The Bubble Game: An Experimental Study of Speculation

Sophie Moinas and Sebastien Pouget[1][2]

Accepted for publication in Econometrica

February 4, 2013

[1]University of Toulouse (Toulouse School of Economics and IAE), Place Anatole France, 31000 Toulouse, France, sophie.moinas@univ-tlse1.fr, spouget@univ-tlse1.fr.

[2]We would like to thank Franklin Allen, Elena Asparouhova, Andrea Attar, Snehal Banerjee, Pierpaolo Battigalli, Milo Bianchi, Bruno Biais, Peter Bossaerts, Christophe Bisière, Georgy Chabakauri, Sylvain Chassang, John Conlon, Tony Doblas-Madrid, James Dow, Xavier Gabaix, Alex Guembel, Jonathan Ingersoll, Guo Kai, Weicheng Lian, Nour Meddahi, Andrew Metrick, John F. Nash Jr., Charles Noussair, Thomas Palfrey, Alessandro Pavan, Gwenael Piaser, Luis Rayo, Jean Tirole, Reinhard Selten, Paul Woolley, Bilge Yilmaz, and especially Thomas Mariotti, as well as seminar participants in Bocconi University, Bonn University, Luxembourg University, Lyon University (GATE), Toulouse University, Paris Dauphine University, the Paris School of Economics, the London Business School, the Midwest Macroeconomics Meetings, the CSIO-IDEI workshop in Northwestern University, The Wharton School of the University of Pennsylvania, Yale University, the New York Fed, Princeton University, Caltech, University of Utah, and the 2011 PWC conference at UTS Sydney, for helpful comments. We are also very grateful to Philippe Jehiel, the Editor, and three anonymous referees for their suggestions that greatly improved the quality of this paper. An earlier version of this paper was circulated under the title "Rational and Irrational Bubbles: an Experiment". Financial support from the Agence Nationale de la Recherche (ANR-09-BLAN-0358-01) is gratefully acknowledged. This research was conducted within and supported by the Paul Woolley Research Initiative on Capital Market Dysfonctionalities at IDEI-R, Toulouse.

Abstract

We propose a bubble game that involves sequential trading of an asset commonly known to be valueless. Because some traders do not know where they stand in the market sequence, the game allows for a bubble at the Nash equilibrium when there is no cap on the maximum price. We run experiments both with and without a price cap. Structural estimation of behavioral game theory models suggests that quantal responses and analogy-based expectations are important drivers of speculation.

Keywords: Rational bubbles, irrational bubbles, experiments, cognitive hierarchy model, quantal response equilibrium, analogy-based expectation equilibrium

1 Introduction

Historical and recent economic developments such as the South Sea, Mississippi, and dot com price run-up episodes suggest that financial markets are prone to bubbles and crashes. However, to the extent that fundamental values cannot be directly observed in the field, it is very difficult to empirically demonstrate that these episodes actually correspond to mispricings.[1] To overcome this difficulty and study bubble phenomena, economists have relied on the experimental methodology: in the laboratory, fundamental values are induced by the researchers and can thus be compared to asset prices. Starting with Smith, Suchanek and Williams (1988), many researchers document the existence of speculative bubbles in experimental financial markets.[2]

We propose a bubble game that complements Smith et al. (1988) and is simple enough to be analyzed using the tools of (behavioral) game theory. Moreover, it enables to control for the number of trading opportunities thus easing the interpretation of experimental data. The bubble game features a sequential market for an asset that generates no cash flow (and this is announced publicly to all market participants). Traders do not know where they stand in the market sequence nor can they observe past traders' actions. The price proposed to the first trader in the market sequence is random and the subsequent price path is exogenous.[3] Traders have limited liability and are financed by outside financiers. At each point in the sequence, an incoming trader has the choice between buying or not at the proposed price. If he declines the offer, the game ends and the current owner is stuck with the asset.

When there is a price cap (consistent with the fact that there is a finite amount of wealth in the economy), only irrational bubbles can form: upon receiving the highest potential price, a trader realizes that he is last in the

[1] In this paper, we define the fundamental value of an asset as the price at which agents would be ready to buy the asset given that they cannot resell it later. See Camerer (1989) and Brunnermeier (2009) for surveys on bubbles.

[2] The design created by Smith, Suchanek and Williams (1988) features a double auction market for an asset that pays random dividends in several successive periods. The subsequent literature refined this design to show that bubbles also tend to arise in call markets (Van Boening, Williams, LaMaster, 1993), with a constant fundamental value (Noussair, Robin, Ruffieux, 2001) and with lottery-like assets (Ackert, Charupat, Deaves, and Kluger, 2006), but tend to disappear when some traders are experienced (Dufwenberg, Lindqvist, and Moore, 2005), when there are futures markets (Porter and Smith, 1995) and when short-sales are allowed (Ackert, Charupat, Church, and Deaves, 2005).

[3] Our set up is inspired by the two-envelope puzzle discussed by Nalebuff (1989) and, especially, Geanakoplos (1992). The Supplementary Appendix available online relates the bubble game to this puzzle as well as to the Saint-Petersburg paradox.

market sequence and, if rational, refuses to buy. Even if not sure to be last in the market sequence, the previous trader, if rational, also refuses to buy because he anticipates that the next trader will know he is last and will refuse to trade. This backward induction argument rules out the existence of bubbles when there is a price cap, if all traders are rational and rationality is common knowledge. By increasing the level of the cap, one increases the number of steps of iterated reasoning needed to rule out the bubble. As a result, varying the level of the cap enables the experimenter to understand how bounded rationality or lack of higher-order knowledge of rationality affect speculative bubbles. This is of interest in light of the theoretical analyses of Morris, Rob, and Shin (1995) and Morris, Postlewaite, and Shin (1995) who show that lack of common knowledge fosters bubble formation.

When the price cap is infinite, bubbles can be rational because no trader is ever sure to be last in the market sequence. Proposing an experimental analysis of rational bubbles is difficult because extant theories in which bubbles are common knowledge involve infinite trading opportunities and infinite losses.[4] [5] The bubble game overcomes these difficulties: there is a finite number of trades and the potentially infinite losses are concentrated in the hands of outside financiers who are consequently not part of the experiment.

Our experiment features various treatments depending on the existence and the level of a price cap. Subjects participate in only one treatment and in a one-shot game.[6] Our experimental results are as follows. First, bubbles arise whether or not there is a cap on prices. Bubbles thus form even if they would be ruled out by backward induction. Second, the propensity for

---

[4]Such an infinite number of trading opportunities may derive from infinite horizon models (see, for example, Tirole (1985) for deterministic bubbles, Blanchard (1979) and Weil (1987) for stochastic bubbles, Abreu and Brunnermeier (2003), and Doblas-Madrid (2012) for clock games), or from continuous trading models (see Allen and Gorton, 1993).

[5]The theoretical analyses of Allen, Morris, and Postlewaite (1993), and Conlon (2004) show that rational bubbles can occur with a finite number of trading opportunities and without exposing participants to potentially infinite losses. These analyses however involve asymmetric information regarding the asset cash flows. In order to be in line with the literature on experimental bubbles, we design an experiment in which there is no asymmetric information on the asset payoff. Because trading is not continuous, asset prices as well as potential gains and losses have to grow without bounds for a bubble to be sustained at equilibrium (see Tirole, 1982).

[6]The Supplementary Appendix reports two robustness experiments. In the first experiment, the same treatments are used but the game is now repeated five times with stranger matching. In the second experiment, a one-shot game experiment is organized with executive MBA students. Experience with the game or in business does not eliminate speculation in the bubble game.

a subject to enter a bubble increases with the distance between the offered price and the maximum price. We refer to this phenomenon as a snowball effect, and show that it is related to a higher probability not to be last and to a higher number of steps of iterated reasoning.

Different explanations based on various generalizations of the Nash equilibrium can be put forward to explain these results. First, because the bubble game involves introspective and iterated reasoning, one could think of the Cognitive Hierarchy (hereafter CH) model of Camerer, Ho, and Chong (2004). This model considers that players underestimate other agents' sophistication (see, for example, Camerer and Lovallo (1999) for evidence consistent with such underestimation in an experimental entry game). Second, because the game requires longer and longer chains of belief formation when a player is further away from the maximum potential price, one could also expect the Quantal Response Equilibrium (hereafter QRE) of Mac Kelvey and Palfrey (1995) to be relevant. The QRE features players who correctly anticipate others' mistakes. This equilibrium concept may thus be viewed as a way to model strategic uncertainty and to study how it propagates across nodes of the game (see, for example, Mc Kelvey and Palfrey (1998) for an application of the QRE to the centipede game). Finally, it might be difficult to form beliefs about the likelihood of speculation at each possible transaction price. Instead, players might simplify the problem by assuming that this likelihood is the same across various transaction prices. The Analogy-Based Expectation Equilibrium (hereafter ABEE) of Jehiel (2005) could thus be pertinent (see Huck, Jehiel, and Rutter (2011) for evidence consistent with players using analogy reasoning in normal form games).[7] Our theoretical analysis shows that each of these three generalizations can account for the main stylized facts from the experiment, namely, the existence of bubbles even when there is a price cap and the presence of a snowball effect. One of the main contribution of the present paper is then to test which of these explanations is most likely from an empirical point of view. This enables us to learn more about the nature of speculation in the bubble game.

We estimate various specifications of two models of bounded rationality: the Subjective Quantal Response Equilibrium (hereafter SQRE) of Rogers, Palfrey, and Camerer (2009), and the Analogy-Based Expectation Equilibrium (hereafter ABEE) of Jehiel (2005). The SQRE is an equilibrium concept in which agents depart from Nash equilibrium due to less than perfect

[7] In the ABEE logic, agents use simplified representations of their environment in order to form expectations. In particular, agents are assumed to bundle nodes of the game into analogy classes. Agents then form correct beliefs concerning the average behavior within each analogy class.

payoff responsiveness (players might not always choose the best response to their beliefs regarding other players' behavior) or due to erroneous expectations regarding other players' payoff responsiveness. The SQRE nests the QRE that features less than perfect payoff responsiveness but rational expectations, and the CH model that features perfect responsiveness but underestimation of other players' one.[8] Estimating SQRE and its various nested models, we show that speculation in the bubble game is related to quantal responses rather than to cognitive hierarchies. Since each subject plays only once, it is somehow surprising that equilibrium considerations embedded in QRE (and also in ABEE, see below for the results) matter to explain behavior in our experiment. The best-fitting specification in this class is a Heterogeneous Quantal Response Equilibrium (hereafter HQRE) that generalizes the QRE to take into account the fact that payoff responsiveness varies across subjects.

The ABEE offers an interesting complementary point of view on the bubble game.[9] Bianchi and Jehiel (2011) indeed suggest that the ABEE logic can generate bubbles in an environment in which they do not arise if all traders are commonly known to be perfectly rational. Our experiment enables us to test what type of analogy classes best explains speculation in the bubble game. Our results show that an ABEE with two analogy classes, one including the traders who know they are not last and the other including the remaining traders, has a fit that is not significantly different at 5% from the HQRE when estimated on the data from treatments in which not all players know their position in the market sequence.[10] The main message of the paper is thus that both stochastic choices and analogy classes are important drivers of speculation in the bubble game.[11]

[8]Both the SQRE and the ABEE as well as their various specifications are introduced more precisely in Section 4 of the paper.

[9]When estimating ABEE, we follow Huck, Jehiel, and Rutter (2011) and consider that agents play noisy best responses to their beliefs regarding other traders' behavior.

[10]The fit of ABEE with two classes is significantly lower than the one of the HQRE when estimated on the entire data that includes an additional centipede-like treatment in which all players know where they stand in the market sequence. Augmenting the ABEE to include potential heterogeneity in individual payoff responsiveness improves the fit of the ABEE but not significantly.

[11]QRE and ABEE offer distinct explanations for the emergence of bubbles. From an empirical point of view, the evidence of categorical-type of thinking during the dot-com bubble (as documented, for example, by Cooper, Dimitrov, and Rau (2001)) suggests that ABEE could be relevant. On the other hand, QRE could be relevant to explain the Chinese warrant bubble to the extent that option prices violated model-free bounds on fundamental values (see, for example, Xiong and Yu (2011) for evidence of put prices being higher than the corresponding strike prices).

The rest of the paper is organized as follows. The next section compares the bubble game to the previous literature. Section 3 presents the bubble game and the Bayesian Nash predictions. Section 4 derives the behavioral game theory predictions using SQRE, and ABEE logics. The empirical results are in Section 5. Section 6 concludes and provides potential extensions.

2 Literature review

The bubble game in which agents trade sequentially can be viewed as a generalization of the centipede game in which not all players know where they stand in the sequence.[12] [13] The bubble game indeed shares some features with the centipede game. On the one hand, because of limited liability, the sum of traders' potential gains increases as the bubble grows. On the other hand, when there is a price cap, the game can be solved by backward induction.

There are however several important differences between the bubble game and the centipede game. First, our generalization enables the existence of a bubble equilibrium when there is no cap on prices, without relying on an infinite horizon game. Second, since traders play only once, there is no reputation building considerations in the bubble game. Third, in the bubble game, traders are offered a price at which they can buy. This price reveals information which enables them to perform inferences regarding their position. This informational ingredient is not present in the centipede game.

These conceptual differences have important consequences from an experimental point of view. First, one can perform a bubble experiment in an environment in which there actually exists a bubble equilibrium. Second, the absence of reputational issues eliminates one potential explanation for behavior that is not really relevant for bubbles from an empirical perspective. Third, the informational aspect of our game opens the scope for behavioral regularities that are not present in the centipede game. In particular, we show that QRE is better at explaining speculative behavior than

[12]See, for example, Mc Kelvey and Palfrey (1992) for an experimental analysis of the centipede game.

[13]This is related to the absent-minded centipede game proposed by Dulleck and Oechssler (1997): in a classic centipede game, some agents suffer from imperfect recall and may not realize that they have reached the end of the game. The bubble game is different in the sense that even traders with perfect recall may not know where they stand in the sequence, and that price information received by traders enables further inference regarding their position.

CH which is the opposite to what has been previously found on the centipede game.[14] The relative importance of quantal responses compared to cognitive hierarchies for bubbles is a novel empirical finding that opens interesting perspectives for the understanding of speculative behavior.

Our experimental analysis is also related to Lei, Noussair and Plott (2001) and to Brunnermeier and Morgan (2010). [15] Lei, Noussair and Plott (2001) use Smith et al. (1988)'s design and show that, even when they cannot resell and realize capital gains, some participants still buy the asset at a price which exceeds the sum of the expected dividends. This behavior is consistent with risk-loving preferences or violation of dominance. We extend Lei et al. (2001)'s analysis in the sense that, in our design, i) risk preferences cannot explain the decision to buy at the maximum price, and ii) one can observe the behavior of traders who need to perform one, two and even more steps of iterated reasoning to find out that speculating is not an equilibrium.

Brunnermeier and Morgan (2010) study clock games both from a theoretical and an experimental standpoint. These clock games can indeed be viewed as metaphors of "bubble fighting" by speculators, gradually and privately informed of the fact that an asset is overvalued. Speculators do not know if others are already aware of the bubble. They have to decide when to sell the asset knowing that such a move is profitable only if enough speculators have also decided to sell. Their experimental investigation and ours share two common features. First, the potential payoffs are exogenously fixed, that is, there is a predetermined price path. Second, there is a lack of common knowledge over a fundamental variable of the environment. In Brunnermeier and Morgan (2010), the existence of a bubble is not common knowledge. In our setting, the existence of the bubble is common knowledge but traders' position in the market sequence is not. There are several differences between our approach and theirs. A first difference is the time dimension. The theoretical results tested by Brunnermeier and Morgan (2010) depend on the existence of an infinite time horizon. They implement this feature in the laboratory by randomly determining the end of the session. By contrast, we design an economic setting in which there could be bubbles

[14] See, for example, the experimental investigations of the centipede game by Mc Kelvey and Palfrey (1992) who apply the QRE logic, and by Kawagoe and Takizawa (2010) who apply the CH logic and argue that it better fits the data than the QRE logic.

[15] A related analysis of bubbles is offered by Palfrey and Wang (2012) who experimentally study speculation due to traders' differential interpretation of public signals. A recent working paper by Asparouhova, Bossaerts, and Tran (2011) study bubbles in a laboratory experiment in which the asset payoff is determined by the result of a centipede game.

in finite time with finite trading opportunities, even if traders act rationally. A second difference is that our experimental design also enables the study of irrational bubbles. A third difference is that we account for the formation of rational and irrational bubbles by showing that bounded rationality models can explain observed behavior.

3 The bubble game

This section proposes a simple experimental design in which bubbles may or may not be ruled out by backward induction. This design features a sequential market for an asset whose fundamental value is commonly known to be 0. There are three traders in the market.[16] Trading proceeds sequentially. Each trader is assigned a position in the market sequence and can be first, second or third with the same probability $\frac{1}{3}$. Traders are not told their position in the market sequence but can infer some information when observing the price at which they are offered to buy.

Prices are exogenously given and are powers of 10.[17] For simplicity, we do not include the issuer of the asset in the present experimental design. The first trader is offered to buy at a price $P_1 = 10^n$. The power n follows a geometric distribution of parameter $\frac{1}{2}$, that is $\mathbb{P}(n = j) = \frac{1}{2}^{j+1}$, with $j \in \{0, 1, 2, ...\}$. The geometric distribution is useful from an experimental point of view because it is simple to explain and implies that the conditional probability to be last in the market sequence is equal to 0 if the proposed price is 1 or 10, and is equal to $\frac{4}{7}$ otherwise.[18] If a trader decides to buy the asset at price P_t, he proposes the asset to the next trader at a price $P_{t+1} = 10P_t$.

In order to prevent participants from discovering their position in the market sequence by hearing other subjects making choices or by measuring the time elapsed since the beginning of the game, subjects play simultaneously. Once P_1 has been randomly determined, the first, second and third

[16]We could have designed an experiment with only two traders per market. However, this would have required higher payments for bubbles to be rational. Indeed, the conditional probability to be last would be higher. We could also have chosen to include more than three traders per market. We decided not to do so in order to have a sufficiently high number of observations at the different price levels.

[17]We have chosen prices to be powers of 10 in order for the profit in case of a successful speculation to compensate for the loss incurred in case of a failed speculation. If there were more traders in the market sequence, the probability to be last would decrease and price explosiveness could be lower.

[18]The probabilities to be first, second or third conditional on each price, which are computed using Bayes' rule, are given to the participants in the Instructions.

traders are simultaneously offered prices of P_1, P_2, and P_3, respectively.[19] If they decide to buy, they automatically try and resell the asset.

Each trader is endowed with 1 unit of capital. Additional capital may be required in order to buy the asset at price $P_t > 1$. This additional capital (that is, $P_t - 1$) is provided by an outside financier. The experimenter plays the role of the outside financier for all players. Payoffs are divided between the trader and the financier in proportion of the capital initially invested: a fraction $\frac{1}{P_t}$ for the trader, and a fraction $\frac{P_t - 1}{P_t}$ for the financier. Consider a trader who decides to buy the asset at price P_t. When he is unable to resell, his final wealth is 0 which corresponds to the fundamental value of the asset. The outside financier also ends up with 0. When the trader is able to resell the asset, he gets $\frac{1}{P_t}$ percent of the proceed $P_{t+1} = 10P_t$ and thus ends up with a final wealth of 10. The outside financier ends up with $10P_t - 10$.[20]

The separation of payoffs between traders and outside financiers allows implementing limited liability in the experiment: the maximum potential loss of a trader is 1. The potentially infinite gains and losses are incurred by financiers. However, financing all the traders and also playing the role of the issuer, the experimenter faces a maximum total payment, per cohort of 3 subjects, of 20. This maximum payment occurs when all subjects decide to enter the bubble. The experimenter is thus not subject to bankruptcy risk.

The timing of the bubble game is depicted in Figure 1.[21] Speculating is profitable for trader j if the following individual rationality (IRj) condition is satisfied:

$$\begin{aligned}
&[1 - \mathbb{P}(\text{last})] \times \mathbb{P}(\text{next trader buys}) \times U_j(10) \\
&+ \{[1 - \mathbb{P}(\text{last})] \times \mathbb{P}(\text{next trader doesn't buy}) + \mathbb{P}(\text{last})\} \times U_j(0) \geq U_j(1),
\end{aligned}$$

[19] This experimental procedure corresponds to the strategy method. When a trader does not accept to buy the asset, subsequent traders end up with their initial wealth whatever their decision. The advantage of this method is that we can observe traders' speculation decision even if a bubble does not actually develop.

[20] When traders are self-financed, payoffs' absolute values are scaled up by P_t. Introducing traders' limited liability and outside financiers undoes this scaling. This change has some relevance from a practical point of view because most traders do have limited liability and invest other people's money. This change can also have some consequences for behavior in our experiment (as well as in practice): the stakes being smaller than when they are self-financed, traders might have more incentive to enter into bubbles.

[21] This timing does not correspond to the extensive form of the game. Indeed, it leaves aside the issue of which player is first, second, or third. The extensive-form game is provided in Appendix A for the two-trader case. When there is no cap on the first price, it includes an infinite number of nodes.

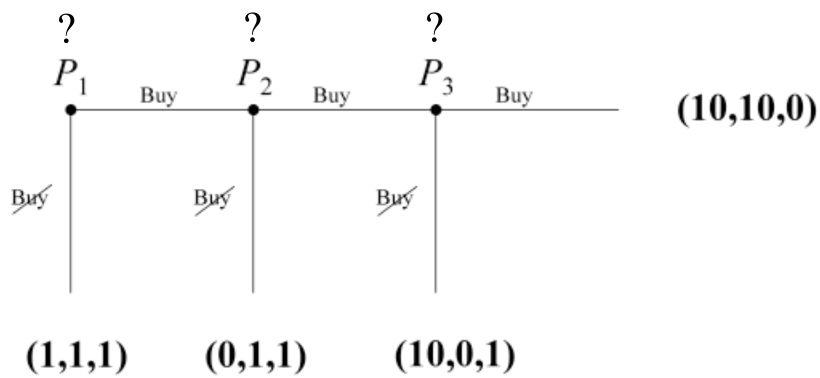


Figure 1: Timing of the bubble game.
The bubble game features a sequential market in which traders are protected by limited liability and financed by outside financiers. Question marks emphasize the fact that traders are equally likely to be first, second or third, and to be offered to buy at prices P_1, P_2, or P_3, respectively. This figure displays traders' payoff only. In case of successful speculation, traders' payoff is 10 because prices are powers of 10 and traders invest one unit of capital. Appendix A provides the extensive-form game for the two-trader case.

where $\mathbb{P}\left(\text{last}\right)$ is the probability that the trader is last in the market sequence conditional on the price he is being proposed, and $U_j\left(.\right)$ is his utility function.

In order to study how traders' rationality influences speculation, we introduce a cap $K = 10^k$ on the first price, with $k \in \{0, 1, 2, ...\}$. With three traders only, the cap K on the first price translates into a cap of $10^{k+2} = 100K$ on the highest potential price in the bubble game. The Bayesian Nash equilibrium is as follows. If K is finite, upon being proposed a price of $100K$, an agent understands that he is last in the market sequence. Consequently, his individual rationality condition cannot be satisfied and he refuses to buy. Anticipating this refusal, agents who are proposed lower prices also refuse to buy, even if they are not sure to be last in the market sequence. At the Bayesian Nash equilibrium, a bubble never forms, being ruled out by backward induction.[22] This establishes a connexion between the bubble game and the previous experiments initiated by Smith et al. (1988) that focus on irrational bubbles.

The bubble game complements this previous literature because we can vary K to study how the number of iterated steps of reasoning needed to reach the Nash equilibrium influences speculation. When the proposed

[22]With a cap on prices, everyone not buying is in fact the unique rationalizable strategy profile.

price is $P = 100K$, an agent knows that he is last and no iterated step of reasoning is needed. When the proposed price is $P = 10K$, a subject knows that he is not first in the market sequence (he can be second or third). At equilibrium, he has to anticipate that the next trader in the market sequence (if any) would not accept to buy the asset. One step of iterated reasoning is thus needed to derive the equilibrium strategy. More generally, when the proposed price is $1 \leq P \leq 100K$, the required number of iterated steps of reasoning is $log_{10}\left(\frac{100K}{P}\right)$. In order to study whether this required number of iterated steps of reasoning affects bubble formation, we have chosen to experimentally study treatments in which K equals 1, 100, and 10,000.

Another interesting aspect of our design is that we can let K go to infinity. A bubble can arise at equilibrium if the (IRj) condition is satisfied for all traders on the market. It is straightforward to show that, if traders anticipate that other traders speculate, there indeed exists increasing and concave utility functions $U_j(.)$ for which this (IRj) condition holds. The bubble game thus offers an economic environment in which rational bubbles can form despite the number of trades being finite and the existence of the bubble being common knowledge. Hence our paper contributes to the literature on rational bubbles by showing that neither infinite trading horizon (see Blanchard (1979) and Tirole (1982)) nor infinite trading speed (see Allen and Gorton (1993) and Abreu and Brunnermeier (2003)) are necessary for common knowledge rational bubbles to exist.[23] The possibility of equilibrium bubbles in our setting arises because no trader is ever sure to be last in the market sequence.[24]

In order to show that the bubble equilibrium is meaningful, we now check whether financiers are willing to fuel the bubble. It is clear that, if the same financier provides capital to all traders (as it is the case in the experiment), his total expected profit would be negative. However, we show below that, if each trader has a different outside financier, these financiers may have an interest in providing capital to traders. Assuming that a financier has a utility function denoted by $U_f(.)$ and an initial wealth denoted by W, his individual rationality condition (IRf) is written as:

[23] In the previous models, two dimensions of infinity were required for bubbles to emerge at equilibrium: infinite trading opportunities and potentially infinite prices. We show that only this last dimension of infinity is necessary for common knowledge rational bubbles to arise at equilibrium.

[24] It is straightforward to show that a no-bubble equilibrium always exists. The Supplementary Appendix proves the existence of a bubble equilibrium for the risk neutral and constant relative risk aversion cases. It also offers a more extensive theoretical analysis of the bubble game including a welfare analysis.

$$
\begin{aligned}
&[1-\mathbb{P}(\text{last})]\,\mathbb{P}(\text{next trader buys})\,U_f(W+10P_t-10)\\
&\qquad+\{[1-\mathbb{P}(\text{last})]\times\mathbb{P}(\text{next trader doesn't buy})+\mathbb{P}(\text{last})\}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\times U_f(W-P_t+1)\geq U_f(W),
\end{aligned}
$$

for all P_t. It is again straightforward to show that, if financiers expect that all traders speculate, there exist functions $U_f(.)$ for which the (IRf) condition holds.

The experimental protocol is as follows. Our baseline experiment includes a total of 234 subjects. Subjects are junior and senior undergraduates in Business Administration at the University of Toulouse. Each subject participates in only one session and receives a 5-euro show-up fee. Each session includes only one replication of the trading game. Subjects' risk aversion is measured thanks to a procedure inspired from Holt and Laury (2002). We adjust their questionnaire to match the set of potential decisions subjects actually face in our experiment. This enables us to measure risk aversion in a context that is in line with the experimental set up as far as payoffs and probabilities are concerned.[25] The minimum, median, maximum, and average gains in the experiment are respectively 0, 1, 10, and 3.35 euros (not including the show-up fee). The instructions for the case where $K=10,000$ are in Appendix B.

Our experimental protocol is summarized in Table I.

Session	# Subjects	cap on initial price, K	Bayesian Nash Equilibrium
1, 5, and 9	60	1	no-bubble
2, 6, and 10	54	100	no-bubble
3, 7, and 11	63	10,000	no-bubble
4, 8, and 12	57	$+\infty$	no-bubble or bubble

Table 1: Experimental protocol.

[25]The questionnaire is composed by a table with 14 decisions. For each decision i, subjects may choose between the riskless option A, which is to receive 1 euro for sure, or the risky option B, which is to receive 10 euros with probability $\frac{i}{14}$, or 0 euro with probability $\frac{14-i}{14}$. This questionnaire features what Harrison, List and Towe (2007) refer to as a higher frame: a risk-neutral agent switches to the risky option B in the upper part of the table. It gives us a precise estimation of the willingness to accept the bets at stake in the bubble game.

4 Behavioral game theory predictions

This section analyzes the game with various behavioral game theory models. The next section structurally estimates these models using data from the bubble game. Two types of models appear relevant in our context: the Subjective Quantal Response Equilibrium (hereafter SQRE) of Rogers, Palfrey, and Camerer (2009), and the Analogy-Based Expectation Equilibrium (hereafter ABEE) of Jehiel (2005). On the one hand, the SQRE is of interest here since i) it is based on the concept of noisy best-response that proved useful to explain failures of backward induction in previous experiments (see Camerer (2003)), and ii) it allows for heterogeneity in agents' rationality that could be an important driver of bubble formation. On the other hand, the ABEE is relevant here because i) belief formation about the likelihood of speculation at each price being complicated, players might simplify the problem as considered in this equilibrium concept, and ii) candidates for analogy classes arise naturally in the bubble game.

4.1 Subjective Quantal Response Equilibrium

According to the SQRE logic, an agent's payoff responsiveness, denoted by $\lambda_{i,s}$, depends on his type i and on his level of sophistication, denoted by s. SQRE then involves a stochastic choice model whereby the agent's propensity to choose an action has a logistic form that depends on the expected payoff of this action given his information, and on his payoff responsiveness. The expected payoff from buying the asset conditional on being proposed a price P is denoted by $u_{i,s}(B|P)$. The expected payoff from not buying the asset is denoted by $u_{\varnothing}$. We adopt a specification with logistic stochastic choice functions so that the probability that agent i buys the asset after being proposed a price P is: $\mathbb{P}_{i,s}(B|P) = \frac{e^{\lambda_{i,s}u_{i,s}(B|P)}}{e^{\lambda_{i,s}u_{i,s}(B|P)}+e^{\lambda_{i,s}u_{\varnothing}}}$.

In the SQRE logic, agent i's payoff responsiveness is given by: $\lambda_{i,s} = \lambda_i + \gamma s$, where it is commonly known that λ_i is uniformly distributed over the interval $[\Lambda - \frac{\epsilon}{2}, \Lambda + \frac{\epsilon}{2}]$, and where γ represents the sensitivity of an agent's payoff responsiveness to his level of sophistication. The level of sophistication s follows a Poisson distribution with density function f. Let τ denote the average level of sophistication.

Finally, in the SQRE logic, agents may not have the same understanding of the overall population of players. In particular, it is assumed that an agent with sophistication s cannot imagine that other agents can have a sophistication greater than $s - \theta$. The agent's truncated beliefs about the fraction of h-level players is thus $g_s(h) = \frac{f(h)}{\sum_{i=0}^{\max(s-\theta,0)} f(i)}$. The expected payoff from buying the asset conditional on being proposed a price P can thus be written as: $u_{i,s}(B|P) = 10\,[1 - \mathbb{P}(\text{last}|P)] \sum_{h=0}^{\max(s-\theta,0)} g_s(h)\mathbb{P}_{i',h}(B|10P)$.[26]

Overall, SQRE has five parameters: Λ, the basic payoff responsiveness; ϵ, the uncertainty surrounding the basic payoff responsiveness; τ, the average level of sophistication; γ, the sensitivity of payoff responsiveness to sophistication; and θ, the imagination parameter. As noted by Rogers, Palfrey, and Camerer (2009), SQRE is useful to incorporate the notion of skill. Skill is captured by the payoff responsiveness parameter $\lambda_{i,s}$. The higher this parameter is, the more responsive the player is to payoffs and the better he perceives other players' skills. This player might misperceive the distribution of skill in the population but, conditional on a skill level, he holds correct beliefs on the choice probabilities.

One advantage of using the SQRE is that it nests various interesting behavioral game theory models. In particular, when $\Lambda = 0$, $\epsilon = 0$, $\gamma = +\infty$ and $\theta = 1$, the SQRE boils down to the Cognitive Hierarchy model (hereafter CH) developed by Camerer, Ho and Chong (2004) with only one free parameter τ. The CH model states that agents best-respond to mutually inconsistent beliefs: they believe that all other agents' sophistication is lower than theirs.[27] Moreover, agents with a level of sophistication $s = 0$ choose each available action with an equal probability. Alternatively, when $\epsilon = 0$, $\theta = +\infty$, and when $\gamma = 0$ or $\tau = 0$, the SQRE corresponds to the Quantal Response Equilibrium (hereafter QRE) of Mc Kelvey and Palfrey (1995) with only one free parameter Λ. The QRE takes into account the fact that players make mistakes but it retains beliefs' consistency. At equilibrium, players are responsive to payoffs to the extent that more profitable actions

[26] Given our experimental design, subjects play a simultaneous-move game in which decisions matter only if previous traders in the market sequence, if any, decide to buy the asset. We calculate expected payoffs conditional on this event in order to simplify equilibrium computations. Our specification of the QRE thus corresponds to Mc Kelvey and Palfrey (1998)'s Agent QRE in which agents play only once. An alternative specification is to compute expected payoffs ex-ante, that is, taking into account the probability that previous players, if any, decide to buy. We checked that, when estimated on the entire data, the quantal response equilibrium has a better fit with our specification than with the ex-ante specification (p-value is 0.00 with a Vuong test).

[27] The level-k model of Stahl and Wilson (1995) and Nagel (1995) also exhibits such downward looking beliefs. The difference between these models and the CH model is that they assume that agents believe that all other agents have a level of sophistication that is just below theirs. See also Costa-Gomes, Crawford, and Broseta, (2001), Costa-Gomes and Crawford (2006), and Crawford and Iriberri (2007) for experimental studies of the level-k model.

are chosen more often. Using the SQRE enables us to study whether noisy best-responses with beliefs consistency or best-responses with beliefs inconsistencies best explain speculation in the bubble game.

Another advantage of the SQRE is that the QRE and CH models can be extended to take into account heterogeneity across agents. The CH model can be extended to a (discretized) Truncated Quantal Response Equilibrium (hereafter TQRE) by setting $\gamma < +\infty$. As a result, we have a CH model in which agents, instead of best-responding, have a payoff responsiveness that increases with their level of sophistication. Likewise, the QRE can be extended to a Heterogeneous Quantal Response Equilibrium (hereafter HQRE) by setting $\epsilon > 0$. We then have a QRE in which agents do not know for sure what the exact level of payoff responsiveness of another agent is.[28] Both of these extensions can prove useful to understand whether heterogeneity across agents plays a role in bubble formation.

A last advantage of the SQRE is that is can be used to estimate whether overconfidence matters for speculative bubbles. The limited imagination parameter θ indeed represents the extent to which agents underestimate the average level of sophistication in the population of players, a phenomenon we refer to overconfidence because it is similar in spirit to the better-than average effect in the psychology literature. Starting from the CH model and freeing the parameter θ, we can estimate the extent to which agents suffer from limited imagination. We call this model the overconfidence CH (hereafter OCH). θ positive corresponds to a high level of overconfidence because agents believe that all other players are less sophisticated than they are. When θ is negative, agents conceive that other players may be more sophisticated than them but they are still underestimating the average level of sophistication due to their truncated beliefs. In the limit, when θ converges to $-\infty$, there is no overconfidence.

We now apply the SQRE to the bubble game. For brevity, we focus here on the QRE and CH models for treatments with a finite price cap K.[29]

We first study the QRE. After being proposed a price $P = 100K$, a trader perfectly infers that he is last in the market sequence and buys with

[28] Uncertainty about other traders' payoff responsiveness can be interpreted as uncertainty about their level of risk aversion.

[29] The computations for the general specification of the SQRE as well as for the case in which $K = +\infty$ are available in the Supplementary Appendix. Extending the CH and QRE models to incorporate traders' heterogeneity as modeled in the TQRE and HQRE models does not change the underlying logic. For the case in which $K = +\infty$, we derive predictions by relying on equilibrium conjectures that are realized at equilibrium as it is the case for the derivation of the Nash equilibrium.

probability $\mathbb{P}\left(B|P=100K\right)=\frac{1}{1+e^{\Lambda}}$. After observing a price $P=10K$, a trader infers that he has a specific probability, denoted by $q(K,P=10K)$, not to be last. He correctly anticipates that the probability to buy of the last trader is not equal to zero. His expected payoffs from buying is $u(B|P=10K)=q(K,P=10K)\times\frac{1}{1+e^{\Lambda}}\times 10$. His probability to buy is therefore $\mathbb{P}\left(B|P=10K\right)=\frac{1}{1+e^{\Lambda\times\left(1-\frac{10q(K,P=10K)}{1+e^{\Lambda}}\right)}}$. Applying this logic backward, we find the predicted probability that a trader buys at all potential prices. This analysis shows that the QRE can predict the existence of bubbles even when there is a price cap.

Consider now the CH model. When proposed a price of $100K$, a trader knows he is last in the sequence. Consequently, only level-0 traders buy, with probability $\frac{1}{2}$. Given that there is a fraction $f\left(0\right)=\frac{\tau^0e^{-\tau}}{0!}$ of such traders in the population, the probability to observe a trader buying at this price is: $\mathbb{P}\left(B|P=100K\right)=\frac{1}{2}e^{-\tau}$.

When a trader is being proposed a price $P=10K$, he infers that he is penultimate in the sequence. If he is a level-0 trader, he buys with probability $\mathbb{P}_{s=0}(B|P=10K)=\frac{1}{2}$. If he is a level-$s$ player with $s\geq 1$, he thinks that the next trader observing the price $100K$ is a level-h with probability $g_s(h)=\frac{f(h)}{\sum_{i=0}^{s-1}f(i)}$. Consequently, his expected profit if he buys is: $u_{s\geq 1}(B|P=10K)=q(K,P=10K)\times\frac{f(0)}{\sum_{i=0}^{s-1}f(i)}\times\frac{1}{2}\times 10$. The trader is strictly better off buying if and only if $\sum_{i=0}^{s-1}\frac{\tau^i}{i!}<5q(K,P=10K)$. Given that $\sum_{i=0}^{s-1}\frac{\tau^i}{i!}$ is strictly increasing in s, there exists a (potentially infinite) threshold $s^*\geq 1$ such that only traders with a level below or equal to s^* buy. This is because higher level traders have a more accurate perception of the distribution of lower-level types. Finally, given the actual distribution of traders' types, the probability to observe a trader buying at a price of $P=10K$ is: $\mathbb{P}\left(B|P=10K\right)=\frac{1}{2}e^{-\tau}+\sum_{s=1}^{s=s^*}\left(\frac{\tau^s}{s!}e^{-\tau}\right)$. As before, the rest of the model is solved backward. The CH model can thus predict the existence of bubbles despite the presence of a price cap.

4.2 Analogy-Based Expectation Equilibrium

According to the ABEE logic, agents use simplified representations of their environment in order to form expectations. In particular, agents are assumed to bundle nodes of the game into analogy classes. [30] Agents then form

[30] Consistent with Jehiel (2005) when he studies endogenous analogy grouping (in page 100), we consider that an agent's bundling includes his own nodes. This specification is chosen for simplicity: it enables us to work with analogy classes that are constant across

correct beliefs concerning the average behavior within each analogy class. Following Huck, Jehiel, and Rutter (2011), we consider that agents apply noisy best-responses to their beliefs.[31]

In the bubble game, two types of analogy classes arise naturally. On the one hand, traders may use only one analogy class, assuming that other traders' behavior is the same across all potential prices. On the other hand, traders may use two analogy classes: one class (Class I) that includes prices at which traders are sure not to be last in the market sequence, the other (Class II) that includes the remaining prices (at which traders think they may be last or know they are last).

We now apply the ABEE to the bubble game. For brevity, we restrict our attention to the case in which the price cap is $K = 1$ (the other cases are addressed in the Supplementary Appendix). Let p_1, p_2, and p_3 denote the actual probability that a trader buys after observing prices equal to 1, 10, and 100, respectively. Let $\mathbb{P}(B|P = 1)$, $\mathbb{P}(B|P = 10)$, and $\mathbb{P}(B|P = 100)$ be the corresponding probabilities as (mis)perceived by traders using analogy classes.

We start by analyzing the one-class ABEE. A trader after observing a price $P = 100$ knows he is last. Consequently, his probability to buy is $p_3 = \frac{1}{1+e^{\Lambda}}$. A trader after observing a price $P = 10$ has the following expected payoff from buying: $u(B|P = 10) = 10 \times \mathbb{P}(B|P = 100)$. Because of the use of one analogy class, we have $\mathbb{P}(B|P = 100) = \frac{p_1+p_2+p_3}{3}$.[32] The probability to buy after $P = 10$ is therefore: $p_2 = \frac{1}{1+e^{\Lambda\left(1-\frac{10}{3}(p_1+p_2+p_3)\right)}}$. A trader after observing a price $P = 1$ has an expected payoff of $u(B|P = 1) = 10 \times \mathbb{P}(B|P = 10) = 10\frac{p_1+p_2+p_3}{3}$. The probability to buy is therefore: $p_1 = p_2$. This analysis leaves us with a system of equations that can be solved numerically to find p_1, p_2, and p_3.

We now turn to the two-class ABEE. As before, a trader after observing a price $P = 100$ buys with a probability $p_3 = \frac{1}{1+e^{\Lambda}}$. A trader after observing a price $P = 10$ has the following expected payoff from buying: $u(B|P = 10) = 10 \times \mathbb{P}(B|P = 100)$. Because Class II is a singleton, we have $\mathbb{P}(B|P =$

players.

[31]The ABEE concept with stochastic choice remains different from the QRE given that, in the ABEE logic, agents have inaccurate expectations regarding other players' behavior. One can thus test whether this inaccuracy helps explaining the data.

[32]The probability $\frac{1}{3}$ corresponds to the ex-ante probability of observing prices of 1, 10 or 100. Given our experimental design in which all agents have to pick an action irrespective of whether previous players have decided to buy or not, using this ex-ante probability is in line with the spirit of ABEE. If we had chosen an experimental design corresponding to the extensive representation of the game, we would have $\mathbb{P}(B|P = 100) = \frac{p_1+p_1p_2+p_1p_2p_3}{1+p_1+p_1p_2}$.

$100) = p_3$. Thus, we have $p_2 = \frac{1}{1+e^{\Lambda(1-10p_3)}}$. A trader after observing a price $P = 1$ uses Class I to form his expectations and thus has an expected payoff of $u(B|P = 1) = 10 \times \mathbb{P}(B|P = 10) = 10\frac{p_1+p_2}{2}$. The probability to buy is therefore: $p_1 = \frac{1}{1+e^{\Lambda(1-5(p_1+p_2))}}$. The system of equations can again be solved numerically.

Overall, one can show that the ABEE can predict the presence of a bubble even when there is a price cap. The intuition is as follows. At the ABEE, traders expect that the probability to buy is constant across different prices. This leads them to overestimate the likelihood that traders buy towards the end of the market sequence. A bubble can also arise at equilibrium in the ABEE logic even if agents best-respond to their beliefs, that is, even if $\Lambda = +\infty$.[33]

4.3 Implications of the behavioral game theory models for speculative bubbles

We use the derivations provided in the previous subsections in order to highlight the implications for speculative bubbles of the various behavioral game theory models we consider. For simplicity, we focus on the case in which $K = 1$ and on the decision to buy after observing $P = 10$ and $P = 100$, the last two consecutive prices.

The QRE predicts that the probability to buy is $\mathbb{P}(B|P = 10) = \left[1 + e^{\Lambda\left(1-\frac{10}{1+e^{\Lambda}}\right)}\right]^{-1}$ after observing $P = 10$, and $\mathbb{P}(B|P = 100) = \left[1 + e^{\Lambda}\right]^{-1}$ after observing $P = 100$. The propensity to buy when $P = 100$ is pretty low because traders can only lose by doing so. It is higher when $P = 10$ because there is a chance that the next trader will buy. This higher chance of reselling induces a higher propensity to speculate.[34] The snowball effect in the QRE model is thus due to the fact that the initial noise due to stochastic choice is amplified when stepping back away from the maximum price. The QRE predicts a snowball effect when $K = +\infty$.

[33] For example, one can check that the following strategy profile is an ABEE with one analogy class: traders buy after observing prices of 1 and 10 and do not buy after observing a price of 100. With two analogy classes, the following profile is an ABEE: traders buy after observing a price of 1 and do not buy after observing prices of 10 and 100.

[34] It is not always true that, in the QRE, $\mathbb{P}\left(B|P = 10^i\right) > \mathbb{P}\left(B|P = 10^{i+1}\right)$. Consider, for example, the general case with a cap K in which the probability that trader i is not to be last after observing $P = 10^i$ is denoted by $q(K, P = 10^i)$. We have $\mathbb{P}\left(B|P = 10^i\right) = \left[1 + e^{\Lambda\left(1-10q(K,P=10^i)\mathbb{P}\left(B|P=10^{i+1}\right)\right)}\right]^{-1}$. If $q(K, P = 10^i)$ is low enough compared to $q(K, P = 10^{i+1})$, it is possible that $\mathbb{P}\left(B|P = 10^i\right) < \mathbb{P}\left(B|P = 10^{i+1}\right)$.

The CH model predicts that $\mathbb{P}(B|P=10)=\frac{1}{2}e^{-\tau}+\sum_{s=1}^{s=s^*}\left(\frac{\tau^s}{s!}e^{-\tau}\right)$ and that $\mathbb{P}(B|P=100)=\frac{1}{2}e^{-\tau}$. s^* is the highest level of sophistication s such that the inequality $\sum_{i=0}^{s-1}\frac{\tau^i}{i!}<5$ is satisfied. This inequality indicates that an agent with sophistication s is better off speculating. The propensity to speculate when $P=100$ is pretty low because only step-0 players buy with probability $\frac{1}{2}$. It is higher when $P=10$ because some agents with higher levels of sophistication also decide to speculate. The snowball effect in the CH model is thus due to an increase in the willingness to speculate of more sophisticated agents. Contrary to the QRE, for $K=+\infty$, the CH model does not predict a snowball effect. Indeed, all traders have the same propensity to buy for all prices: traders with $s>0$ always speculate while traders with $s=0$ speculate with probability $\frac{1}{2}$. Our experiment thus constitutes a stringent test for the CH model.

To analyze the implications of ABEE for the bubble game, we focus on the one-class ABEE. This model predicts that a trader speculates with probability $p_2=\left[1+e^{\Lambda\left(1-\frac{10}{3}\left[\left(1+e^{\Lambda}\right)^{-1}+2p_2\right]\right)}\right]^{-1}$ after observing $P=10$, and $p_3=\frac{1}{1+e^{\Lambda}}$ after observing $P=100$ (p_1, the probability to buy after observing $P=1$, is equal to p_2). For example, for $\Lambda=1$, numerical computations yield $p_1=p_2=0.999$ and $p_3=0.269$. Because of analogy-based expectations, the trader who observes $P=10$ believes that the probability that the next trader buys is $\frac{p_1+p_2+p_3}{3}$ instead of p_3. In the case we consider, such erroneous beliefs induces him to overestimate the probability that the next trader buys which increases his propensity to speculate.[35] This reinforces the snowball effect above what would be predicted by the QRE, even in the case in which $K=+\infty$.

5 Empirical Results

5.1 The determinants of speculation in the bubble game

We study individual decisions to buy the overvalued asset. Figure 2 plots, for each treatment, the proportion of buy decisions for each price level. The number of times a given price has been proposed is indicated at the bottom of the bar. Below the horizontal axis, we explicitly indicate, for each price,

[35] It is not always the case that, in ABEE, traders overestimate the likelihood that the next trader speculates. Consider, for example, the two-class ABEE with $K=100$. The trader observing $P=10$ anticipates that the next trader is in Class II and that his probability to buy is $\frac{p_3+p_4+p_5}{3}$. If $p_3>p_4$ and $p_3>p_5$, then the actual probability that the next trader buys p_3 is higher than the expected probability $\frac{p_3+p_4+p_5}{3}$.

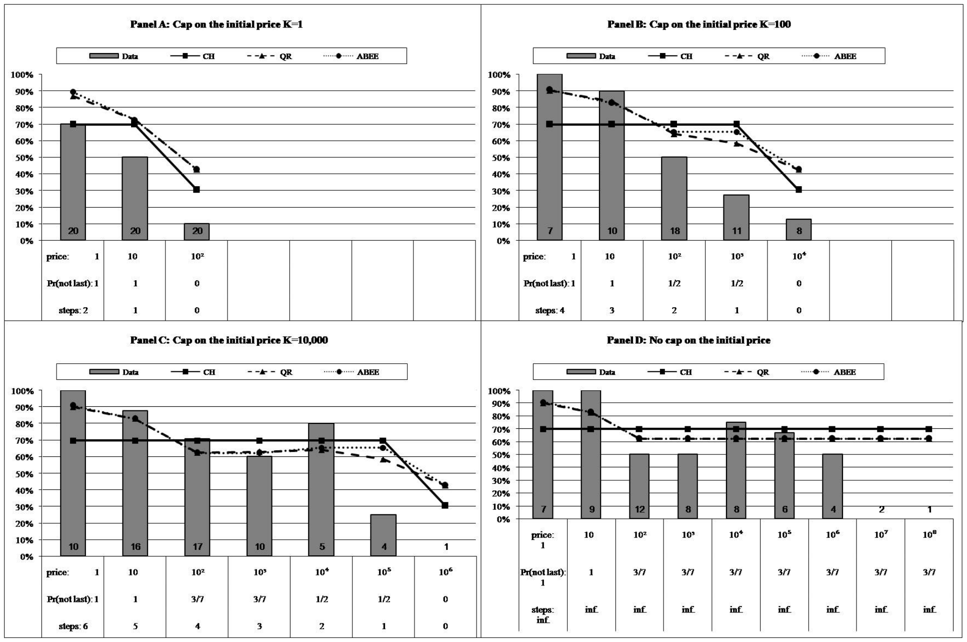


Figure 2: Data and predictions from behavioral game theory models.
Experimental data on the probability of a Buy decision, depending on the initial price, the probability not to be last and the number of steps of iterated reasoning. Predictions from behavioral game theory models use the parameters of interest estimated on the entire data set.

the number of steps of reasoning required to reach equilibrium as well as the conditional probability not to be last.

Let's first focus on the treatments with a cap on the first price. The rather high probabilities to buy in these treatments (Figure 2, Panels A, B and C) are inconsistent with Nash equilibrium. Keeping the probability not to be last constant, it seems that traders are more likely to buy when more steps of reasoning are required (for example, compare the proportion of buy decisions when traders are sure not to be last in Figure 2, Panels A, B, and C). This is in line with previous experimental results on the centipede game. [36]

[36] When the price proposed to subjects is 1, the propensity to buy is very high (100% of subjects choose to buy when the price cap is greater than 1). This propensity to deviate from the Nash equilibrium strategy is in line with the results reported by Mc Kelvey and Palfrey (1992) on the six-move centipede game regarding the very low proportion of games that end up in the first node. A high propensity to buy when being proposed a price of 1 is also observed in the robustness experiments discussed in the Supplemental Appendix.

Also, keeping the number of steps of reasoning constant, it seems that traders are more likely to buy when their probability not to be last increases (for example, compare the proportion of buy decisions for 3 and 4 steps of reasoning in Figure 2, Panel B and C). This is a new empirical result that could not meaningfully be obtained in the centipede game because the probability not to be last is equal to 1 for each node except the last one at which it is 0. This result indicates that there is some elements of rationality in subjects' decisions.

These results uncover a snowball effect: when there is a price cap, the propensity to enter bubbles seems to increase with the required number of reasoning steps and with the probability not to be last.

We now turn to the treatment in which there is no cap on the first price (Figure 2, Panel D), that is, $K = +\infty$. First, subjects who are sure not to be last always buy the asset which indicates a higher propensity to speculate than when there is a price cap (for example, compare the proportion of buy decisions when traders are sure not to be last in Figure 2, Panels D, and C). A Wilcoxon rank sum test indicates that the proportion of buy decision when subjects are offered a price of 1 or 10 is significantly higher when $K = +\infty$ (100%) than when there is one (77%) (the p-value is 0.034). [37] This is in line with the fact that bubbles are rational when $K = +\infty$.

Second, focusing exclusively on data from $K = +\infty$, the probability to buy appears to be statistically higher when subjects are sure not to be last (100%) than when they can be last (54%) (the p-value is 0.001 according to a Wilcoxon rank sum test). There is thus a snowball effect in the treatment with $K = +\infty$. This is in contrast with the prediction of the CH model.

Third, when there is no cap and when prices are 100 or above, if participants coordinate on the same equilibrium, their decisions should be the same for all price levels. In line with this hypothesis, using a Wilcoxon rank sum test, we cannot reject the fact that the probability to buy is the same after observing prices of 100, 1,000, and 10,000 (57%), and after observing higher prices (46%) with a p-value of 0.52 (this test keeps the required number of steps constant and equal to infinity).

These three results on the treatment with no cap cannot be obtained in an experiment with the centipede game, and underline the interest of our design for the study of speculation.

In order to deepen our statistical analysis, we run a logit regression of

[37] This result however does not hold if we compare the cases $K = +\infty$ (100%) and $K = 10,000$ (92%) (the p-value is 0.261). Besides, there is no difference in the probability to buy when a subject has a probability not to be last equal to $\frac{3}{7}$ or $\frac{1}{2}$ between the cases $K = +\infty$ (54%) and $K = 10,000$ (64%) (the p-value is 0.367).

the propensity to buy the overvalued asset.[38] As can be seen in Table II, the first four explanatory variables are constructed by interacting variables that measure the number of steps of reasoning required to reach equilibrium and the probability to be last. The last two variables are the individual degree of risk aversion and the offered price.[39] The constant reflects the propensity to speculate of subjects who are proposed to buy at the maximum price ($100K$), if any.[40] This is useful because, since these subjects are expected not to buy, their probability to buy can be viewed as the incompressible level of noise in our data.

The results are in Table II. We first focus on the subjects who know they are last in the market sequence. We can reject the hypothesis that these subjects never enter the bubble. Out of the 29 subjects who knew they were last, three bought the asset.[41] This number is low but it is not zero. This result is in line with the findings of Lei, Noussair and Plott (2001) that subjects were buying an overvalued asset even when prohibited to resell. In our framework, these agents correspond to "step 0" subjects. Lei et al. (2001) report in page 853 that 6 out of 36 subjects made at least one dominated transaction.[42] This proportion (16.7%) is slightly higher than our proportion of dominated choices (10.3%). We now complement their results by studying the behavior of subjects who are further away from the maximum price and who have more chances not to be last.

The regression analysis provides other interesting results. First, subjects who have a chance not to be last buy significantly more than the ones who know they are last. Indeed, the coefficients of Variables 1 through 4 are significantly positive at the one percent level. Second, keeping constant the probability to be last, subjects are more likely to buy when there are

---

[38] The results are the same if we run a probit regression.

[39] The coefficient of risk aversion is computed assuming a constant relative risk aversion utility function as in Holt and Laury (2002).

[40] The regression also includes control variables that reflect the level of the cap in order to test for treatment effects. These variables are omitted for the sake of brevity. None of their coefficient is statistically significant and the results do not change if we omit these variables from the analysis.

[41] None of the behavioral game theory models we consider can explain this phenomenon in a way more satisfying than stating that people make random errors. However, as explained above, the various models have different implications regarding the impact of these errors on the market.

[42] We focus here on the experiment of Lei, Noussair and Plott (2001) during which subjects could participate in several markets, the so-called TwoMarket/NoSpec treatment. This is because, in this treatment, subjects could participate actively in the experiment without being forced to participate in the bubble. This provides a lower bound for the number of subjects who make mistakes in the market.

	Coefficient	p-value
Constant	-1.84	0.005
1: $I_{Step=1\ or\ 2}$ x $I_{0<P(last)<1}$	1.80	0.013
2: $I_{Step=1\ or\ 2}$ x $I_{P(last)=0}$	2.60	0.000
3: $I_{step>=3}$ x $I_{0<P(last)<1}$	2.82	0.000
4: $I_{step>=3}$ x $I_{P(last)=0}$	5.65	0.000
Degree of risk aversion	- 0.62	0.249
Price	-0.00	0.451
Log likelihood	-116.22	
Number of observations	234	

Table 2: Logit Regression of the Purchase Decision.
I_x is an indicator variable that takes the value 1 if condition x is true. The variable Step represents the number of steps of reasoning needed to rule out bubbles. The variable P(last) is the probability to be last in the market sequence.

3 or more steps of reasoning than when there are 1 or 2 steps. Wald tests indicate that the differences between the coefficients of Variables 1 and 3 and of variables 2 and 4 are statistically significant with a p-value of 0.07 and 0.01, respectively. Third, keeping constant the required number of reasoning steps, subjects are more likely to buy when they are less likely to be last. Wald tests indicate that the difference between the coefficients of Variables 1 and 2 is marginally significant with a p-value of 0.11, and that the difference between the coefficients of Variables 3 and 4 is statistically significant with a p-value of 0.01.

These results statistically establish the presence of the snowball effect: the propensity to enter bubbles increases with the required number of reasoning steps and with the probability not to be last. This snowball effect is not present at the Bayesian Nash equilibrium (whether there is a cap or not) but it can be displayed by the behavioral game theory models discussed in the previous section. The next subsection proposes a structural estimation of these models in order to better understand the nature of speculation in the bubble game.

5.2 Estimating behavioral game theory models of speculation

Our results so far suggest that some players have bounded rationality and that the formation of bubbles is related to a snowball effect. To account for these phenomena, we estimate models that explicitly incorporate bounded rationality: the Subjective Quantal Response Equilibrium of Rogers, Palfrey, and Camerer (2009), and the Analogy-Based Expectation Equilibrium of Jehiel (2005).[43] For each model, we estimate the parameters of interest using maximum likelihood methods for the entire data set as well as for each treatment separately.[44] Confidence intervals are computed using a bootstrapping procedure: using the empirical distribution of the observed data, we resample 10,000 data sets on which the parameters of interest are re-estimated. We then choose the 2.5 and 97.5 percentile points values to construct 95% confidence intervals. When comparing the fits of two models, we use a likelihood ratio test when the models are nested, and Vuong (1989)'s test when they are not. [45]

Table III reports our estimation results, and Figure 3 displays the statistical tests for various models' comparisons. The first two lines of Table III describe our data, namely, the number of observations, and the observed average probability to buy. The next two lines show the predictions and log-likelihoods of the Nash equilibrium under risk neutrality.[46] The mean choices are generally far away from the Nash equilibrium; the observed probability to buy is too low when there exists a bubble-equilibrium, and too high when it does not exist.

Table III then provides the predictions and log-likelihoods of the various

[43]We initially planned to estimate the quantal response equilibrium and the cognitive hierarchy model. Following suggestions from the referees, we decided to estimate the Subjective Quantal Response Equilibrium which nests these two models, and the Analogy-Based Expectation Equilibrium.

[44]For simplicity, when estimating the Truncated Quantal Response Equilibrium and the general Subjective Quantal Response Equilibrium models, we set $\theta = 1$ as in the CH model.

[45]Under the null hypothesis, the probability distribution of the log-likelihood ratio statistic used to test nested models is approximated by a Chi-squared distribution with degrees of freedom equal to the difference between the numbers of parameters in the two models, while the probability distribution of the Vuong's statistic used to test non-nested models is a standard normal distribution.

[46]We consider that traders coordinate on the bubble equilibrium when there is no cap on the initial price. The no-bubble Nash equilibrium has a lower log-likelihood. In order to compute the likelihoods, we assume that players choose non-equilibrium strategies with a probability of 0.0001.

		Session				
		All	No cap	Cap K=10,000	Cap K=100	Cap K=1
Data						
	Sample size	234	57	63	54	60
	Av. probability buy	60%	67%	75%	54%	43%
Nash Equilibrium						
	Av. probability buy	24%	100%	0%	0%	0%
	Log L	-1 114.46	-175.00	-432.89	-267.10	-239.47
CH						
	Tau	0.5	0.4	1.1	3.9	0.5
	Av. probability buy	65%	66%	82%	48%	57%
	Log L	-143.61	-36.28	-34.36	-30.02	-36.65
	95% CI	[0.3 - 0.7]	[0.1 - 0.8]	[0.4 - 1.9]	[1.2 - 4.0]	[0.2 - 4.1]
TQRE						
	Tau	3.7	4.4	5.3	3.8	0.5
	Gamma	0.1	0.1	0.1	4.8	34.3
	Av. probability buy	68%	72%	78%	53%	57%
	Log L	-137.45	-31.86	-30.64	-29.77	-36.65
	95% CI Tau	[2.7 - 4.7]	[3.2 - 6.0]	[1.5 - 7.5]	[2.2 - 12.1]	[0.2 - 4.3]
	95% CI Gamma	[0.1 - 0.2]	[0.1 - 0.2]	[0.1 - 50.0]	[0.1 - 33.1]	[4.1 - 37.2]
QRE						
	Lambda	0.3	0.4	0.4	2.7	0.1
	Av. probability buy	69%	75%	78%	45%	56%
	Log L	-135.12	-31.25	-30.17	-29.84	-39.79
	95% CI	[0.2 - 0.4]	[0.3 - 7.8]	[0.3 - 0.8]	[0.3 - 3.0]	[0.0 - 3.2]
HQRE						
	Lambda	1.1	0.4	0.4	2.3	1.8
	Epsilon	0.4	0	0	1.4	0.4
	Av. probability buy	64%	75%	78%	53%	46%
	Log L	-128.91	-31.25	-30.17	-25.88	-33.21
	95% CI Lambda	[0.4 - 1.9]	[0.3 - 0.5]	[0.3 - 3.1]	[0.5 - 3.1]	[1.1 - 2.9]
	95% CI Epsilon	[0.0 - 0.6]	[0.0 - 0.2]	[0.0 - 2.8]	[0.0 - 4.8]	[0.4 - 1.0]
SQRE						
	Tau	0	0.7	0	0	0.5
	Gamma	0	0.1	0	0	0.8
	Lambda	1.1	0.3	0.4	2.3	1.7
	Epsilon	0.4	0	0	1.4	0.4
	Av. probability buy	64%	72%	78%	53%	47%
	Log L	-128.91	-31.20	-30.17	-25.88	-33.14
	95% CI Tau	[0 - 3.0]	[0 - 0.7]	[0 - 3.1]	[0 - 4.7]	[0 - 4.1]
	95% CI Gamma	[0 - 2]	[0 - 0.1]	[0 - 2]	[0 - 2]	[0 - 2]
	95% CI Lambda	[0.4 - 1.9]	[0.3 - 0.4]	[0.3 - 3.0]	[0.4 - 4.1]	[0 - 2.3]
	95% CI Epsilon	[0.2 - 0.6]	[0.0 - 0.2]	[0 - 2]	[0 - 5]	[0 - 0.8]
ABEE - 1 class						
	Lambda	0.2	0.3	0.3	0.2	0.1
	Av. probability buy	64%	72%	75%	63%	57%
	Log L	-141.42	-31.77	-31.67	-32.92	-40.03
	95% CI	[0.1 - 0.3]	[0.2 - 0.4]	[0.2 - 0.4]	[0.2 - 0.4]	[0.0 - 0.2]
ABEE - 2 classes						
	Lambda	0.3	0.4	0.4	0.3	0.1
	Av. probability buy	69%	75%	79%	68%	56%
	Log L	-137.54	-31.32	-30.97	-31.98	-39.77
	95% CI	[0.2 - 0.4]	[0.3 - 0.4]	[0.3 - 0.5]	[0.2 - 0.5]	[0.0 - 0.3]

Table 3: Estimations and goodness of fit of behavioral game theory models.
In order to compute log-likelihoods for the Nash equilibrium, we assume that there is one chance out of 10,000 that players choose the non equilibrium strategy. In the no-cap case, there are two Nash equilibria: this table focuses on the bubble equilibrium which has the highest likelihood.

SQRE and ABEE models including the Cognitive Hierarchy model and its extension, the Truncated Quantal Response Equilibrium, and the Quantal Response equilibrium and its extension, the Heterogeneous QRE (hereafter HQRE), and the one- and two-classes ABEE. For brevity, the estimation of the Overconfidence Cognitive Hierarchy model is given in the Supplementary Appendix.

The results are as follows. The average level of sophistication τ of the CH, estimated on the entire data set, is 0.5. This is in line with the median estimates reported by Camerer, Ho and Chong (2004) that lie between 0.7 and 1.9, but is a little low. This low estimated τ suggests a high proportion of level-0 players, around 60%. Interestingly, what drives this result is not really the fact that traders enter too much into bubbles when they should not. Indeed the fact that there is only around 10% of subjects who buy when they know they are last in the market sequence suggests a proportion of level-0 players equal to 20%. What explains the high estimated proportion of level-0 players is rather the fact that subjects do not buy as much as expected by the cognitive hierarchy model (with a higher average sophistication level) when there is no cap on the initial price or when the cap is large. This can be seen on Figure 2 that plots the predictions of the CH model using the best-fitting value of τ estimated on the entire data set. CH embeds the better than average effect to the extent that traders believe that no other agent does as many steps of reasoning as them. Our estimations of OCH (reported in the Supplementary Appendix) show that, when the parameter θ can be identified, we cannot reject the hypothesis that $\theta = 1$, its CH restriction. Overconfidence is thus an important feature that enables the CH logic to fit our data pretty well.

The payoff responsiveness Λ of the QRE, estimated on the entire data set, is 0.3. This is consistent with the results of Mc Kelvey and Palfrey (1995) who report estimates between 0.15 and 3.3. As was also the case for the CH model, such a low value of the responsiveness parameter is required not so much to explain why subjects buy when they should not (that is, when the offered prices are close to the price cap, if any) but to explain why they do not buy that much when they should (that is, when the offered prices are low).

As shown in Figure 3 that displays the statistical tests for various models' comparisons, the QRE fits our data better than the CH model (p-value=0.01 for the overall data). This result is interesting because Camerer, Ho and Chong (2004) show that CH fits better than QRE for a wide variety of games. This suggests that there is a specific aspect to the nature of speculation in the bubble game. Moreover, this result is also at odd with those

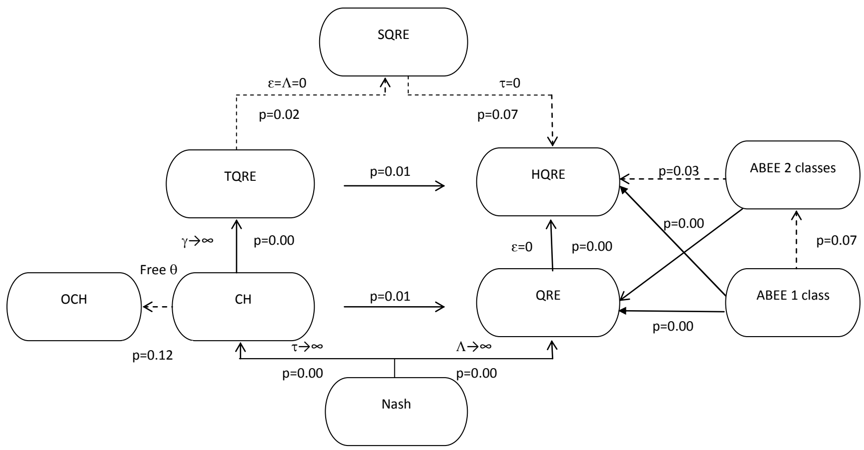


Figure 3: Goodness of fit comparisons.
Statistical tests for various models' comparisons using parameters estimated across all treatments. The arrows point towards the best model. Plain and dotted lines indicate, respectively, significant and insignificant differences in likelihood at the 1% confidence level.

of Kawagoe and Takizawa (2010), who compare the goodness of fit of both models in laboratory experiments of the centipede game. In order to understand why QRE fits better than CH in the bubble game, it is interesting to focus on the treatment with $K = 1$. Indeed, this treatment corresponds to a specific centipede game with three agents playing once. In this treatment, CH appears to fit better than QRE (p-value=0.046). The other treatments with $K > 1$ are not centipede games because some agents do not know what their position is. In this case, the information revealed by prices enable traders to better infer their chances not to be last and affect their expected payoffs. For these treatments, QRE fits better than CH most of the time (p-values are 0.49, 0.06, and 0.00 for K equals 100, 10,000, and $+\infty$, respectively). Given that the informativeness of prices is a relevant feature from an empirical point of view, this result demonstrates again the interest of our design in better understanding the nature of speculation.

Looking at Figure 2, it appears that QRE better captures the fact that the probability to buy when the price is higher than 100 is lower than the one when the price is 1 or 10. In the QRE, since costlier mistakes are less likely, this model is able to capture the drop in players' expected utility from buying: when they are proposed a price $P \geq 100$, the conditional probability to be third is greater than or equal to $\frac{1}{2}$, whereas, when they are proposed a price of 1 or 10, the conditional probability to be third is zero. This informational feature is present in our design but not in the centipede game, and has behavioral consequences in the bubble game.

The most striking difference between the fit of CH and QRE arises in the treatment with $K = +\infty$. Indeed, in this case, as explained in the theory section and as can be seen in Figure 2, the CH model does not capture the snowball effect while QRE does account for it.[47]

We then estimate generalizations of the CH and QRE models. First, as Figure 3 indicates, TQRE and HQRE improve on CH and QRE models, respectively. This suggests that taking into account heterogeneity in payoff responsiveness enables the models to better fit data from the bubble game. Second, the fact that HQRE better fits the data than TQRE confirms that cognitive hierarchies are not necessary to explain speculative behavior in the bubble game. This result is further confirmed by the SQRE estimations. On the one hand, as shown in Table 3, the CH parameter τ is strictly positive in only two treatments (those with $K = 1$ and $K = +\infty$) and is rather small. In the two other treatments, we estimate that $\tau = 0$, so the SQRE model boils down to the HQRE. On the other hand, even in the former cases, a Vuong test indicates that the difference between the fits of the HQRE and of the SQRE models is not significant. Figure 3 also shows that, when these models are estimated on the overall data, the likelihood ratio test favors the HQRE over the SQRE. These results confirm the fact that cognitive hierarchies are not crucial to understand speculation in the bubble game. We thus conclude that the nature of speculation in the bubble game is related to less than perfect payoff responsiveness.

We finally estimate the ABEE models that assume traders form expectations within analogy classes. We find that the two-class ABEE fits the bubble game data better than the one-class ABEE. Figure 3 indicates that a Vuong test favors the two-class ABEE model with a p-value of 7%. This suggests that informational aspects related to the inference on the probability not to be last are important for speculation in our game. When estimated on the entire data, the fit of the two-class ABEE appears lower than the one of the QRE and of the HQRE.[48] However, when excluding

[47] In the No cap treatment, the CH model predicts a probability to buy that is constant across prices. The maximum likelihood procedure can thus not do better than choosing a parameter τ such that the predicted probability to buy equals the unconditional probability (up to numerical approximation errors). The QRE has a higher maximum likelihood than the CH model despite the fact that it overestimates the average probability to buy because it displays a snowball effect and thus minimizes the loss of log-likelihood at prices of 1 and 10.

[48] The Supplementary Appendix reports estimations of an heterogeneous ABEE model referred to as HABEE. Allowing for heterogeneity in payoff responsiveness in the ABEE model increases the log-likelihood but this increase is not significant.

the treatment in which $K = 1$ that corresponds to a centipede game, the fit of the two-class ABEE is not significantly different at 5% than the one of the QRE and of the HQRE. Furthermore, focusing on individual treatments, the fit of the two-class ABEE is never significantly different even at 10% than the one of QRE. Compared to HQRE, it is significantly better at 1% for $K = +\infty$ and not significantly different at 5% for the other three treatments. This suggests that analogy classes play an important role in understanding speculation as hypothesized by Bianchi and Jehiel (2011).

Moreover, the ABEE models appear better in terms of estimates' stability. Indeed, for the SQRE model and its various specifications, the parameters of interest, estimated separately for each treatment, display some variability: point estimates in Table III often vary by an order of magnitude.[49] For example, the estimates of Λ vary from 0.1 to 2.7 in the QRE model and from 0.4 to 2.3 in the HQRE model. In contrast, the estimates from the ABEE models appear quite stable across treatments.

6 Conclusion

This paper proposes a novel experimental design to study speculative behavior in laboratory experiments: a bubble game in which agents trade sequentially and do not always know where they stand in the sequence. Our game has a no bubble Nash equilibrium when there is a finite price cap, and an additional bubble equilibrium when there is no price cap.

Analyzing our experimental data, some descriptive statistics show that speculation increases with the number of steps of iterated reasoning needed to reach equilibrium and with the probability that a subject is not last in the market sequence. We then offer maximum likelihood estimations of various behavioral game theory models including, but not limited to, cognitive hierarchy (Camerer, Ho and Chong, 2004), quantal response equilibrium (Mc Palfrey and Palfrey, 1995), and analogy-based expectation equilibrium (Jehiel, 2005). The main finding of the paper is that speculation in the bubble game is related to quantal responses and analogy classes.

This finding echoes the results of Carrillo and Palfrey (2009, 2011) and Camerer, Nunnari, and Palfrey (2012) on other Bayesian games. Carrillo and Palfrey (2009) create and study the compromise game, and show that

[49] Such parameter estimates' variability is not unusual in structural estimations of game theory models. For example, in his Table 3, Weizsäcker (2003) reports estimates of average payoff responsiveness that lie between 1.75 and 11.33 for player j. As indicated above, there is also some variability in the estimates of τ and Λ offered by Camerer, Ho, and Chong (2004) and Mc Kelvey and Palfrey (1995), respectively.

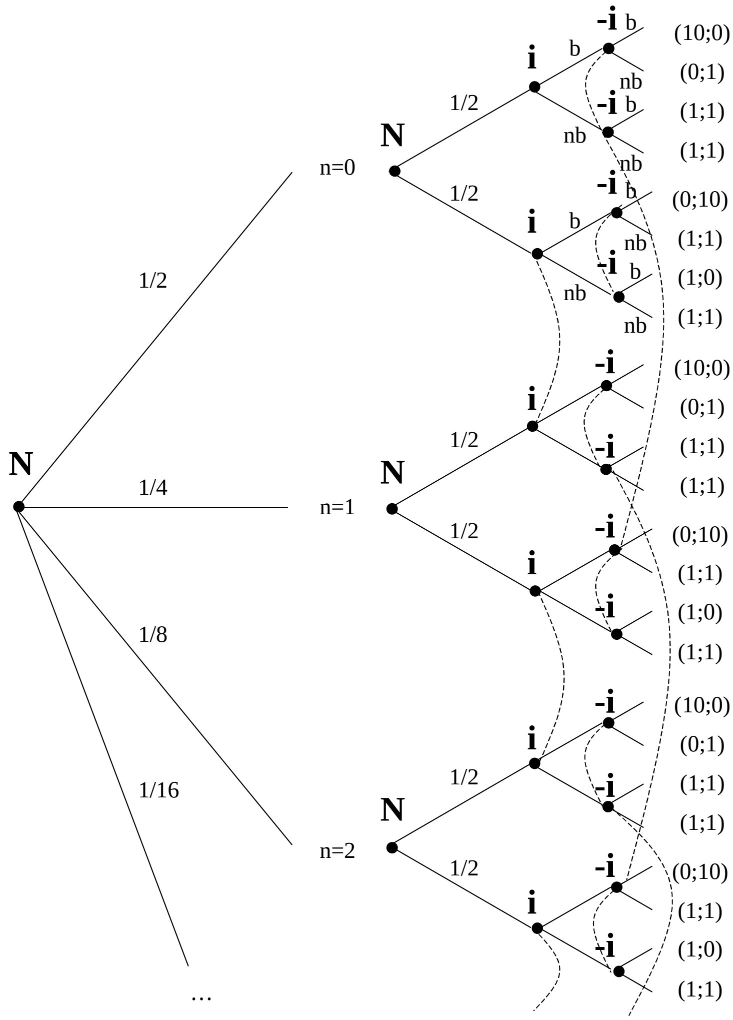
a mix between the cursed equilibrium of Eyster and Rabin (2005) and the quantal response equilibrium best fits behavior in the compromise game.[50] Carrillo and Palfrey (2011) and Camerer, Nunnari, and Palfrey (2012) further find that this cursed quantal response equilibrium also fits quite well data from experiments with private information on bilateral trading games and auctions, respectively. Quantal responses and coarse thinking, an ingredient of both the cursed equilibrium and the analogy-based expectation equilibrium, thus appears to be relevant to understand strategic behavior in Bayesian games.

The experimental setting proposed in the present paper opens several avenues of research. It would be interesting to study whether the occurrence of bubbles (rational and irrational) vary with the number of traders, the introduction of risk in the underlying asset payoff, and the level of transparency (one could proxy for transparency by setting a non-null probability that a trade is publicly announced). It would also be interesting to test whether the structural estimations of behavioral game theory models would be different after allowing for a high number of repetitions of the bubble game. Finally, it could be fruitful to extend the experimental setting to cases in which the price path and the timing are left at the discretion of traders. This would allow testing whether traders are able to coordinate on a price path and a timing that sustains rational bubbles.

[50]In a cursed equilibrium, agents underestimate the correlation between players' information and action. As noted by Jehiel and Koessler (2008), the fully cursed equilibrium of Eyster and Rabin (2005) is equivalent to an ABEE in which analogy-classes correspond to the private information partition. Both equilibrium concepts incorporate a form of coarse thinking according to which agents reason in terms of categories. In the context of ABEE, agents reason in terms of categories of game nodes (the analogy classes) in which they expect behavior to be identical. In the context of cursed equilibrium, agents reason in terms of categories of types that are assumed to have the same behavior.

7 Appendix

Appendix A: Extensive form of the game with two players



At each node, Nature (N), player i or player $-i$ choose an action. $(x;y)$ represents the payoffs; x for player i, and y for player $-i$. Dotted lines relate nodes that are observationally equivalent. b refers to the buy decision, nb to the refusal decision.

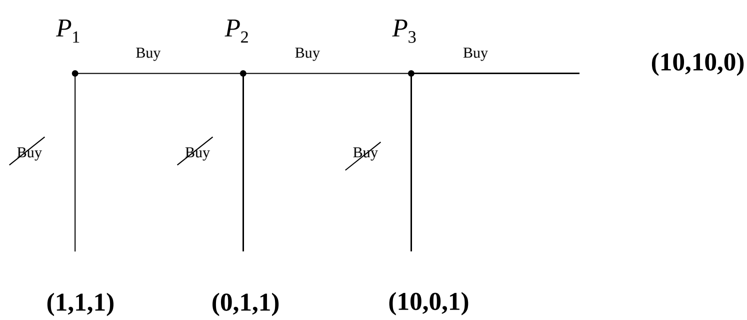
Appendix B: Instructions for the case where $K = 10,000$

Welcome to this market game. Please read carefully the following instructions. They are identical for all participants. Please do not communicate with the other participants, stay quiet, and turn off your mobile phone during the game. If you have questions, please raise your hand. An instructor will come and answer.

As an appreciation for your presence today, you receive a participation fee of 5 euros. In addition to this amount, you can earn money during the game. The game will last approximately half an hour, including the reading of the instructions.

Exchange process

To play this game, we form groups of three players. Each player is endowed with one euro which can be used to buy an asset. Your task during the game is thus to choose whether you want to buy or not the asset. This asset does not generate any dividend. If the asset price exceeds one euro, you can still buy the asset. We indeed consider that a financial partner (who is not part of the game) provides you with the additional capital and shares profits with you according to the respective capital invested. The market proceeds sequentially. The first player is proposed to buy at a price P_1. If he buys, he proposes to sell the asset to the second player at a price which is ten times higher, $P_2 = 10 \times P_1$. If the second player accepts to buy, the first player ends up the game with 10 euros. The second player then proposes to sell the asset to the third trader at a price $P_3 = 10 \times P_2 = 100 \times P_1$. If the third player buys the asset, the second player ends up the game with 10 euros. The third player does not find anybody to whom he can sell the asset. Since this asset does not generate any dividend, he ends up the game with 0 euro. This game is summarized in the following figure



At the beginning of the game, players do not know their position in the market sequence. Positions are randomly determined with one chance out of three for each player to be first, second or third.

Proposed prices

The price P_1 that is proposed to the first player is random. This price is a power of 10 and is determined as follows:

Price P_1	Probability that this price is realized
1	1/2 (50%)
10	1/4 (25%)
100	1/8 (12.5%)
1,000	1/16 (6.3%)
10,000	1/16 (6.3%)

Players decisions are made simultaneously and privately. For example, if the first price $P_1 = 1$ has been drawn, the prices that are simultaneously proposed to the three players are: $P_1 = 1$ for the first player, $P_2 = 10$ for the second player, and $P_3 = 100$ for the third player. Identically, if the first price $P_1 = 10,000$ has been drawn, the prices that are simultaneously proposed to the three players are: $P_1 = 10,000$ for the first player, $P_2 = 100,000$ for the second player, and $P_3 = 1,000,000$ for the third player.

The prices that you are been proposed can give you the following information regarding your position in the market sequence:

- if you are proposed to buy at a price of 1, you are sure to be first;

- if you are proposed to buy at a price of 10, you have one chance out of three to be first and two chances out of three to be second in the market sequence;

- if you are proposed to buy at a price of 100 or 1,000, you have one chance out of seven to be first, two chances out of seven to be second, and four chances out seven to be last in the market sequence;

- if you are proposed to buy at a price of 10,000, you have one chance out of four to be first, one chance out of four to be second, and two chances out four to be last.

- if you are proposed to buy at a price of 100,000, you have one chance out of two to be second, and one chance out of two to be third.

- if you are proposed to buy at a price of 1,000,000, you are sure to be last.

In order to preserve anonymity, a number will be assigned to each player. Once decision will be made, we will tell you (anonymously) the group to which you belong, your position in the market sequence, if you are proposed to buy, and your final gain.

Do you have any question?

References

[1] Abreu D., and M. K. Brunnermeier, 2003, Bubbles and Crashes, *Econometrica*, 71(1), 173-204.

[2] Ackert L. F., N. Charupat, B. K. Church, and R. Deaves, 2005, Bubbles in Experimental Asset Markets: Irrational Exuberance No More, *mimeo*.

[3] Ackert L. F., N. Charupat, R. Deaves, and B. Kluger, 2006, The Origins of Bubbles in Laboratory Asset Markets, *mimeo*.

[4] Allen, F., and G. Gorton, 1993, Churning Bubbles, *Review of Economic Studies*, 60, 813-836.

[5] Allen, F., S. Morris, and A. Postlewaite, 1993, Finite Bubbles with Short-Sale Constraints and Asymmetric Information, *Journal of Economic Theory*, 61, 206-229.

[6] Asparouhova, E., P. Bossaerts, and A. Tran, 2011, Market Bubbles and Crashes as an Expression of Tension between Social and Individual Rationality: Experiments, *mimeo*.

[7] Bianchi, M., and P. Jehiel, 2011, Bubbles and Crashes with Partially Sophisticated Investors, *mimeo*.

[8] Blanchard O., 1979, Speculative Bubbles, Crashes, and Rational Expectations, *Economic Letters*, 3, 387-389.

[9] Brunnermeier M. K., 2009, Bubbles, Entry in *The New Palgrave Dictionary of Economics*, edited by Steven Durlauf and Lawrence Blume, 2nd edition.

[10] Brunnermeier M. K., and J. Morgan, 2010, Clock Games: Theory and Experiments, *Games and Economic Behavior*, 68, 532-550.

[11] Camerer C., 1989, Bubbles and Fads in Asset Prices: A Review of Theory and Evidence, *Journal of Economic Surveys*, 3, 3-41.

[12] Camerer C., 2003, *Behavioral Game Theory: Experiments in Strategic Interaction*, Princeton University Press.

[13] Camerer C., T.-H. Ho, and J.-K. Chong, 2004, A cognitive hierarchy model of one-shot games, *Quarterly Journal of Economics*, vol. 119, 861-898.

[14] Camerer C. and D. Lovallo, 1999, Overconfidence and Excess Entry: An Experimental Approach, *American Economic Review*, 89, 306-318.

[15] Camerer C., S. Nunnari, and T. Palfrey, 2012, Quantal Response and Nonequilibrium Beliefs Explain Overbidding in Maximum-Value Auctions, Caltech Social Science Working Paper no. 1349.

[16] Carrillo J., and T. Palfrey, 2009, The Compromise Game: Two-Sided Adverse Selection in the Laboratory, *American Economic Journal: Microeconomics*, 1, 151-181.

[17] Carrillo J., and T. Palfrey, 2011, No Trade, *Games and Economic Behavior*, 71, 66-87.

[18] Conlon J., 2004, Simple Finite Horizon Bubbles Robust to Higher Order Knowledge, *Econometrica*, 72, 927-936.

[19] Cooper M.J., O. Dimitrov and R. Rau, 2001, A rose.com by Any Other Name, *Journal of Finance*, 56, 2371-2388.

[20] Costa-Gomes M., and V. Crawford, 2006, Cognition and Behavior in TwoPerson Guessing Games: An Experimental Study, *American Economic Review*, 96, 1737-1768.

[21] Costa-Gomes M., V. Crawford, and B. Broseta, 2001, Cognition and Behavior in Normal-Form Games: An Experimental Study, *Econometrica*, 69, 1193-1235.

[22] Crawford V., and N. Iriberri, 2007, Level-k Auctions: Can Boundedly Rational Strategic Thinking Explain the Winner's Curse and Overbidding in Private-Value Auctions?", *Econometrica*, 75, 1721-1770.

[23] Doblas-Madrid A., 2012, A Robust Model of Bubbles with Multidimensional Uncertainty, *Econometrica*, 80, 1845-1893.

[24] Dufwenberg M., T. Lindqvist, and E. Moore, 2005, Bubbles and Experience: An Experiment, *American Economic Review*, vol. 95, 1731-1737.

[25] Dulleck, U., and J. Oechssler, 1997, The absent-minded centipede, *Economics Letters*, vol. 55, 309-315.

[26] Eyster E. and M. Rabin, 2005, Cursed Equilibrium, *Econometrica*, 73, 1623-1672.

[27] Geanakoplos, J., 1992, Common Knowledge, *Journal of Economic Perspectives*, 6, 52-83.

[28] Harrison G., J. List and C. Towe, 2007, Naturally Occurring Preferences and Exogenous Laboratory Experiments: A Case Study of Risk Aversion, *Econometrica*, 75, 433-458.

[29] Holt, C. A., and S. K. Laury, 2002, Risk Aversion and Incentive Effects, *American Economic Review*, 92, 1644-1655.

[30] Huck, S., P. Jehiel, and T. Rutter, 2011, Feedback Spillover and Analogy-based Expectations: a Multi-Game Experiment, *Games and Economic Behavior*, 71, 351-365.

[31] Jehiel, P., 2005, Analogy-based expectation equilibrium, *Journal of Economic Theory*, 123, 81-104.

[32] Jehiel, P., and F. Koessler, 2008, Revisiting Games of Incomplete Information with Analogy-Based Expectations, *Games and Economic Behavior*, 62, 533-557.

[33] Kawagoe, T. and H. Takizawa, 2010, Level-K Analysis of Experimental Centipede Games, *mimeo.*

[34] Lei V., C. Noussair and C. Plott (2001), Nonspeculative Bubbles in Experimental Asset Markets: Lack of Common Knowledge of Rationality vs. Actual Irrationality, *Econometrica*, 69(4), 831.859.

[35] Mc Kelvey, R. D., and T. R. Palfrey, 1992, An Experimental Study of the Centipede Game, *Econometrica*, vol. 60, 803-836.

[36] Mc Kelvey, R. D. and T. R. Palfrey, 1995, Quantal Response Equilibrium for Normal Form Games, *Games and Economic Behavior*, 10, 6-38.

[37] Mc Kelvey, R. D. and T. R. Palfrey, 1998, Quantal Response Equilibrium for Extensive Form Games, *Experimental Economics*, 1, 9-41.

[38] Moinas, S. and S. Pouget, 2012, Supplementary Appendix to "The Bubble Game: An Experimental Study of Speculation", *mimeo.*

[39] Morris, S., A. Postlewaite, and H. Shin, 1995, Depth of Knowledge and the Effect of Higher Order Uncertainty, *Economic Theory*, 6, 453-467.

[40] Morris, S., R. Rob, and H. Shin, 1995, p-Dominance and Belief Potential, *Econometrica*, 63, 145-157.

[41] Nagel, R., 1995, Unraveling in Guessing Games: An Experimental Study, *American Economic Review*, 85, 1313-1326.

[42] Nalebuff, B., 1989, The Other Person's Envelope is Always Greener, *The Journal of Economic Perspectives*, Vol. 3, 171-181.

[43] Noussair, C., S. Robin, and B. Ruffieux, 2001, Price Bubbles in Laboratory Asset Markets with Constant Fundamental Values, *Experimental Economics*, 4, 87-105.

[44] Palfrey, T., and S. Wang, 2012, Speculative Overpricing in Asset Markets with Information Flows, *Econometrica*, 60, 1937-1976.

[45] Porter, D. P & Smith, V. L, 1995, Futures Contracting and Dividend Uncertainty in Experimental Asset, University of Chicago Press, vol. 68, 509-41.

[46] Rogers, B., T. Palfrey, and C. Camerer, 2009, Heterogeneous Quantal Response Equilibrium and Cognitive Hierarchies, *Journal of Economic Theory*, 144, 1440-1467.

[47] Smith V., G. Suchanek, and A. Williams, 1988, Bubbles, Crashes, and Endogenous Expectations in Experimental Spot Asset Markets, *Econometrica*, 56, 1119-1151.

[48] Stahl, D., and P. Wilson, 1995, On Players Models of Other Players: Theory and Experimental Evidence, *Games and Economic Behavior*, 10, 213-254.

[49] Tirole J., 1982, On the possibility of speculation under rational expectations, *Econometrica*, 50, 1163-1181.

[50] Tirole J., 1985, Asset Bubbles and Overlapping Generations, *Econometrica*, 53, 1499-1528.

[51] Van Boening, M.V., Williams, A.W., and LaMaster S. 1993. Price bubbles and crashes in experimental call. markets. *Economic Letters*, 41, 179-185.

[52] Vuong, Q., 1989, Likelihood Ratio Tests for Model Selection and non-nested Hypotheses, *Econometrica*, 57, 307-333.

[53] Weil P., 1987, Confidence and the Real Value of Money in an Overlapping Generations Economy, *Quarterly Journal of Economics*, 102, 1-22.

[54] Weizsäcker G., 2003, Ignoring the Rationality of Others: Evidence from Experimental Normal-form Games, *Games and Economic Behavior*, 44, 145-171.

[55] Xiong W. and J. Yu, 2011, The Chinese Warrants Bubble, *American Economic Review*, 101, 2723-2753.