Contracting and Ideas Disclosure in the Innovation Process

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Abstract

We analyze the contract between an innovator and a developer, when the former has private information on his idea and the latter must exert efforts but may also quit the relationship after having been informed. We show that the equilibrium contracts distort downwards the developer's incentives but in different ways according to the strength of intellectual property rights (IPR). For example, with intermediate IPR, only pooling contracts arise with a limited amount of information revealed.

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1 Introduction

How inventors manage to appropriate the returns from their research, ideas and discoveries is a central topic in the economics of innovation. This is also an important issue as diffusion of knowledge and industrial R&D have been widely identified as fundamental drivers of growth in our modern economies.[1] Arrow (1962) made a significant breakthrough in understanding this question and argued that the public good nature of knowledge and uncertainties around of the innovative process are key factors that limit incentives to innovate. Before bringing an innovation to the market, some disclosure of core knowledge is needed upstream to transform ideas and raw materials into prototypes. Depending on whether ideas, concepts and knowledge can be easily protected, information transmission within and across R&D units will occur without the threat of expropriation. In case of maverick ideas, *intellectual property rights* (hereafter IPR) appear to be poorly protected so the value of the disclosure is generally offset by the threat that ideas could be stolen. Thus, the process by which information and knowledge spread through the economy is shaped by the way inventions are produced and innovative information disseminated.

This paper studies the diffusion of ideas throughout the innovative process. We analyze the relationship between innovators who have private information on the quality of their ideas and developers who provide complementary but essential skills and expertise helping to transform those ideas into final products. More precisely we study how contracts between innovators and developers are designed and how such arrangements facilitate or not information disclosure during the innovation process. This process is hindered by contractual hazards which can be at the root of distortions in the transmission of knowledge: asymmetric information on the value of the innovator's idea, moral hazard on the developer's side and the protection of IPR when the negotiation reveals information an opportunistic developer might use to start his own business.

Consider an innovator who is privately informed on the value of his idea. This innovator proposes a contract to a developer to share profits. This contract plays three different roles. First, this offer signals the quality of the idea. Second, it provides incentives for the developer. Finally, the contract must also protect the innovator against the developer's opportunistic behavior.

In this signaling environment, bad innovators might overstate the quality of their idea to reap more of the developer' surplus. To credibly convince the developer, good innovators must thus keep enough royalties on the project. They limit thereby the stake of the developer. As a drawback, such contract exacerbates moral hazard on the developer'

[1] A majority of empirical studies found a strong link between R&D capital and national output. As quoted by Cameron (1998) in his empirical survey, *"a 1% increase in the R&D capital stock is found to lead to a rise in output of between 0.05% and 0.1%"*.

side and his incentives to steal ideas. Equilibrium contracts result from this trade-off between inducing knowledge disclosure, providing incentives to developers and avoiding the steal of ideas.

In such environment, the amount of information disclosed at equilibrium depends crucially on the regime of property rights that prevails. With strong IPR, the developer's incentives to renege are limited and the innovator's royalties can be significantly increased, reducing thereby the developer's bonus, without inducing the steal of ideas. With weak IPR, reducing the developer's bonus is no longer so attractive since disclosing information exacerbates developer's incentives to steal the idea for all possible qualities of the idea. Hence, although getting surplus from the developer is harder, the innovator has always some incentives to overstate the quality of his idea. In the two previous situations, there is full disclosure. For intermediate levels of IPR instead, i.e., when the developer is tempted to steal only ideas which are good enough, *countervailing incentives* arise. Indeed, the incentives to exaggerate the value of the idea are now countered by the innovator's desire to make his ideas less attractive to steal. Secrecy endogenously arises.

One of our key findings is that innovators may use information withholding when IPR are weak. With information withholding, contract design does not reveal enough information to let the innovator infer precisely the quality of the idea. This aspect distinguishes significantly our paper from an earlier literature which has instead emphasized the strategic choice made by innovators when hiding pieces of hard evidence on the quality of their ideas. For example, Anton and Yao (2004) showed that, when property right are weak, secrecy must be chosen only for very profitable cost-reducing inventions. Following this "patents versus secrecy" debate,[2] Denicolò and Franzoni (2004) pointed out that the disclosure motive alone suffices to justify patents as IPR and to dimension their length. However, they showed that innovation disclosure through patenting is always socially preferable to secrecy. In our paper instead, withholding of ideas does not necessary harm the diffusion of innovations.

Our paper is more broadly related to a literature addressing information transmission in R&D environments. Contracting hazards associated to the possibility of stealing ideas under asymmetric information have been explored by Gallini and Wright (1990). The trade-off between disclosing licensor's information and facilitating imitation explains the shape of licensing contracts. Macho-Stadler and Perez-Castrillo (1991) and Beggs (1992) also analyzed how menus of licensing contracts stipulating different levels of royalties reveal information on the quality of the innovation. Those papers pointed out that licensing contracts are sufficient tools to communicate information if no other credible device exists.

Our paper differs from this earlier literature by stressing the important trade-off be-

[2]For an extensive economic survey on this debate see Friedman et al. (1991).

tween signaling and downstream moral hazard. In this respect, we build on our companion paper (Martimort, Poudou and Sand-Zantman, 2009) where private information on the value of the innovator's idea is binary. The present paper extends this analysis to consider a continuum of possible values of the innovator's idea. Enriching the model that way allows us to highlight the countervailing incentives that may emerge with intermediate levels of IPR. Partial disclosure has also been explored by Anton and Yao (2002) who analyzed the mechanism by which ideas can be sold on final competitive markets. Similarly, Bhattacharya and Ritter (1983) highlighted a trade-off between the cost from public disclosure of valuable information to potential product market competitors and the gains of signaling to the capital market. More recently, Bhattacharya and Guriev (2006) focused on the impact of information revelation on the kind of market competition between innovators and imitators that takes place. Most of those models rely on the possibility that innovators could disclose hard evidence that could partially reveal their knowledge. In our paper, such partial disclosure is not possible as hard pieces of evidence (software codes, blue prints or generally maverick technical concepts) cannot be gathered.

Section 2 presents the model. Section 3 characterizes conditions on intellectual property such that all ideas are revealed at the equilibrium contract. In Section 4, we focus on the possibility of information withholding and its impact on the diffusion of knowledge. Section 5 concludes. Proofs are relegated to an Appendix.

2 The Model

We consider the contractual relationship between an innovator (the principal), or research unit who owns an idea but has no expertise to develop it and a developer, or customer (the agent) who has such expertise. Both parties are risk-neutral and have unlimited liability.

• **The Innovator.** The quality θ of the innovator's idea belongs to a set $\Theta = [\underline{\theta}, \bar{\theta}]$ with better ideas corresponding to greater values of θ. The innovator has private information on θ. The common knowledge cumulative distribution of ideas is $F(\cdot)$ with an everywhere non-negative density $f = F'$. Let $E(\cdot|\cdot)$ denote the conditional expectation operator. θ is a piece of soft information which can be manipulated both upwards and downwards. Therefore, the principal may either exaggerate or somewhat conceal the true quality of his idea.

• **The Developer.** The developer's effort e is a non-verifiable input which is key to the innovation process. Both the quality of the idea and the effort made by the developer are complements at the extrinsic margin. However, they both enter additively in the (normalized) probability $p(\theta, e) = \theta + e$ that an innovation worth π is realized.[3] Exerting effort e has a non-monetary cost $\psi(e)$ for the developer. For tractability, we assume a quadratic functional form, namely $\psi(e) = \frac{e^2}{2}$.

If the developer chooses to leave the relationship, he gets a state-dependent reservation utility $u(\theta)$ in state θ. Indeed, once he has learned whether the idea is good enough, the developer can renege on the contract and use this information to start his own business. Of course, doing so may require duplicating infrastructures, investments in human capital, losing reputation for keeping trade secrets, paying fines for breach of contract and so on. All those extra costs of setting up a new business, in particular those due to the strength of the IPR, are captured by introducing a fixed-cost I incurred by the agent once he reneges from the contract. Stronger IPR correspond to a larger cost.

Finally, the value of the innovation $\gamma\pi$ that accrues to the developer in case he steals the idea is strictly less than its value inside the relationship, i.e., $\gamma \in [0, 1]$. This captures the fact that, on top of his idea, the innovator's human capital is key to the development process and this capital gets lost if the developer decides to run his own business.[4] Therefore, the developer's state-dependent utility is given by:

$$u(\theta) = \max\left\{0, \max_e (\theta + e)\gamma\pi - \frac{e^2}{2} - I\right\} = \max\left\{0, \theta\gamma\pi + \frac{(\gamma\pi)^2}{2} - I\right\}.$$

Three cases of interest will be analyzed.

- **Strong Property Rights**. $I \geq \bar{I} \equiv \gamma\pi\bar{\theta} + \frac{(\gamma\pi)^2}{2}$. In this first case, it is impossible to initiate a new business when stealing the innovator's idea. The use of knowledge which has been disclosed inside the contractual relationship is easily prevented outside.

- **Weak Property Rights**. $I \leq \underline{I} \equiv \gamma\pi\underline{\theta} + \frac{(\gamma\pi)^2}{2}$. In this second case, it is always easy to renege, leave the relationship and start a valuable business with the stolen idea.

- **Intermediate Property Rights**. $\underline{I} \leq I \leq \underline{I}$. In this last case, only good ideas are worth being stolen by the developer. We can thus define θ_* as the cut-off type which uniquely solves $I = \gamma\theta_*\pi + \frac{(\gamma\pi)^2}{2}$. Therefore, $u(\theta) = \gamma\pi\left(\theta - \theta_*\right) > 0$ for $\theta \geq \theta_*$ while $u(\theta) = 0$ otherwise.

[3] Under full information, the probability of innovation would be given by $\theta + \pi$ so we need to assume that $\bar{\theta} + \pi < 1$.

[4] Note that the lower return on the developer's effort outside the relationship ensures that the innovator and the developer may find some benefits from contracting altogether in the first place

• **The Contract.** Our contracting environment entails bilateral asymmetric information. There is both adverse selection (only the principal knows the value of the technology) and moral hazard (the agent exerts a non-verifiable effort). A contract between the innovator and the developer consists of an up-front fixed-fee a paid by the developer to get access to the innovator's idea, and a bonus w paid to the developer in case the innovation succeeds. Alternatively, the innovator keeps the royalties $\pi - w$ in case of success. Moreover, the mere offer of such contract gives some information about the principal's quality of the idea through the choice of the fixed-fee and the bonus.

The contractual relationship between the innovator and the developer unfolds as follows:

- First, the innovator discovers θ.
- Second, the innovator offers the contract $\mathcal{C} = (a, w)$ to the developer.
- Third, the developer uses the information he has learned from observing the choice of the contract $\mathcal{C}$. He updates accordingly his beliefs about the quality of the innovator's idea. The developer assesses the benefits from stealing the idea and walking away from the relationship with these updated beliefs. If he leaves, the game ends with payoff $E(u(\theta)|\mathcal{C})$ to the developer and 0 to the innovator.
- Fourth, the developer pays an up-front payment a to the principal if he has not opted out. He then exerts a non-verifiable effort e.
- Fifth, the innovation might be successful and the bonus w is paid to the developer.

A first obvious remark is that, because $u(\theta) \geq 0$ for all θ in Θ, the binding participation constraint is obtained when the agent is prevented from reneging on the contract. Therefore, the agent's ex ante participation constraint plays no role in our analysis.

• **Complete Information.** When θ is common knowledge, the only contractual issue stems from the non-verifiability of the developer's effort. It is well-known that this moral hazard problem can easily be solved by making the risk-neutral agent residual claimant for the overall profit of the bilateral coalition he forms with the innovator.

Consider thus the simple "sell-out" contract $\mathcal{C}^*(\theta) = (w^*(\theta), a^*(\theta))$ such that

$$w^*(\theta) = \pi \quad \text{and} \quad a^*(\theta) = \theta\pi + \frac{\pi^2}{2} - u(\theta).$$

Since he enjoys the full gain from increasing the probability of an innovation, the developer exerts the first-best level of effort $e^*(\theta) = \pi$. The innovator can extract the agent's gains from trade by asking for an up-front payment that leaves the developer just indifferent between walking away or not. Moral hazard on the agent's side is not on issue when the quality of the idea θ is common knowledge so the complete information outcome can easily be achieved. As a result, innovations are more likely to be brought to the market as ideas become better since the probability of an innovation is strictly increasing with quality of the idea.

Since the up-front payment $a(\theta)$ increases with the type announced, the innovator would have some incentives to exaggerate this quality if the same scheme $\{w^*(\theta), a^*(\theta)\}$ were still offered once the principal is privately informed on θ.

• **Incomplete Information.** We now turn to the design of the contract under bilateral asymmetric information. To find the equilibrium allocations, we study the family of profit-sharing linear schemes $\{a(\hat{\theta}), w(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ that are Perfect Bayesian equilibria of the R&D game describe above and we take the undominated one ("least-costly") from the innovator's viewpoint. First, a contract is feasible if it is incentive compatible and prevents the agent from leaving the relationship upon inferring θ from the choice of the contract $\mathcal{C}(\hat{\theta})$. Second, among these feasible contracts, we take the one that maximizes the innovator's expected payoff. As shown in the Appendix, an optimal contract $\mathcal{C}(\theta)$ gives to the innovator with type θ the expected payoff $V(\theta)$ such that:

$$V(\theta) = (\theta + w(\theta))\pi - \frac{1}{2}w^2(\theta) + (E(\theta|\mathcal{C}(\theta)) - \theta)w(\theta) - E(u(\theta)|\mathcal{C}(\theta)). \tag{1}$$

with the fixed-fee satisfying $a(\theta) = E(\theta|\mathcal{C}(\theta))w(\theta) + \frac{w^2(\theta)}{2} - E(u(\theta)|\mathcal{C}(\theta))$. Relation (1) states that the principal gets the whole surplus of the relationship net of the developer's opportunity cost.

Moreover, the following incentive compatibility constraints must hold:

- $V(\theta)$ is also a.e. differentiable with

  $$\dot{V}(\theta) = \pi - w(\theta) \geq 0 \tag{2}$$

  where the inequality follows from the fact that the innovator's royalties $\pi - w$ are positive;

- $w(\theta)$ is non-increasing and thus a.e. differentiable with

  $$\dot{w}(\theta) \leq 0. \tag{3}$$

Constraints (2) and (3) indicate that to send an incentive compatible and credible signal on the quality of his idea, the innovator should get an increasing expected payoff. Moreover, incentive compatibility implies that, as his idea gets better, the innovator should be ready to keep more royalties to credibly signal such information. The drawback of such design is that the developer's marginal incentives to exert effort necessarily decrease.

3 Full Disclosure of Ideas and Strength of IPR

We first focus on cases where the contract is fully separating. Innovators with different quality of idea end up choosing different ways of sharing profit with the developer. The developer infers from this choice the exact value of the innovator's idea. On the equilibrium path, the developer has point mass beliefs on the true quality of the idea, i.e., $E(\theta|\mathcal{C}(\theta)) = \theta$ for all θ.

To characterize the equilibrium contract, let us define the variable $w(\theta|y, z, x)$ as the unique solution to the differential equation:[5]

$$\dot{w}(\theta) = \frac{x\pi - w(\theta)}{\pi - w(\theta)}, \tag{4}$$

where $x \in [0, 1]$ is a positive real parameter and $(y, z) \in [0, \pi] \times \Theta$ are real parameters defining the generic boundary condition $w(z) = y$. Define also the following paths $w_0(\theta) = w(\theta|\pi, \underline{\theta}, 0)$, $w_\gamma(\theta) = w(\theta|\pi, \underline{\theta}, \gamma)$ and $w_*(\theta) = w(\theta|y, \theta_*, \gamma)$ obtained when $y = w_0(\theta_*)$. The least-costly equilibrium contract exhibits the following features.

Proposition 1 *Assume that $\bar{\theta} - \underline{\theta} \leq \frac{1-\gamma}{2}\pi$,[6] the least-costly separating contract entails a bonus $w(\theta)$ which is everywhere continuous, and non-increasing. Moreover, it satisfies the following properties.*

- $\star$ $\dot{u}(\theta) \leq w(\theta) \leq \pi$ *with* $w(\bar{\theta}) = \pi$.
- $\star$ *If* $I \geq \bar{I}$, $w(\theta) = w_0(\theta), \forall\theta \in \Theta$.
- $\star$ *If* $\min\{I_*, \bar{I}\} > I > \underline{I}$,*where* $I_* = \underline{I} + \gamma\pi^2[\gamma - \log\gamma - 1]$, *we have*

$$w(\theta) = \begin{cases} w_0(\theta) & on\ [\underline{\theta}, \theta_*] \\ w_*(\theta) & on\ [\theta_*, \bar{\theta}]. \end{cases}$$

- $\star$ *If* $I \leq \underline{I}$, $w(\theta) = w_\gamma(\theta), \forall\theta \in \Theta$.
- $\star$ *Finally, the fixed-fee* $a(\theta)$ *satisfies*

$$a(\theta) = \theta w(\theta) + \frac{w^2(\theta)}{2} - u(\theta).$$

To understand these results, remember that, when choosing the complete information contract $\{a^*(\theta), w^*(\theta)\}$, the innovator has some incentives to overstate the quality of his

[5]See Appendix for details.

[6]This restriction is sufficient to ensure that out-of-equilibrium offers are never profitable for any type. See the Appendix for details.

idea in order to increase the developer's up-front payment. To credibly convince the developer and separate himself from bad innovators, an innovator with a good quality idea must be ready to take more royalties on a project which is more likely to succeed. Therefore, the equilibrium contract calls for reducing the developer's bonus for all ideas, except the worst one for which the innovator asks for no royalties.

As IPR become weaker (i.e., as I decreases), the downward distortion of the developer's bonus decreases and royalties increase. This reflects the fact that the developer's incentives inside and outside the relationship go hand in hand. Indeed, when IPR are weak enough, reducing the developer's contractual bonus for signaling reasons is no longer as easy since it might induce him to leave. This limits the ability for the best innovators to signal their types and reduces the amount of royalties they can get upon success. Signaling is made harder with weak IPR and bonuses are above their market value $w_\gamma(\theta) > \gamma\pi$ for all θ.

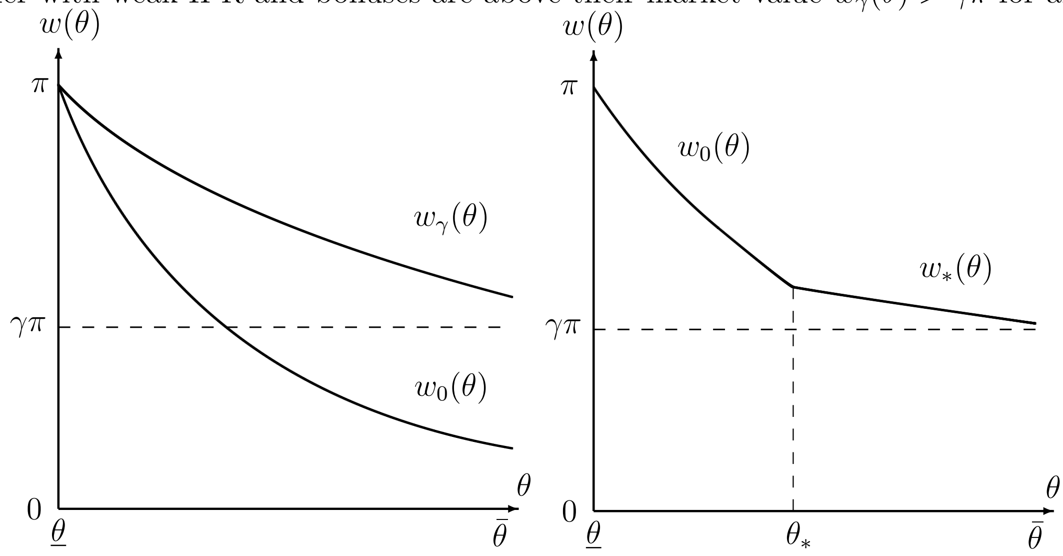

Figure 1: Disclosure for Strong and Weak IPR

The optimal bonuses are depicted in Figure 1. The left-hand side of the figure represents the cases with either strong or weak IPR which correspond respectively to large and small distortions.

The right-hand side corresponds instead to the case of where imitation costs are low-intermediate (i.e. $\min\{I_*, \bar{I}\} > I > \underline{I}$). As the idea becomes sufficiently good, precisely when $\theta \geq \theta^*$, the developer gets now a positive profit by leaving the relationship. Innovators with types just above θ^* still needs to separate themselves from less efficient ones but cannot do it so easily without making valuable for the developer to run away. This explains the kink in the profile $w(\theta)$ at θ^*.

Proposition 1 depicts[7] all regimes for the equilibrium contract when $I_* \geq \bar{I}$. The case $I_* < \bar{I}$, i.e., intermediate values of IPR, is studied in the next section.

Under asymmetric information, the probability of innovation is lower than the first-best due to the developer's reduced effort. Of course, this effect matters more when bonus distortions are important, i.e., for strong IPR.

Corollary 1 *Assume that $I_* \geq \bar{I}$, the probability of innovation is U-shaped in θ, with a slope strictly lower than one.*

When $I_* \geq \bar{I}$, information on the value of the idea can be fully disclosed. However, this separation has a significant impact on the probability of innovation. Innovators with ideas close to the worst type need to increase the amount of royalties they keep to be differentiated from slightly more inefficient innovators. This separation is increasingly harder as one moves closer to this worst type. This means that the developer's effort also sharply decreases around that point. The probability of innovation necessarily decreases with θ in that neighborhood. Thus, moral hazard matters a lot around the worst ideas.

Instead, an innovator with a much better type does not need to stifle as much the developer's incentives to separate from nearby less efficient types. The driving force in the probability of innovation is due to the increase in the quality of his idea and, overall, the probability increases. The intrinsic quality of the idea matters more as the idea becomes better.

Finally, it is interesting to see the impact of stronger IPR on the probability of innovation. Quite surprisingly, strengthening IPR makes it easier to signal the quality of ideas by keeping royalties which has a detrimental impact on the developer's effort and thus on the probability of innovation.

4 Countervailing Incentives and Information Withholding

Suppose now that I takes intermediate values, i.e., a regime with intermediate IPR. The type space can be divided into two distinct intervals $[\underline{\theta}, \theta_*]$ and $[\theta_*, \bar{\theta}]$. Upon knowing that θ belongs to $[\underline{\theta}, \theta_*]$, the developer's best outside option is to refuse contracting but running a business on his own with the stolen idea is not valuable, $u(\theta) = 0$. Instead, when θ lies in $(\theta_*, \bar{\theta}]$, running a new business yields a positive payoff, $u(\theta) > 0$.

[7] This is the case if $\bar{\theta} - \underline{\theta} < \pi \min\{\frac{1}{2}(1-\gamma), \gamma - \log\gamma - 1\}$, which requires in turns that $\gamma \in (0, \gamma_0)$ where γ_0 is the unique value of γ such that $\frac{1}{2}(1-\gamma) = \gamma - \log\gamma - 1$ and worth $\gamma_0 \simeq 0.4172$.

The existence of these two distinct intervals creates some *countervailing incentives* for the innovator. Indeed, an innovator having a rather bad idea, say just below the cut-off value θ_*, still wants to exaggerate the quality of his idea to increase the fee paid by the developer. He can be prevented from doing so by keeping most of the returns on innovation but also by taking an upfront payment large enough that extracts all the developer's surplus. However, an innovator having an idea just above the cut-off value θ_* has now also some incentives to understate this idea to get this large up-front payment, making thereby the developer believe that stealing the idea to run a new business is not as attractive. The innovator is therefore torn between his desire to exaggerate his idea to get extract more surplus from the developer and his willing to pretend that his idea is not as good so that this up-front payment gets larger.

When the cut-off between those two intervals is close enough to $\underline{\theta}$, this problem can be solved by decreasing slightly the slope of the marginal returns w but keeping a full separation between types. This corresponds to the second case highlighted in Proposition 1. When the cut-off type is larger, this is no longer possible since the marginal productivity of effort inside the contract at point θ_* is smaller than the marginal productivity outside. More precisely, if the optimal contract were still defined by Proposition 1, the bonus for types just below θ_* would be lower than the marginal incentives outside for ideas just above θ_*. We could then have an upward jump in the bonus scheme which would contradict incentive compatibility.

This line of argument shows that there cannot be any fully separating equilibrium in that case. The equilibrium contract exhibits necessarily some pooling over some range with different types of the innovator in that subset choosing the same contract and revealing only rough information on ideas. In other words, the innovator has now some incentives to conceal the exact quality of his ideas using the same level of bonus on an upper tail of the distribution. This can be interpreted as an information withholding strategy in the contract.

In that case of countervailing incentives,[8] the equilibrium contract that entails maximal separation of types has the following features.

Proposition 2 *Assume that* $I \in]I^*, \bar{I}[$, $\pi[\gamma - \log\gamma - 1] < \bar{\theta} - \underline{\theta} \leq \frac{1-\gamma}{2}\pi$,[9] *and define* $\bar{E}(t) = E(\theta|\theta \geq t)$. *The equilibrium contract entails:*

[8]Gertner, Gibbons and Scharstein (1988), Noldeke and Van Damme (1990) and Spiegel and Spulber (1997) have also analyzed signaling games with countervailing incentives in other contexts.

[9]This requires that $\gamma \in (\gamma_0, 1)$ where γ_0 is defined in footnote 7 .

⋆ *A bonus* $w(\theta)$ *which is continuous, such that* $w(\theta) \leq \pi$ *and*

$$w(\theta) = \begin{cases} w_0(\theta) & \text{on } [\underline{\theta}, \tau] \\ \bar{w} & \text{on } [\tau^*, \bar{\theta}] \end{cases}$$

where $\bar{w} = w_0(\tau)$ *and* $\tau < \theta_*$ *is defined implicitly as the unique solution to*

$$(1 - F(\tau))\,(\bar{E}(\tau) - \tau)w_0(\tau) = (1 - F(\theta_*))\,(\bar{E}(\theta_*) - \theta_*)\gamma\pi \tag{5}$$

and such that $\bar{w} < \gamma\pi$*;*

⋆ *The agent's conditional expectations about the quality of the idea are defined by*

$$E(\theta|\mathcal{C}(\theta)) = \begin{cases} \theta & \text{on } [\underline{\theta}, \tau] \\ \bar{E}(\tau) & \text{on } [\tau, \bar{\theta}] \end{cases} \tag{6}$$

with $\bar{E}(\tau) > \theta_*$*;*

⋆ *Finally, the fixed-fee* $a(\theta)$ *satisfies*

$$a(\theta) = E(\theta|\mathcal{C}(\theta))w(\theta) + \frac{w^2(\theta)}{2} - E(u(\theta)|\mathcal{C}).$$

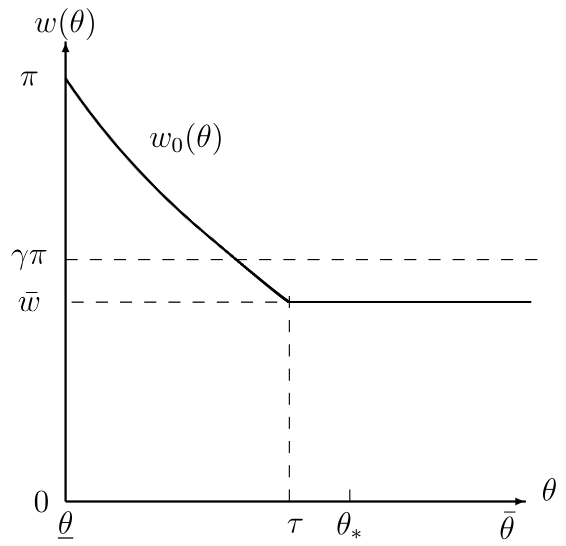

Figure 2: High-Intermediate IPP and Non-disclosure

With countervailing incentives, the innovator conceals the innate quality of his idea when this quality is good enough. This strategy of non-disclosure appears as an endogenous remedy to the weakness of IPR in this context.[10] Over the bunching area and given

[10]Indeed reducing IPR increases information withholding i.e $\frac{d\tau}{dI} > 0$ as shown in Appendix, proof of Proposition 2.

his posterior beliefs on the innovator's type following an offer, the developer is just indifferent *on average* between stealing the idea, whose true quality is not yet revealed, and still contracting with the innovator. As a result, with those strong countervailing incentives, the developer may face some potential losses.

An interesting question is whether information withholding has a depressing effect on the the probability of success of an innovation.

Corollary 2 *At the equilibrium contract described in Proposition 2, and for ideas such that $\theta > \tau$, the probability of innovation is strictly increasing with θ and strictly higher than if information had been disclosed.*

One could think that information withholding, since it reduces the diffusion of knowledge between parties, could slow down innovation. Corollary 2 shows that information withholding has a positive effect on the probability of innovation. Indeed, the depressive effect of bonus reduction used by the innovator for incentives purposes when ideas are disclosed is now limited. Since the bonus is constant for undisclosed ideas, the developer's effort remains the same. Therefore, only the innate quality of the idea has a marginal effect on the probability of innovation and this probability is increasing at its first-best pace (i.e. $\dot{p}() = 1$). Finally, this probability is also higher than if pooled ideas have been disclosed.

5 Conclusion

This paper has analyzed the design of contracts between an informed inventor and a developer and its implications on the diffusion of ideas. The main insight is that, as the contract must simultaneously induce the innovator to reveal information on the quality of his idea, induce effort from the developer and secure IPR on knowledge, innovators must keep more royalties on innovative projects, reducing in turn the developer's effort and lowering the probability of innovation. Moreover, the better the quality of the idea, the larger royalties are.

However, the strength of IPR - which is related to the difficulty for the developer to replicate the idea - mitigates these contractual features. When IPR are weak, reducing the bonus in case of success is less attractive for the innovator since idea's disclosure exacerbates the developer's incentives to steal the idea.

The fact that the developer may bypass the inventor once the quality of an idea becomes common knowledge might induce countervailing incentives. When IPR are not so strong, an innovator with a bad idea is not afraid of stealing ideas but would be if the ideas were better. Therefore, information withholding may occur for the best ideas and equilibrium contracts call for a fixed bonus. Surprisingly, this behavior has positive effect on the probability of success of the innovative project and therefore is not harmful for the success of innovations.

The main insight of our analysis is that full disclosure of knowledge inside the innovative process is not a guarantee of success of innovations. When IPR are not perfectly secured, information withholding about core knowledge in the R&D relationship can enhance innovation.

Our analysis could be extended along at least two lines. First other technologies aggregating effort and ideas could be studied. In the case of complementarity between effort and ideas at the intrinsic margin, we might suspect that bonus distortions are less valuable because a lower effort the developer feeds back negatively on the innovator's incentives to signal himself. Second, in our analysis all information revelation takes place through contract design. However, one could think that contracts cannot be agreed upon by both parties without being relatively specific on the idea brought by the innovator. Hence a more realistic information transmission device would be to assume that the innovator should also provide the developer with hard evidences (i.e., blue-print or running prototypes for instance). Innovators could forge favorable evidence on their idea in order to fool developers and this would relax the signalling problem and mitigate much of the effect we have stressed above.[11] In particular, the amount of information withholding is likely to be reduced in such environments.

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Appendix

• **Incentive compatibility and enforcement constraints.** We denote $\mathcal{C}(\hat{\theta}) = \{w(\hat{\theta}), a(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ feasible contracts offered in a separating perfect Bayesian equilibrium. For any $\theta \in \Theta$, let $\tilde{V}(\theta, \hat{\theta})$ the type θ innovator's expected payoff when he proposes a contract $\mathcal{C}(\hat{\theta})$ and $V(\theta) = \tilde{V}(\theta, \theta)$ his payoff when following a truthtelling strategy. We have:

$$\tilde{V}(\theta, \hat{\theta}) = (\theta + e(\hat{\theta}))(\pi - w(\hat{\theta})) + a(\hat{\theta}) \tag{A.1}$$

where $e(\hat{\theta})$ is the agent's effort when the contract $\mathcal{C}(\hat{\theta})$ has been chosen by the principal and the developer has put mass beliefs on the innovator's type being $\hat{\theta}$. This effort maximizes the developer's expected payoff:

$$e(\hat{\theta}) = \arg\max_{e} \left\{ (E(\theta|\mathcal{C}(\hat{\theta})) + e)w(\hat{\theta}) - \frac{e^2}{2} - a(\hat{\theta}) \right\} = w(\hat{\theta}). \tag{A.2}$$

At a separating equilibrium, the innovator with idea θ prefers to offer $(a(\theta), w(\theta))$ rather than other equilibrium contract $(a(\hat{\theta}), w(\hat{\theta}))$ or any other unexpected offer that may be accepted by the agent. Let us first focus on the first set of constraints. The corresponding incentive compatibility constraints can be written as

$$V(\theta) = \max_{\hat{\theta} \in \Theta} \tilde{V}(\theta, \hat{\theta}) \tag{A.3}$$

Standard arguments show that - see Laffont and Martimort (2002, Chapter 3) for instance-

- $w(\theta)$ is non-increasing and thus a.e. differentiable with

$$\dot{w}(\theta) \leq 0, \tag{A.4}$$

- $V(\theta)$ is also a.e. differentiable with

$$\dot{V}(\theta) = \pi - w(\theta). \tag{A.5}$$

Moreover, a contract is feasible if it also prevents the agent from leaving the relationship upon inferring θ from the choice of the contract in $\mathcal{C}(\hat{\theta})$. The following enforcement constraint must thus be satisfied:

$$E(\theta|\mathcal{C}(\hat{\theta}))w(\hat{\theta}) + \frac{1}{2}w^2(\hat{\theta}) - a(\hat{\theta}) \geq E(u(\theta)|\mathcal{C}(\hat{\theta})), \quad \forall \hat{\theta} \in \Theta.$$

using (A.5) and substituting for $a(\hat{\theta})$, this last constraint becomes:

$$\tilde{V}(\theta, \hat{\theta}) \leq (\theta + w(\hat{\theta}))\pi - \frac{1}{2}w^2(\hat{\theta}) + (E(\hat{\theta}|\mathcal{C}(\hat{\theta})) - \theta)w(\hat{\theta}) - E(u(\hat{\theta})|\mathcal{C}(\hat{\theta})) \quad \forall \hat{\theta} \in \Theta. \tag{A.6}$$

As there are many possible feasible contracts, we shall focus on the undominated ones from the principal viewpoint. For those allocations, the developer's enforcement constraint (A.6) is in fact binding, hence at any equilibrium, the incentive scheme $\mathcal{C}(\theta)$ yields an innovator's expected payoff $V(\theta)$ such that:

$$V(\theta) = (\theta + w(\theta))\pi - \frac{1}{2}w^2(\theta) + (E(\theta|\mathcal{C}(\theta)) - \theta)w(\theta) - E(u(\theta)|\mathcal{C}(\theta)) \quad \forall \theta \in \Theta. \tag{A.7}$$

with (A.4) and (A.5) to hold. The innovator should also be deterred to propose any unexpected contract. Formally, a contract $(w, a) \notin \{w^*(\theta), a^*(\theta)\}_{\theta \in \Theta}$ should not be preferred to the equilibrium contract. Specifying the out-of-equilibrium beliefs following such a proposal, we choose the most pessimistic beliefs which put all weight on $\underline{\theta}$ type to relax incentive constraints. See Mailath (1987) among many others.

• **Proof of Proposition 1**. We neglect the monotonicity constraint (A.4). Letting $E(\theta|\mathcal{C}(\theta)) = \theta$ for all $\theta \in \Theta$ in (A.7), the optimal bonus is such that

$$V(\theta) = (\theta + w(\theta))\pi - \frac{1}{2}w^2(\theta) - u(\theta).$$

Differentiating in θ and using (A.5) leads directly to:

$$\dot{w}(\theta) = \frac{\dot{u}(\theta) - w(\theta)}{\pi - w(\theta)}, \forall \theta \in \Theta. \tag{A.8}$$

Letting $\dot{u}(\theta) = x\pi$ and $x \in \{0, \gamma\}$, one can see that (A.8) is (4) in the text. Notice that from equation (A.8), whenever $\dot{u}(\theta) = 0$, $w(\theta)$ is decreasing since $w(\theta) \in [0, \pi]$, but when $\dot{u}(\theta) = \gamma\pi$ it is necessary that $w(\theta) \geq \gamma\pi$. Hence under these conditions, all candidate equilibria are characterized by the initial value $w(\underline{\theta}) \leq \pi$.

The equilibrium offer $(a(\theta), w(\theta))$ is preferred by the innovator with type θ to any out-of-equilibrium contract if and only if

$$a(\theta) + (\pi - w(\theta))(\theta + w(\theta)) \geq \max_{\{w \in [0,\pi], a \leq \underline{\theta} w + \frac{w^2}{2} - u(\underline{\theta})\}} \{a + (\pi - w)(\theta + w)\} \tag{A.9}$$

Resolving the program in RHS of (A.9) easily leads to the optimal out-of-equilibrium offer $w^{\text{off}}(\theta) = \pi - (\theta - \underline{\theta})$ and $a^{\text{off}}(\theta) = \underline{\theta} w^{\text{off}}(\theta) + \frac{w^{\text{off}}(\theta)^2}{2} - u(\underline{\theta})$. In fact, given the out-of-equilibrium pessimistic beliefs and using the incentive compatibility constraints $V(\theta) \geq \tilde{V}(\theta, \hat{\theta}), \forall \hat{\theta} \in \Theta$, the best contract that an innovator with the worst idea can offer must be such that

$$w(\underline{\theta}) = \arg \max_{w \in [0,\pi]} \{\underline{\theta}\pi + w(\pi - \frac{w}{2}) - u(\underline{\theta})\} = \pi \tag{A.10}$$

Hence with (A.10), the monotonicity constraint (A.5) is checked for $\dot{u}(\theta) \in \{0, \gamma\pi\}$ for *all* θ, that is to say when $I \leq \underline{I}$ or when $I \geq \bar{I}$ (these fixed-cost levels have been defined in section 2, p.5). Then just to denote $w_0(\theta) = w_0(\theta|\pi, \underline{\theta}, 0)$ the solution of (A.8) with $w(\underline{\theta}) = \pi$ whenever $u(\theta) = 0$ and $w_\gamma(\theta) = w(\theta|\pi, \underline{\theta}, \gamma)$ whenever $u(\theta) > 0$. Looking at (A.8) as a separable differential equation, one can implicitly define $w(\theta|y, z, x)$ as the unique solution in w to:

$$y - w + \pi(1 - x) \log \left[\frac{w - x\pi}{y - x\pi}\right] = z - \theta. \tag{A.11}$$

where $w(z) = y$ is a boundary condition and $x \in \{0, \gamma\}$.

Finally, to check condition (A.9) for all types, define then the out-of-equilibrium payoff of type θ as $V^{\text{off}}(\theta) = (\pi - w^{\text{off}}(\theta))(\theta + w^{\text{off}}(\theta)) + \underline{\theta} w^{\text{off}}(\theta) + \frac{(w^{\text{off}}(\theta))^2}{2} - \bar{u}(\underline{\theta})$, then $\dot{V}^{\text{off}}(\theta) = \pi - w^{\text{off}}(\theta)$. Since $V^{\text{off}}(\underline{\theta}) = V(\underline{\theta})$, then (A.9) is satisfied as long as $w(\theta) \leq w^{\text{off}}(\theta)$. This inequality is satisfied for $\underline{\theta}$ so for the first type $z \in \Theta$ such that $w(z) = w^{\text{off}}(z)$, we must have $\dot{w}(z) \geq \dot{w}^{\text{off}}(z)$. What is the condition such that, $\dot{w}(\theta) \leq \dot{w}^{\text{off}}(\theta)$, $\forall \theta \in \Theta$?

1. If $I \geq \bar{I}$, then, for $w(\theta) = w^{\text{off}}(\theta)$,

$$\begin{aligned} \dot{w}(\theta) &\leq \dot{w}^{\text{off}}(\theta) \Leftrightarrow \frac{-w(\theta)}{(\pi - w(\theta))} = \frac{-w^{\text{off}}(\theta)}{(\pi - w^{\text{off}}(\theta))} \leq \dot{w}^{\text{off}}(\theta) = -1 \\ &\Leftrightarrow w^{\text{off}}(\theta) \geq \frac{\pi}{2} \end{aligned}$$

Therefore, if $\bar{\theta} - \underline{\theta} \leq \frac{\pi}{2}$, then $w(\theta) \leq w^{\text{off}}(\theta)$, $\forall \theta \in \Theta$.

2. For $I \leq \underline{I}$, we replicate the previous proof with the new separating scheme defined implicitly by $w_\gamma(\theta)$. The sufficient condition is then $\bar{\theta} - \underline{\theta} \leq \frac{(1-\gamma)\pi}{2}$. This latter condition is more stringent than the previous condition thus valid for both cases.

1. Now consider the case where $\underline{I} < I < \bar{I}$, then we can proceed exactly as apart from the fact that (A.8) is discontinuous for $\theta = \theta_*$ so its solution $w(\theta)$ has a kink for this value. However if I is such that $w_0(\theta_*) < \gamma\pi$ then a suboptimal upward discontinuity occurs at $\theta = \theta_*$. So define I_* such that $w_0(\theta_*) = \gamma\pi$ and using (A.11) one can find $I_* = \underline{I} + \gamma\pi^2[\gamma - \log\gamma - 1]$ which is greater than $\bar{I}$ if $\bar{\theta} - \underline{\theta} \geq \pi[\gamma - \log\gamma - 1]$. Therefore to accept the separating solution, we restrict I to be less than $\min\{I_*, \bar{I}\}$. The restriction $\bar{\theta} - \underline{\theta} \leq \frac{(1-\gamma)\pi}{2}$ is then again sufficient to sustain out-of-equilibrium pessimistic beliefs. ■

• **Proof of Corollary 1**. At the equilibrium depicted in Proposition 1, the probability of innovation at is given by $p(\theta, w(\theta)) = \theta + w(\theta)$. Differentiating in θ and using (A.8) leads to

$$\dot{p}(\theta, w(\theta)) = 1 + \frac{\dot{u}(\theta) - w(\theta)}{\pi - w(\theta)} \text{ and } \ddot{p}(\theta, w(\theta)) = -\dot{w}(\theta)\frac{\pi}{(\pi - w(\theta))^2} > 0 \tag{A.12}$$

Convexity of $p(\cdot)$ is then readily checked. If $\dot{u}(\theta) = 0$ then a minimum for $p(\cdot)$ is reached for $w_0(\theta) = \frac{\pi}{2}$ that is, using (A.11), for $\tilde{\theta} = \underline{\theta} + \frac{\pi}{2}(2\ln 2 - 1)$. When $\dot{u}(\theta) = \gamma\pi$, the minimum is reached for $w(\theta) = \frac{(1+\gamma)\pi}{2}$ that is in $\tilde{\theta}_\gamma = \underline{\theta} + (1-\gamma)\frac{\pi}{2}(2\ln 2 - 1)$. Therefore $\tilde{\theta}_\gamma < \tilde{\theta}$. Consequently the probability of innovation is decreasing in θ below these values and increasing above. If $\theta_* \in (\tilde{\theta}_\gamma, \tilde{\theta})$, which is the case if $(1-\gamma)I_0 < I < I_0$ with $I_0 = \underline{I} + \gamma\frac{\pi^2}{2}(2\log 2 - 1)$, the probability of success is minimum for $\theta = \theta_*$. Moreover from (A.12), we easily check that for all $\theta \in \Theta, \dot{p}(\theta, w(\theta)) \in (-\infty, 1)$ for any $w(\theta) \in \{w_0(\theta), w_*(\theta), w_\gamma(\theta)\}$. Finally for a given quality of the idea $\theta > \underline{\theta}$, from Proposition 1 one can easily see that $w_0(\theta) < w_*(\theta) < w_\gamma(\theta)$ so it is true that $p(\theta, w_0(\theta)) < p(\theta, w_*(\theta)) < p(\theta, w_\gamma(\theta))$. ■

• **Proof of Proposition 2:** If $I \in (I_*, \bar{I})$ that is if $\bar{\theta} - \underline{\theta} < \pi[\gamma - \log\gamma - 1]$, the incentive constraint (A.5) is violated in the case of a separating equilibrium since $w(\theta)$ defined by (A.11) is increasing on $[\theta_*, \bar{\theta}]$. The optimal solution must include an area of pooling. Let us denote τ the lower bound of this pooling area and $\bar{w}$ the corresponding bonus. It must be clear that $\tau < \theta_*$, to avoid violating again the incentive compatibility condition on $[\theta_*, \tau]$. If $V^b(\cdot)$ is the innovator's payoff with a pooling contract and $V^s(\cdot)$ his payoff in case of separation, and denoting $\bar{E}(t) = E(\theta|\theta \geq t) = \frac{\int_t^{\bar{\theta}} \theta f(\theta) d\theta}{1-F(t)}$, then

$$\begin{aligned} V^s(\theta) &= \theta\pi + w_0(\theta)\pi - \frac{w_0(\theta)^2}{2} - u(\theta) \\ V^b(\theta) &= \theta\pi + \overline{w}\pi - \frac{\overline{w}^2}{2} + \overline{w}(\bar{E}(\tau) - \theta) - E(u(\theta)|\theta \geq \tau) \end{aligned}$$

Moreover $E(u(\theta)|\theta \geq \tau) = \frac{\int_\tau^{\theta_*} 0 f(\theta)d\theta + \int_{\theta^*}^{\bar{\theta}} \gamma\pi(\theta - \theta_*) f(\theta) d\theta}{1-F(\tau)} = \frac{1-F(\theta_*)}{1-F(\tau)} \left(\bar{E}(\theta_*) - \theta_*\right)\gamma\pi$
At $\theta = \tau$, the optimal pair $(\tau, \overline{w})$ must satisfy the incentive constraint (A.4) which writes $\overline{w} \leq w_0(\tau)$. If $\overline{w}$ is an undominated incentive contracts, then $\overline{w} = \arg\max_{\overline{w} \leq w_0(\tau)} V^b(\tau)$. $V^b(\tau)$ is clearly increasing at $\overline{w} = w_0(\tau)$ since $w_0(\tau) < \pi$ and $\bar{E}(\tau) > \tau$:

$$\frac{\partial V^b(\tau)}{\partial \overline{w}}_{|\overline{w} = w_0(\tau)} = \pi - w_0(\tau) + \bar{E}(\tau) - \tau > 0$$

so $\overline{w} = w_0(\tau)$ at the equilibrium. Since $\tau < \theta_*$, $u(\tau) = 0$. Clearly for $\theta = \tau$, incentive compatibility implies continuity of payoffs, this writes:

$$V^s(\tau) = V^b(\tau) \Leftrightarrow w_0(\tau)(\pi - \frac{w_0(\tau)}{2}) = \overline{w}\pi - \frac{\overline{w}^2}{2} + \overline{w}(\bar{E}(\tau) - \tau) - \frac{1-F(\theta_*)}{1-F(\tau)}\left(\bar{E}(\theta_*) - \theta_*\right)\gamma\pi$$

and with $\overline{w} = w_0(\tau)$ this implies

$$w_0(\tau) = \frac{(1-F(\theta_*))\left(\bar{E}(\theta_*) - \theta_*\right)}{(1-F(\tau))\left(\bar{E}(\tau) - \tau\right)}\gamma\pi$$

The RHS should be strictly positive hence $\bar{E}(\tau) > \theta_*$. This RHS is increasing with respect to τ (starting from zero) so the optimal τ is characterized by

$$(1 - F(\tau))\,(\bar{E}(\tau) - \tau) w_0(\tau) = (1 - F(\theta_*))\,(\bar{E}(\theta_*) - \theta_*)\gamma\pi. \tag{A.13}$$

Last, let us check that no out-of-equilibrium offer can be preferred by any principal to this equilibrium. We have shown in the preceding proof that the optimal out-of-equilibrium offer is $w^{\text{off}}(\theta) = \pi - (\theta - \underline{\theta})$ and that (A.9) is satisfied as soon long as $w(\theta) \leq w^{\text{off}}(\theta)$. Up to τ, (A.9) is satisfied if $\bar{\theta} - \underline{\theta} \leq \frac{\pi}{2}$ as shown in the previous proof. From τ, since $w_0(\tau) < \gamma\pi$, one has simply to check that $w^{\text{off}}(\theta) > \gamma\pi$. Using the characterization of

$w^{\text{off}}(\theta)$ leads to the following restriction: $\bar{\theta} - \underline{\theta} \leq (1-\gamma)\pi$.
We finally prove that $\frac{d\tau}{dI} > 0$. One can define τ as a function of I in the interval $[I_*, \bar{I}]$ and shows it is increasing since differentiating (A.13) leads to

$$\frac{d\tau}{dI} = \frac{1 - F(\theta_*)}{(1 - F(\tau))\left[w_0(\tau) - (\bar{E}(\tau) - \tau)\dot{w}_0(\tau)\right]} > 0.$$

Moreover from definition (A.13) $\tau = \theta_*$ if $I = I_*$ and $I = \bar{I}$. ■

• **Proof of Corollary 2**. From the equilibrium in Proposition 2 and for $\theta > \tau$, one can see that $\dot{p}(\theta, \bar{w}) = 1 > 0$. If disclosure were ensured for these ideas and either $w(\theta) = w_0(\theta)$ or $w(\theta) = w_\gamma(\theta)$ were optimal solutions for all θ, then (A.12) would hold and this immediately implies that $\dot{p}(\theta, \bar{w}) = 1 > \dot{p}(\theta, w(\theta))$. Moreover at $\theta = \tau$, by definition of $\bar{w}$, $p(\tau, \bar{w}) = p(\tau, w_0(\tau))$ and for $\theta > \tau$, $p(\theta, \bar{w})$ is increasing at a faster pace than $p(\theta, w_0(\theta))$ or $p(\theta, w_\gamma(\theta))$. ■