will be such that consumption before retirement is encouraged. Pre retirement consumption is decreasing with the life span while the post retirement consumption is increasing. Moreover, every additional labor income earned after the optimal age of retirement is taxed at a positive rate while it is not before retirement.

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Appendix

A The second best optimum

The second best problem can be transformed into a standard optimal control problem. Using the local incentive constraints $\dot{V}(c_T, z_{T,}T) = u\left(c_T^2\right)$ and $\dot{c}_T^2 \geq 0$, the problem can be rewritten as :

$$\begin{array}{l} \underset{c_T^1, c_T^2,\, z_T}{Max} \int_{T_{\min}}^{T_{\max}} G\left(V\left(c_T^1, c_T^2, z_{T,}T\right)\right) f(T)\, dT \\ s.to: \int_{T_{\min}}^{T_{\max}} \left\{z_T - z_T c_T^1 - (T - z_T)\, c_T^2\right\} f(T)\, dT \ \geq 0, \\ V_T(T) = z_T u\left(c_T^1\right) + (T - z_T)\, u\left(c_T^2\right) - R(z_T), \\ \dot{V}_T(T) = u\left(c_T^2\right), \\ \dot{c}_T^2 \geq 0 \end{array} \tag{SB'}$$

The state variables are $V_T(T)$, c_T^2 and control variables are y_T, c_T^1 and z_T where $y_T \equiv \dot{c}_T^2$. The Hamiltonian is:

$$\begin{aligned} H &= G\left(V\left(c_T^1, c_T^2, z_{T,}T\right)\right) f(T) - \alpha(T)\left(V_T(T) - z_T u\left(c_T^1\right) - (T - z_T) u\left(c_T^2\right) + R(z_T)\right) \\ &\quad + \gamma\left\{z_T - z_T c_T^1 - (T - z_T) c_T^2\right\} f(T) + \lambda(T) u\left(c_T^2\right) + \mu(T) y_T + \pi(T) y_T \end{aligned}$$

where $\lambda(T)$ is the co-state variable associated with $\dot{V}_T(T) = u\left(c_T^2\right)$, $\mu(T)$ is the co-state variable associated with $\dot{c}_T^2 \equiv y_T$. $\alpha(T)$ is the shadow value of the constraint $V_T(T) = z_T u\left(c_T^1\right) + (T - z_T) u\left(c_T^2\right) - R(z_T)$, $\pi(T)$ is the shadow value of the constraint $\dot{c}_T^2$ and γ is the Lagrange multiplier associated with the resource constraint.

From the Pontryagin principle,

$$\dot{\lambda}(T) = -\frac{\partial H}{\partial V_T(T)} = \alpha(T) - G'(V_T(T)) f(T) \tag{5}$$

$$\dot{\mu}(T) = -\frac{\partial H}{\partial c_T} = -\left[(T - z_T)\alpha(T) + \lambda(T)\right] u'\left(c_T^2\right) + (T - z_T)\gamma f(T) \tag{6}$$

where $\lambda(T_{\min}) = \lambda(T_{\max}) = \mu(T_{\min}) = \mu(T_{\max}) = 0$ are given by the transversality conditions provided that $z_{T_{\min}} > 0$.

Optimizing with respect to z_T, c_T^1 and y_T also yields:

$$\frac{\partial H}{\partial z_T} = -\alpha(T)\left\{r(z_T) + u\left(c_T^2\right) - u\left(c_T^1\right) +\right\} + \gamma\left(1 - c_T^1 + c_T^2\right) f(T) = 0 \tag{7}$$

$$\frac{\partial H}{\partial c_T^1} = \alpha(T) z_T u'\left(c_T^1\right) - \gamma z_T f(T) = 0 \tag{8}$$

$$\pi(T) > 0 \rightarrow \dot{c}^2(T) = 0; \quad \pi(T) = 0 \rightarrow \dot{c}^2(T) > 0 \tag{9}$$

In what follows, we only consider interior solutions for z_T, c_T^1 and c_T^2 for every T.

A.1 First Order approach

Assume first that $\dot{c}^2(T) > 0$ for every T so that $\mu(T) = \pi(T) = 0$. We study the case where $\dot{c}^2(T) = 0$ is binding in the following section.

A.1.1 Arbitrage conditions

This section proves propositions 4 and 5.

We first find the trade off between pre retirement consumption c_T^1 and retirement age z_T. Replacing $\alpha(T)$ from (8) in (7), we find the marginal rate of substitution between pre retirement consumption and retirement to be equal to its marginal rate of transformation:

$$MRS_{c_T^1,z_T} = -\frac{u\left(c_T^1\right) - u\left(c_T^2\right) - r\left(z_T\right)}{z_T u'\left(c_T^1\right)} = MRT_{c_T^1,z_T}\frac{\left(1 - c_T^1 + c_T^2\right)}{z_T} \quad (10)$$

Secondly, in finding the trade off between post retirement consumption c_T^2 and the retirement age z_T, we rearrange (6) to obtain :

$$(T - z_T)\gamma f(T) = u'\left(c_T^2\right)\{(T - z_T)\alpha(T) + \lambda(T)\} \quad (11)$$

Substituting $\gamma f(T)$ by $\alpha(T)\left(r(z_T) + u\left(c_T^2\right) - u\left(c_T^1\right)\right)/\left(1 - c_T^1 + c_T^2\right)$ obtained with (7) and dividing by$-(T - z_T)^2\alpha(T)u'\left(c_T^2\right)$ on each side of the equality yields:

$$\begin{aligned} MRS^T_{c_T^2,z_T} &= \frac{r(z_T) + u\left(c_T^2\right) - u\left(c_T^1\right)}{(T - z_T)u'\left(c_T^2\right)} = \frac{\left(1 - c_T^1 + c_T^2\right)}{T - z_T} + \frac{\lambda(T)}{\alpha(T)}\frac{\left(1 - c_T^1 + c_T^2\right)}{(T - z_T)^2} \\ &= MRS^T_{c_T^2,z_T} + \frac{\lambda(T)}{\alpha(T)}\frac{\left(1 - c_T^1 + c_T^2\right)}{(T - z_T)^2} \end{aligned}$$

Denoting the last term of the right-hand-side W for "wedge" between the marginal rate of transformation between c_T^2 and z_T (i.e. $\left(1 - c_T^1 + c_T^2\right)/(T - z_T)$) and the marginal rate of substitution, we have:

$$W = \frac{\lambda(T)}{\alpha(T)}\frac{\left(1 - c_T^1 + c_T^2\right)}{(T - z_T)^2}$$

and using again (7) to substitute for $\alpha(T)$, this is equivalent to:

$$W = \frac{\lambda(T)}{\gamma f(T)}\frac{u\left(c_T^2\right) + r(z_T) - u\left(c_T^1\right)}{(T - z_T)^2} \quad (12)$$

Finally, we find an expression for $\lambda(T)$ using (5) and the transversality conditions:

$$\lambda(T) = \int_{T}^{T^{\max}} \left[G'(V_x(x))f(x) - \alpha(x)\right] dx$$

Using again (7) to substitute for $\alpha(T)$, this gives:

$$\lambda(T) = \int_{T}^{T^{\max}} \left[G'(V_x(x)) - \gamma \frac{1 - c_x^1 + c_x^2}{r(z_x) + u(c_x^2) - u(c_x^1)}\right] f(x)\, dx$$

Substituting this expression in (12) yields:

$$W = \left(\frac{r(z_T) - u\left(c_T^1\right) + u\left(c_T^2\right))}{(T - z_T)^2 f(T)}\right)\left(\int_{T}^{T^{\max}} \left[\frac{G'(V_x(x))}{\gamma} - \frac{1 - c_x^1 + c_x^2}{r(z_x) - u(c_x^1) + u(c_x^2)}\right] f(x)\, dx\right) \tag{13}$$

In interpreting this wedge formula, let us begin with the last term. It measures the social net gain associated with a marginal decrease of the utility of every individuals who live more than T years. The gain in increased revenue is $1 - c_x^1 + c_x^2/\left(r(z_x) - u\left(c_x^1\right) + u\left(c_x^2\right)\right)$ per person while the cost is a loss of welfare measured in units of revenue, $G'(V_x(x))/\gamma$. By the transversality conditions, one has

$$\int_{T_{\min}}^{T^{\max}} \frac{G'(V_x(x))}{\gamma} f(x)\, dx = \int_{T_{\min}}^{T^{\max}} \frac{1 - c_x^1 + c_x^2}{r(z_x) - u(c_x^1) + u(c_x^2)} f(x)\, dx$$

This simply means that at the optimum a unit increase in the utility of *all* individuals implies a zero net benefit. This implies that the marginal (life cycle) income taxes are nil at the bottom and at the top of the distribution. Let us denote T_k the life duration for which $\left(1 - c_x^1 + c_x^2\right) / \left(r(z_x) - u\left(c_x^1\right) + u\left(c_x^2\right)\right) = G'(V_{T_k}(T_k))/\gamma$. Because the utility is increasing with the life duration (from local incentive constraints, $\dot{V}_T(T) = u\left(c_T^2\right))$ and G is concave (the social planner put more preference towards those who have a short life), the integral in (13) is decreasing up to T_k and then increasing. Still, we cannot infer how the first term in (13) varies with T. Because $\lambda(T)$ is negative, it turns out that W is negative everywhere which represents a downward distortion in the trade off between c_T^2 and z_T. There is then an implicit tax on continued activity after the optimal age of retirement.

Similarly, we find the trade off between before and after retirement consumptions. Using (8) and the expression of $\gamma f(T)$ from (6), we find that :

$$\begin{aligned} MRS_{c_T^2,c_T^1} &= -\frac{z_T u'\left(c_T^1\right)}{\left(T-z_T\right)u'\left(c_T^2\right)} = -\frac{z_T}{T-z_T} - \frac{z_T}{\left(T-z_T\right)^2}\frac{\lambda\left(T\right)}{\alpha\left(T\right)} \\ &= MRT_{c_T^2,c_T^1} - \frac{z_T}{\left(T-z_T\right)^2}\frac{\lambda\left(T\right)}{\alpha\left(T\right)} \end{aligned}$$

Since $z_T\lambda\left(T\right)/\left(T-z_T\right)^2\alpha\left(T\right) < 0$, there is a positive wedge between $MRS_{c_T^2,c_T^1}$ and the marginal rate of transformation between c_T^2 and c_T^1.

Rearranging the last equation, this yields:

$$u'\left(c_T^1\right) = u'\left(c_T^2\right)\left(1+\frac{\lambda\left(T\right)}{\alpha\left(T\right)\left(T-z_T\right)}\right)$$

Since $\lambda\left(T\right) < 0$ and $\alpha\left(T\right) > 0$, one gets $u'\left(c_T^1\right) < u'\left(c_T^2\right)$ so that by concavity of $u\left(.\right)$, $c_T^1 > c_T^2$.

A.1.2 Optimal bundles

This section proves proposition 3.

We already know from the local incentive constraint that $\dot{c}_T^2 > 0$, we now determine how pre retirement consumption and optimal retirement age vary with the individual's type T, i.e. $\dot{c}_T^1$ and $\dot{z}_T$. Differentiating totally (10) yields:

$$r'\left(z_T\right)\dot{z}_T + \left[u'\left(c_T^2\right)-u'\left(c_T^1\right)\right]\dot{c}_T^2 = u''\left(c_T^1\right)\dot{c}_T^1\left(1-c_T^1+c_T^2\right) \qquad (14)$$

Moreover, in response to functions $\left\{c_T^1, c_T^2, z_T\right\}$, individuals with type T maximize their utility such that :

$$\underset{T'}{Max} V\left(c^1_{T'}, c^2_{T'}, z_{T'}, T\right) = z_{T'} u\left(c^1_{T'}\right) + \left(T - z_{T'}\right) u\left(c^2_{T'}\right) - R\left(z_{T'}\right)$$

First order condition is:

$$\dot{z}_{T'} u\left(c^1_{T'}\right) + z_{T'} u'\left(c^1_{T'}\right) \dot{c}^1_{T'} + \left(T - z_{T'}\right) u'\left(c^2_{T'}\right) \dot{c}^2_{T'} - \dot{z}_{T'} u\left(c^2_{T'}\right) - r\left(z_{T'}\right) \dot{z}_{T'} = 0$$

Since revealing his true type is an optimal strategy this yields:

$$\dot{z}_T\left[u\left(c^1_T\right) - u\left(c^2_T\right) - r\left(z_T\right)\right] + z_T u'\left(c^1_T\right) \dot{c}^1_T + \left(T - z_T\right) u'\left(c^2_T\right) \dot{c}^2_T = 0$$

From the expression above ,we get:

$$\dot{z}_T = -\frac{z_T u'\left(c^1_T\right) \dot{c}^1_T + \left(T - z_T\right) u'\left(c^2_T\right) \dot{c}^2_T}{u\left(c^1_T\right) - u\left(c^2_T\right) - r\left(z_T\right)} \tag{15}$$

Substituting (15) in (14) yields:

$$-\left[\frac{z_T u'\left(c^1_T\right) r'\left(z_T\right)}{u\left(c^1_T\right) - u\left(c^2_T\right) - r\left(z_T\right)} + u''\left(c^1_T\right)\left(1 - c^1_T + c^2_T\right)\right] \dot{c}^1_T = \left[\frac{\left(T - z_T\right) u'\left(c^2_T\right) r'\left(z_T\right)}{u\left(c^1_T\right) - u\left(c^2_T\right) - r\left(z_T\right)} - \left(u'\left(c^2_T\right) - u'\left(c^1_T\right)\right)\right] \dot{c}^2_T$$

where $u\left(c^1_T\right) - u\left(c^2_T\right) - r\left(z_T\right)$ is the marginal disutility of working one additional year and is negative. $1 - c^1_T + c^2_T$ is the marginal productivity of work and is positive. So both terms in brackets are negative and $\dot{c}^1_T < 0$.

A.2 Second Order approach

We infer how c^1_T and z_T vary with T when one has $\dot{c}^2_T = 0$. Using equations (14) and (15) one has:

$$\begin{aligned}
\dot{z}_T &= \dot{c}^1_T\left(\frac{u''\left(c^1_T\right)\left(1 - c^1_T + c^2_T\right)}{r'\left(z_T\right)}\right) \\
\dot{z}_T &= \dot{c}^1_T\left(-\frac{z_T u'\left(c^1_T\right)}{u\left(c^1_T\right) - u\left(c^2_T\right) - r\left(z_T\right)}\right)
\end{aligned}$$

The terms in brackets of the first and second equations have a different sign so that the only possible solution is $\dot{c}_T^1 = \dot{c}_T^2 = \dot{z}_T = 0$.

Assume more generally that the second order condition is not satisfied over a range $[T_{00}, T_0]$ so that there is bunching on this interval. Therefore we have that pre and post retirement consumptions as well as retirement age are the same for all individuals with a type $T \in [T_0, T_{00}]$. Over this range, denote c_0^1, c_0^2 and z_0 the constant levels of consumption and retirement age. To determine T_{00}, T_0 and c_0^2, first note that $\mu(T_0) = \mu(T_{00}) = 0$ but $\mu(T) \neq 0$ for $T_{00} < T < T_0$.

Integrating the RHS of (6) and using $\mu(T_{00}) = 0$, one has

$$\mu(T) = \int_{T_{00}}^{T} \left[-u'\left(c_0^2\right)\left((x - z_0)\,\alpha(x) + \lambda(x)\right) + (x - z_0)\,\gamma f(x)\right] dx$$

Using (11) and $\mu(T_0) = 0$, we obtain

$$\int_{T_{00}}^{T_0} \frac{(T - z_0) f(T)}{u'\left(c_0^2\right)} dT = \int_{T_{00}}^{T_0} \frac{(T - z_T) f(T)}{u'\left(c_T^2\right)} dT \tag{16}$$

Values for T_{00}, T_0 and c_0^2 are jointly determined by (16) and by the requirement that c_T^2 satisfies the first order approach at $T = T_o$ and $T = T_{00}$. Equation (16) stands that the mean of the reciprocal of the marginal utility of consumption c_0^2 should equal the mean of the reciprocal of the marginal utility obtained with the first order approach over the bunching interval.

Note finally that by continuity, propositions 4 and 5 still hold when a type T individual is bunched over the interval $[T_0, T_{00}]$.