Abuse of Dominance and Licensing of Intellectual Property

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Abstract

We examine the impact of the licensing policies of one or more upstream owners of *essential* intellectual property (*IP* hereafter) on the downstream firms requiring access to that IP, as well as on consumers and social welfare. We consider a model with downstream product differentiation. License fees and fixed entry costs determine the number of downstream competitors and thus variety. In the case of a single upstream owner of essential IP, increasing the number of licenses enhances product variety, which adds to consumer value, but also intensifies downstream competition, and dissipating profits. We derive conditions under which the upstream IP monopoly will want to provide an excessive or insufficient number of licenses, relative to the number that maximizes consumer surplus or social welfare. With multiple owners of essential IP, royalty stacking can reduce both the number of the downstream licensees, as well as downstream equilibrium prices facing consumers. We derive conditions determining whether these reductions in downstream prices and variety is beneficial to consumers or society. Finally, the paper explores the impact of alternative licensing policies. With fixed license fees or royalties expressed as a percentage of the price, an upstream IP owner cannot control the intensity of downstream competition. In contrast, per-unit license fees permit an upstream owner to control downstream competition and to replicate the outcome of complete integration. We also show that vertical integration can have little impact on downstream competition and licensing terms when IP owners charge fixed or volume-based access fees.

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1 Introduction

In many high technology industries, the development of any new product or service often involves hundreds and thousands of patents. Of particular concern is the so-called patent thicket problem,[1] where independent licensing policies by the owners of complementary intellectual property may give rise to *royalty stacking* – a "horizontal" form of the double marginalization problem identified by Cournot (1838)[2] and result in prohibitively high licensing fees, potentially creating immense obstacles for the introduction or diffusion of any new technology.[3] This patent thicket problem is often presented as a compelling rationale for significant reform of the patent system and/or licensing policies.[4] This patent thicket problem has also prompted standard setting organizations (SSOs) to adopt such principles as *fair, reasonable and non-discriminatory* (FRAND),[5] the interpretation of which is the source of many disputes,[6] and it has led some competition authorities to apply "abuse of dominance"

[1]See e.g. Shapiro (2001) for further discussion. Empirical studies of the effects of patent thickets include Heller and Eisenberg (1998), Kiley (1992) and Kitch (2003) in bio-medical research, and Geradin, Layne-Farrar, and Padilla (2007), Schankerman and Noel (2006), Walsh, Arora and Cohen (2003) and Ziedonis (2003) in technology intensive industries.

There is a related literature analyzing hold-up problems in standard setting and joint licensing agreements. See Shapiro (2006), Lichtman (2006), Lemley and Shapiro (2007). See also Farrell et al. (2007) for a comprehensive discussion.

[2]Such double marginalization problems arise whenever complementary inputs are involved; following Schmidt (2008), the "horizontal" form refers to situations where the inputs are bought by the same customer (e.g., when a product developer needs several pieces of IP), whereas the "vertical" form arises when the inputs involve different stages of a vertical chain (e.g., when a consumer buys from a retailer, who in turn buys from a manufacturer; addressing the consumer needs thus requires both "production" and "distribution" services).

[3]See for example SCM v Xerox: Paper Blizzard for $1.8 Billion," New York Times, June 27, 1977. As technology has become increasingly complex, this concern has drawn both judicial and legislative scrutiny – see Business Week Online http://www.businessweek.com/magazine/content/07_20/b4034049.htm (May 14, 2007) and http://www.businessweek.com/smallbiz/content/may2007/sb20070523_462426.htm (May 23, 2007), as well as http://www.house.gov/apps/list/press/ca28_berman/berman_patent_bill.pdf and http://www.ip-watch.org/weblog/index.php?p=427.

[4]For opposing views, see for example Geradin, Layne-Farrar, and Padilla (2007), who argue that the theoretical conclusion lacks empirical support. Elhauge (2008) argues that previous analyses tend to start with too low a benchmark for royalties and that other factors can offset the adverse effects (if any) of patent thickets on royalties.

[5]Most SSOs, such as the *European Telecommunications Standards Institute* (ETSI), indeed impose FRAND (or RAND, in the US) licensing obligations upon their members.

[6]US cases include for example ESS Technology, Inc. v. PCTel, Inc., No. C-99-20292 (N.D. Cal. Nov. 4, 1999); Agere Sys. Guardian Corp. v. Proxim, Inc., 190 F. Supp. 2d 726 (D. Del. 2002);

laws in order to reduce licensing fees.[7]

In practice, however, there are several reasons why patent thickets need not present an insurmountable obstacle for a new technology. First, the reality is often not of thousands of patent owners, but of thousands of patents with a few owners; moreover, patents are often licensed in groups and not individually.[8] To be sure, even a few patent owners will tend to set royalties which in aggregate exceed monopoly levels, when acting independently. This type of double marginalization can result in excessive royalties from the patent owners' standpoint and tends to reduce the number of firms in the product market. When only prices matter in that market, this reduction in competition unambiguously hurts consumers and society. The impact is less clear when variety matters; as some of the customers buying from a new entrant are switching away from rivals, the revenue they generate may exceed the social value created by entry. Excessive entry can involve inefficient duplication of fixed costs, and, as a result, excessive differentiation can increase prices and hurt consumers as well as society.[9] In such situations royalty stacking can potentially have beneficial effects.[10]

Broadcom Corp. v. Qualcomm, Inc., No. 05-3350 (D. N.J. Aug. 31, 2006); and Nokia Corp. v. Qualcomm, Inc., No. 06-509 (D. Del Aug. 16, 2006). Together with Ericsson, NEC, Panasonic and TI, Broadcom and Nokia lodged a similar complaint in the EU.

[7]For example, in July 2007 the European Commission sent Rambus a *Statement of Objections*, stating that Rambus may have infringed then Article 82 of the EC Treaty (now Article 102) by abusing a dominant position in the market for DRAMs. After eighteen months of procedure, in December 2009 the European Commission accepted Rambus' offer – making it a binding commitment – to put a five-year worldwide cap on its royalty rates for products compliant with the standards set by the Electron Device Engineering Council (JEDEC).

The US Federal Trade Commission (FTC) had similarly ordered Rambus to reduce its licensing rates on the basis of Section 2 of the Sherman Act (monopolization) and of Section 5 of the FTC Act (unfair competition) – see the FTC Final order and Opinion of 2 February 2007 in Docket No. 9302. However, the Court of Appeals for the District of Columbia repelled the order, and the US Supreme Court denied to review this ruling, which led the FTC to abandon the complaint.

[8]Goodman and Myers (2005) break down the composition of portfolios for the patents declared essential to 3G PP2 technology; they find that the largest IP holder owns approximately 65% of these patents, and that the three largest portfolios account for 80% of the total number. Parchomovsky and Wagner (2005) stress the importance of patent portfolios over individual patents.

[9]See for example Lancaster (1975), Spence (1976), Dixit and Stiglitz (1977), Vickrey (1964) and Salop (1979), and Mankiw and Whinston (1986) for detailed analyses of monopolistic or spatial competition, and Katz (1980) for that case of a multiproduct monopolist; Tirole (1988, chapter 7) offers a good overview of this literature. More recently, Chen and Riordan (2007) show that the market may again provide too many or too few products in a spokes model of nonlocalized spatial competition.

[10]The literature on variety has primarily focused on the polar cases of free-entry by mono-product

To study this issue, we develop a framework based on a standard oligopoly model of competition with product differentiation, in which entry requires essential intellectual property (*IP* hereafter) from several IP holders. The terms under which downstream firms can access this IP affects entry decisions,[11] and thus product diversity as well as prices and welfare. Keeping constant the number of products, increasing the number of IP holders can only raise prices and thus reduce consumer surplus as well as total welfare. Accounting for the impact on entry decisions and variety as well as prices alters the picture. We find that independent licensing policy of the owners of complementary intellectual policy indeed does not always have adverse effects on consumers and social welfare.[12] While independent licensing tends to restrict entry and thus variety, this may also reduce prices, which benefits consumers and can more than offset the adverse effect on variety.

We study how the form of licensing policies – flat rate access fees, royalty percentages or per unit fees – and market structure – including vertical as well as horizontal integration (e.g., through patent pools and cross-licensing arrangements) – affect the impact of independent licensing on consumers and society.

We first consider, as a benchmark, the case of a single owner of essential IP. In this case, the IP holder faces a trade-off between two conflicting forces. Increasing the number of licenses enhances product variety, which allows downstream firms to better meet consumer demand, thus creating added value. However, it also intensifies downstream competition, which dissipates profits. We adopt a framework that reflects this trade-off, in which the IP owner can have an incentive to sell either fewer or more licenses than is socially desirable.

Specifically, we suppose that downstream firms compete in price and other non-price attributes, modeled as the firms' locations on a circular market, as in Vickrey (1964) and Salop (1979). The number of downstream competitors is endogenous, and

firms (with either oligopolistic or monopolistic competition) and of a multi-product monopolist; we revisit this literature by studying instead the case where a few upstream firms (the IP owners) can affect entry and variety through their licensing terms. Also, while for expositional purposes we develop our analysis using a particular model of oligopolistic competition, our main insights would apply in other models where entry can be excessive.

[11] We will assume that any entry takes place at once and thus ignore the positive externalities that early adopters may exert on later ones; see Glachant and Meniere (2010) for an exploration of the role of patents on technology adoption when such externalities are present.

[12] In a different vein, Scotchmer (1991), Green and Scotchmer (1995), and Scotchmer and Menell (2005) stress that when early investors cannot capture the benefits accruing to subsequent investors, patent protection for complementary products should be strengthened. A key assumption for this result is that investment is sequential - different firms invest at different dates.

depends on license fees as well as on entry costs. Spence (1975) pointed out that a key factor determining the desired number of licenses is the effect on the downstream market price, rather than on overall consumer surplus.[13] This market price, in turn, depends on the value of the marginal consumer served by each downstream firm. A higher density of firms means lower transportation costs on average, and even more so for the marginal consumer. Since marginal consumers benefit most from increased variety, an integrated monopolist would typically wish to have too many downstream outlets. An unintegrated IP owner may however wish to have too many or too few downstream firms competing against each other, due to concern about profit dissipation from downstream competition.

The IP owner can better control the intensity of downstream competition with per unit fees than with either royalty percentages or fixed access fees. As a result, volume-based access fees encourage the IP owner to issue more (and possibly too many) licenses; in our framework, per unit fees actually allow the IP owner to replicate the fully integrated outcome; in contrast, flat rate access fees, which yield the same outcome as percentage royalties, can exhibit reduced variety and lower prices and profits than the fully integrated outcome.

We then consider the case in which there are two independent owners of complementary and essential IP. We find that the "patent thicket" can reduce variety downstream relative to the case of monopoly. More precisely, by relying on per unit licensing fees, the IP owners can still replicate the fully integrated outcome; however, when they rely instead on flat access fees or percentage royalties, horizontal double marginalization leads to higher access charges and fewer downstream firms than does monopoly or joint licensing. This reduction in variety is accompanied by a reduction in consumer prices, and the net effect benefits consumers – but may either increase or decrease social welfare – when an IP monopolist (or a patent pool) would sell too many licenses.

Vertical integration – namely, the acquisition of a downstream competitor by an upstream IP holder – appears to have little impact on the IP owner's ability to control competition; in particular, it has no impact on equilibrium prices, profits and variety in the case of flat rate and per unit access fees. In contrast, patent pools and cross-licensing agreements allow the IP owners to replicate the same outcome as an upstream monopoly controlling all the IP.

The literature on IP licensing initially focused on the case of a single owner of

[13] Spence focused on quality choice, but the same insight applies to other dimensions such as variety, which an IP owner can control through the number of licenses.

(inessential) innovation that allows a reduction in cost in a downstream market. Arrow (1962) studied the impact of competition in that downstream market on the incentives to innovate, while most of the other pioneering work focused on specific modes of licensing such as the auctioning of a given number of licenses, flat rate licensing or per unit fees. Katz and Shapiro (1985,1986) focus on the use of flat rate licensing and study the incentive to share or auction an innovation. Kamien and Tauman (1986) show that flat rate licensing is indeed more profitable (for non-drastic, and thus inessential IP) than volume-based royalties in the case of a homogenous Cournot oligopoly.[14] This is partly a consequence of the fact that the licensing agreement offered to one firm affects its rivals' profits if they do not buy a license, and thus their bargaining position vis-à-vis the IP owner; such strategic effects do not arise in the case of essential (or, in their context, of drastic) innovation, since firms get no profit if they do not buy a license - whatever the agreements offered to their rivals. This optimality of flat rate licensing is somewhat at odds with what is observed in practice. This paradox triggered a number of authors to seek explanations for the use of royalties. For example, Muto (1993) shows that per unit fees can be more profitable in the case of Bertrand oligopoly with differentiated products;[15] Wang (1998) obtains a similar result in the original context of a Cournot oligopoly when the IP owner is one of the downstream firms, while Kishimoto and Muto (2008) extend this insight to Nash Bargaining between an upstream IP owner and downstream firms; and Sen (2005) shows that lumpiness, too, can provide a basis for the optimality of volume-based royalties.[16]

In a recent paper Schmidt (2008) provides an analysis of the patent thicket problem that is closely related to ours. He, too, considers a model with upstream IP owners and downstream competitors needing access to the IP. He finds that, when licensing agreements involve a simple per unit fee, vertical integration between an upstream IP owner and a downstream producer solves a "vertical" double mark-up problem – of successive monopolies – but gives the integrated firm an incentive to increase the licensing fees charged to others, so as to "raise rivals' costs".[17] Schmidt also finds that horizontal integration of IP owners is always beneficial, and reduces

[14] See Kamien (1992) for an overview of this early literature.

[15] Hernandez-Murillo and Llobet (2006) consider monopolistic competition with differentiated products and introduce private information on the value of the innovation for the downstream firms.

[16] Faulli-Oller and Sandonis (2002) and Erutku and Richelle (2006) look at two part licensing policy when there is a differentiated product downstream duopoly and the upstream IP owner is vertically integrated with one of the downstream firms.

[17] See also Layne-Farrar and Schmidt (2009).

the "horizontal" double mark-up problem of complementary monopolies. While the model is in many respects more general (e.g., by allowing for more general demand specifications or alternative forms of oligopolistic competition), it does not consider the impact of horizontal integration of IP owners or patent pools on downstream market variety. In contrast, we show that horizontal integration or patent pools are not always beneficial when accounting for such impact.[18]

2 Framework

Upstream firms own a technology, protected by an IP right, which is a key input to be active in a downstream market. Initially, a single IP owner does not use the technology itself but licences it to downstream competitors (we discuss the impact of vertical integration and of multiple complementary IP firms and rights later on). The licensing fees then affect downstream entry decisions; this affects consumers in two ways: directly, through enhanced product variety, and indirectly, through increased competitive pressure on prices.

2.1 The role of variety

Suppose first that downstream competitors are all identical, and produce the same product at the same cost. Entry has then no intrinsic value and, if there is any set-up cost, it would clearly be socially as well as privately optimal to have the market served by a single downstream firm.[19] Yet a regulator might wish to stimulate entry in order to encourage downstream competition. If for example the IP owner charges a fixed license fee and the downstream firms compete imperfectly in a Cournot fashion, then the IP owner would maximize and appropriate all the industry profit by charging a fee equal to the downstream monopoly profit, whereas a regulator might want to

[18] For further analyses of the impact of licensing policy and vertical integration on downstream markets, see e.g. Fosfuri (2006), who stresses that competition among licensors triggers more aggressive licensing, Lerner and Tirole (2005), who study the choice among open licenses, and Rockett (1990), who notes that the licensor may choose a weak licensee, to avoid tough competition once the patent expires.

[19] Suppose for example that the downstream firms have the same cost function $C(q) = f + cq$, and let $U(q)$ denote consumers' gross surplus, and $P(q) = U'(q)$ the associated inverse demand function. The social optimum maximizes $U(q) + (P(q) - c)q - nf$, possibly subject to a budget constraint $(P(q) - c)q \geq nf$, whereas the private optimum maximizes $(P(q) - c)q - nf$. The social and private interests then lead to different pricing rules (marginal or average cost versus monopoly price) but agree on the optimal number of firms, $n = 1$.

impose a cap on the license fee, in order to reduce consumer prices and allocative inefficiency, even if this inefficiently duplicates entry costs.[20]

When instead variety is valuable, increasing the number of firms can have an ambiguous impact on consumer surplus: enhancing product variety tends to benefit consumers, but it may also lead to higher prices, since firms' offerings then better respond to consumer needs. As a result, the IP holder may want to issue either too many or too few licenses.

If for example the IP owner can extract all downstream profits through licensing fees, it will choose the number of licenses, n, so as to maximize equilibrium profits, $\Pi^*(n)$, whereas total welfare would also include consumer surplus: $W(n) = S^*(n) + \Pi^*(n)$, where $S^*(n)$ denotes the surplus that consumers obtain when n firms compete in the downstream market. When entry in the downstream market overall benefits consumers (i.e., $S^*(.)$ increases with n), the IP owner will tend to restrict entry, compared to what would be desirable for consumers or society – and in the case of multiple complementary IPs, royalty stacking will further restrict entry and hurt consumers as well as society. This is the case in the situation discussed above, where consumers do not care about variety; when consumers enjoy variety, this remains the case as long as increasing the number of downstream firms still yields lower prices or generates only moderate price increases, despite the positive impact of variety on demand. A regulator would then wish to foster entry, e.g., by imposing a cap on licensing fees.

In contrast, when additional entry generates price increases that dominate the direct impact on demand (i.e., $S^*(.)$ decreases as n increases), then the IP owner will tend to issue too many licenses, by setting the licensing fee too low. When there are multiple complementary IPs, royalty stacking may come as a blessing since it counterbalances the bias towards excessive entry – provided there is no "overshooting": double marginalization may also lead to a number of licensees that is lower than socially desirable, to an extent such that social welfare is reduced. We now explore these issues in more detail, using a standard model of downstream competition with horizontal product differentiation.

[20] Consider the example described in the previous footnote and let $q^C(n)$ denote the aggregate quantity produced when n downstream firms compete à la Cournot. A regulator would seek to maximize

$$U\left(q^C(n)\right) - cq^C(n) - nf$$

and would thus choose $n > 1$ whenever (ignoring integer problems) $P\left(q^C(1)\right) > c + f/\left(q^C\right)'(1)$.

2.2 Downstream competition with differentiated products

We adopt the model proposed by Vickrey (1964) and Salop (1979), extending the Hotelling model of horizontal differentiation to allow for any number of downstream competitors. There is a continuum of consumers of total mass 1, uniformly distributed along a circle of length one. A consumer buying from a firm "located" at a distance d gets a utility r but incurs a "transportation cost" td, reflecting the disutility from not having a unit corresponding to that consumer's ideal characteristics.

As before, there is an infinite number of potential entrants in the downstream market, and any firm with access to the technology can enter the market by incurring a fixed cost f; for expositional simplicity, we will suppose that downstream firms can then produce at no cost (introducing a constant marginal cost would not affect the analysis, rescaling the reservation and the equilibrium prices by the same amount). For the sake of exposition, we will also ignore integer problems and treat the number of entrants as a continuous variable.

This simple and well-known model, which relies on a standard discrete choice approach, moreover allows us to focus on variety (i.e., entry) since, as long as the market is served, prices do not affect total welfare directly (the terms of the licensing agreements may and will however have an indirect impact, through their effect on entry). It is also flexible enough to reflect the benefits of entry for consumers (directly through increased variety, but also indirectly through more intense competition), as well as potential adverse effects (through increased local market power, as discussed below). As a result, increasing the number of competitors may have either a positive or a negative impact on consumers.

Before studying the impact of access terms on downstream competition, it is useful to characterize first the optimal degree of variety, both from the private standpoint of a fully integrated company, who would own and control the IP as well as the downstream firms, and from the social (i.e., total welfare) standpoint.

Lemma 1 *The industry is viable if consumers' reservation price is large enough, compared with production and transportation costs, namely, if $r^2/tf > 2$. In that case, ignoring divisibility problems, an integrated monopolist would obtain a positive profit, $\Pi^M \equiv r - \sqrt{2tf}$, by issuing a number of licenses, $n^M \equiv \sqrt{\frac{t}{2f}}$, which exceeds the socially desirable number of downstream firms, $n^W \equiv \sqrt{\frac{t}{4f}}$.*

Proof. An integrated monopolist would serve the entire market (or none) and distribute the downstream outlets uniformly along the circle in order to minimize transportation costs and thus maximize demand. Setting up n outlets then allows the integrated monopolist to charge $p(n) = r - t/2n$, and the resulting profit $p(n) - nf$ is maximal for $n^M = \sqrt{t/2f}$. By contrast, total welfare is equal to $r - T(n) - nf$, where $T(n) \equiv 2n \int_0^{1/2n} tx dx = t/4n$ denotes total transportation costs, and is maximal for $n^W \equiv \sqrt{t/4f}$. ■

When deciding whether to add a downstream outlet, an integrated monopolist – who fully internalizes the additional entry cost f – focuses on its impact on marginal consumers (since they are the ones that determine prices), which are the farthest away from the existing outlets and thus benefit most from the introduction of additional outlets. In contrast, total welfare takes into consideration the impact on all consumers, including inframarginal ones.[21] As a result, a fully integrated monopolist has here an incentive to introduce excessively many downstream subsidiaries.

3 Licensing arrangements and downstream competition

We now study the IP holder's optimal licensing policy, given its impact on the downstream market. We consider the following timing:

- First, the IP owner sets the terms for its licenses; these terms are non-discriminatory and available to any firm wishing to enter the downstream market.[22]
- Second, potential entrants decide whether to buy a license or not; for the sake of exposition, we assume that firms entering the market locate themselves uniformly along the circle; that minimizes total transportation costs and is thus desirable for consumers as well as for the upstream firm.
- Third, licensees compete in prices on the downstream market.

3.1 Fixed license fees

We first consider the case where the IP holder charges a *fixed fee* ϕ per license.

[21] See Spence (1975).

[22] Allowing for secret, possibly discriminatory licensing terms might give the IP owner an incentive to behave opportunistically and issue more licenses than it would otherwise. See Hart and Tirole (1990), O'Brien and Shaffer (1992) and McAfee and Schwartz (1994), or Rey and Tirole (2007) for an overview of this literature.

3.1.1 Downstream equilibrium

We first describe the downstream equilibrium price and profits as a function of the number of firms, n, assuming that they uniformly distribute themselves along the circle:

Lemma 2 *Suppose that n firms are uniformly distributed along the circle. There then exists a symmetric equilibrium, where the price and aggregate profit are as follows:*

$$\begin{cases} \text{for } n < \underline{n} \equiv \frac{t}{r}, & p^*(n) = p^m \equiv \frac{r}{2} & \text{and } \Pi^*(n) = n\pi^m \equiv n\left(\frac{r^2}{2t} - f\right), \\ \text{for } \underline{n} \leq n \leq \overline{n} \equiv \frac{3}{2}\frac{t}{r}, & p^*(n) = \hat{p}(n) \equiv r - \frac{t}{2n} & \text{and } \Pi^*(n) = \hat{\Pi}(n) \equiv r - \frac{t}{2n} - nf, \\ \text{for } n > \overline{n}, & p^*(n) = p^H(n) \equiv \frac{t}{n} & \text{and } \Pi^*(n) = \Pi^H(n) \equiv \frac{t}{n} - nf. \end{cases}$$

Proof. See Salop (1979).[23] ■

When there are few firms in the market ($n < \underline{n}$), each one acts as a local monopolist and charges the local monopoly price p^m. When instead many competitors are present ($n > \overline{n}$), the standard "Hotelling" equilibrium arises, in which the firms' margin reflects the degree of differentiation, t/n, and thus increases with the differentiation parameter t but decreases as the number of firms increases. By contrast, in the intermediate range ($\underline{n} < n < \overline{n}$), the firms charge the maximal price that marginal consumers (located at equal distance between two firms) are willing to pay; as the entry of an additional firm increases variety, it also leads to higher prices and aggregate profits ($\hat{p}(n)$ increases with n), at the expense of (inframarginal) consumers. As a result, as n increases:

- The profit of a downstream firm (gross of the license fee ϕ),

$$\pi^*(n) \equiv \frac{\Pi^*(n)}{n},$$

  first remains constant at the local monopoly level, π^m (as long as n remains below $\underline{n}$) and then strictly decreases: $\hat{\pi}(n)$ decreases with n when $n > \underline{n}$, and $\pi^H(n)$ always decreases with n.

- Consumer surplus first increases proportionally to the number of firms, then decreases when n lies between $\underline{n}$ and $\overline{n}$, before increasing again.[24]

[23] Vickrey (1964) provides a first analysis of the last case.

[24] Consumer surplus is equal to $2n\int_0^{x^m} txdx = nr^2/4t$ where x^m is the distance to the marginal consumer, i.e., $x^m = \frac{r}{2t}$, for $n < \underline{n}$, to $2n\int_0^{1/2n} txdx = t/4n$ for $\underline{n} \leq n \leq \overline{n}$ and to $r - t/4n - p^*(n) = r - 5t/4n$ for $n > \overline{n}$.

- Total welfare increases as long as $n < n^W$ (which exceeds $\underline{n}$ but can lie either above or below $\overline{n}$), and decreases afterwards.

This model of product differentiation reflects the features discussed in the introduction: an increase in the number of competitors benefits consumers and dissipates profit when Hotelling-like competition prevails (when $n > \overline{n}$, the aggregate profit $\Pi^H(n)$ decreases with n), but it can also allow firms to extract a bigger share of consumers' benefit from variety, resulting in higher prices that reduce consumer surplus but increase aggregate profit.[25]

3.1.2 Optimal and equilibrium licensing fees

We now characterize the privately optimal licensing fee, ϕ^Π, which maximizes the IP holder's profit. As the individual downstream profit, $\pi^*(n)$, decreases with n, the IP owner can control the number of downstream firms through the licensing fee ϕ:

- if the upstream IP owner sets $\phi > \pi^m$, no firm enters the market;
- if instead the IP owner sets $\phi = \pi^m$, any $n \leq \underline{n}$ firms are willing to enter; since $\pi^m > 0$, it is then optimal for the IP owner to issue as many licenses as possible (i.e., $n = \underline{n}$, offering if needed an arbitrarily small discount);
- last, if the IP owner sets $\phi < \pi^m$, then there exists a unique n such that $\pi^*(n) = \phi$; this licensing fee thus triggers a unique continuation equilibrium where, at stages 2 and 3, n downstream firms enter (ignoring again integer problems) and sell at price $p^*(n)$.

Thus, by setting the licensing fee to $\phi^*(n) = \pi^*(n)$, the IP owner can induce exactly n firms to enter, and moreover recover all of their profits. It will thus choose the fee so as to maximize the industry profit:

$$\max_n n\phi^*(n) = \Pi^*(n).$$

Without loss of generality, we can furthermore restrict attention to $n \geq \underline{n}$. Conversely, too intense downstream competition dissipates profit: since $\Pi^H(n)$ decreases with n, the IP holder will thus never choose $n > \overline{n}$. In the range $[\underline{n}, \overline{n}]$, the industry profit coincides with the integrated monopoly profit ($\Pi^*(n) = \hat{\Pi}(n)$), which is concave and

[25] The spokes model of Chen and Riordan (2007) has similar features.

maximal for $n = n^M$. Therefore, the industry profit is globally quasi-concave and the upstream firm will find it optimal to induce the entry of n^Π downstream firms, where

$$n^\Pi \equiv \min\left\{n^M, \overline{n}\right\}.$$

Indeed, if it could control prices as well as the number of the downstream firms, the IP owner would choose to let n^M firms enter the market. However, having that many firms in the downstream market can trigger intense price competition and dissipate profits: this occurs when $n^M > \overline{n}$, in which case the IP owner prefers to have only $\overline{n}$ firms in the market.

It can be checked that $n^M \geq \overline{n}$ if and only if $r^2/tf \geq 9/2$. Therefore, the IP holder makes positive profits whenever the industry is viable (i.e., $r^2/tf > 2$) and:

- when $2 < r^2/tf \leq 9/2$, $n^\Pi = n^M \leq \overline{n}$ and $\Pi^*\left(n^\Pi\right) = \Pi^M > 0$;
- when instead $r^2/tf > 9/2$, $n^\Pi = \overline{n} < n^M$ and $0 < \Pi^*\left(n^\Pi\right) < \Pi^M$.

In both cases, the IP owner generates the (constrained) optimal number n^Π of downstream firms by setting

$$\phi^\Pi \equiv \pi^*\left(n^\Pi\right).$$

We can now compare the privately optimal number of downstream firms, n^Π, with the socially desirable one, n^W, which could be achieved by setting the licensing fee to $\phi^W \equiv \pi^*\left(n^W\right)$. As long as $\overline{n} > n^W$, the IP owner issues too many licenses: either n^M, if $\overline{n} > n^M > n^W$, or $\overline{n}$, if $n^M > \overline{n} > n^W$. If instead $n^W > \overline{n}$, downstream competition would dissipate profits not only with n^M competitors, but also with the (smaller) number of competitors that would be socially desirable, n^W; in that case, the IP owner excessively restricts entry, in order to limit downstream competition. Even in this case, though, a positive fee is required to induce the socially desirable number of downstream firms, since:

$$\phi^W = \pi^*\left(n^W\right) = \pi^H\left(n^W\right) = \frac{t}{\left(n^W\right)^2} - f = 3f > 0.$$

This positive license fee is needed to prevent the "excessive entry" that would otherwise derive from a "business stealing" effect, each downstream firm failing to take into account that (some of) the customers it serves would otherwise be served anyway by other firms.

It can be checked that $n^W \geq \overline{n}$ if and only if $r^2/tf \geq 9$, which leads to:[26]

[26] In the second case ($2 < r^2/tf < 9$), the socially desirable number of downstream firms may yield negative industry profits; taking into account a budget constraint ($\Pi \geq 0$) would then call for a higher number of firms, $\hat{n}^W > n^W$, which would however remain larger than n^Π.

Proposition 3 *Suppose that the market is viable: $r^2/tf \geq 2$. Then:*

- *if in addition $r^2/tf > 9$, the IP owner lets too few firms enter the downstream market;*
- *if instead $r^2/tf < 9$, the IP owner lets too many firms enter the downstream market.*

Thus, when variety is "cheap" (i.e., the fixed cost f is small) and/or "not highly regarded" (i.e., the transportation cost t is small, implying that variety is not very valuable) compared with the intrinsic value of the good (as measured by r), the upstream IP holder issues too few licences: it would be desirable in that situation to have more firms in the downstream market, but competition would dissipate the profits that the IP owner can recover. When instead variety is costly and/or particularly viable (i.e., f and t are substantial), the IP holder issues too many licenses: increasing variety raises the price that marginal consumers are willing to pay, which then increases industry profit in spite of competition. This ambiguity in the comparison between the privately and socially desirable numbers of firms reflects a similar ambiguity for the licensing fees: the IP owner charges an excessively high fee when $r^2/tf > 9$, but charges instead too low a fee when $r^2/tf < 9$.

Finally, it can be noted that the IP owner's inability to fully control the downstream firms' pricing policies limits the risk of excessive entry. In the present set-up, where a fully integrated industry would generate more variety than is socially desirable (i.e., $n^W < n^M$), the IP owner's inability to prevent profit dissipation through Hotelling-like product market competition leads it to somewhat limit the number of downstream firms, which, in turn, reduces the scope for excessive entry (e.g., when $n^\Pi < n^W < n^M$).

3.2 Alternative licensing arrangements

We have so far focussed on fixed licensing fees. We now briefly discuss alternative arrangements, such as volume-based fees or royalty percentages.

3.2.1 Royalties

We first note that replacing fixed fees with *profit-based royalties* does not affect the analysis.[27] Suppose indeed that, instead of a fixed licensing fee ϕ, the IP holder asks

[27] Since we normalized downstream unit costs to zero, there is no distinction here between profits and revenues; when costs are taken into consideration, however, the analysis applies to profit-based

for a percentage τ of downstream profits. If n firms enter and the others charge the same price $\widetilde{p}$, a downstream firm then maximizes

$$(1-\tau)\,pD\left(p,\widetilde{p};n\right)-f,$$

which leads to the same best response as before and thus to the same equilibrium price $p^*(n)$. In equilibrium, each firm thus pays $\tau p^*(n)/n = \tau\left[\pi^*(n)+f\right]$ and gets:

$$\pi^*(n)-\tau\left[\pi^*(n)+f\right],$$

which decreases as τ or n increases. Free entry thus implies an inverse relation between τ and the equilibrium number of firms, and the IP holder can therefore again control n, by setting here the rate to $\tau^*(n) \equiv \pi^*(n)/\left[\pi^*(n)+f\right]$. Furthermore, since free entry drives downstream profits to zero, the IP holder recovers as before the aggregate profit: $n\tau\left[\pi^*(n)+f\right] = n\pi^*(n) = \Pi^*(n)$. The IP holder will thus issue n^{Π} licenses, by charging a rate $\tau^{\Pi} \equiv \tau\left(n^{\Pi}\right)$, which is again too low or too high, according to whether $r^2 < 9tf$ or $r^2 > 9tf$.

3.2.2 Unit fees

The IP holder could better control price and variety through the use of more complex licensing arrangements. In particular, two-part tariffs often allow an upstream monopolist to replicate the fully integrated monopoly outcome; we now note that, since demand is inelastic, the same outcome can be achieved in this model with *volume-based royalties*, when downstream firms simply pay a *per-unit fee*, γ. We show this informally here, and provide a formal proof in the Appendix. As before, the IP owner will serve the entire market (or none); otherwise, downstream firms would be viable local monopolists, and issuing additional licenses would increase coverage and profit. Conversely, as long as the entire market is served, the IP owner obtains a profit equal to γ, and thus increases γ as much as possible, up to the point where a local monopolist would just break even. Since the local monopoly price and profit, based on a unit cost γ, are respectively equal to:

$$p^m(\gamma) \equiv \arg\max_p\,(p-\gamma)\,D^m(p) = \frac{r+\gamma}{2},\ \pi^m(\gamma) = \frac{(r-\gamma)^2}{2t}-f,$$

the IP owner will thus set γ so that $\pi^m(\gamma)=0$, or:

$$\gamma = \bar{\gamma} \equiv r-\sqrt{2tf}.$$

rather than revenue-based licensing fees, as the latter would yield different equilibrium prices – denoting by c the downstream unit cost, fixed fees and profit-based royalties would yield the same equilibrium price $p^*(n;c)$, whereas a revenue-based royalty would yield $p^*\left(n;\frac{c}{1-\tau}\right)$.

But the market price is then at the optimal level:

$$p^m(\bar{\gamma}) = r - \sqrt{\frac{tf}{2}} = p^M,$$

and the market shares yield the optimal number of firms: marginal consumers are located at a distance $\hat{x}$ such that $p^M + t\hat{x} = r$, and market shares are thus equal to

$$2\hat{x} = 2\frac{r - p^M}{t} = \sqrt{\frac{2f}{t}};$$

covering the entire market thus requires a number of firms equal to:

$$n = \frac{1}{2\hat{x}} = \sqrt{\frac{t}{2f}} = n^M.$$

Therefore:

Proposition 4 *Profit-based royalties yield the same outcome as fixed licensing fees. In contrast, volume-based royalties (i.e., per unit fees) allow the IP owner to replicate the fully integrated outcome and thus yield excessive entry.*

Proof. See Appendix A. ■

Unit fees thus constitute here a better licensing arrangement, which allows the IP holder to control both price and variety.[28] Interestingly, unit fees cannot sustain the social optimum. Appendix A shows that the equilibrium per-firm profit (net of unit fees), $\pi^*(n;\gamma)$, decreases as n or γ increases. Therefore:

- any $\gamma > \bar{\gamma}$ deters entry, since then, for any n, $\pi^*(n;\gamma) \leq \pi^m(\gamma) < 0$;
- $\gamma = \bar{\gamma}$ triggers any $n \leq n^M$ firms, but the market is entirely served for $n = n^M$; therefore, while it is possible to sustain exactly n^W firms, only part of the market would then be served;
- and any $\gamma < \bar{\gamma}$ triggers a continuation equilibrium in which all the market is served, since $\pi^m(\gamma) > (\pi^m(\bar{\gamma}) =) \, 0$, but in which more than n^M downstream firms enter the market, since then $\pi^*(n^M;\gamma) > 0$[29] and $\pi^*(n;\gamma)$ further increases (up to $\pi^m(\gamma)$) as n decreases below n^M.

[28] This result relies on the specifics of the model and in particular on the assumption of unit demands. When demand is elastic, Appendix A shows that unit fees yield excessive entry and higher prices than what would maximize the industry profit.

[29] It is shown in Appendix A that $\underline{n}(\gamma)$ increases with γ. Therefore, $\gamma < \bar{\gamma}$ implies $n^M > \underline{n}(\gamma)$, and thus either $\pi^*(n^M;\gamma) = \hat{\pi}(n^M;\gamma) > \pi^*(n^M;\bar{\gamma}) = 0$, or $\pi^*(n^M;\gamma) = \pi^H(n^M) = f > 0$.

These observations moreover indicate that, while unit fees cannot sustain the social optimum, the IP owner however chooses the "second-best" level for such a fee: *conditional on relying on volume-based fees*, the (second-)best fee γ^W coincides with $\bar{\gamma}$, since any higher level would generate no entry and any lower level would generate additional entry, from a point where there is already excessive entry (since $n^M > n^W$).

3.2.3 Regulating licensing terms

We can use this analysis to address the following question: suppose that one cannot "regulate" the actual level of the licensing terms (i.e. the amount of the fee or the royalty rate), but still dictate the type of licensing arrangement (e.g., fixed fees versus profit-based or volume-based royalties); which type of arrangement works best for society? Insisting on fixed licensing fees or profit-based royalties leads the IP owner (with or without vertical integration in the first case, and without integration in the second case) to issue $n^\Pi = \min\left\{\overline{n}, n^M\right\}$ licenses, whereas allowing for alternative (e.g., volume-based fees) and more profitable licensing schemes leads instead the IP owner (with or without vertical integration) to issue $n^M > n^W$ licenses. As a result, allowing for more flexible licensing schemes has no impact on the number of licensees or welfare when $n^\Pi = n^M$ $(> \overline{n})$, and instead increases the number of licensees (from $\overline{n}$ to n^M) when $n^\Pi = \overline{n} < n^M$; in that case, this can decrease welfare (e. g., if $n^W < \overline{n}$, since in that case n^Π is already excessively high) but may increase it as well when $\overline{n}$ is low enough.[30]

4 Complementary technologies

We now consider a situation where two upstream firms, U_1 and U_2, each control an essential technology. These two technologies are perfect complements: combined together, they allow firms to compete in the downstream market, and each of them is necessary to be active in that market. If the two technologies were owned by the same IP holder, then the IP owner would provide a joint license to both technologies and the above analysis of the single IP owner case would apply. We now compare this joint licensing outcome with the outcome of independent licensing, for the same types of licensing fees.

[30] This is for example the case for $t = r = 2f$.

4.1 Fixed license fees

Since the two IP holders independently market their rights, we adjust the previous timing as follows:

- First, each IP owner, $i = 1, 2$, simultaneously and independently sets a license fee, ϕ_i.
- Second, potential downstream entrants decide whether or not to buy the licenses; as before, those that enter locate themselves uniformly along the circle.
- Third, downstream competitors set their prices.

As already noted, independent licensing creates a "horizontal" double marginalization problem and leads to higher total fees. It may even trigger a "coordination breakdown" where both IP owners charge prohibitively high fees, thereby discouraging any downstream firm from entering the market: any pair of fees satisfying $\phi_i \geq \pi^m$, for $i = 1, 2$, constitutes an equilibrium. Such equilibria rely however on weakly dominated strategies; we will therefore focus our discussion on equilibria in which each IP owner charges a fee below the monopoly profit π^m.

Given its rival's equilibrium fee $\phi^e < \pi^m$, U_i can induce the entry of n_i firms by setting its own fee to $\phi_i^* (n_i)$, such that

$$\pi^* (n_i) = \phi_i + \phi^e. \tag{1}$$

Each U_i will thus will want to choose n_i (or ϕ_i) so as to maximize:

$$\Pi_i = n_i \phi_i^* (n_i) = n_i (\pi^* (n_i) - \phi^e) = \Pi^* (n_i) - n_i \phi^e.$$

We show in the Appendix that the unique equilibrium (excluding weakly dominated strategies) is symmetric ($\phi_1 = \phi_2 = \phi^D$, where the superscript D stands for "Double marginalization") and leads to a number of firms equal to:

$$n^D \equiv \frac{r}{2f} \left(\sqrt{1 + 6\frac{tf}{r^2}} - 1 \right),$$

which is such that $\underline{n} \leq n^D < n^\Pi = \min \left\{ n^M, \overline{n} \right\}$. We thus have:

Proposition 5 *When the IP holders rely on fixed licensing fees, independent licensing leads to higher fees and fewer downstream firms than joint licensing.*

Proof. See Appendix B. ■

Because of double marginalization, independent licensing reduces the number of licenses that are eventually issued. This may however enhance social welfare here, since joint licensing can lead to excessively many firms. Yet, independent licensing can also result in too few licenses. We show in the Appendix that, indeed, $n^D < n^W$ when

$$\frac{r^2}{tf} > \frac{25}{4}.$$

This therefore only happens when variety is cheap (f small) and/or not very interesting (t small), compared with the intrinsic value of the good (as measured by r). More precisely:

- When $r^2/tf > 9$, joint licensing would already generate too few licenses $\left(n^\Pi = \overline{n} < n^W\right)$; independent licensing then reduces welfare, since double marginalization further reduces the number of licenses below the optimal level $\left(n^D < \overline{n} < n^W\right)$.

- In contrast, when $r^2/tf < 25/4$ (but $r^2/tf > 2$, to ensure the viability of the market), even independent licensing generates too many licenses; the associated double marginalization then brings the number of licensees closer to what is socially desirable and improves welfare $\left(n^W < n^D < n^\Pi\right)$.

- In the intermediate range where $9 > r^2/tf > 25/4$, double marginalization still reduces the number of licensees, but joint licensing would lead to too many licenses; independent licensing may thus improve welfare.[31]

Remark: Consumer surplus. A monopoly IP owner chooses n^Π, while a duopoly results in $n^D < n^\Pi$ firms. Double marginalization thus reduces variety, but it also results in lower downstream prices; furthermore, the benefit of lower prices more than offsets the effect of reduced variety and generates greater consumer surplus. Indeed, whenever the market is viable (i.e., $r^2/tf > 2$), we have: $\underline{n} \leq n^D < n^\Pi \leq \overline{n}$ and, in this range, the consumer price is equal to $\hat{p}(n) = r - \frac{t}{2n}$ and increases with n, while consumer surplus is given by

$$CS\left(n\right) = r - (r - \frac{t}{2n}) - 2n \int_0^{\frac{1}{2n}} txdx = \frac{t}{4n},$$

and thus decreases with n. Therefore:

[31] By continuity, there is a threshold $\hat{\rho}$ for $\rho = r^2/tf$, such that $\hat{\rho} \in (25/4, 9)$, such that, compared with joint licensing, independent licensing and the associated double marginalization reduces welfare if $\rho > \hat{\rho}$, but instead enhances welfare if $\rho < \hat{\rho}$.

- compared with the case of an IP monopoly, an IP duopoly results in *higher upstream fees* but *lower downstream prices*;
- the increase in the upstream fees implies *fewer downstream firms* and *lower industry profits*, but *higher consumer surplus* – consumers may even prefer this double marginalization situation to *royalty-free* licenses, unless the royalty-free equilibrium results in significantly more than $\overline{n}$ firms;[32]
- the duopoly outcome is however less efficient than the monopoly outcome when the reduction in profit exceeds consumer benefits.

4.2 Alternative licensing arrangements

4.2.1 Royalties

Suppose now that the IP holders ask for *royalty percentages* τ_1 and τ_2 on downstream profits, so that the total royalty rate is $\tau = \tau_1 + \tau_2$. Given its rival's equilibrium rate τ^e, each U_i will set τ_i so as to maximize $\tau_i n^* p^* D(p^*) = \tau_i [\Pi^*(n) + n^* f]$, which, using the free-entry condition $(\tau_i + \tau^e)[\Pi^*(n) + n^* f] = \Pi^*(n)$, amounts to maximizing:

$$\Pi_i = \Pi^*(n) - \tau^e [\Pi^*(n) + nf] = (1 - \tau^e) \left[\Pi^*(n) - \frac{\tau^e}{1 - \tau^e} nf \right].$$

Therefore, there is again some double marginalization (reflected in the last term of the right-hand side in the above equation) which lead the IP owners to limit the number of licenses. It it shown in Appendix B that this double marginalization is less severe here than with flat rate licensing fees; letting n^R denote the number of licenses generated by independent licensing and percentage royalties, we have:

Proposition 6 *When relying on profit-based royalties, independent licensing creates again double marginalization problems, which are however less severe than in the case of fixed licensing fees: we have*

$$n^D < n^R \leq n^\Pi,$$

[32] Consumer surplus decreases with n in the range $[\underline{n}, \overline{n}]$ and then increases with n for $n > \overline{n}$. Let denote by n^f the number of firms entering the downstream market when licenses are free (i.e., such that $\pi^*(n^f) = 0$) and by $\hat{n} > \overline{n}$ the number of firms that yields as much surplus as n^D. Then, as long as $n^f \leq \hat{n}$ (that is, when f is "large enough"), the outcome of IP duopoly and double marginalization is better for consumers than the free-entry equilibrium – in that case, the number of firms that maximize consumer surplus, subject to non-negative profit constraint, is $\underline{n}$; when $n^f > \hat{n}$, however, consumers would prefer to have "as many firms" as possible and free-entry would work beter for them.

with strict inequalities whenever $n^{\Pi} < n^{M}$.

Proof. See Appendix B. ■

4.2.2 Unit fees

Suppose now that the IP holders charge instead *unit fees* (i.e., volume-based royalties) γ_1 and γ_2, so that the total fee is $\gamma = \gamma_1 + \gamma_2$. As long as $\gamma < \overline{\gamma}$, the entire market market is served; therefore, each IP holder i gets

$$\Pi_i = \gamma_i,$$

which clearly increases with γ_i. In contrast, when $\gamma > \overline{\gamma}$, no entry occurs and thus $\Pi_1 = \Pi_2 = 0$; last, when $\gamma = \overline{\gamma}$, there are enough firms willing to enter to serve the entire market, and the total profits are

$$\Pi_1 + \Pi_2 = \Pi^M.$$

As a result, the equilibrium is such that $\gamma = \overline{\gamma}$, and the two IP holders share the integrated monopoly profit.[33] In other words, double marginalization does not preclude here the IP holders from maximizing their joint profits, and they issue as many licenses as is privately optimal $\left(n = n^M\right)$.

We thus have:

Proposition 7 *When relying on unit-fees, the IP holders can replicate the fully integrated outcome, whether they license their technologies jointly or independently.*

5 Extensions

We consider here alternative organizations and market structures. We first show that patent pools and cross-licensing agreements can replicate "horizontal integration" and solve double marginalization problems, before noting that vertical integration appears to have little impact on the equilibrium outcome.

[33] There is actually an infinity of equilibria, which only differ in the way the profit Π^M is shared among the two IP holders: any couple of fees γ_1 and γ_2 adding-up to $\bar{\gamma}$ constitutes an equilibrium.

5.1 Pool

The two IP holders can replicate joint licensing by assigning their IP rights to a pool that sells the technology for them and retrocedes the profits, say on a fifty-fifty basis. To see this, consider for example the case of fixed licensing fees. The pool manager controls again the number of firms n and recover downstream profits by setting the fee of the pool to $\phi^*(n) = \pi^*(n)$. The pool manager then seeks to maximize each owner's profit, equal to

$$n\frac{\phi^*(n)}{2} = \frac{\Pi^*(n)}{2},$$

and thus maximizes again total profit by selling n^Π licenses for a fee $\phi = \phi^\Pi$.

Likewise, in the case of profit-based royalties the pool manager would issue the same number of licenses, n^Π, by setting the royalty rate to $\tau = \tau^\Pi$ – and in the case of unit fees, the pool manager would still replicate the fully integrated outcome by charging a unit fee $\gamma = \bar{\gamma}$.

5.2 Cross-licensing

The IP holders could instead opt for cross-licensing agreements, allowing them to issue "complete" licenses covering both technologies, subject to paying the other IP holder a fee per license issued. We first consider in Appendix C the case of a *reciprocal* cross-licensing agreement that allows *both IP holders* to issue complete licenses, by paying the other a fee equal to ψ. As long as the reciprocal fee ψ is not too large (namely, $\psi \leq \phi^D$), Bertrand competition between the two upstream firms then leads them to set their fees (for "full licenses") to

$$\Phi_1 = \Phi_2 = \Phi \equiv 2\psi,$$

each U_i being then indifferent between issuing a license and earning $\Phi - \psi = \psi$, or letting the other IP holder issue the license and learning ψ again (if $\psi \leq \phi^D$, the IP owners would instead have an incentive to undercut each other). Clearly, as long as this equilibrium prevails, it is optimal for the IP holders to adjust the *upstream* cross-licensing fee ψ to $\phi^\Pi/2$, so as to drive the *downstream* licensing fee $\Phi = 2\psi$ to ϕ^Π and share the integrated monopoly profit. Conversely, $n^\Pi > n^D$ ensures that $\phi^\Pi/2 < \phi^D$, implying that setting ψ to $\phi^\Pi/2$ indeed yields the desired outcome. Such a cross-licensing arrangement thus formally achieves the same outcome as a merger or patent pool.

If instead each U_i independently sets its upstream fee ψ_i, then cross-licensing may again mitigate double marginalization problems and result in more downstream firms than n^D, but does not eliminate them entirely and still results in fewer firms than n^Π (see Appendix C).

5.3 Vertical integration

Vertical integration has little impact on the above analysis when IP holders charge either fixed or per-unit fees, irrespective of whether there is one or more than one IP owner. To see this, consider the case in which one IP holder owns a single downstream firm, which competes with the other downstream firms; the logic extends to the case in which multiple IP holders are vertically integrated. Note that, in both cases, vertical integration does not affect the behavior of non-integrated downstream firms.

We first consider the case of fixed licensing fees. In that case, vertical integration does not affect the behavior of the subsidiary either since, once it has sold its licenses, the variable profit of the integrated firm coincides with that of its downstream subsidiary. Therefore, as before, a total licensing fee $\phi(n) = \pi^*(n)$ will again induce the entry of exactly n downstream competitors (integrated or not).

When the integrated firm is the sole IP holder, it then wants to set n so as to maximize:

$$\pi^*(n) + (n-1)\,\phi(n) = n\pi^*(n),$$

and again chooses to let n^Π firms (including its own subsidiary) enter the downstream market.

When instead there is another IP holder, who sets a licensing fee ϕ^e, the integrated IP holder U_i will again seek to let n_i firms so as to maximize:

$$\pi^*(n_i) - \phi^e + (n_i - 1)\left(\pi^*(n_i) - \phi^e\right) = \Pi^*(n_i) - n_i\phi^e,$$

and thus its licensing behavior is thus the same as if it was not integrated. As a result, the equilibrium outcome is the same, whether the IP holders are vertically integrated or not (the same observation carries over to the case where both IP holders are vertically integrated with distinct subsidiaries).

We next turn to the case of unit fees, and first consider the case in which the integrated firm is the only IP holder. In setting its downstream market price p, it then takes into account that it loses its unit fee γ on any unit taken away from its rivals; it will thus maximize:

$$pD(p, \widetilde{p}; n) + \gamma\left[1 - D(p, \widetilde{p}; n)\right],$$

which amounts to maximizing

$$(p-\gamma)\,D(p, \widetilde{p}; n).$$

Similarly, in the case of multiple IP holders, the integrated firm will set its price p_i so as to maximize (with $j \neq i = 1, 2$, and as long as the entire market is served):

$$\left(p_i - \gamma_j\right) D\left(p_i, \widetilde{p}; n\right) + \gamma_i \left[1 - D\left(p_i, \widetilde{p}; n\right)\right],$$

which again boils down to maximizing (letting $\gamma = \gamma_1 + \gamma_2$ denote the total unit fee):

$$\left(p_i - \gamma\right) D\left(p_i, \widetilde{p}; n\right).$$

Therefore, whatever the number of IP owners, all firms, vertically integrated or not, behave in the same way in the downstream market, and thus vertical integration again has no impact on the downstream equilibrium. More specifically, an integrated IP monopolist can then obtain the monopoly profit Π^M by setting $\gamma = \gamma^M$.[34] And in the case of multiple IP holders, the integrated firm will maximize:

$$\frac{p^* - \gamma_j}{n} + \gamma_i \left(1 - \frac{1}{n}\right) = \frac{p^* - \gamma}{n} + \gamma_i = f + \gamma_i,$$

and will thus seek to increase its own fee, γ_i, as much as possible, as if it were not integrated. Therefore, vertical integration has no impact on the equilibrium, which remains such that $\gamma = \overline{\gamma}$ and the two IP holders share the fully integrated monopoly profit.

The same reasoning applies to both IP holders when they are each integrated with a single distinct downstream subsidiary. We thus have:

Proposition 8 *Vertical integration by one or more IP holders, each with a single downstream firm, does not affect the equilibrium outcome when the licensing terms consist of either fixed or per unit fees.*

The neutrality of vertical integration relies here on the fact that the final demand is inelastic (and in equilibrium the IP holder has always an incentive to issue sufficiently many licenses to cover the market). As Schmidt (2008) observed, when the final demand is elastic, vertical integration can alleviate (vertical) double mark-up

[34] It is always optimal for the IP holder to let enough downstream firms enter to cover the entire market. For a given fee γ and associated number of firms n, the total profit of the integrated IP holder is then equal to (using the free-entry condition)

$$\frac{p^*(n)}{n} + \gamma\left(1 - \frac{1}{n}\right) = \frac{p^*(n) - \gamma}{n} + \gamma = f + \gamma.$$

The integrated IP holder thus wishes to maximize γ, as when there is no vertical integration.

problems, enhancing coordination between upstream and downstream pricing decisions within the integrated firm, as well as providing the integrated IP owner an incentive to increase its licensing (unit) fees, in order to "raise rivals' costs" and benefit from the resulting foreclosure effect;[35] vertical integration may also allow the IP owner to better exert its market power,[36] or induce the downstream subsidiary to become less aggressive.[37]

Remark: profit-based royalties. Even within our framework, vertical integration affects the outcome when the IP owner relies on profit-based royalty percentages: for example, in the case where it is an IP monopolist, the integrated firm then maximizes:

$$pD(p, \widetilde{p}; n) + \tau \widetilde{p}\left[1 - D(p, \widetilde{p}; n)\right],$$

and is thus less aggressive than the others. The downstream equilibrium is then more complex to characterize, since the reduction in the competitive pressure is greater for the integrated subsidiary's immediate neighbors than for the other unintegrated firms, and is asymmetric, the integrated firm's downstream subsidiary having a lower market share than the unintegrated firms. Such royalty schemes thus lead to an inefficient allocation of consumers among the existing firms; they can moreover lead to an inefficient distribution of firms along the circle, since locations closer to the integrated firm downstream subsidiary are more profitable.

Remark: joint licensing. Proposition 8 extends to joint licensing.[38] If for example the pool sets a fixed licensing fee ϕ and redistributes half of the profit to each IP owner, the pool manager will pick the total number of firms n (by setting $\phi = \pi^*(n)$)

[35] See Ordover, Salop and Saloner (1990) and Salinger (1988). More recently, Allain, Chambolle and Rey (2010) show that vertical integration can discourage downstream innovation when downstream firms must exchange sensitive information with their suppliers in order to implement an innovation.

[36] In case of secret contracting, vertical integration may help limiting the risk of opportunistic behavior that would otherwise lead the IP owner to issue too many licenses (see See Hart and Tirole (1990) and the discussion in footnote 22), since issuing an additional license then hurts the integrated subsidiary as well as the other downstream competitors.

[37] See Chen (2001), who stresses that the downstream subsidiary will internalize the impact of its behavior on the sales of the integrated supplier.

[38] Vertical integration still does not affect downstream behaviour when IP owners license their rights jointly. This is obvious in the case of fixed license fees, and in the case of unit fees an integrated U_i will set its price p_i so as to maximize:

$$(p_i - \gamma) D(p_i, \widetilde{p}; n) + \frac{\gamma}{2},$$

which again amounts to maximize $(p_i - \gamma) D(p_i, \widetilde{p}; n)$.

so as to maximize:

$$\pi^*(n) + \frac{(n-2)\,\phi}{2} = \pi^*(n) + \frac{(n-2)\,\pi^*(n)}{2} = \frac{n\pi^*(n)}{2} = \frac{\Pi^*(n)}{2}.$$

The pool manager thus again maximizes total profits and chooses $n = n^{\Pi}$. Similarly, in the case of unit fees the pool manager will set the fee γ so as to maximize each IP holder's total profit, equal to (using the free-entry condition $(p^* - \gamma)/n = f$):

$$\frac{p^* - \gamma}{n} + \frac{\gamma}{2} = f + \frac{\gamma}{2},$$

and will thus choose the maximal acceptable value for γ $(\gamma = \bar{\gamma})$.

6 Conclusion

Patent thickets have long been a concern due to the potential for delaying deployment of products and adversely affecting consumers. We examine the implications of such patent thickets for downstream market structure and product variety as well as prices and welfare. In the absence of vertical licensing agreements, it is well known that there can be excessive entry, due e.g. to business stealing effects, or insufficient entry, if firms entering the market appropriate only part of the surplus they generate. We revisit this issue, taking into account the gatekeeper role that upstream IP owners play through their licensing policies.

We first consider the case in which a single owner of essential IP controls entry in the downstream market and can appropriate the resulting profits through licensing fees. While the IP holder then internalizes any business stealing effect, we show that it can still choose to sell either a larger or smaller number of licenses than is socially optimal. Granting too many licenses never occurs when the downstream licensees offer homogeneous products, but can occur when products are sufficiently differentiated, in which case additional licensees extract a substantial share of the surplus that consumers derive from enhanced variety; when instead downstream products are close substitutes, competition dissipates profits and the IP holder tends to issue too few licenses or, equivalently, charges too high fees for these licenses.

When there are two or more upstream IP owners, royalty stacking also tends to reduce the number of downstream licensees. But when a single IP owner (or multiple IP owners jointly licensing their technologies) would issue too many licenses, the reduction in the number of downstream competitors and product variety can result in lower prices, and higher consumer surplus and social welfare. By contrast, royalty stacking always reduces the IP holders' profits. They therefore have an incentive to develop licensing arrangements, such as patent pools, that allow them to solve the double marginalization problems. We show that reciprocal cross-licensing agreements can have the same effect.

We examine how the form of licensing fees affects the outcome. We find that the IP owner(s) may sell fewer licenses than would be offered by a fully integrated monopolist when licensing fees assume the form of a fixed license fee or a profit-based royalty percentage. This is because, in that case, the IP owner cannot control its licensees' pricing policies; the fear of profit dissipation through downstream competition then tends to reduce the risk of excessive entry. By contrast, the IP holder(s) may replicate the fully integrated outcome by charging per-unit fees. Finally, when IP owners charge fixed or unit fees, vertical integration does not alter the behavior of affiliated downstream subsidiaries, and as a result vertical integration has no effect on the equilibrium outcome.

Products offered in high technology industries are often quite differentiated and embody (sometimes extensive) patent portfolios of a few firms. Our analysis indicates that, in such industries, royalty stacking may have a less clear impact than the patent thicket literature suggests.

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Appendix

A Proof of Proposition 4

We first study the impact of a unit fee γ on the downstream equilibrium. Charging a unit fee γ pushes the Hotelling price by the same amount:

$$p^H(n;\gamma) \equiv \gamma + \frac{t}{n};$$

as a result, downstream profits (net here of payments to the IP owner) are not affected by this fee:

$$\pi^H(n;\gamma) \equiv \pi^H(n) = \frac{t}{n^2} - f.$$

In contrast, when downstream firms act as local monopolists, they pass only part of the fee γ on to consumers; their prices and (net) profits are then equal to:

$$\begin{aligned} p^m(\gamma) &\equiv \arg\max_p (p-\gamma) D^m(p) = \frac{r+\gamma}{2}, \\ \pi^m(\gamma) &\equiv \frac{(r-\gamma)^2}{2t} - f. \end{aligned}$$

The unit fee also affects the conditions under which the various competition regimes prevail. The Hotelling competitive regime now prevails when

$$p^H(n;\gamma) + \frac{t}{2n} = \gamma + \frac{3t}{2n} < r,$$

that is,

$$n > \overline{n}(\gamma) \equiv \frac{3}{2}\frac{t}{r-\gamma}, \tag{2}$$

whereas the local monopoly regime prevails when

$$p^m(\gamma) + \frac{t}{2n} = \frac{r+\gamma}{2} + \frac{t}{2n} \geq r,$$

or

$$n \leq \underline{n}(\gamma) \equiv \frac{t}{r-\gamma}. \tag{3}$$

In the intermediate range $[\underline{n}(\gamma), \overline{n}(\gamma)]$, the entire market is served at a price as before equal to $\hat{p}(n) = r - t/2n$, so that each downstream firm earns

$$\hat{\pi}(n;\gamma) \equiv \frac{1}{n}\left(r - \gamma - \frac{t}{2n}\right) - f,$$

which decreases in n in that range;[39] the downstream equilibrium is thus now such that (profits being again expressed here net of licensing fees):

- for $n < \underline{n}(\gamma) \equiv \dfrac{t}{r-\gamma}$:

$$p^*(n;\gamma) = p^m(\gamma) = \frac{r-\gamma}{2} \quad \text{and} \quad \pi^*(n;\gamma) = \pi^m(\gamma) = \frac{(r-\gamma)^2}{2t} - f.$$

- for $\underline{n}(\gamma) < n < \overline{n}(\gamma) \equiv \dfrac{3}{2}\dfrac{t}{r-\gamma}$:

$$p^*(n;\gamma) = \hat{p}(n) = r - \frac{t}{2n} \quad \text{and} \quad \pi^*(n;\gamma) = \hat{\pi}(n;\gamma) = \frac{1}{n}\left(r - \gamma - \frac{t}{2n}\right) - f,$$

- for $n > \overline{n}(\gamma)$:

$$p^*(n;\gamma) = p^H(n;\gamma) = \gamma + \frac{t}{n} \quad \text{and} \quad \pi^*(n;\gamma) = \pi^H(n) = \frac{t}{n^2} - f,$$

Setting the maximal fee $\gamma = \bar{\gamma} \equiv r - \sqrt{2tf}$ thus allows the IP owner to replicate the fully integrated monopoly outcome: it leads indeed to $p^m(\gamma) = p^M = \hat{p}\left(n^M\right)$ and $\pi^m(\bar{\gamma}) = 0$, and thus $n = n^M$.

Note: elastic demand. Suppose that consumers have a variable demand $D(p)$, with an elasticity $\varepsilon(p) = -pD'(p)/D(p)$ that is positive and increasing, to ensure that revenues and profits are quasi-concave; assuming that transportation cost are not volume-sensitive, consumers located at a distance $\hat{x}$ such that $v(p) \equiv \int_p^{+\infty} D(x)\,dx = t\hat{x}$ are marginal. For a given price, total industry profit is maximal for a number of firms that "just" covers the market, $n = t/2v(p)$, and is then equal to:

$$\pi_T(p) \equiv pD(p) - \frac{tf}{2v(p)}.$$

We thus have:

$$\pi_T'(p) = D(p) + pD'(p) - \frac{tfD(p)}{2v^2(p)} = \left(1 - \varepsilon(p) - \frac{tf}{2v^2(p)}\right)D(p).$$

This profit is therefore quasi-concave (since the term within bracket is increasing),[40] and the optimal price, p^M, is solution to $1 - \varepsilon(p) = tf/2v(p)$.

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[39]

$$\hat{\pi}'(n;\gamma) = -\frac{(r-\gamma)}{n^2} + \frac{t}{n^3} = \frac{r-\gamma}{n^3}\left(\underline{n}(\gamma) - n\right) < 0$$

as long as $n > \underline{n}(\gamma)$.

[40] Since $(\pi_T'/D)' = -\varepsilon' - \frac{tfD}{2v^3} < 0$, the profit $\pi_T(p)$ increases with p for $p < p^M$, and decreases afterwards.

By contrast, if the IP holder charges a unit fee γ, it will issue as many licenses as needed to cover the market and seek to maximize $\gamma p(\gamma) D(p(\gamma))$, where the price $p(\gamma)$ still increases with γ. If the IP holder then induces a price exceeding the level p^R that maximizes the revenue $R(p) = pD(p)$, such a price would clearly exceed p^M since the above optimality condition implies $p^M < p^R$. If instead the IP holder induces a price lower than p^R, than in that range it will again set γ to the maximal level that a local monopolist could bear, since γ and $R(p(\gamma))$ increase in that range. In that case, the price that will emerge will maximize the profit of a local monopoly, $(p-\gamma) D(p) 2v(p)/t$, and thus satisfy the first-order condition:

$$(D(p) + (p-\gamma) D'(p)) v(p) - (p-\gamma) D^2(p) = 0,$$

or:

$$\left(1 - \varepsilon(p) \frac{p-\gamma}{p}\right) v(p) = \frac{p-\gamma}{p} pD(p).$$

This local monopoly profit must moreover be zero, or:

$$(p-\gamma) D(p) \frac{2v(p)}{t} = f.$$

Combining these two conditions yields

$$\left(1 - \varepsilon(p) \frac{f}{R(p)}\right) v^2(p) = \frac{tf}{2},$$

implying that $\pi_T'(p) < 0$ and thus that the price again exceeds p^M. Thus, when consumer demand is elastic, the use of a unit fee generates higher prices than what would maximize the industry profit, and the number of firms therefore also exceeds the privately optimal one.

B Proof of Propositions 5 and 6

Consider first the case of fixed license fees. Given the two IP owners' fees ϕ_1 and ϕ_2, the number of downstream firms entering the market is given by $n^*(\phi_1 + \phi_2)$, where

$$n^*(\phi) \equiv \begin{cases} (\pi^*)^{-1}(\phi) & \text{when} \quad \phi < \pi^m, \\ \text{any } n \leq \underline{n} & \text{when} \quad \phi = \pi^m, \\ 0 & \text{when} \quad \phi > \pi^m. \end{cases}$$

Each U_i then obtains a profit equal to

$$\Pi_i = n^*(\phi_1 + \phi_2) \phi_i.$$

As already noted, independent licensing may trigger a "coordination breakdown", where both IP owners charge fees higher than the monopoly profit π^m and no downstream firm enters the market. These equilibria however involve weakly dominated strategies, and we now focus instead on equilibria in which both upstream firms charge a fee lower than π^m.

Fix the rival's fee $\phi_j < \pi^m$ and suppose first that U_i chooses to induce a number $n_i > \overline{n}$ of downstream firms, by setting a fee ϕ_i such that $\phi_i + \phi^e = \pi^* (n_i) = \pi^H (n_i)$; U_i would then rather increase ϕ_i in order to reduce n_i to $\overline{n}$, since its profit, given by

$$\Pi_i = n_i \phi_i = n_i \left(\pi^H (n_i) - \phi^e \right) = \Pi^H (n_i) - n_i \phi^e,$$

decreases in n_i (since $\Pi^H (n)$ decreases in n). Therefore, the upstream firms will never choose to have more than $\overline{n}$ downstream firms. Similarly, setting $\phi_i = \pi^m - \phi_j$ induces any $n \leq \underline{n}$ firms to enter and gives U_i a profit

$$\Pi_i = n_i \left(\pi^m - \phi_j \right),$$

which is positive and proportional to the number of firms; hence U_i will never choose to induce less than $\underline{n}$ downstream firms.

Thus, without loss of generality, we can assume that U_i sets a fee ϕ_i such that $\phi_i + \phi_j \in [\overline{\pi}, \pi^m]$, where

$$\overline{\pi} \equiv \pi^* (\overline{n}) = \frac{4r^2}{9t} - f,$$

so as to induce a number of firms $n_i \in [\underline{n}, \overline{n}]$, given by $\phi_i + \phi_j = \pi^* (n_i) = \hat{\pi} (n_i)$, that maximizes

$$\Pi_i = n_i \phi_i = n_i \left(\hat{\pi} (n_i) - \phi_j \right) = \hat{\Pi} (n_i) - n_i \phi_j = r - \frac{t}{2n_i} - n_i \left(f + \phi_j \right). \quad (4)$$

Ignoring the constraint $n_i \in [\underline{n}, \overline{n}]$ would lead U_i to choose

$$n_i = n^M \left(f + \phi_j \right) = \sqrt{\frac{t}{2 \left(f + \phi_j \right)}}, \quad (5)$$

which is larger than $\underline{n}$ when $\phi_j \leq \pi^m$ and is also smaller than $\overline{n}$ as long as

$$\phi_j \geq \hat{\phi} \equiv \frac{2r^2}{9t} - f,$$

where $\hat{\phi} < \pi^m$ and $\hat{\phi} > 0$ is equivalent to $\overline{n} < n^M$. Therefore:

- if $\overline{n} < n^M$, U_i's best response to $\phi_j \leq \pi^m$ is to induce a number of firms equal to $\overline{n}$ for $\phi_j \leq \hat{\phi}$ and to $n^M \left(f + \phi_j \right)$ otherwise, where $n^M (f + \phi)$ denotes the integrated monopoly outcome for a fixed cost equal to $f+\phi$ instead of f; the corresponding fee is then $\phi^R\left(\phi_j\right)$, where:

$$\phi^R(\phi) \equiv \begin{cases} \overline{\pi}-\phi & \text{when} \quad \phi \leq \hat{\phi}, \\ \hat{\pi}\left(n^M(f+\phi)\right)-\phi & \text{when} \quad \hat{\phi} \leq \phi \leq \pi^m. \end{cases}$$

- if $\overline{n} \geq n^M$, U_i's best response to $\phi_j \leq \pi^m$ is always to induce a number of firms $n_i = n^M\left(f+\phi_i\right)$ with a fee equal to $\hat{\pi}\left(n^M\left(f+\phi_j\right)\right)-\phi_j$.

In both cases, in the range $\phi \in [0, \pi^m]$ the resulting number of firms is $n^R(\phi) = \min\left\{\overline{n}, n^M(f+\phi)\right\}$, which weakly decreases from n^Π to $\underline{n}$ as ϕ increases, whereas the best response ϕ^R is continuous and decreases from $\phi^R(0) = \phi^\Pi$ to $\phi^R(\pi^m) = 0$: the slope is equal to -1 for $\phi > \hat{\phi}$ and, for $\phi > \hat{\phi}$, using

$$\hat{\pi}\left(n^M(f+\phi)\right)-\phi = \frac{r}{\sqrt{\frac{t}{2(f+\phi)}}} - \frac{t}{2\frac{t}{2(f+\phi)}} - (f+\phi) = r\sqrt{\frac{2(f+\phi)}{t}} - 2(f+\phi).$$

we have:

$$\frac{d\phi^R}{d\phi}(\phi) = \frac{r}{t\sqrt{\frac{2(f+\phi)}{t}}} - 2 = \frac{n^M(f+\phi)}{t/r} - 2,$$

where $n^M(f+\phi)$ decreases from $\frac{3t}{2r}$ to $\frac{t}{r}$ as ϕ increases from $\hat{\phi}$ to π^m; the slope thus lies between $-1/2$ and -1. Therefore, the best responses $\phi_i = \phi^R\left(\phi_j\right)$, for $i \neq j = 1, 2$, cross once and only once in the range $[0, \pi^m]$. Therefore, there is unique equilibrium in this range, which is moreover symmetric: $\phi_1 = \phi_2 = \phi^D$ and $n_1 = n_2 = n^D$. Furthermore, since $\overline{\pi} - 2\hat{\phi} = f > 0$, we have $\overline{\pi} - \hat{\phi} > \hat{\phi}$, as illustrated by Figure 1; the equilibrium thus satisfies $\phi^D > \hat{\phi}$ and $n^D < \overline{n}$, and is therefore characterized by:

$$n^D = n^M\left(f+\phi^D\right) = \sqrt{\frac{t}{2\left(f+\phi^D\right)}} \text{ and } 2\phi^D = \hat{\pi}\left(n^D\right) = \frac{1}{n^D}\left(r - \frac{t}{2n^D}\right) - f.$$

These two conditions imply:

$$2\phi n^2 = t - 2fn^2 = rn - \frac{t}{2} - fn^2,$$

and thus:

$$fn^2 + rn - \frac{3t}{2} = 0, \tag{6}$$

which has a unique non-negative solution:

$$n^D \equiv \frac{r}{2f}\left(\sqrt{1+\frac{6tf}{r^2}} - 1\right).$$

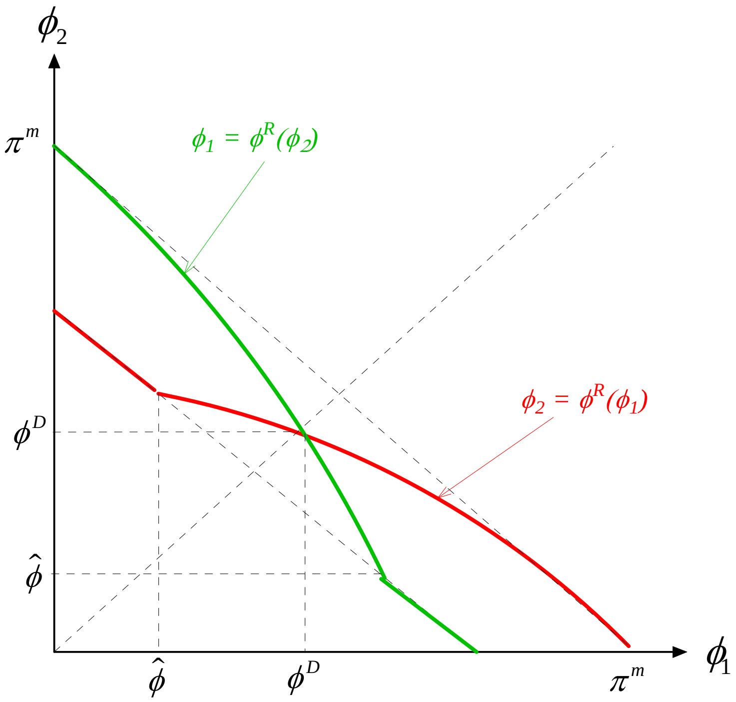

Figure 1: Best response fees for complementary technologies

Last, it is straightforward to confirm that double marginalization leads to fewer licenses being issued. This is obvious in the case of coordination breakdown, where no license is issued; and when the upstream firms coordinate on the above equilibrium, $\phi^D > \hat{\phi}$ implies $n^D = n^R\left(\phi^D\right) < n^R(0) = n^\Pi$. Double marginalization can actually excessively reduce the number of licenses: it can be checked that $n^D < n^W$ whenever $r^2/tf > 25/4$.

We now turn to the case of percentage royalties. We have seen that each U_i seeks to maximize

$$\Pi_i = (1-\tau_j)\left[\Pi^*(n) - n\frac{\tau_j f}{1-\tau_j}\right] = (1-\tau_j)\,\Pi^*\left(n;\hat{f}_j\right), \tag{7}$$

where $\Pi^*\left(n,\hat{f}\right)$ denotes the industry equilibrium profit based on a fixed entry cost equal to $\hat{f}$, and $\hat{f}_j \equiv f + \dfrac{\tau_j f}{1-\tau_j} = \dfrac{f}{1-\tau_j}$. Each U_i would therefore seek to induce a number of firms $n_i = n^\Pi\left(\hat{f}_j\right)$, which implies that the equilibrium is again symmetric: $n^\Pi\left(\hat{f}_1\right) = n^\Pi\left(\hat{f}_2\right) = n^R$ implies $\tau_1 = \tau_2 = \tau^R$. Double marginalization results again in fewer firms (at least weakly so), since the equilibrium number of firms, n^R, satisfies:

$$n^R = n^\Pi\left(\frac{f}{1-\tau^R}\right) = \min\left\{\overline{n}, n^M\left(\frac{f}{1-\tau^R}\right)\right\} \le n^\Pi = n^\Pi(f).$$

We now check that the double marginalization problem is less severe with royalties than with fixed license fees. In the case of royalties, the equilibrium number of firms is either $\overline{n}$, in which case it exceeds indeed n^{Π}, or $\hat{n}^R$, which maximizes the expression in (7) for $\Pi^* = \hat{\Pi}$ and $\tau_j = \tau^R$ and thus satisfies:

$$\hat{\Pi}'\left(\hat{n}^R\right) = \frac{\tau^R f}{1-\tau^R}.$$

Using the free entry condition $\hat{\pi}\left(\hat{n}^R\right) = 2\tau^R\left[\hat{\pi}\left(\hat{n}^R\right) + f\right]$, this condition implies

$$\hat{\Pi}'\left(\hat{n}^R\right) = \frac{1-2\tau^R}{1-\tau^R}\frac{\hat{\pi}\left(\hat{n}^R\right)}{2} < \frac{\hat{\pi}\left(\hat{n}^R\right)}{2}. \tag{8}$$

By contrast, n^D maximizes the expression in (4) for $\phi_j = \phi^D$ and thus satisfies:

$$\hat{\Pi}'\left(n^D\right) = \phi^D = \frac{\hat{\pi}\left(n^D\right)}{2}. \tag{9}$$

Comparing (8) and (9) yields:

$$\varphi\left(\hat{n}^R\right) < \varphi\left(n^D\right) = 0, \tag{10}$$

where:

$$\varphi(n) \equiv \hat{\Pi}'(n) - \frac{\hat{\pi}(n)}{2}.$$

Furthermore,

$$\varphi'(n) = \hat{\Pi}''(n) - \frac{\hat{\pi}'(n)}{2} = -\frac{t}{n^3} - \frac{1}{2}\left(-\frac{r}{n^2} + \frac{t}{n^3}\right) = \frac{r}{2n^2}\left(1 - \frac{3t}{rn}\right),$$

which is negative for $n \leq \overline{n}$. Therefore, (10) implies $n^D < n^R$.

C Cross licensing

We analyze here the situation where the upstream firms allow each other to license their own technology. We will denote by ψ_i the (upstream) fee that U_i charges to U_j for each license it issues, and by Φ_j the (downstream) fee charged by U_j for a "complete" license covering both technologies. The timing is as follows:

- first, the IP owners set the upstream fees ψ_1 and ψ_2 (more on this below);
- second, the IP owners set their downstream fees Φ_1 and Φ_2; the downstream firms then decide whether to buy a license and enter the market.

We first characterize the continuation equilibria of the second stage, for given upstream fees ψ_1 and ψ_2. We then consider two scenarios for the first stage: in the first scenario, the IP owners jointly agree on a reciprocal fee $\psi_1 = \psi_2 = \psi$; in the second scenario, the two IP owners sets their fees simultaneously and independently.

C.1 Downstream IP competition

We take here the upstream fees ψ_1 and ψ_2 as given and consider the second stage, where the two IP owners charge fees Φ_1 and Φ_2 for "complete" licenses; any downstream entrant then buys a license from the cheapest licensor and, given $\Phi = \min\{\Phi_1, \Phi_2\}$, the number of entrants is equal to $n^*(\Phi)$.

Note first that each U_i is unwilling to sell a complete license for a fee Φ_i lower than U_j's upstream fee ψ_j. Therefore, if $\min\{\psi_1, \psi_2\} > \pi^m$, then no license is issued and both IP owners get zero profit. If $\min\{\psi_1, \psi_2\} = \pi^m$, there are multiple continuation equilibria, in which the upstream firms set downstream fees exceeding π^m or serve up to $\underline{n}$ licences at a fee $\Phi = \pi^m$, thereby sharing up to $\underline{n}\pi^m$. If $\psi_i \geq \pi^m > \psi_j$ then, anticipating that U_j is unwilling to issue any license, U_i will set Φ_i so as to maximize

$$n^*(\Phi_i)\left(\Phi_i - \psi_j\right),$$

which using $\phi_i \equiv \Phi_i - \psi_j$ as the decision variable, amounts to maximize

$$n^*\left(\phi_i + \psi_j\right)\phi_i$$

and thus leads U_i to choose

$$\phi_i = \phi^R\left(\psi_j\right),$$

or, equivalently: $\Phi_i = \Phi^R\left(\psi_j\right)$, where

$$\Phi^R(\phi) \equiv \phi^R(\phi) - \phi,$$

which results in a number of downstream firms equal to $n^R\left(\psi_j\right)$. The two firms then obtain

$$\begin{aligned} \Pi_i &= n^R\left(\psi_j\right)\left(\Phi^R\left(\psi_j\right) - \psi_j\right) = n^R\left(\psi_j\right)\phi^R\left(\psi_j\right), \\ \Pi_j &= n^R\left(\psi_j\right)\psi_j. \end{aligned}$$

It is straightforward to check that U_j has indeed no incentive to undercut U_i, since this would require selling at a loss.

We now consider the case where both IP owners set fees lower than π^m, and consider first a candidate equilibrium where $\Phi_1 = \Phi_2 = \Phi$. Each U_i can then obtain $n(\Phi)\psi_i$ by increasing its fee (and letting the other IP owner sell its license to all downstream entrants) and can also obtain $n(\Phi)\left(\Phi - \psi_j\right)$ by slightly undercutting its rival. Therefore, it must be the case that $\Phi = \psi_1 + \psi_2$. Conversely, $\Phi_1 = \Phi_2 = \psi_1 + \psi_2$ constitutes an equilibrium as long as no U_i benefits from undercutting its rival; this is the case when

$$\Phi_i < \psi_1 + \psi_2 \implies n^*(\Phi_i)\left(\Phi_i - \psi_j\right) < n^*(\psi_1 + \psi_2)\psi_i,$$

that is, using $\phi_i \equiv \Phi_i - \psi_j$, when

$$\phi_i < \psi_i \implies n^* \left(\phi_i + \psi_j\right) \phi_i < n^* \left(\psi_1 + \psi_2\right) \psi_i. \tag{11}$$

Since the profit function $n \left(\psi_1 + \psi_2\right) \psi_i = n \left(\Phi_i\right) \left(\Phi_i - \psi_j\right)$ is strictly quasi-concave in Φ_i,[41] (11) is equivalent to:

$$\psi_i \leq \phi^R \left(\psi_j\right).$$

Consider now a candidate equilibrium in which $\Phi_i < \Phi_j$, implying that the two IP owners obtain respectively (posing $\phi_i = \Phi_i - \psi_j$):

$$\begin{aligned} \Pi_i &= n^* \left(\Phi_i\right) \left(\Phi_i - \psi_j\right) = n^* \left(\phi_i + \psi_j\right) \phi_i, \\ \Pi_j &= n^* \left(\Phi_i\right) \psi_j = n^* \left(\phi_i + \psi_j\right) \psi_j. \end{aligned}$$

U_i should not be able to gain from small deviations, which implies $\phi_i = \phi^R \left(\psi_j\right)$ (and thus $\Phi_i = \Phi^R \left(\psi_j\right)$, $n = n^R \left(\psi_j\right)$) and should not gain either from letting U_j sell at Φ_j, which requires $\Pi_i = n^R \left(\psi_j\right) \phi^R \left(\psi_j\right) \geq n^* \left(\Phi_j\right) \psi_i$; Φ_j must therefore be "large enough" (any $\Phi_j > \pi^m$, for which $n^* \left(\Phi_j\right) = 0$, would do). In addition, U_j should not gain from undercutting U_i, that is:

$$\Pi_j = n^R \left(\psi_j\right) \psi_j \geq \max_{\Phi \leq \Phi^R \left(\psi_j\right)} n^* \left(\Phi\right) \left(\Phi - \psi_i\right). \tag{12}$$

In particular, this implies (considering a deviation to just below $\Phi_i = \Phi^R \left(\psi_j\right)$)

$$\Pi_j = n^R \left(\psi_j\right) \psi_j \geq n^R \left(\psi_j\right) \left(\Phi^R \left(\psi_j\right) - \psi_i\right),$$

that is:

$$\psi_j \geq \Phi^R \left(\psi_j\right) - \psi_i$$

or

$$\psi_i \geq \Phi^R \left(\psi_j\right) - \psi_j = \phi^R \left(\psi_j\right).$$

Building on these insights, we have for $\psi_1, \psi_2 < \pi^m$:

- If $\psi_i \leq \phi^R \left(\psi_j\right)$ for $i \neq j = 1, 2$, there is a unique continuation equilibrium, $\Phi_1 = \Phi_2 = \psi_1 + \psi_2$; each U_i then obtains:

$$\Pi_i = n^* \left(\psi_1 + \psi_2\right) \psi_i.$$

[41] It coincides with the industry profit, which is strictly concave, for $\Phi \in [\overline{\pi}, \pi^m]$, drops to zero for $\Phi > \pi^m$ (and lies anywhere between 0 and $\underline{n}\pi^m$ for $\Phi = \pi^m$), and is equal to $\Pi^H \left(n^* \left(\Phi\right)\right)$ for $\Phi < \overline{\pi}$, in which case it strictly increases with Φ.

- If $\psi_i > \phi^R\left(\psi_j\right)$ but $\psi_j \leq \phi^R\left(\psi_i\right)$, there is a unique continuation equilibrium, in which U_j charges a prohibitively high fee while U_i sells $n^R\left(\psi_j\right)$ complete licenses at a fee $\Phi^R\left(\psi_j\right)$; the two IP owners then obtain respectively:

$$U_i = n^R\left(\psi_j\right)\phi^R\left(\psi_j\right), U_j = n^R\left(\psi_j\right)\psi_j.$$

Note that condition (12) is indeed satisfied, as $\psi_i > \phi^R\left(\psi_j\right)$ and $\psi_j \leq \phi^R\left(\psi_i\right)$ imply $\psi_i > \psi_j$ (see Figure 1) and thus $\Phi^R\left(\psi_i\right) \geq \Phi^R\left(\psi_j\right)$;[42] therefore:

$$\max_{\Phi \leq \Phi^R\left(\psi_j\right)} n\left(\Phi\right)\left(\Phi - \psi_i\right) = n^R\left(\psi_j\right)\left(\Phi^R\left(\psi_j\right) - \psi_i\right) > n^R\left(\psi_j\right)\psi_j,$$

where the last inequality follows from $\psi_i > \phi^R\left(\psi_j\right) = \Phi^R\left(\psi_j\right) - \psi_j$.

- Finally, if $\psi_i > \phi^R\left(\psi_j\right)$ for $i \neq j = 1, 2$, suppose without loss of generality that $\psi_i \geq \psi_j$. A similar reasoning then shows that there always exists an equilibrium in which U_j charges a prohibitively high fee while U_i sells $n^R\left(\psi_j\right)$ complete licenses at a fee $\Phi^R\left(\psi_j\right)$. In addition, there may exist an equilibrium in which U_i charges a prohibitively high fee while U_j sells $n^R\left(\psi_i\right)$ complete licenses at a fee $\Phi^R\left(\psi_i\right)$; for this to be an equilibrium, it must however be the case that

$$\Pi_i = n^R\left(\psi_i\right)\psi_i \geq \max_{\Phi \leq \Phi^R\left(\psi_i\right)} n^*\left(\Phi\right)\left(\Phi - \psi_j\right).$$

C.2 Upstream interaction

We now turn to the first stage and start with the scenario where the two IP owners jointly determine a reciprocal upstream fee $\psi_1 = \psi_2 = \psi$. By setting this fee to:

$$\psi^\Pi \equiv \frac{\pi^*\left(n^\Pi\right)}{2},$$

they can ensure that the second stage leads to $\Phi_1 = \Phi_2 = \pi^*\left(n^\Pi\right)$ and thus to the entry of n^Π downstream firms, and share equally the profit that an integrated IP owner could generate. In the light of the above analysis, it suffices to note that $n^D < n^\Pi$ implies $\phi^D = \pi^*\left(n^D\right)/2 > \psi^\Pi = \pi^*\left(n^\Pi\right)/2$, which in turn implies $\psi^\Pi < \phi^R\left(\psi^\Pi\right)$.

Finally, consider the alternative scenario where the two IP owners set their upstream fees simultaneously and independently. It is easy to check that, in the range $\psi_1, \psi_2 \leq \pi^m$:

[42] $\Phi^R\left(\phi\right) = \pi^*\left(\min\left\{\overline{n}, n^M\left(f + \phi\right)\right\}\right)$, where $\pi^*\left(n\right)$ decreases with n and $n^M\left(f + \phi\right)$ decreases with ϕ; therefore, $\Phi^R\left(\phi\right)$ wealky increases with ϕ.

- There is no equilibrium in which $\psi_1 < \phi^R(\psi_2)$ and $\psi_2 < \phi^R(\psi_1)$: each U_i would obtain a profit $\Pi_i = n^*(\psi_1 + \psi_2)\psi_i$ and would thus deviate and increase its fee.

- There is no equilibrium in which $\psi_i \geq \phi^R(\psi_j)$ but $\psi_j < \phi^R(\psi_i)$ (for either $j = 1$ or 2 and $i \neq j$): U_j would then obtain a profit $\Pi_j = n^R(\psi_j)\psi_j$, which increases with ψ_j, and would thus deviate and increase its fee.

- There is no equilibrium in which $\psi_1 > \phi^R(\psi_2)$ and $\psi_2 > \phi^R(\psi_1)$, and in addition $\psi_j > \phi^D$ while U_i sells some licenses (for either $j = 1$ or 2 and $i \neq j$); this would require $\Phi_i < \Phi_j$ and $\Pi_i = n^R(\psi_j)\phi^R(\psi_j)$, but then U_i would profitably deviate by setting a fee ψ_i' just below $\phi^R(\psi_j)$, which would prompt U_j to sell $n^R(\psi_i') > n^R(\psi_j)$ ($\psi_j > \phi^D$ implies $\phi^R(\psi_j) < \phi^D < \psi_j$) and give U_i a greater profit $\Pi_i' = n^R(\psi_i')\psi_i' = n^R(\psi_i')\phi^R(\psi_j)$.

- There exist equilibria such that $\psi_1 > \phi^R(\psi_2)$ and $\psi_2 > \phi^R(\psi_1)$, in which (for either $j = 1$ or 2) $\psi_j \leq \phi^D$ (which then implies $\psi_i > \phi^D \geq \psi_j$ for $i \neq j$) and U_j sells complete licenses; in each such equilibrium the two IP owners obtain respectively:
$$\Pi_i = n^R(\psi_j)\phi^R(\psi_j), \Pi_j = n^R(\psi_j)\psi_j.$$
In principle, U_j would want to deviate and increase its fee ψ_j, but such deviations can be deterred by "reverting" to a continuation equilibrium where U_j, rather than U_i sells the licenses for a fee $\Phi_j = \Phi^R(\psi_i)$, since in that case U_j obtains $\Pi_j' = n^R(\psi_i)\phi^R(\psi_i)$, which is lower than Π_j since $\phi^R(\psi_i) < \psi_j$ and $\psi_i > \psi_j$ moreover implies $n^R(\psi_i) < n^R(\psi_j)$. This however requires that such continuation equilibrium exists, which in turn requires (see condition (12)):
$$n^R(\psi_i)\psi_i \geq \max_{\Phi \leq \Phi^R(\psi_i)} n^*(\Phi)(\Phi - \psi_j).$$
The right-hand side decreases with ψ_j whereas the left-hand side increases with ψ_i, and they coincide for $\psi_i = \psi_j = \phi^D$. Therefore this condition determines a curve that goes through (ϕ^D, ϕ^D) in the (ψ_1, ψ_2) plane and above which the two continuation equilibria coexist. The equilibrium that generates the greater joint profit is the one for which ψ_j is the lowest, and thus for which ψ_i is maximal: $\psi_i = \pi^m$ and ψ_j such that $n^R(\psi_j)\phi^R(\psi_j) = \underline{n}\pi^m$. This equilibrium gives both IP owners a larger total profit than the "double marginalization" outcome but only one IP owner benefits from it: $\psi_j < \phi^D$ and $\phi^R(\phi^D) = \phi^D$ indeed imply:
$$\begin{aligned} \Pi_i &= n^R(\psi_j)\phi^R(\psi_j) > \Pi^D = n^R(\phi^D)\phi^R(\phi^D), \\ \Pi_j &= n^R(\psi_j)\psi_j < \Pi^D = n^R(\phi^D)\phi^D. \end{aligned}$$