

# Economic growth and the design of search engines

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## ABSTRACT

The Internet plays a growing role in the economy. This paper extrapolates this trend and analyses a world where most transactions take place in "cyberspace". We ask the following question: how does the design of the search engine affect the incentives to innovate and the economy's long run growth rate? This is done in the context of a "qualitative" model where growth occurs because the number of varieties grows and consumers select a shrinking fraction of the available goods, of growing quality. They must use a search engine to locate goods. The search engine affects the market size of a good over its life cycle, and thus the incentives to innovate. Its structure has two conflicting effects. A *visibility* effect by which a greater hit score increases market size. A *selection* effect by which consumers are more picky and select higher quality goods, thus reducing the life span of any given good.

While these two effects on growth cancel out for simple specifications, that is no longer the case if a firm's score is variable along its life cycle or if the search process uses resources.

It is shown that the discount effect of gradual recognition of popularity tends to reduce growth. Hence, growth is enhanced if the search engine is less sensitive to popularity. Also, growth is lower when the search engine rewards "web page quality" better because of the resources diverted away from R and D into advertising. But these mechanisms generate opposite level effects on the average quality selected by consumers. As a result the net effect on welfare is ambiguous.

Keywords: Endogenous Growth, Search Engines, Selection, Quality Ladders, Advertising, Internet, R and D

JEL: O3

# 1 Introduction

The Internet plays a growing role in the economy. This paper extrapolates this trend and analyses a world where most transactions take place in "cyberspace". One key difference between "cyberspace" and the "real world" is the role of search engines. There is ample anecdotal evidence that changes in the algorithm of Google, for example, has substantial influence on the fate of businesses.

Here we take a macroeconomic perspective and ask the following question: how does the design of the search engine affect the incentives to innovate and the economy's long run growth rate?

This is done in the context of a "qualitative" growth model in which there are physical limits to the number of goods that can be produced (there is no physical productivity growth) and consumed (one can only consume 0 or 1 unit of each good), but goods differ in their quality and the introduction of new blueprints, by increasing the total number of available goods, allows consumers to select higher quality goods. If the distribution of quality levels is unbounded, horizontal innovation may lead to sustained qualitative growth. That assumption is meant to capture a modern feature of the "new economy": given that it is pointless to buy the same CD, videogame, etc, twice and given that there consuming these goods is time intensive, the only scope for growth is indeed an improvement in their quality. In the model, existing goods cannot raise their quality and growth is associated with "creative destruction" in that at some point consumers stop buying a good as they switch to higher quality products.

In the model, consumers must use a search engine to locate goods, and then consume a subset of the goods that have been effectively located. The search engine has a specific design which at any time relates the "score" of a good – the probability that it is located – to the good's characteristics, which may consist of its vintage, its quality, and how much it has invested in its web page. The search engine therefore crucially affects the market size

of a good over its life cycle, which in turn determines its present value and thus the incentives to innovate. I show that the search engine has two major conflicting effects on the incentives to innovate. A *visibility* effect which simply means that a greater score increases one's market size and therefore the returns to innovation. A *selection* effect which means that when more goods are located, consumers are more picky and select higher quality goods. As a result an improvement in the performance of the search engine increases the market size of a good but reduces its life span.

The net effect on growth of the search engine's performance thus depends on the relative size of these two effects. Under our assumption of an exponential quality distribution, in many cases these two effects exactly cancel each other, implying that the overall quality of the search engine has no effect on growth, but does increase welfare by allowing consumers to select better goods – a level effect, not a growth effect.

There are circumstances under which one departs from that neutrality result. In particular:

- A firm's score is variable along its life cycle. These variations alter the firm's value and therefore the visibility effect in a discounted fashion, whereas the selection effect is determined by the cross-sectional distribution of scores at a point in time and does not involve such discounting. This discrepancy introduces non neutralities. We analyse such non neutralities in the "popularity" version of the model (section 4), where the search engine rewards product quality indirectly through its observed popularity among consumers. Because of this indirect channel, a firm's score gradually improves over time.
- The search process needs resources. This is so in the "advertising" version of the model (section 5) where there is a sunk investment in "web page quality" which increases a firm's score. The search engine affects the resources used for such advertising, which in turn has an impact on what is left for alternative uses, including R and D. As a

result total growth is affected.

In the popularity model, the discount effect of gradual recognition of popularity tends to reduce growth. Therefore, growth is enhanced if the search engine is less sensitive to popularity. This conclusion does not apply to welfare, as a reduction in the engine's sensitivity to popularity forces consumers to be less selective – but that is a level effect and not a growth effect.

In the advertising model, the resource effect generates a negative trade-off between advertising and growth; as a result growth is lower when the search engine rewards "web page quality" better. But that feature again has a positive level effect on welfare since higher quality goods invest more in their web page quality – because they expect to stay longer in business and therefore reap the benefits over a longer time period – which helps consumers select higher quality goods.

To my knowledge, this paper is the first one to recognize that search engines will have macroeconomic significance and to accordingly embody them in a growth model. The closest equivalents in the existing literature are papers that embody search and matching frictions in the labor market in endogenous growth models, but these papers are not concerned with search frictions in finding the best possible variety, nor with the effect of the characteristics of the search technology on growth and welfare. See Aghion and Howitt (1994), Laing et al. (1995), Moreno-Galbis (2004), Chen et al. (1999), Postel-Vinay (1998). Most of the existing (small) literature on search engines is in IO and is concerned with their effect on competition, as in Gandal (2001), Pollock (2008), White (2008), and Ellison and Fisher Ellison (2004).

## **2 A simple model of growth through selection**

I now describe the simple endogenous growth model on which the analysis is based. While this model has the unusual feature that the only source

of growth is the consumer's ability to become more selective as the range of available goods increase, this can be viewed as a special case of a model of creative destruction with quality ladders in the fashion of Aghion and Howitt (1992) and Grossman and Helpman (1991), where varieties are perfect substitutes but one can only consume one unit of each good<sup>1</sup>.

The economy is populated by  $L$  individuals, each endowed with one unit of labor and an equal claim on profits. At each date  $t$  there is a continuum of goods available for consumption. The total mass of goods is  $N_t$ . Goods differ by their quality  $q$ . The quality distribution is invariant over time and given by the c.d.f.

$$F(q) = 1 - e^{-\lambda q}; \quad (1)$$

the corresponding density is therefore

$$f(q) = \lambda e^{-\lambda q}. \quad (2)$$

As is standard in the literature, goods are introduced by innovators. While there is no intellectual property, the original innovator has a trade secret which allows it to produce the good at a unit cost equal to 1 in terms of labor. Competitors can only produce it at a cost of  $p$  units of labor, with  $p > 1$ . Accordingly, I assume limit pricing. Normalizing the wage to 1, this implies that all goods will be charged at price  $p$ .

For each available good, consumers can consume either one or zero units. They get a utility flow equal to the quality of the good  $q$ . Their total flow of utility at  $t$  is

$$U_t = \int_0^{+\infty} \omega(q)g_t(q)dq,$$

where  $g_t(\cdot)$  is the local mass of goods of quality  $q$  being consumed, which can never exceed  $N_t f(q)$ , and  $\omega(\cdot)$  is an increasing function.

At each date  $t$  a research sector produces new goods. One unit of labor employed in the research sector produces  $\gamma N_t$  new blueprints per unit of

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<sup>1</sup>Models of endogenous growth with indivisibilities and non homothetic preferences include Matsuyama (2001), Foellmi and Zweimüller (2006), and Saint-Paul (2006).

time. When a new good is introduced, its quality is drawn randomly from the distribution  $f()$ .

At date  $t$  we denote by  $C_t$  the total quantity of goods being consumed. Each representative individual then consumes  $C_t/L$  goods. Since one unit of each good is consumed, and all goods have the same price while higher quality goods generate higher utility, consumers will consume one unit of all the available goods above some quality threshold  $q_t^*$  which satisfies

$$\frac{C_t}{L} = N_t(1 - F(q_t^*)). \quad (3)$$

This yields a utility flow equal to

$$U_t = N_t \int_{q_t^*}^{+\infty} \omega(q) f(q) dq. \quad (4)$$

Because of unit prodductivity, the total labor force devoted to production at date  $t$  is  $C_t$ . We assume that the labor market clears, so that  $L - C_t$  people must be working in the research sector, implying that

$$\dot{N}_t = \gamma N_t (L - C_t). \quad (5)$$

In what follows I confine the analysis to balanced growth path where  $N_t$  grows at a constant rate and therefore  $C_t$  is constant through time. As long as the growth rate of  $N$  is strictly positive, it must be that  $q_t^*$  grows with time and tends to infinity so that the RHS of (3) stays constant. Consequently, each good of quality  $q$  eventually becomes obsolete at a critical date  $T(q)$  such that  $q_{T(q)}^* = q$ . After this critical date consumers no longer consume that good as they can spend all their money on higher quality goods.

At  $t$ , a newly invented good of quality  $q$  will actually be produced provided  $q > q_t^*$ ; otherwise it is immediately obsolete. Denoting by  $r$  the exogenous real interest rate, the PDV of inventing a new good at  $t$  can then be computed as

$$V_t = \left( \int_{q_t^*}^{+\infty} f(q) \int_t^{T(q)} e^{-r(u-t)} du \right) (p - 1)L. \quad (6)$$

As long as the good is not obsolete, all consumers purchase one unit of it, yielding a profit flow to the firm equal to  $(p - 1)L$ .

In equilibrium, the value of introducing a new good at  $t$  must be equal to its cost:

$$V_t = \frac{1}{\gamma N_t}. \quad (7)$$

We are now in a position to use the convenient exponential distribution to further characterize the BGP. Let  $g$  be the constant growth rate of  $N$ , and  $\bar{C}$  be the constant value of  $C$ . Substituting (1) into (3) yields

$$\frac{\bar{C}}{L} = N_0 e^{gt} e^{-\lambda q_t^*}, \quad (8)$$

i.e.

$$q_t^* = a + gt/\lambda,$$

with

$$a = \frac{1}{\lambda} \ln \left( \frac{N_0 L}{\bar{C}} \right).$$

Next, we have that  $T(q) = \frac{\lambda}{g}(q - a)$ . Substituting this and (2) into (6), computing the integrals and rearranging, yields

$$V_t = \frac{(p - 1)L}{r} e^{-\lambda q_t^*} \left[ \frac{r}{r + g} \right],$$

which, after substituting (8) is simply equal to

$$V_t = \frac{p - 1}{r + g} \frac{\bar{C}}{N_t}. \quad (9)$$

Intuitively, this formula tells us that future profits have to be discounted at a higher rate, the higher the growth rate, as faster growth speeds obsolescence. This is the usual "creative destruction" effect.

To compute the equilibrium growth rate, we eliminate  $V_t$  between (9) and (7) and then use (5) to eliminate  $\bar{C}$ . We get

$$\frac{(p - 1)(\gamma L - g)}{r + g} = 1,$$



i.e.

$$g = \frac{(p-1)\gamma L - r}{p}. \quad (10)$$

As in other endogenous growth models, growth depends positively on the monopoly markup  $(p-1)/p$ , on the productivity of research  $\gamma$ , on the economy's size  $L$ , and negatively on the discount rate  $r$ .

### 3 'Random' search engines

In the preceding model, consumers encounter no frictions when looking for the highest quality goods. As the number of goods grows, they consume the same absolute number of goods. Hence the fraction of the total number of available goods that they consume falls over time and converges to zero.

If frictions are present in locating the high quality goods, we expect the analysis to be changed. We also expect the technology of locating goods – the search engine – to have an effect on growth. In what follows we analyse the impact of that technology on economic growth.

In this section, we start with two very simple search engines. These are "random" in that neither the quality of the good nor the inputs into the search process matter. Thus they are rather thought of as simple search frictions rather than search engines. Nevertheless analyzing them will provide us with useful benchmark.

#### 3.1 Constant hit probability

The first search engine that we consider is a "constant hit probability" one. That is, consumers are able to locate a good with some constant probability  $\rho < 1$ . As the economy grows, the number of "hits" that consumers get increases proportionally to the total number of goods. The greater  $\rho$ , the greater the efficiency of the search engine. The preceding model where all goods are reachable corresponds to the special case where  $\rho = 1$ .

How is the analysis modified when  $\rho < 1$ ? In (3), the total pool of goods from which a consumer can consume now has a mass equal to  $\rho N_t$ . Thus we

now have instead of (3),

$$\frac{C_t}{L} = \rho N_t (1 - F(q_t^*)). \quad (11)$$

With our exponential distribution, that is equivalent to

$$\frac{\bar{C}}{L} = \rho N_0 e^{gt} e^{-\lambda q_t^*}, \quad (12)$$

For a given  $C_t$  and  $N_t$ ,  $q_t^*$  is lower, the lower  $\rho$  : consumers are now less selective as they can access fewer goods.

The other equation which is changed is (6). As before, a good with quality  $q$  becomes obsolete at  $T(q)$  such that  $q_{T(q)}^* = q$ . Until that date, its producer reaches only a fraction  $\rho$  of consumers. Thus for  $u < T(q)$  its profit is now  $(p - 1)\rho L$ . Equation (6) must then be modified as follows:

$$V_t = \left( \int_{q_t^*}^{+\infty} f(q) \int_t^{T(q)} e^{-r(u-t)} du \right) (p - 1)\rho L. \quad (13)$$

Let us again compute the growth rate in a BGP. Using (12), we see that the evolution of  $q_t^*$  is now

$$q_t^* = a' + gt/\lambda,$$

with

$$a' = \frac{1}{\lambda} \ln \left( \frac{\rho N_0 L}{\bar{C}} \right).$$

Thus  $T(q) = (q - a')\lambda/g$ . The same computations as above now yield

$$\begin{aligned} V_t &= e^{-\lambda q_t^*} \left[ \frac{\rho(p-1)L}{r+g} \right] \\ &= \frac{p-1}{r+g} \frac{\bar{C}}{N_t}. \end{aligned}$$

Using again (7) and (5) we find that the growth rate is independent of  $\rho$  and again given by (10).

Therefore, the efficiency of search has no effect the growth rate. It does affect welfare, though, since a lower  $\rho$  reduces the average quality of goods

being consumed, which harms consumers. But this is a level effect, not a growth effect.

Growth is unchanged when  $\rho$  changes because two conflicting effects cancel each other. On the one hand, firms get a lower share of consumers when  $\rho$  falls. On the other hand, they expect to stay longer in business as consumers are less selective.

### 3.2 Constant number of hits

We now move to the other extreme and consider a random search friction such that each consumer gets a constant number of hits  $K$ .

The relationship between the threshold quality level and total consumption at  $t$  now no longer depends on the total number of goods  $N_t$ , since an increase in that number does not raise the number of goods that consumers can access:

$$\frac{C_t}{L} = K(1 - F(q_t^*)). \quad (14)$$

As the number of goods grows relative to the number of hits, the market share of each goods falls with time as its chances of being located fall. The probability of being located at date  $t$  is  $K/N_t$ . Therefore, the PDV of introducing a new good at  $t$  is now given by

$$V_t = \left( \int_{q_t^*}^{+\infty} f(q) \int_t^{T(q)} \frac{K}{N_u} e^{-r(u-t)} du \right) (p-1)L. \quad (15)$$

In a balanced growth path,  $C_t$  is constant, and so is  $q_t^*$ : The obsolescence process is shut down and  $T(q) = +\infty$ . Integrating (15), we find that

$$V_t = \frac{K(p-1)L e^{-\lambda q_t^*}}{(r+g)N_t}.$$

Using the same steps as above we can show again that the growth rate is independent of  $K$  and still given by (10).

While growth is strictly positive, utility does not grow as consumers see the same number of goods and do not get more selective over time. It is profitable to introduce new products because they "steal" business from existing ones, but a central planner would allocate all resources to production and none to innovation.

The two polar cases we have just discussed can be generalized by assuming a time-varying probability  $\rho_t$  of locating a good. In Appendix 1, it is shown that if  $\rho_t$  decreases exponentially over time, the equilibrium growth rate is given by (10) and is therefore independent of both the level and rate of decay of  $\rho_t$ .

## 4 Popularity

I now study the impact on growth of a search engine which rewards quality. Clearly, any redistribution of hits away from low-quality goods and in favor of high-quality goods will typically improve welfare. However, I am assuming that the only way for the search engine to reward quality is by gradually observing some measure of the popularity of a web site. Popularity builds up over time gradually, and there is a trade-off: The more the search engine wants to reward quality, the more it must rely on popularity which gives more hits to old goods relative to new goods at any given quality level. If on the other hand the search engine wants to reduce the advantage of older goods it can only do so at the cost of making the number of hits less sensitive to quality.

I represent this trade-off by assuming that a good has a 'score' given by the following function

$$\pi_t(q, s) = \rho(1 - ke^{-\varepsilon(q-q_t^*)})(1 - e^{-\alpha(t-s)}).$$

This score is the probability of being located by a consumer. In the preceding formula,  $t$  is the current date,  $q$  is the quality of the good,  $s$  is the date at which the good was introduced, and  $q_t^*$  is the threshold quality

at  $t$ , as in the preceding analysis. The structure of the search engine is then characterized by four parameters:  $\rho, k, \varepsilon$  and  $\alpha$ .

- $\rho$  is a measure of the overall efficiency of the search engine. An increase in  $\rho$  increases the number of hits proportionally for all goods. For convenience I assume  $\rho < 1$ .
- $\alpha$  is the speed of convergence to the target level of hits, which reflects the fact that it takes time to build popularity. The higher  $\alpha$ , the lower the popularity advantage of older goods over newer goods.
- $\varepsilon$  is the sensitivity of the target level of hits to quality. The higher  $\varepsilon$ , the greater the number of hits of the higher quality sites over the lower quality sites.
- $k$  is a weight which captures the importance of quality; it will be treated as a fixed parameter.

Note that quality enters not in absolute terms, but relative to the marginally obsolete quality  $q_t^*$ . This is mostly for convenience (it helps ensuring the existence of a balanced growth path), but it also captures the idea that popularity is a relative concept. A given good introduced at a given time gets fewer hits if consumers are more selective.

We model the trade-off discussed above by assuming that it is not possible to increase the speed of convergence  $\alpha$  without reducing the sensitivity to quality  $\varepsilon$ . Thus we assume that these two parameters must satisfy the following constraint:

$$\alpha + b\varepsilon \leq \delta.$$

How does such an engine affect the growth rate? In a balanced growth path with growth rate  $g$ , at any date  $t$  the density of birth dates for goods of

any quality is given by  $ge^{g(s-t)}$ . Thus the average score of a good with quality  $q$  is given by

$$\begin{aligned}\pi_t(q) &= \int_{-\infty}^t ge^{g(s-t)}\pi_t(q, s)ds \\ &= \frac{\alpha\rho}{g+\alpha}(1 - ke^{-\varepsilon(q-q_t^*)}).\end{aligned}$$

Consumers can only access a fraction  $\pi_t(q)$  of goods of quality  $q$ . Their threshold level is therefore determined by

$$\begin{aligned}\frac{C_t}{L} &= N_t \int_{q_t^*}^{+\infty} \pi_t(q)f(q)dq \\ &= N_te^{-\lambda q_t^*} \frac{\alpha\rho}{g+\alpha} \left(1 - \frac{\lambda k}{\lambda + \varepsilon}\right).\end{aligned}\quad (16)$$

The next step is to compute the value of innovation. A good of quality  $q$  introduced at date  $t$  yields a present discounted value equal to

$$R_t(q) = (p-1)L\rho \int_t^{T(q)} e^{-r(u-t)}(1 - ke^{-\varepsilon(q-q_u^*)})(1 - e^{-\alpha(u-t)})du.$$

In a balanced growth path,  $C_t$  is again constant and  $q_t^*$  grows linearly over time:

$$q_t^* = a'' + gt/\lambda,$$

$$a'' = \frac{1}{\lambda} \ln \left( \frac{\rho N_0 L}{\bar{C}} \frac{\alpha}{g+\alpha} \left(1 - \frac{\lambda k}{\lambda + \varepsilon}\right) \right).$$

This allows us to compute  $R_t(q)$  by straightforward integration. We eventually get

$$\begin{aligned}R_t(q) &= (p-1)L\rho \left[ \frac{1 - e^{-r\lambda(q-q_t^*)/g}}{r} - \frac{k}{r-\varepsilon g/\lambda} \left( e^{-\varepsilon(q-q_t^*)} - e^{-\frac{r\lambda}{g}(q-q_t^*)} \right) \right. \\ &\quad \left. - \frac{1 - e^{-(r+\alpha)\lambda(q-q_t^*)/g}}{r+\alpha} + \frac{k}{r+\alpha-\varepsilon g/\lambda} \left( e^{-\varepsilon(q-q_t^*)} - e^{-\frac{(r+\alpha)\lambda}{g}(q-q_t^*)} \right) \right] \quad (17) \\ &= (p-1)L\rho \left[ \frac{\alpha}{r(r+\alpha)} - \frac{k\alpha}{(r-\varepsilon g/\lambda)(r+\alpha-\varepsilon g/\lambda)} e^{-\varepsilon(q-q_t^*)} \right. \\ &\quad \left. + \frac{\varepsilon g/\lambda - (1-k)r}{r(r-\varepsilon g/\lambda)} e^{-r\lambda(q-q_t^*)/g} - \frac{\varepsilon g/\lambda - (1-k)(r+\alpha)}{(r+\alpha)(r+\alpha-\varepsilon g/\lambda)} e^{-\frac{(r+\alpha)\lambda}{g}(q-q_t^*)} \right].\end{aligned}$$

The expected value of an innovation is equal to

$$\begin{aligned} V_t &= \int_{q_t^*}^{+\infty} R_t(q)f(q)dq \\ &= e^{-\lambda q_t^*} (p-1)\rho L H(g), \end{aligned} \quad (18)$$

where  $H(g)$  is a function that can be computed by direct integration of (17), yielding

$$\begin{aligned} H(g) &= \frac{\alpha}{r(r+\alpha)} - \frac{\lambda k \alpha}{(r-\varepsilon g/\lambda)(r+\alpha-\varepsilon g/\lambda)(\lambda+\varepsilon)} + \frac{\varepsilon g - \lambda(1-k)r}{r(r-\varepsilon g/\lambda)(\lambda+r\lambda/g)} \\ &\quad - \frac{\varepsilon g - \lambda(1-k)(r+\alpha)}{(r+\alpha)(r+\alpha-\varepsilon g/\lambda)(\lambda+(r+\alpha)\lambda/g)}. \end{aligned}$$

This formula can be considerably simplified and we get

$$H(g) = \frac{\alpha}{(r+g)(r+g+\alpha)} \left( 1 - \frac{\lambda k}{\lambda + \varepsilon} \right). \quad (19)$$

The derivation of the equilibrium growth rate is then similar as in the preceding sections. We substitute (16) into (18) and use (7) to get

$$(p-1)H(g)\bar{C} \frac{(g+\alpha)(\lambda+\varepsilon)}{\alpha(\lambda(1-k)+\varepsilon)} = 1/\gamma.$$

We then eliminate  $\bar{C}$  using (5) and then substitute (19) to get<sup>2</sup>

$$(p-1)(\gamma L - g) = \frac{(r+g)(r+g+\alpha)}{g+\alpha}. \quad (20)$$

This condition determines the equilibrium growth rate. The only parameter of the search engine which affects the growth rate is  $\alpha$ , the speed of convergence to the target hit level.

Thus, an engine which is more sensitive to quality – i.e. a higher  $\varepsilon$  – has no effect on growth. The reason is again the trade-off between selectivity and visibility: while innovators know that they are more likely to sell if their good has a higher quality, which is captured by the term in  $(1 - \frac{\lambda k}{\lambda + \varepsilon})$  in

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<sup>2</sup>We can check that for  $\alpha \rightarrow \infty$  and  $k \rightarrow 0$  this condition boils down to (10).

(19), that is offset by the associated increase in  $q^*$ , as captured by the same term in (16), which makes it less likely that they are selected by consumers given their quality, since consumers have an easier access to higher quality goods. The two effects again exactly cancel each other when the net effect on growth is computed.

Why does then  $\alpha$  matters? The same visibility/selectivity trade-off comes into play when  $\alpha$  is increased. However, in (16)  $\alpha$  enters through the term  $\frac{\alpha}{g+\alpha}$ . That expression is proportional to the average probability of getting a hit across firms in a balanced growth path, which is what is relevant for the consumers' selection decision. The greater that number, the more selective the consumers and the greater  $q^*$ . On the other hand, the corresponding term in (19) is  $\frac{\alpha}{r+g+\alpha}$ , which captures the fact that from the point of view of an innovator, the benefits of convergence are future and therefore subject to discounting. As a result, an increase in  $\alpha$  has a stronger effect on the an innovator's present discounted profit than on the consumer's selection decision, because the benefits of visibility come sooner.

The result is that the equilibrium growth rate is increasing in  $\alpha$ . This is illustrated on Figure 1, which plots the LHS of (20) against its RHS. The LHS must cross the RHS from above if the equilibrium has to be locally stable.<sup>3</sup> An increase in  $\alpha$  unambiguously lowers the RHS, so that the economy grows faster. Note that the effect becomes nil at  $r = 0$ , since  $\alpha$  then disappears from the RHS of (20).

## 5 Advertising

In this section I consider a radically different search engine. In the previous example firms were passive and could not affect the number of hits that they get. This is what would happen if firms were posting 'candid' web pages that reflect the true quality of their product. I now assume that instead the number of hits that they get depends on the resources they spend on

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<sup>3</sup>Otherwise, a small increase in growth would increase profits in such a way that one would innovate more, so that growth would increase further.



advertising. This can capture two phenomena:

- Direct spending on advertising, say to appear on the right panel of google
- Manipulating one's web page to make it more friendly to the engine (e.g. google bombs, inclusion of popular search terms, etc)

I assume that while these strategies increase visibility, consumers still observe the true quality of the good once they have located the web page. Thus they cannot erroneously buy a low quality good.

Clearly, innovators who come up with higher quality goods will have want to invest more in their web page, because they expect to stay longer in business<sup>4</sup>.

I formalize these ideas as follows. When a good is introduced, a firm spends  $s$  units of labor investing in the visibility of its web page. Its score is then given by

$$\pi(\sigma) = 2\alpha\sigma^{0.5} + \rho.$$

The parameters  $\rho$  and  $\alpha$  capture the design of the search engine. The greater  $\alpha$ , the more sensitive the search engine to the firm's investment.

While R and D is undertaken prior to observing the quality of the good that will be invented, advertising decisions are made after that is observed. Therefore, denoting again by  $T(q)$  the obsolescence date of a good with quality  $q$ , a firm entering the market at date  $s$  sets  $\sigma$  so as to maximize

$$\max_{\sigma} \pi(\sigma)(p-1)L \int_s^{T(q)} e^{-r(u-s)} du - \sigma$$

The optimal  $\sigma$  is

$$\sigma(q, s) = \left( \alpha(p-1)L \frac{1 - e^{-r(T(q)-s)}}{r} \right)^2. \quad (21)$$

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<sup>4</sup>Alternatively, one could spend more resources ex-ante at the R and D stage so as to make it more likely that the resulting good is of high quality. This would bring us back to the models of directed innovation as studied by Acemoglu (1998).

The corresponding score is thus constant through the life of the good and equal to<sup>5</sup>

$$\pi(q, s) = 2\alpha^2(p-1)L \frac{1 - e^{-r(T(q)-s)}}{r} + \rho$$

The resulting present discounted profit is equal to

$$R_t(q) = \left( \alpha(p-1)L \frac{1 - e^{-r(T(q)-t)}}{r} \right)^2 + \rho(p-1)L \frac{1 - e^{-r(T(q)-t)}}{r}. \quad (22)$$

At any date  $t$ , the average score of a good with quality  $q$  is in steady state given by

$$\begin{aligned} \pi_t(q) &= \int_{-\infty}^t g e^{g(s-t)} \pi(q, s) ds \\ &= \frac{2\alpha^2(p-1)L}{r} + \rho - \frac{2\alpha^2(p-1)Lg}{r(g+r)} e^{-r(T(q)-t)}. \end{aligned} \quad (23)$$

In equilibrium, the growth rate is given by

$$g = \gamma(L - C_t - M_t), \quad (24)$$

where

$$M_t = \dot{N}_t \int_{q_t^*}^{+\infty} \sigma(q, t) f(q) dq \quad (25)$$

is total employment in advertising.

The cut-off quality level is determined by

$$\frac{C_t}{L} = N_t \int_{q_t^*}^{+\infty} \pi_t(q) f(q) dq. \quad (26)$$

To compute the growth rate, we need again to derive a formula for the value of an innovation. Since  $C$  and  $M$  must be constant in a balanced growth path, it is easy to see that  $q_t^*$  is again affine in  $t$

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<sup>5</sup>This will always be lower than 1 provided  $\alpha^2 \leq (1 - \rho) * r / (2(p - 1)L)$ .

$$q_t^* = a''' + \frac{g}{\lambda}t.$$

Consequently,  $T(q) - t = \frac{\lambda}{g}(q - q_t^*)$ , which we can substitute into (22) and we then have that

$$\begin{aligned} V_t &= \int_{q_t^*}^{+\infty} R_t(q)f(q)dq \\ &= e^{-\lambda q_t^*} \frac{(p-1)L}{r+g} \left[ \rho + 2\alpha^2(p-1)L \frac{1}{2r+g} \right]. \end{aligned} \quad (27)$$

Substituting (23) into (26) and integrating yields

$$\frac{C_t}{L} = N_t e^{-\lambda q_t^*} \left[ \rho + 2\alpha^2(p-1)L \frac{r+2g}{(r+g)^2} \right], \quad (28)$$

implying, in particular, that

$$a''' = \frac{1}{\lambda} \ln \left( \frac{N_0 L}{\bar{C}} \left( \rho + 2\alpha^2(p-1)L \frac{r+2g}{(r+g)^2} \right) \right). \quad (29)$$

Eliminating  $e^{-\lambda q_t^*}$  between (27) and (28) and using (7) again, we get

$$\frac{1}{\gamma} = \frac{(p-1)\bar{C}}{r+g} \left[ \frac{\rho + \frac{2\alpha^2(p-1)L}{2r+g}}{\rho + \frac{2\alpha^2(p-1)L(r+2g)}{(r+g)^2}} \right] \quad (30)$$

To compute  $\bar{C}$  we first compute the steady state level of advertising  $\bar{M}$  by using (25). Substituting (21) into (25) and integrating, we get

$$\bar{M} = gN_t e^{-\lambda q_t^*} \frac{\alpha^2(p-1)^2 L^2}{r^2} \left( 1 - \frac{rg}{(r+g)(2r+g)} \right) \quad (31)$$

Substituting (28) into (31) and then reporting the resulting expression along with (30) into (24) we eventually get an equation which determines the equilibrium growth rate:

$$pg + r - \gamma L(p-1) = \eta H_1(g), \quad (32)$$

where

$$\eta = \frac{\alpha^2}{\rho}$$

and  $H_1(g)$  is a function determined by

$$H_1(g) = -2(p-1)L\frac{r+2g}{r+g} + (p-1)^2L \left[ \frac{2\gamma L}{2r+g} - \frac{g}{r} \left( \frac{2r-g}{2r+g} + \frac{g+r}{r} \right) \right]. \quad (33)$$

It can be shown straightforwardly that  $H_1'(g) < 0$ , therefore there exists a unique solution to (32).<sup>6</sup> Furthermore, the parameters of the search engine only enter through the ratio  $\eta$ , and it multiplies the RHS of (32). The equilibrium growth rate  $g^*$  increases (resp. falls) with  $\eta$  if and only if  $H_1(g^*) > 0$  (resp.  $H_1(g^*) < 0$ ). We can show<sup>7</sup> that  $H_1(g^*) < 0$ , therefore a search engine which is more elastic to the innovator's search input always reduces the long-run growth rate of the economy.

What is the intuition? As shown by (27), given the selectivity level  $q_t^*$ , an increase in the search sensitivity parameter  $\alpha$  increases the value of the firm, as firms take advantage of the possibility to advertise more should they come up with a higher quality product (the profit effect). However, this increased advertising effort is offset by the consumer's greater selectivity, as shown in (28) (the selectivity effect). The weight of  $\alpha^2$  relative to  $\rho$  in (28) is  $\frac{r+2g}{(r+g)^2}$ , to be compared with  $\frac{1}{2r+g}$  in (27), which is lower. This means that selectivity increases relatively more than the direct effect of advertising on the value of

<sup>6</sup>We have  $H_1'(g) = -\frac{2\gamma L}{(2r+g)^2} - \frac{1}{r} \frac{2r-g}{2r+g} - 2\frac{g}{r^2} + \frac{4g}{(2r+g)^2} - \frac{2(p-1)L}{(r+g)^2}$ .

Therefore, a sufficient condition for  $H_1'(g) < 0$  is  $4g < 2\gamma L + \frac{1}{r}((2r-g)(2r+g) + 2\frac{g}{r}(2r+g)^2)$

$= 2\gamma L + \frac{1}{r}(4r^2 - g^2 + 8rg + 8g^2 + 2\frac{g^3}{r})$   
 $= 2\gamma L + 4r + 8g + 7g^2/r + 2g^3/r^2$ .

This last expression clearly exceeds  $4g$ .

<sup>7</sup>To see this, let  $A(g) = 2\frac{(p-1)L(r+2g)}{r+g} + \frac{(p-1)^2L}{r^2}g(1 - \frac{rg}{(r+g)(2r+g)})$  and  $B(g) = 2\frac{(p-1)L}{2r+g} < A(g)$

Call  $\hat{g}$  the root of the LHS of (32);  $\hat{g} = \frac{(p-1)\gamma L - r}{p}$ .

Note that  $H_1(g) = B(g)(p-1)(\gamma L - g) - (r+g)A(g)$ .

We have that  $H_1(\hat{g}) < B(\hat{g})((p-1)(\gamma L - \hat{g}) - (r+\hat{g})) = 0$ .

Therefore, one must have  $g^* < \hat{g}$ . Since the LHS of (32) is increasing in  $g$ , it follows that both sides must be negative at  $g^*$ .

the firm when  $\alpha$  goes up. The end result is a net reduction in that value and lower incentives to innovate.

This is due to a rather subtle mechanism. The selectivity effect is driven by the effect of the engine's design on the total number of hits (or total score of all firms). The larger that total score, the more selective the firms given  $C$ , and the higher  $q^*$ . The firm's revenue is less sensitive to  $\alpha$ , relative to  $\rho$ , than its score, because when  $\alpha$  goes up, so does  $\sigma$ . This effect increases advertising costs, which dampens the increase in revenues. On the other hand, an increase in  $\rho$  has no impact on the search intensity, so that this cost deduction effect is not present.<sup>8</sup> This explains why, as compared to its score, a firm's revenue is less elastic to  $\alpha$  relative to  $\rho$ . Consequently, the selectivity effect dominates the profit effect.

In addition to this net effect on the incentives to innovate, the increased employment in advertising reduces the human resources available for innovation which has a direct negative effect on the equilibrium growth rate. This is apparent from (31) whose RHS increases with  $\alpha$ , which yields a lower growth rate in (24) for any value of  $C$ .

## 6 Welfare

The preceding examples do not exhibit trade-offs between different dimensions of the search engine in terms of long-run growth. The reason is that many of these parameters only have levels effect on the number of goods, not growth effects. But these levels effects are relevant when one looks at welfare. In this section, I derive the expression for welfare and discuss how it is affected by the design of the search engine.

One should note that in this class of models, the functional form of the value of quality  $\omega(q)$  matters. In particular, for most specifications, the

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<sup>8</sup>This is further compounded by a discount effect: At any date  $t$ , there are firms of high quality that were created much before  $t$ . While current profits weighed little in their decision to innovate because of discounting, they advertised their web page a lot and still generate many hits, thus increasing current selectivity substantially.

optimal growth rate will depend on the initial value of  $N$ ,  $N_0$ , meaning that the optimal growth path is not a balanced growth path. To avoid these complications, I assume that  $\omega()$  is an exponential:

$$\omega(q) = e^{\omega q}, \omega < \lambda.$$

The utility flow at  $t$  is then given by

$$U_t = N_t \int_{q_t^*}^{+\infty} \pi_t(q) f(q) e^{\omega q} dq, \quad (34)$$

where  $\pi_t(q)$  is the average score of goods with quality  $q$ . We can now compute total utility (the PDV of  $U_t$ ) and we do so in both the popularity model and the advertising model.

## 6.1 The popularity model

We have seen that in the popularity model,

$$\pi_t(q) = \frac{\alpha \rho}{g + \alpha} (1 - k e^{-\varepsilon(q - q_t^*)}).$$

We then get that

$$U_t = N_0 e^{gt} e^{-(\lambda - \omega)q_t^*} \frac{\alpha \rho}{g + \alpha} \left[ \frac{\lambda}{\lambda - \omega} - k \frac{\lambda}{\lambda + \varepsilon - \omega} \right].$$

Total utility is then

$$\begin{aligned} \bar{U} &= \int_0^{+\infty} U_t e^{-rt} dt \\ &= \frac{\alpha \rho}{g + \alpha} \left[ \frac{\lambda}{\lambda - \omega} - k \frac{\lambda}{\lambda + \varepsilon - \omega} \right] N_0 \frac{e^{-(\lambda - \omega)a''}}{r - g\omega/\lambda}. \end{aligned}$$

Using the derivations of Section 4 to compute  $a''$ , we get<sup>9</sup>

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<sup>9</sup>We have that  $\bar{C} = L - g/\gamma = \frac{(r+g)(r+g+\alpha)}{\gamma(p-1)(g+\alpha)}$  and therefore  $a'' = \frac{1}{\lambda} \ln\left(\frac{\rho N_0 L \alpha}{(g+\alpha)(L-g/\gamma)} \left(1 - \frac{\lambda k}{\lambda + \varepsilon}\right)\right)$ .

$$\bar{U} = \frac{\lambda}{\lambda - \omega} \frac{(L - g/\gamma)^{1-\omega/\lambda}}{L(r - g\omega/\lambda)} \frac{1 - k \frac{\lambda - \omega}{\lambda + \varepsilon - \omega}}{\left(1 - \frac{\lambda k}{\lambda + \varepsilon}\right)^{1-\omega/\lambda}} \left[ \rho N_0 L \frac{\alpha}{g + \alpha} \right]^{\omega/\lambda}.$$

This formula shows that utility varies in a non monotonic fashion with  $\alpha$  (both through the terms in  $\alpha$  and the terms in  $g$ ), and also goes up with  $\varepsilon$ .<sup>10</sup> Thus in the zone where  $\bar{U}$  goes up with  $\alpha$  there is a trade-off between increasing  $\varepsilon$  – making the search engine more sensitive to popularity – and increasing  $\alpha$  – increasing the speed at which firms converge to their target score.

## 6.2 The advertising model

To compute welfare in the advertising model, we now use (23) and substitute it into (34). We get

$$U_t = U_t = N_0 e^{gt} e^{-(\lambda - \omega)q_t^*} \left[ \frac{\lambda}{\lambda - \omega} \left( \rho + \frac{2\alpha^2(p-1)L}{r} \right) - \frac{2\alpha^2(p-1)L}{r(r+g)} g^2 \frac{\lambda}{\lambda g + \lambda r - \omega g} \right]$$

Next, we have

$$\begin{aligned} \bar{U} &= \int_0^{+\infty} U_t e^{-rt} dt \\ &= \frac{\lambda}{\lambda - \omega} \left[ \rho + \frac{2\alpha^2(p-1)L}{(r+g)} \frac{r + 2g - \omega g/\lambda}{r + g - \omega g/\lambda} \right] N_0 \frac{e^{-(\lambda - \omega)a'''}{r - g\omega/\lambda}} \end{aligned}$$

And finally, substituting the value of  $a'''$  from (29) while using (30):

$$\bar{U} = \frac{\lambda}{\lambda - \omega} \left( \frac{r + g}{(p-1)\gamma L} \right)^{1-\omega/\lambda} \frac{\left[ \rho + \frac{2\alpha^2(p-1)L}{(r+g)} \frac{r + 2g - \omega g/\lambda}{r + g - \omega g/\lambda} \right]}{\left[ \rho + \frac{2\alpha^2(p-1)L}{2r+g} \right]^{1-\omega/\lambda}} N_0^{\omega/\lambda}.$$

---

<sup>10</sup>Let  $Z = \frac{1 - k \frac{\lambda - \omega}{\lambda + \varepsilon - \omega}}{\left(1 - \frac{\lambda k}{\lambda + \varepsilon}\right)^{1-\omega/\lambda}} = \left(1 - k \frac{1-z}{1-z+x}\right) \left(1 - \frac{k}{1+x}\right)^{z-1}$ , with  $z = \omega/\lambda$  and  $x = \varepsilon/\lambda$ .

$d \ln Z / dx = \frac{k(1-z)/(1-z+x)^2}{1-k(1-z)/(1-z+x)} - \frac{k(1-z)/(1+x)^2}{1-k/(1+x)}$

$\propto \frac{1}{(1-z+x)^2 - k(1-z)(1-z+x)} - \frac{1}{(1+x)^2 - k(1+x)}$ , and it can be checked that this expression is  $> 0$ , since  $z > 0$  and the quantity  $(1-z+x)^2 - k(1-z)(1-z+x)$  is a decreasing function of  $z$ .

While this expression is not tractable, numerical simulations (see Appendix II) suggest that for low values of  $\omega$ ,  $\bar{U}$  is a decreasing function of  $\alpha$ . This makes sense, since at  $\omega = 0$  consumers do not value quality and their utility is just  $\frac{\bar{C}}{rL}$ , the PDV of per capita consumption. This falls with  $\alpha$  since advertising crowds out production of goods when  $\alpha$  goes up. As  $\omega$ , the preference for quality, goes up,  $\bar{U}$  becomes U-shaped in  $\alpha$ . Furthermore, its upward sloping portion is steeper and delivers higher values of  $\bar{U}$  relative to its level at  $\alpha = 0$ , the greater  $\omega$ . Consequently, for  $\omega$  large enough consumers end up preferring the highest possible value of  $\alpha$ , everything else equal, while they prefer  $\alpha = 0$  for lower values of  $\omega$ .

## 7 Conclusion

In this paper I have studied a model of growth through selection of higher quality goods which is suited to analyze the role of search engines. The model has been applied to two specific search engines: one which rewards popularity as it gradually builds up over time, another which rewards ex-ante investment in advertising.

This is obviously a first step which opens the door for many variants and extensions. In particular, the modelling of search engines could be enriched to take into account ranking of hits, commercial links, etc. On the consumer side, one may want to introduce heterogeneity in tastes, an endogenous consumer search effort, and limited cognitive capacity in handling hits.

So far, the use of the model is so far only normative, since I look at the effects of the search engine on growth and welfare. The model can potentially be used to endogenize the structure of the search engine(s) by specifying an adequate objective function and a competitive environment for this type of business. The resulting equilibrium design can then be compared to the one that maximizes growth or welfare.



## APPENDIX I

A time-varying random hit probability.

I assume that the score of any firm at  $t$  is now

$$\rho_t = \rho_0 e^{-\eta t}.$$

In steady state, we must have

$$\frac{\bar{C}}{L} = \rho_0 N_0 e^{gt} e^{-\lambda q_t^*} e^{-\eta t}. \quad (35)$$

Therefore,

$$q_t^* = a'''' + (g - \eta)t/\lambda,$$

where

$$a'''' = \frac{1}{\lambda} \ln \left( \frac{\rho_0 N_0 L}{\bar{C}} \right).$$

The value of an innovation is

$$\begin{aligned} V_t &= \left( \int_{q_t^*}^{+\infty} f(q) \int_t^{T(q)} \rho_u e^{-r(u-t)} du dq \right) (p-1)L \\ &= e^{-\eta t} \left( \int_{q_t^*}^{+\infty} \lambda e^{-\lambda q} \rho_0 \frac{1 - e^{-(r+\eta)(T(q)-t)}}{r+\eta} dq \right) (p-1)L \\ &= \frac{e^{-\eta t} (p-1)L e^{-\lambda q_t^*}}{r+\eta} \rho_0 \left( 1 - \int_{q_t^*}^{+\infty} \lambda e^{-\lambda(q-q_t^*)} e^{-\lambda \frac{r+\eta}{g-\eta}(q-q_t^*)} dq \right) \\ &= \frac{e^{-\eta t} (p-1)L e^{-\lambda q_t^*}}{r+g} \rho_0 \\ &= \frac{(p-1)\bar{C}}{N_t(r+g)}, \end{aligned}$$

and the rest of the analysis follows as in Section 3, yielding the same equilibrium growth rate.

## APPENDIX II

Numerical computations of growth and welfare in the advertising model.

Figures 1a, 1b and 1c represent the evolution of the consumer's present discounted utility  $\bar{U}$  as a function of  $\alpha$  for  $\omega = 0.1, 0.2$  and  $0.3$  respectively. These simulations were run for  $\rho = 0.5, \lambda = 1, L = 1, \gamma = 0.1, r = 0.05, p = 2$ .

The GAUSS source file for generating these simulations is:

```

library pgraph;
output file = results.asc reset;
screen off;
ro=0.5;
la=1;
ll=1;
ga=0.1;
r=0.05;
p=2;
om=0.3;
for i(1,100,1);
al=i/100;
gs=solve();
q1=ro+2*al^2*(p-1)*ll/(r+gs)*(r+2*gs-om*gs/la)/(r+ga-om*gs/la);
q2=ro+2*al^2*(p-1)*ll/(2*r+gs);
ubar=la/(la-om)*((r+gs)/(p-1)/ga/ll)^(1-om/la)*q1/q2^(1-om/la);
print al gs ubar;
endfor;
output off;
screen on;
stop;
proc h1(g);
local s;
s=0;
s=s-2*(p-1)*ll*(r+2*g)/(r+g);
s=s+(p-1)*ll*(2*ga*ll/(2*r+g)-(g/r)*((2*r-g)/(2*r+g)+(g+r)/r));

```

```

retp(s);
endp;
proc funct(g);
retp(p*g+r-ga*ll*(p-1)-al^2/ro*h1(g));
endp;
proc solve();
local i0,i1,i2,t;
if funct(0)>0;
retp(0);
endif;
i0=0;
i1=ga*ll;
do until abs(i0-i1)<0.0000001;
i2=(i0+i1)/2;
t=funct(i2);
if t>0;
i1=i2;
else;
i0=i2;
endif;
endo;
retp(i2);
endp;
end;

```

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Figure 1a:  $\omega = 0.1$

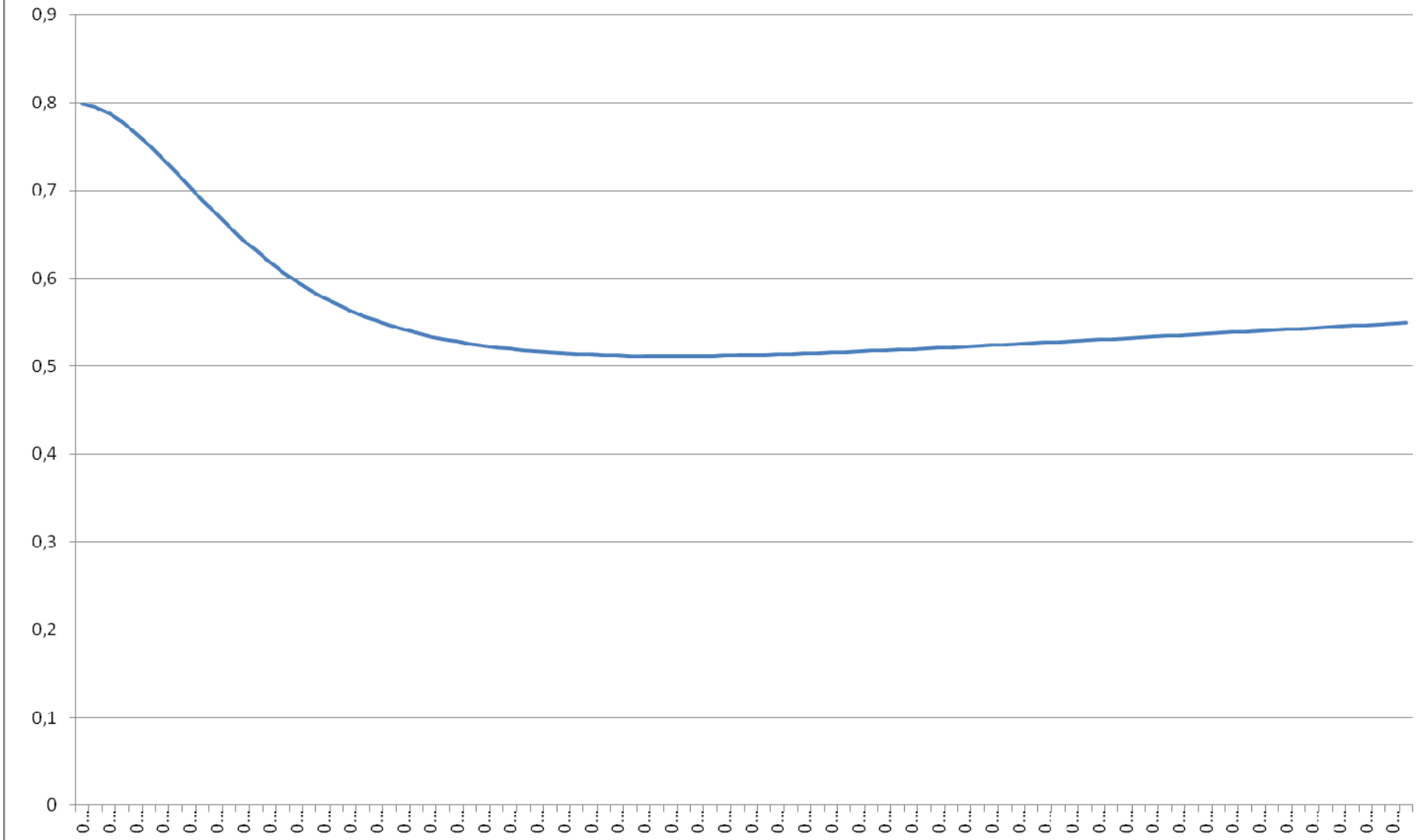


Figure 1b:  $\omega = 0.2$

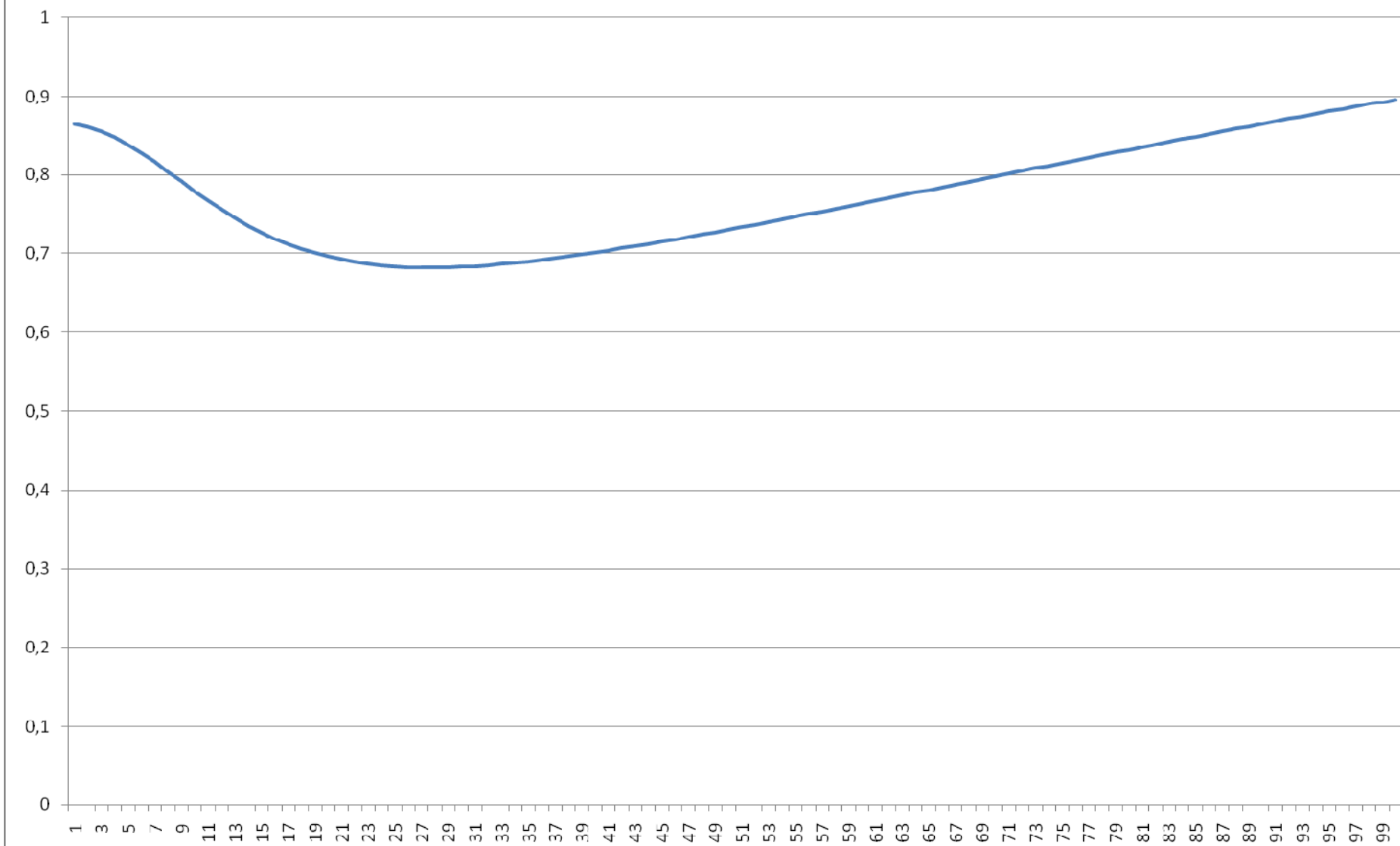


Figure 1c:  $\omega = 0.3$

