Selecting Equilibria in Common Agency Games

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Abstract: We characterize equilibrium payoffs of a delegated common agency game in a public good context where principals use smooth contribution schedules. We prove that under complete information, payoff vectors of equilibria with truthful schedules coincide with the set of smooth equilibrium payoffs, including non-truthful schedules. We next consider whether the presence of arbitrarily small amounts of asymmetric information is enough to refine this payoff set. Providing that the extensions of the equilibrium schedules beyond the equilibrium point are flatter than truthful schedules, the set of equilibrium payoffs is strictly smaller than the set of smooth (equivalently, truthful) equilibrium payoffs. Interestingly, some forms of asymmetric information do not sufficiently constrain the slopes of the extensions and fail to refine the payoff set. In the case of a uniform distribution of types and arbitrary out-of-equilibrium contributions, the refinement has no bite. If, however, one restricts out-of-equilibrium behavior in a natural way, the refinement is effective. Alternatively, we may consider an exponential distribution with unbounded support (and hence no out-of-equilibrium choices) and we find that the refinement selects a unique equilibrium payoff vector equal to Lindahl prices.

As a separate contribution, equilibria with forcing contracts are also considered both under complete and asymmetric information.

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1 Introduction

In a public good common-agency game, several principals non-cooperatively design payment schedules for a common agent who chooses the level of a public good. Under complete information, Bernheim and Whinston [1] argued that one should focus on the class of *truthful Nash equilibria* of such games. In those equilibria, each principal offers a contribution schedule which reflects his own marginal preferences for the decision taken by the agent. Truthful Nash equilibria are attractive for several reasons. First, they yield an efficient outcome. Since his marginal contribution reflects a principal's marginal preferences, the agent is made residual claimant for the surplus of the grand-coalition he forms with all principals. The multiplicity of equilibria, if any, comes from the possibility to sustain different payoff distributions. Second, truthful equilibria are generally coalition-proof; that is, the equilibria are immune to deviations by coalitions of principals, which would themselves also be robust to further deviations by subcoalitions, and so on.[1] Finally, truthful Nash equilibria have been found attractive in applied research since the distribution of truthful payoffs is easily characterized by means of simple inequalities.[2]

Although truthful contributions schedules have attractive properties, their use raises two sets of issues. First, one may wonder which minimal properties of contribution schedules really ensure that efficiency is achieved in equilibrium. In other words, can we find other equilibrium schedules (for instance by specifying other out-of-equilibrium extensions that are not globally truthful or by considering non-differentiable schedules) and still maintain efficiency? What is the impact of alternative choices of extensions for the characterization of the principals' payoffs? Second, given that the choice of extensions might have a significant impact on the distribution of equilibrium payoffs, can we find any rationale for relying on some particular extensions and what are the consequences?

This paper addresses these two sets of questions. In section 3, we address the first issue and show that the differentiability of the schedules at the equilibrium point is enough to ensure efficiency. We fully describe the set of equilibria allocations when the principals and their agent's optimal choices are characterized with first-order conditions. All such equilibria have the property that the principals' marginal contributions at the equilibrium point reflect their marginal valuations, i.e., a requirement of *local truthfulness* is satisfied. Even though the schedules that support the efficient outcome may differ out of equilibrium, the set of equilibrium payoffs remains the same as those achievable in truthful

[1] This property is hardly surprising given the fact that any such subcoalition turns out to jointly offer a schedule reflecting the aggregate preferences of the coalition.

[2] Indeed, because truthful contributions not only reflect the principals' preferences at the equilibrium output but also for any out-of-equilibrium one, it becomes easy to compute the agent's reservation payoffs when he deals only with a subset of principals. Those reservation payoffs determine in turn how much can be extracted by any principal, and thus his equilibrium payoff. Laussel and Lebreton [5, 6].

equilibria. In all cases, a principal gets at most his incremental contribution to welfare and at least his stand-alone payoff.

The set of principals payoffs in smooth equilibria is generally quite large. In applications, we often want to predict the particular payoff vector that emerges, particularly when the common agency game is part of a larger game with prior stages of investments or other costly actions. To this end, for the remainder of the paper, we turn our attention to a simple class of asymmetric-information refinements. The motivation for this form of refinement is that in many real-world settings the principals are offering nonlinear contribution schedules because they are attempting to extract surplus from (or otherwise to provide incentives to) the agent. In particular, it is well-known from the incentive literature in monopolistic screening environments in which the agent has private information, that nonlinear schedules serve an important role in extracting surplus. With this competing use of nonlinear schedules as our backdrop, we can characterize the equilibrium contribution schedules and payoff vectors for some distribution of agent type. In such a game with competing principals, the requirement of truthfulness is no longer imposed a priori but is instead replaced by the more natural incentive compatibility constraint of the agent at any equilibrium point.[3] Then, taking the limit as the support of the agent's private information converges to a point mass, we can ask what is the limiting set of payoffs to the players. If the limit payoff set is strictly smaller than the set of all smooth-equilibrium payoffs under complete information, we have a refinement that potentially has desirable properties and predictive appeal.

In section 4 we solve for the differentiable equilibria of the delegated common agency game when the agent is privately informed on the cost of producing the public good and both principals can contract on the same screening variable.[4] This analysis is of independent value for readers interested in common agency games under adverse selection with common screening devices. Second, we investigate the limit of equilibrium payoffs and strategies as the types distribution converges towards a mass point. We demonstrate that the limit set of equilibrium payoffs under asymmetric information when the support of the distribution (supposed to be uniform) converges to a mass point corresponds to the set of smooth (equivalently, truthful) equilibrium payoffs. Asymmetric information alone does not refine the complete-information equilibrium payoffs set because in the case of the uniform distribution it fails to impose sufficient constraints on out-of-equilibrium contributions. There is still much latitude in selecting out-of-equilibrium contributions while maintaining the efficient outcome in the limit. This points to the source of the refinement failure.

[3]For a similar discussion, see Martimort and Stole [9].

[4]This is distinct from the analysis of Martimort and Stole [10] in which principals are restricted to writing contracts on mutually exclusive subsets of the screening variables.

To select among equilibrium payoffs, asymmetric information must provide a rationale for focusing on differentiable equilibrium schedules which are less steep than truthful strategies, and this degree of curvature must be preserved when the types distribution converges towards a Dirac mass. Flatter schedules constrain the set of equilibrium payoffs. In this respect, there are two strategies for removing the large degrees of freedom inherent in the uniform distribution case. First, we can directly refine the out-of-equilibrium contributions in the asymmetric game while maintaining our assumption of a uniform distribution. Second, we can turn our attention to a type distribution that is unbounded (and hence leaves nothing out-of-equilibrium). We consider each in section 6.

In section 6.1, we focus on a reasonable restriction to out of equilibrium contributions when types are uniformly distributed. We require simply that the out-of-equilibrium schedules are extensions of the equilibrium schedules using the same analytical expression. This quite natural extension allows an easy computation of the agent's reservation payoffs when refusing one of the principal's contracts.[5] We obtain thereby the so-called *natural-extension equilibria* and the corresponding distributions of equilibrium payoffs.[6]

To understand how natural extensions help select among equilibrium outcomes it is useful to review the logic underlying asymmetric information distortions and how those distortions affect payoffs. Under adverse selection, each principal attempts to extract rent from the privately-informed agent taking as given the other principal's contribution. To reduce the agent's information rent, each principal reduces his own marginal contribution to reduce the agent's output. The principals' marginal contributions no longer reflect their marginal valuations as with truthful schedules but are strictly lower for screening purposes. This screening effect causes naturally-extended contribution schedules to be flatter than truthful schedules. Consider now the case where the support of the type distribution shrinks to zero. Even in the limit, natural schedules keep track of the rent-extraction externality which arises under adverse selection and remain flatter than truthful schedules. The set of natural equilibrium payoffs when the support of the adverse selection parameter shrinks becomes a strict subset of the set of truthful payoffs of the complete information game.

Pushing this logic to the extreme, a very powerful selection device can be obtained if we were able to find an asymmetric information game with equilibrium schedules being almost linear. In section 6.2, we obtain this equilibrium by having types exponentially distributed and assuming symmetric payoffs for the principals. Adding some mild technical conditions, there exists a unique equilibrium of the asymmetric information game.

[5]In this respect, natural extensions preserve the tractability offered by truthful schedules.

[6]These equilibria are quite robust. Enlarging the support of the uniform distribution of types would indeed keep the principals' marginal contributions unchanged. The fixed-fees in the equilibrium contributions are derived from the condition that the least efficient type's information rent is fully extracted. Hence, those fixed-fees change as the support is modified.

As the exponential distribution converges towards a Dirac mass, there always remains a positive, albeit small, probability that any positive output below the first-best may arise in equilibrium. This eliminates any freedom in choosing extensions and is enough to have a unique equilibrium under asymmetric information. This equilibrium is such that contribution schedules are almost linear. It converges towards the Lindahl outcome of the complete information game as the type distribution converges towards a Dirac mass. This provides a powerful refinement of the set of all smooth, efficient equilibria in the common agency game.

The analysis for most of the paper is restricted to the class of continuous, weakly increasing contribution schedules that are differentiable everywhere except at possibly the highest output for which a principal's transfer is zero. There is another class of equilibria, however, based with forcing contracts that are not differentiable. Truthful equilibria have been criticized by Kirchsteiger and Prat [3] who ran experiments to confirm that simple forcing contracts may be quite attractive. For these reasons we extend our analysis to the class of non-differentiable forcing contracts in section 7. Here, we characterize the set of forcing equilibria which we hope is of independent interest to those doing applied work. Although inefficient outcomes may now be sustained in equilibrium, any equilibrium outcome under complete information achieved with forcing contracts is also the limit of equilibrium outcomes of the game under asymmetric information. This confirms again that asymmetric information has little power as a selection device unless the modeler is ready to focus on differentiable schedules and make an argument to use schedules in the asymmetric information game which are "flat enough."

Let us now review the relevant literature. In the framework of private common agency that differs from our public common agency game, Chiesa and Denicolo [2] describe the whole set of payoffs achieved under complete information, showing that truthful strategies are not necessarily very attractive. Laussel and Lebreton [5] have instead rationalized the use of truthful schedules by introducing uncertainty on the agent's cost function with a finite support for the cost parameter and ex ante contracting. One issue there is that contributions may be negative for some realizations. This makes it hard to enforce contributions ex post, once the agent has learned about his cost. Accordingly, we consider adverse selection and ex post contracting so that negative transfers can always be rejected by the privately informed agent. With that alternative timing, non-cooperating principals exert on each other a contractual externality which modifies their contributions away from truthfulness. Martimort and Moreira [7] replace truthfulness by an incentive compatibility constraint in a model where principals are instead privately informed. Finally, our paper is also related to Klemperer and Meyer [4]'s analysis of equilibria with supply functions. Those authors introduced uncertainty on demand to select among those equilibria. Although the complete information game analyzed below has also multiple equilibria, our paper differs from theirs along several lines, most noticeably the strategic play of the common agent and our focus on adverse selection instead of ex ante uncertainty.

2 The Model

Consider two principals ($i = 1, 2$) whose utility functions are respectively given by $V_i(q, t_i) = S_i(q) - t_i$ where t_i is the monetary transfer made to a common agent. This agent produces q units of public good on the principals' behalf at cost θq, and gets a payoff $U(q, t_1 + t_2) = t_1 + t_2 - \theta q$ of doing so. We assume that $S_i(\cdot)$ is strictly increasing, concave and satisfies the Inada conditions $S_i'(0) = +\infty$, $S_i'(\infty) = 0$ with $S_i(0) = 0$ to ensure interior optima under all circumstances. We will also impose the condition $2S_i'' < S_{-i}''$ for $i = 1, 2$, which restricts the degree of payoff asymmetry between the principals.[7]

We characterize payoffs both under complete and asymmetric information. Under asymmetric information, principals ignore the cost parameter θ which is private information to the agent. For most of the paper, this parameter is drawn from a uniform distribution on $[\underline{\theta}, \bar{\theta}]$.[8] We are interested in the limit of equilibrium payoffs of the asymmetric information game when the distribution converges towards a Dirac mass at $\underline{\theta}$.

The common agency game unfolds as follows. First, principals simultaneously and non-cooperatively offer their nonlinear contribution schedules $t_i(q)$ ($i = 1, 2$) to the agent who may be privately informed or not depending on whether the game has asymmetric information or not.[9] Second, the agent chooses which contracts to accept. He gets an exogenous payoff of zero if he refuses all offers. Third, the agent chooses the quantity $q(\theta)$ and receives payments $t_i(q(\theta))$ from those principals whose offers are accepted.

Without loss of generality, we restrict each principal's strategy space to the set of non-negative contribution schedules, $t_i(q) \geq 0$; thus, all principals' offers are accepted by the agent in equilibrium. Throughout sections 3 to 6, we restrict our attention to equilibria in which principals offer schedules that are continuous, weakly increasing and differentiable everywhere except possibly at the maximal output for which $t_i(q) = 0$; at this point there may be a kink with well-defined left and right derivatives. We call such contribution schedules *smooth* or *admissible* in what follows and refer to such equilibria as either smooth or admissible. Note that this is not a restriction on strategy spaces, but rather a restriction

[7] This condition is satisfied when principals have symmetric preferences but also for relatively small asymmetries as well. It ensures that each equilibrium schedule is concave under adverse selection when the agent takes both principal's contracts; an important step to compute the optimal payoff of the agent if he chooses to contract only with one principal.

[8] An exception is section 6.2 where the distribution is exponential.

[9] Following Peters [12] and Martimort and Stole [8], there is indeed no loss of generality in looking at competition between principals with menus as long as pure strategy equilibria are concerned.

on the set of equilibria. In section 7, we will discard our smoothness and continuity requirement and examine non-differentiable equilibria with forcing contracts. For further references, denote

$$W_{12}(\theta) = \max_{q\geq 0}\left\{\sum_{i=1}^{2} S_i(q) - \theta q\right\} \text{ and } W_i(\theta) = \max_{q\geq 0}\{S_i(q) - \theta q\}$$

as the greatest payoffs achievable by any coalition involving the agent and either two principals or only principal i, respectively. By assumptions made on $S_i(\cdot)$, those payoffs are all positive. The optimal outputs for these coalitions are respectively given by the Lindahl-Samuelson outcome $q^*(\theta)$ such that $\sum_{i=1}^{2} S_i'(q^*(\theta)) = \theta$ and by the bilaterally efficient output $q_i^*(\theta)$ such that $S_i'(q_i^*(\theta)) = \theta$.[10]

3 Complete Information

This section characterizes equilibrium payoffs under complete information assuming differentiability of the schedules at the equilibrium point so that the principals' and the agent's best-responses are characterized by first-order conditions. Our main result is that the whole set of such payoffs can be sustained with truthful schedules.

General Properties: First, we provide a few general properties of equilibria with admissible schedules.

Principal 1's best-response to an admissible schedule $t_2(q)$ offered by principal 2 is a transfer t_1 and an output q together solving:[11]

$$(P_1) : \max_{\{q,t_1\}} \; S_1(q) - t_1$$

$$\text{subject to } t_1 + t_2(q) - \theta q \geq \max_{\tilde{q}\geq 0}\{t_2(\tilde{q}) - \theta\tilde{q}\}. \tag{1}$$

Because $t_2(q) \geq 0$, the right-hand side of (1) is necessarily nonnegative. The participation constraint (1) ensures that the agent prefers taking both contracts rather than contracting only with principal 2 or not participating at all and getting his reservation payoff of zero.

We are interested in equilibria where the principals' and the agent's optimal choices are characterized by first-order conditions. To characterize these equilibria, let us propose the following definition:

Definition 1 *A schedule $t_i(q)$ is locally truthful if, at the equilibrium point $q(\theta)$, the principal's marginal contribution is equal to his marginal valuation:*

$$t_i'(q(\theta)) = S_i'(q(\theta)).$$

[10]Note that $q^*(\theta) > q_i^*(\theta)$ since $S_{-i}'(\cdot) > 0$.

[11]Provided its solution gives to principal 1 a positive payoff, a condition that will be checked ex post once we have the full description of equilibrium payoffs.

The next proposition provides necessary conditions that must be satisfied by any equilibrium with admissible schedules.

Proposition 1 *In any equilibrium with admissible schedules of the common agency game under complete information such that the principals' and the agent's optimal choices are characterized by first-order conditions, the following necessary conditions hold:*

- *The efficient level of public good $q^*(\theta)$ is produced;*
- *There is full extraction of the agent's rent;*
- *Contributions schedules are locally truthful:*

$$S_i'(q^*(\theta)) = t_i'(q^*(\theta)). \tag{2}$$

The pair $\{q^*(\theta), t_1(q^*(\theta))\}$ in the support of principal 1's best-response is determined by the solution of (P_1). However, many admissible schedules $t_1(q)$ may support this outcome as long as these schedules reflect principal 1's marginal valuation at the equilibrium point (condition (2)), i.e., such schedules are locally truthful. Much of our analysis below focuses on the role of these extensions for out-of equilibrium outputs in characterizing equilibrium payoffs.

Denote $\Sigma(\theta)$ the set of equilibrium payoff vectors (V_1, V_2) achieved with admissible schedules. Consider also the non-empty set $\Gamma(\theta)$[12] of all pairs (V_1, V_2) satisfying the linear constraints:

$$V_1 + V_2 = W_{12}(\theta) \tag{3}$$

$$V_i \geq W_i(\theta) \text{ for } i = 1, 2. \tag{4}$$

Proposition 2 *Under complete information, the set of equilibrium payoffs achieved with admissible schedules $\Sigma(\theta)$ is such that:*

$$\Sigma(\theta) \subseteq \Gamma(\theta).$$

From Proposition 1, the equilibrium output is always efficient from the grand-coalition's viewpoint and the agent's rent if fully extracted. The only indeterminacy comes from the

[12]Non-emptyness is proved in the Appendix.

distribution of the efficient surplus between principals. To understand the possible lower bound on any principal's payoff, observe that one principal, say i, can always deviate and offer the following admissible schedule:

$$t_i(q) = \max\{S_i(q) - W_i(\theta), 0\}. \tag{5}$$

When positive, this *truthful schedule* perfectly reflects principal i's marginal valuation for the public good not only at the equilibrium output but also elsewhere.[13] Since it is non-negative, this schedule is always accepted by the agent whatever principal $-i$'s offer. Moreover, when contracting with principal i only, the agent produces an output $q_i^*(\theta)$. This yields his stand-alone payoff $W_i(\theta)$ to principal i. When contracting also with principal $-i$, the agent might increase production above the stand-alone output $q_i^*(\theta)$ since, by assumption, $t_{-i}(\cdot)$ is non-decreasing without affecting principal i's payoff. Hence, truthful schedules have the attractive property that they always ensure to a given principal at least his stand-alone payoff.

Truthful Equilibria: We now investigate the set of equilibrium payoffs that can be sustained with truthful schedules. Let us now suppose that principal 2 offers such a non-negative truthful schedule: $t_2(q) = \max\{S_2(q) - V_2, 0\}$ where V_2 is a constant. Within principal 1's best-response correspondence, we may as well select, as suggested by [1], a truthful schedule which (when positive) reflects principal 1's preferences, namely $t_1(q) = \max\{S_1(q) - V_1, 0\}$ for some constant V_1. This truthful extension determines also the reservation payoff that the agent gets when refusing principal i's contract (see the right-hand side of (1)).

By definition, the fixed-fees (V_1, V_2) correspond to the principals' equilibrium payoffs. Using the binding participation constraint (1) for principal 1 and a similar condition coming from computing principal 2's best-response, these payoffs solve the system:

$$\max_{q\geq 0}\left\{\sum_{i=1}^{2} S_i(q) - \theta q\right\} - \sum_{i=1}^{2} V_i = \max\left\{0, \max_{q\geq 0}\{S_j(q) - \theta q\} - V_j\right\}, \text{ for } j = 1, 2. \tag{6}$$

The left-hand side above is the agent's payoff when taking both contracts whereas the right-hand side is his payoff when either not contracting at all or contracting only with one of the principals.

Denote by $\Sigma^T(\theta)$ the set of truthful equilibrium payoffs for the principals. [1, 6, 11 p.319] have shown that solutions to (6) describe in fact the non-empty core of a cooperative game with a super-additive characteristic function $W_K(\theta)$ defined for any subset $K \in \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ (using the convention $W_\emptyset(\theta) = 0$). This core is described by the linear constraints (3) and (4) which are obtained when using the definition of

[13] Note that $t_i(q)$ is non-negative, piecewise differentiable and thus admissible.

$W_K(\theta)$ in (6). From Proposition 1, full extraction of the agent's rent under complete information implies that principals share the aggregate surplus (Eq. (3)). The fact that the agent prefers taking both contracts to taking only one immediately implies (4).

From (3) and (4), in any truthful equilibrium, principal i gets at most his marginal contribution to the aggregate surplus of the grand-coalition and at least what he could gain by standing alone with the agent.

This characterization of $\Sigma^T(\theta)$ together with Proposition 2 yields immediately.

Proposition 3 *Under complete information, any payoff in $\Gamma(\theta)$ can be implemented as a truthful equilibrium payoff and thus:*

$$\Sigma^T(\theta) = \Sigma(\theta) = \Gamma(\theta).$$

Focusing on truthful schedules is enough to generate the entire set of equilibrium payoffs with admissible schedules under complete information. Truthful schedules both ensure efficiency at the equilibrium point and help each principal to guarantee himself his stand-alone payoff.

Beyond Truthful Equilibria: One may be interested in characterizing the complete set of equilibrium payoffs achievable with schedules having a fixed curvature, for instance linear schedules or, more generally, schedules that are less concave than the truthful ones. Indeed the concavity of the equilibrium schedules affects how sensitive the agent's output choice is to the decision to contract with either one or two principals, i.e., it affects how much a given principal can request from the agent without inducing him to contract only with the other principal. In that respect, it is important to understand how moving away from the truthful schedules may affect the set of equilibrium payoffs.

One such interesting and simple departure is to consider linearized contribution schedules – what we will see is equivalent to Lindahl pricing equilibria. To this end, notice then that any non-negative equilibrium schedule $t_i(q)$ which is concave on its non-negative part must also satisfy, when $t_i(q^*(\theta)) > 0$:

$$t_i(q) \leq \max\{t_i(q^*(\theta))+t_i'(q^*(\theta))(q-q^*(\theta)), 0\} = \max\{S_i(q^*(\theta))+S_i'(q^*(\theta))(q-q^*(\theta))-V_i, 0\}.$$

Consider thus the piecewise linear equilibrium schedules which support the efficient output and have constant slopes on their positive part defined as:

$$t_i(q) = \max\{0, S_i(q^*(\theta)) + S_i'(q^*(\theta))(q - q^*(\theta)) - V_i\} \tag{7}$$

where V_i is again principal i's payoff at $q = q^*(\theta)$. These schedules are linear around $q^*(\theta)$ and locally truthful. If he accepts both schedules, the agent is indifferent between producing any output and may as well produce $q^*(\theta)$. These schedules are also best-responses to each other when, from the binding participation constraint (1) (and a similar condition coming from principal 2's best-response), the principals' payoffs (V_1, V_2) satisfy:

$$W_{12}(\theta) - \sum_{i=1}^{2} V_i = \max\left\{0, \max_{q \geq 0}\{S_j(q^*(\theta)) - S_j'(q^*(\theta))q^*(\theta) + (S_j'(q^*(\theta)) - \theta)q - V_j\}\right\}. \quad (8)$$

The right-hand sides above achieve maxima at $q = 0$ since $S_j'(q^*(\theta)) < \theta$ for $j = 1, 2$: The agent produces nothing if he contracts only with one principal. The unique solution $(V_1^L(\theta), V_2^L(\theta))$ to (8) gives to principals the same payoffs as a non-strategic Lindahl equilibrium, namely $V_i^L(\theta) = S_i(q^*(\theta)) - q^*(\theta)S_i'(q^*(\theta))$. The Lindahl payoffs belong to the core of this public good economy and $(V_1^L(\theta), V_2^L(\theta)) \in \Sigma(\theta)$. Inserting these payoffs into (7) yields the final expression of the linear Lindahl prices in this strategic environment as $t_i^L(q) = S_i'(q^*(\theta))q$ for all $q \geq 0$.

As schedules come closer to the Lindhal prices above, principals become less "strategic" and end up getting a payoff that corresponds to their rent in the Lindhal equilibrium where they would adopt a price-taking behavior. Thus, the refinement of Lindahl (i.e., linear) contribution extensions is appealing both in simplicity and its strategic implications.

4 Asymmetric Information

Turning now to asymmetric information, we assume that the agent knows the value of θ before contracting with principals. Principals only know that this parameter is uniformly distributed on $[\underline{\theta}, \bar{\theta}]$.

Let $U_i(\theta)$ be the agent's information rent and $q_i(\theta)$ the corresponding optimal output when contracting only with principal i:

$$U_i(\theta) = \max_{q \geq 0}\{t_i(q) - \theta q\} \text{ and } q_i(\theta) = \arg\max_{q \geq 0}\{t_i(q) - \theta q\}$$

where $t_i(\cdot)$ is the nonnegative schedule offered by principal i. We assume for now the objective function is strictly concave, i.e., that $t_i(\cdot)$ is concave on its positive domain.[14] Given that admissible contributions are non-negative, we necessarily have $U_i(\theta) \geq 0$.

Let $U(\theta)$ be the agent's rent when taking both contracts and $q(\theta)$ be the corresponding

[14]Concavity holds for small values of $\bar{\theta} - \underline{\theta}$ or, more generally (as we can check below by looking at the expression of the equilibrium schedules) when $2S_i'' < S_{-i}''$, as assumed above.

output:

$$U(\theta) = \max_{q \geq 0} \left\{ \sum_{i=1}^{2} t_i(q) - \theta q \right\} \text{ and } q(\theta) = \arg\max_{q \geq 0} \left\{ \sum_{i=1}^{2} t_i(q) - \theta q \right\}.$$

Using the Envelope Theorem, we get:

$$\dot{U}(\theta) = -q(\theta), \tag{9}$$

where $q(\theta)$ is given by the first-order condition

$$\sum_{i=1}^{2} t_i'(q(\theta)) = \theta, \tag{10}$$

with the second-order condition

$$q(\theta) \text{ non-increasing.} \tag{11}$$

Therefore, $q(\cdot)$ is a.e. differentiable with $\dot{q}(\theta) \leq 0$ at any differentiability point.[15]

Principal 1's problem under asymmetric information becomes:[16]

$$(P_1)^{AS} : \max_{\{q(\cdot), U(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} \left[S_1(q(\theta)) + t_2(q(\theta)) - \theta q(\theta) - U(\theta) \right] \frac{d\theta}{\bar{\theta} - \underline{\theta}}$$

$$\text{subject to (9)-(11) and}$$

$$U(\theta) \geq U_2(\theta), \quad \forall \theta \in \Theta. \tag{12}$$

The ex post participation constraints (12) indicate that, whatever his type θ, the agent should prefer to take both contracts rather than only that offered by principal 2 or, again, refusing all contracts.[17]

A priori, solving $(P_1)^{AS}$ is difficult due to the type-dependent participation constraint (12). Finding this solution is facilitated by making two observations. First, because contributions are non-negative, the agent is always weakly better off contracting with both principals. Second, because marginal contributions are also positive, at least when $\bar{\theta} - \underline{\theta}$ is small enough,[18] the agent always chooses to produce more when contracting with both principals than with only one. Hence, t he slope of $U(\theta)$ is strictly greater than the slope of $U_i(\theta)$ and (12) binds only at $\bar{\theta}$. We thus have:

$$U(\theta) = \int_{\theta}^{\bar{\theta}} q(x)dx + U_2(\bar{\theta}). \tag{13}$$

[15]Monotonicity, together with (9) (or (10)), is also sufficient for incentive compatibility.

[16]Implicit in writing this problem is the fact that principal 1 gets a positive payoff when dealing with any type of the agent, even the worst one. This condition is made explicit in the Appendix.

[17]This latter option yields zero and is weakly dominated since $U_2(\theta) \geq 0$.

[18]See Footnote 21 below.

Taking expectations and integrating by parts, we obtain:

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)\frac{d\theta}{\bar{\theta}-\underline{\theta}} = \int_{\underline{\theta}}^{\bar{\theta}} (\bar{\theta}-\theta)q(\theta)\frac{d\theta}{\bar{\theta}-\underline{\theta}} + U_2(\bar{\theta}). \tag{14}$$

Inserting this expression of the agent's expected information rent into the maximand of $(P_1)^{AS}$ and optimizing pointwise with respect to $q(\theta)$ yields the necessary first-order condition satisfied by the equilibrium output $q^c(\theta)$:[19]

$$S_1'(q^c(\theta)) + t_2'(q^c(\theta)) = 2\theta - \underline{\theta}. \tag{15}$$

Summing this condition with a similar one obtained from analyzing principal 2's best-response and using the agent's incentive constraint (10), one finds that the equilibrium output $q^c(\theta)$ is the unique solution to:[20]

$$\sum_{i=1}^{2} S_i'(q^c(\theta)) - \theta = 2(\theta - \underline{\theta}) \geq 0. \tag{16}$$

Because $S_i''(\cdot) < 0$, $q^c(\theta)$ is monotonically decreasing with $\dot{q}^c(\theta) = \frac{3}{\sum_{i=1}^{2} S_i''(q^c(\theta))} < 0$. Let denote by $\theta^c(q)$ the inverse function defined over the range $[q^c(\bar{\theta}), q^c(\underline{\theta})]$:

$$\theta^c(q) = \frac{1}{3}\left(\sum_{i=1}^{2} S_i'(q) + 2\underline{\theta}\right).$$

Finally, using (10) and (15) yields the expression of principal 1's marginal contribution:[21]

$$t_1'(q^c(\theta)) = S_1'(q^c(\theta)) - (\theta - \underline{\theta}) \quad \forall \theta \in \Theta. \tag{17}$$

From (17), contributions are not everywhere truthful, i.e., $t_1'(q) \neq S_1'(q)$ at any q except for $q = q^c(\underline{\theta})$. Principal 1's marginal contribution is less than his marginal valuation for screening purposes except, of course, for the highest equilibrium output: a standard "no distortion at the top" result.

Adverse selection rationalizes the use of a nonlinear contribution schedule. Any output q in the range of $q^c(\cdot)$ corresponds to a principal 1's marginal contribution given by:

$$t_1'(q) = S_1'(q) - \theta^c(q) + \underline{\theta} = \frac{1}{3}(2S_1'(q) - S_2'(q) + \underline{\theta}). \tag{18}$$

Principal 1's marginal contribution in any differentiable equilibrium is uniquely defined by (18). Integrating (18) yields for any equilibrium output q:

$$t_1(q) = \frac{1}{3}(2S_1(q) - S_2(q) + \underline{\theta}q) - c_1 \tag{19}$$

[19] Sufficiency is checked in the Appendix.

[20] Different equilibrium payoffs may be sustained by varying the fixed-fees in each principal's schedule but all these equilibria correspond to the same output $q^c(\theta)$.

[21] Note that, for $\bar{\theta} - \underline{\theta}$ small enough, $t_1'(q) > 0$ for any output q in the range of $q^c(\cdot)$.

for some constant c_1 as long as this contribution remains positive,[22] i.e., for any equilibrium output as long as $\bar{\theta}-\underline{\theta}$ is small enough. Although equilibrium schedules may differ through their fixed-fees, they all have the same margin.

The set of possible values of the fixed-fee c_1 is determined by the reservation payoff $U_2(\bar{\theta})$ which in turn depends on how $t_2(q)$ is extended below the set of equilibrium outputs, i.e., for $q \leq q^c(\bar{\theta})$. Indeed, only outputs $q \leq q^c(\bar{\theta})$ matter to compute $U_2(\bar{\theta})$ since $t_1'(q) > 0$ for all q in the equilibrium range and thus $q_2(\bar{\theta}) \leq q(\bar{\theta})$ necessarily. Different extensions yield different values of $U_2(\bar{\theta})$ and may thus generate different divisions of the principals' equilibrium payoffs.

Our next proposition summarizes the common features of equilibria leaving to section 5 the characterization of equilibrium payoffs in the limit of a small uncertainty. Because we shall soon be interested in studying the convergence of the equilibrium schedules and payoffs as $\bar{\theta}$ goes to $\underline{\theta}$, we make explicit the dependence of the equilibrium schedules on the upper bound of the type support and denote $\{t_i(q|\bar{\theta})\}_{i\in\{1,2\}}$ these schedules.

Proposition 4 *Assume that $\bar{\theta}-\underline{\theta}$ is sufficiently small and θ is uniformly distributed on $[\underline{\theta},\bar{\theta}]$.*

- *At any equilibrium of the common agency game under asymmetric information with admissible schedules, principals offer nonlinear contribution schedules $\{t_i(q|\bar{\theta})\}_{i\in\{1,2\}}$ which are not everywhere truthful:*

$$t_i(q|\bar{\theta}) = \frac{1}{3}(2S_i(q) - S_{-i}(q) + \underline{\theta}q) - c_i > 0, \forall q \in [q^c(\bar{\theta}), q^c(\underline{\theta})] \tag{20}$$

- *In any such equilibrium, the agent with type θ produces an output $q^c(\theta)$.*
- *There is full extraction of the agent's rent at $\bar{\theta}$. More efficient types get a positive rent.*

From (20), one can see that principal 1's contribution is lower as principal 2's valuation increases. Equilibrium contributions reflect now both principals' preferences. This contrasts sharply with the case of complete information where truthful contributions depend only on own preferences. To understand this point, notice that each principal designs his own contribution with some concerns on extracting the agent's information rent. Starting from the cooperative outcome where principals would share equally the cost of getting information from the agent, a given principal has an incentive to induce less production to reduce this rent because he does not fully internalize the cost of underproduction for the other principal. This results in an excessively low output compared with a cooperative design of incentives. To induce less production, each principal reduces thus his marginal

[22] A similar expression is obtained for principal 2.

contribution below what he would offer under complete information for any equilibrium output q below $q^c(\underline{\theta}) = q^*(\underline{\theta})$. Formally, we have:

$$\begin{aligned} S_1'(q) - t_1'(q|\bar{\theta}) &= \frac{1}{3}(S_1'(q) + S_2'(q) - \underline{\theta}) \quad \forall q \in [q^c(\bar{\theta}), q^c(\underline{\theta})], \\ &\geq \frac{1}{3}(S_1'(q^c(\underline{\theta})) + S_2'(q^c(\underline{\theta})) - \underline{\theta}) = 0. \end{aligned}$$

Principal 1's marginal contribution is closer to his own marginal valuation when principal 2's marginal valuation is lower. When principal 2's preferences are more pronounced, principal 1 has indeed less incentives to contribute at the margin.

5 Equilibrium Sets and Their Limits

Fix the lower bound $\underline{\theta}$ of the support of the type distribution and let $\Sigma(\bar{\theta}, \underline{\theta})$ be the set of principals equilibrium' payoffs $(V_1^{AS}(\bar{\theta}), V_2^{AS}(\bar{\theta}))$ in admissible equilibria of the asymmetric information game when types are uniformly distributed on $[\underline{\theta}, \bar{\theta}]$. We are interested in describing the limit of the set of equilibrium payoffs as $\bar{\theta}$ converges towards $\underline{\theta}$.

Proposition 5 *Any limit of equilibrium payoffs of the asymmetric information game as the uniform distribution converges towards a Dirac mass at $\underline{\theta}$ is a truthful payoff. Reciprocally, any such payoff is the limit of equilibrium payoffs under asymmetric information:*

$$lim_{\bar{\theta} \to \underline{\theta}} \Sigma(\bar{\theta}, \underline{\theta}) = \Gamma(\underline{\theta}).^{23}$$

Equilibrium payoffs of the asymmetric information game converge towards complete information payoffs which are themselves fully described with truthful schedules. Reciprocally, when asymmetric information is small enough, one can modify truthful schedules for off the equilibrium outputs in such a way that equilibrium payoffs under asymmetric information converge towards any given truthful payoffs. This result does not tell us anything on whether the equilibrium schedules under asymmetric information also converge towards truthful schedules. To get sharper predictions on the form of equilibrium schedules, we will need to describe in more details their shapes under asymmetric information and show how this shape may be preserved as one converges towards the complete information outcome. This exercice is undertaken in the next section. It allows us to compute

[23] We define the limit set as

$$lim_{\bar{\theta} \to \underline{\theta}} \Sigma(\bar{\theta}, \underline{\theta}) = \{(W_1, W_2) | \exists (W_1(\bar{\theta}), W_2(\bar{\theta})) \in \Sigma(\bar{\theta}, \underline{\theta}) \text{ such that } (W_1, W_2) = lim_{\bar{\theta} \to \underline{\theta}} (W_1(\bar{\theta}), W_2(\bar{\theta}))\}.$$

the agent's reservation payoff when contracting with a single principal, an off-the equilibrium path, and this in turns affects how much each principal can claim from the agent and the other principal's equilibrium payoff.

6 Refinements Beyond "Truthfulness"

This section provides more positive results showing that asymmetric information is an effective device to select equilibrium outcomes when schedules are conveniently extended under asymmetric information. In both cases investigated below, truthful payoffs are not all achieved and more stringent selections are obtained when the type distribution converges towards a Dirac mass at $\underline{\theta}$.

The basic intuition behind this result is as follows. Remember first that, with truthful schedules, principals can at most get their incremental contributions to the overall surplus and get at least their stand-alone payoffs. Suppose that asymmetric information makes equilibrium schedules become flatter than truthful ones and that this property is preserved when looking at how equilibrium schedules converge as the distribution becomes closer to a Dirac. Then, very extreme distributions of the surplus between the principals are no longer sustained following the logic of section 3.

6.1 Natural-Extension Equilibria

We now specify a "natural" downward extension of the schedules in the asymmetric information game for $q \leq q^c(\bar{\theta})$ and show that such extension helps refining the set of equilibrium payoffs. Equation (19) delivers the expression of the contribution only for equilibrium outputs but its analytic formula can also be used off the equilibrium, especially for outputs less than $q^c(\bar{\theta})$, as long as the non-negativity constraint $t_i(q) \geq 0$ is preserved. With such *natural* extensions, contribution schedules keep track of the incentive externality that arises under asymmetric information.[24]

Definition 2 *A schedule of the common agency game under asymmetric information for θ being uniformly distributed on $[\underline{\theta}, \bar{\theta}]$ has a natural extension when:*[25]

$$t_i^N(q|\bar{\theta}) = \max\left\{\frac{1}{3}(2S_i(q) - S_{-i}(q) + \underline{\theta}q) - c_i, 0\right\} \quad \forall q \geq 0 \tag{21}$$

[24]To justify this focus on *natural* schedules and equilibria, one may think of the principals as insisting on mechanisms which are robust to perturbations of the information structure. Indeed, if the uniform distribution of types is extended over an increasingly large support (by increasing $\bar{\theta}$ and keeping $\underline{\theta}$ fixed), the marginal price given by (18) remains the same and does not depend on the support.

[25]Again, we make the dependence on the upper bound of the support explicit.

for some constant c_i.

In the limit, as $\bar{\theta}$ converges towards $\underline{\theta}$ (i.e., when one converges towards complete information at $\underline{\theta}$), natural-extension contributions are also best-responses to each other in the complete information game. Their expressions are now given by:

$$t_i^N(q) = \max\left\{\frac{1}{3}(2S_i(q) - S_{-i}(q) + \underline{\theta}q) + \frac{1}{3}W_{12}(\underline{\theta}) - V_i, 0\right\}, \forall q \text{ for } i = 1, 2 \qquad (22)$$

where V_i denotes now principal i's payoff in the complete information game.

When facing a pair of natural-extension contributions, the agent with type $\underline{\theta}$ still chooses the efficient output $q^*(\underline{\theta})$, just as with truthful schedules. Formula (22) shows that, contrary to truthful equilibria, natural-extension schedules only reflect the principals' preferences at the equilibrium point, and not elsewhere. Indeed, because of the incentive externality under asymmetric information, natural-extension marginal contributions are everywhere below the principals' valuations. For instance, principal 1's marginal contribution is $S_1'(q)$ minus an amount $\frac{1}{3}(S_1'(q)+S_2'(q)-\underline{\theta})$. Since this last term represents a fraction of marginal welfare which is concave, natural-extension contributions are flatter below the equilibrium outputs than truthful ones. Following the intuition given at the beginning of that section, relying on such flat schedules restricts the set of equilibrium payoffs.

Let denote by $\Sigma^N(\bar{\theta}, \underline{\theta})$ the set of equilibrium payoffs (V_1, V_2) achieved with natural-extension schedules under asymmetric information when θ is uniformly distributed on $[\underline{\theta}, \bar{\theta}]$. The set $\Sigma^N(\underline{\theta})$ of natural-extension equilibrium payoffs in the complete information game is defined as the limit of that set when $\bar{\theta}$ converges towards $\underline{\theta}$:

$$\Sigma^N(\underline{\theta}) = lim_{\bar{\theta}\to\underline{\theta}}\Sigma^N(\bar{\theta}, \underline{\theta}).$$

Proposition 6 *The set of natural-extension equilibrium payoffs is a strict subset of truthful payoffs:*

$$\Sigma^N(\underline{\theta}) \subsetneq \Sigma^T(\underline{\theta}).$$

6.2 Exponential Distribution

Looking now for an even more powerful selection, we build a common agency game under asymmetric information with equilibrium schedules which are almost linear so that, when the type distribution converges towards a Dirac mass at $\underline{\theta}$, the corresponding schedules and payoffs come close to the linear Lindahl prices and payoffs presented in section 3.

Consider the case where the agent's type is distributed on an unbounded interval $[\underline{\theta}, +\infty)$ with an exponential density $f(\theta) = \lambda \exp(-\lambda(\theta - \underline{\theta}))$ and cumulative distribution $F(\theta) = 1 - \exp(-\lambda(\theta - \underline{\theta}))$. λ close to zero corresponds to an improper uniform measure over $[\underline{\theta}, +\infty)$ which generalizes the uniform distribution on a bounded support. When λ goes to infinity, instead, the distribution is closer to a Dirac mass at $\underline{\theta}$. The benefit of working with such an unbounded support is that any positive output below $q^*(\underline{\theta})$ turns out to be an equilibrium output for some type (may be an arbitrarily large and very unlikely one however). The drawback is a loss in tractability since equilibrium schedules no longer have a closed form expression as in section 6.1 although their limits as the distribution converges towards a Dirac mass can be easily found.

Assume also that both principals are symmetric with $S_1(q) = S_2(q) = S(q)$. For further references, let denote by $q^c_\lambda(\theta)$ the monotonically decreasing solution to

$$2S'(q^c_\lambda(\theta)) = \theta + \frac{2}{\lambda}(\exp(\lambda(\theta - \underline{\theta})) - 1). \tag{23}$$

$q^c_\lambda(\theta)$ is simply the equilibrium output that arises in the common agency game under asymmetric information. Denote $\theta^c_\lambda(q)$ its inverse. Note that $q^c_\lambda(\underline{\theta}) = q^*(\underline{\theta})$ for all λ and $q^c_\lambda(\theta)$ converges towards zero as λ goes to infinity for any $\theta \neq \underline{\theta}$, i.e., as λ goes to $+\infty$.

Proposition 7 *Assume that* $\int_{\underline{\theta}}^{+\infty} q^c_\lambda(x)dx < +\infty$. *Then the following properties hold:*

- *There exists a unique equilibrium of the common agency game under asymmetric information. It corresponds to a symmetric contribution* $t_\lambda(q)$ *which is increasing and concave:*

$$t^c_\lambda(q) = S(q) - \frac{1}{\lambda}\int_0^q (\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx, \quad \forall q \in [0, q^c(\underline{\theta})]. \tag{24}$$

- *An agent with type* θ *produces an output* $q^c_\lambda(\theta)$ *and gets an information rent* $U^c_\lambda(\theta) = \int_\theta^{+\infty} q^c_\lambda(x)dx$.

With an exponential distribution, any output below $q^*(\underline{\theta})$ might be chosen in equilibrium by some type even though it might be with some tiny probability. This pins down fully the equilibrium schedule and avoids the indeterminacy problem that arose under asymmetric information with a uniform distribution on a bounded interval since the downward extension was left unspecified. Finally, note that, since equilibrium outputs can be arbitrarily small, schedules do not specify any positive fee at zero production.

Denote V^{AS}_λ the principals' (common) payoff in that symmetric equilibrium under asymmetric information for a fixed parameter λ. We are now ready to analyze the limit of such equilibria as the distribution converges towards a Dirac mass at $\underline{\theta}$:

Proposition 8 *The following properties hold:*

$$\lim_{\lambda \to +\infty} t^c_\lambda(q) = t^L(q) = S'(q^*(\underline{\theta}))q \text{ and } \lim_{\lambda \to +\infty} V^{AS}_\lambda = V^L(\underline{\theta}). \tag{25}$$

Lindahl prices and payoffs emerge as limits of strategic behavior between symmetric principals when uncertainty on the agent's type diminishes. Indeed, when the distribution puts more mass around $\underline{\theta}$ (i.e., when λ increases), it becomes almost optimal to reduce the output of all (very unlikely) types above $\underline{\theta}$ to zero as it can be seen by passing to the limit in the right-hand side of (23). To induce any type $\theta > \underline{\theta}$ to produce almost no output whereas type $\underline{\theta}$ produces the first-best amount, an equilibrium schedule in the asymmetric information game should have a slope close to $\underline{\theta}$ everywhere at least when λ is large enough. Therefore, the limit of such schedule as the distribution converges towards a Dirac mass at $\underline{\theta}$ is the symmetric linear Lindahl price stressed in section 3. In the limit, our procedure selects thus the Lindahl outcome.[26]

7 Non-Differentiable Equilibria

In this section, we first show that differentiability is key to get efficiency in equilibrium. Inefficient outcomes can be implemented with forcing contracts which are Nash equilibria. We characterize below the set of outputs and payoffs for such equilibria. Second, we show that equilibria with forcing contracts exist also under asymmetric information and that the whole set of complete information equilibria with forcing contracts is the limit of equilibria with forcing contracts under asymmetric information as the support of the distribution shrinks. Hence, the negative results of section 5 carries over to the non-differentiability case: Asymmetric information again fails to provide a selection device.

7.1 Complete Information

Consider the following non-negative forcing contracts:

$$\hat{t}_i(q) = \begin{cases} \hat{t}_i \geq 0 & \text{for } q = \hat{q} \\ 0 & \text{for } q \neq \hat{q} \end{cases} \tag{26}$$

[26] This model with an unbounded support is gives also some rationale for the natural extension of the schedules seen above for the case of a bounded support distribution. Indeed, note that (23) implies $2S'(q^c_\lambda(\theta)) \geq 3\theta - 2\underline{\theta}$ and thus $t'_\lambda(q) \leq t^{N'}(q)$ for all q. Moreover, $lim_{\lambda \to 0} t'_\lambda(q) = t^{N'}(q)$. The slope of schedules in natural-extension equilibria is thus an upper bound for the slope of the nonlinear schedules in equilibria corresponding to an exponential distribution with unbounded support. This upper bound is obtained when the exponential distribution converges towards the improper uniform density function over $[\underline{\theta}, +\infty)$.

with the added conditions

$$\sum_{i=1}^{2} \hat{t}_i - \theta\hat{q} = 0 \geq \max_i\{\hat{t}_i - \theta\hat{q}\}. \tag{27}$$

The set of such pairs $(\hat{t}_1, \hat{t}_2)$ is trivially non-empty. When both principals offer such contracts, the agent accepts both and produces $q = \hat{q}$ and principal i gets payoff $V_i = S_i(\hat{q}) - \hat{t}_i$.

Fix any output $\hat{q}$ such that

$$\sum_{i=1}^{2} S_i(\hat{q}) - \theta\hat{q} > \sum_{i=1}^{2} W_i(\theta). \tag{28}$$

The set of outputs $\hat{q}$ satisfying (28) contains at least $q^*(\theta)$ and it has a non-empty interior containing inefficient outputs.

Denote $\Gamma^{nd}(\theta, \hat{q})$ the set of payoff vectors (V_1, V_2) corresponding to a given output $\hat{q}$ satisfying (28) and defined as:

$$V_1 + V_2 = \sum_{i=1}^{2} S_i(\hat{q}) - \theta\hat{q} \tag{29}$$

$$V_i \geq W_i(\theta) > S_i(\hat{q}) - \theta\hat{q}, \quad i = 1, 2. \tag{30}$$

Proposition 9 *Assume that (28) holds. $\Gamma^{nd}(\theta, \hat{q})$ is the set of equilibrium payoffs achieved under complete information with the forcing contracts (26) and the equilibrium output $\hat{q}$.*

7.2 Asymmetric Information

Consider now the case of asymmetric information with types being uniformly distributed on a support $[\underline{\theta}, \bar{\theta}]$. Let denote by $\Sigma^{nd}(\bar{\theta}, \underline{\theta}, \hat{q})$ the set of equilibrium payoffs of the asymmetric information game achieved with forcing schedules implementing output $\hat{q}$. Under weak conditions on $\hat{q}$, any payoff achieved in the complete information game with forcing schedules implementing $\hat{q}$ is also the limit of payoffs achieved under asymmetric information with forcing schedules implementing the same output $\hat{q}$ for all types. First, the fact that $\Sigma^{nd}(\bar{\theta}, \underline{\theta}, \hat{q})$ is non-empty shows that asymmetric information and the associated requirement of incentive compatibility do not imply the differentiability of the equilibrium schedules. Contrary to the case of monopolistic screening environments where the (a.e.) differentiability of schedules follows from incentive compatibility, that differentiability is now a modeler's choice. If one principal offers a forcing non-differentiable contract, there is no reason to expect that the other principal's schedule will be differentiable at a best-response. Second, this result also confirms that asymmetric information does not offer any bite as a refinement even in the non-differentiable case.

Proposition 10 *Fix any* $\hat{q} \geq \max\{q_1^*(\underline{\theta}), q_2^*(\underline{\theta})\}$ *and assume that (28) holds at* $\underline{\theta}$. *We have:*

$$\Sigma^{nd}(\underline{\theta}, \hat{q}) = lim_{\bar{\theta} \to \underline{\theta}} \Sigma^{nd}(\bar{\theta}, \underline{\theta}, \hat{q}).$$

8 Conclusion

We showed that the set of equilibrium payoffs of the delegated common agency game under complete information that can be achieved when the principals' and the agent's choices are obtained at a differentiability point is entirely described by only considering equilibria with truthful schedules. The refinement of truthfulness can be replaced by the weaker requirement of local truthfulness at the equilibrium point only without changing the description of the equilibrium payoffs and without invalidating the efficiency result.

Asymmetric information can sometimes provide a rationale to select within that set of truthful payoffs. It has some power as a selection device if one insists on natural-extension equilibria or if types are exponentially distributed on an infinite support. In both cases, the limit equilibria obtained as the distribution of types becomes more concentrated is only a strict subset of all truthful payoffs. Roughly, those limit equilibria correspond to schedules which are flatter than truthful ones. These extensions make the most extreme distributions of payoffs between the principals impossible to sustain.

Finally, relaxing the differentiability requirement, inefficient equilibria may be sustained and asymmetric information does not provide any rationale for focusing on differentiable schedules or restrict the equilibrium set either.

Those results on the significant role of the extensions of contribution schedules on equilibrium characterization and their possible impact for applied works suggest that more research should be devoted to pin down what are the most relevant conjectures if one wants to build a robust theory of common agency with some predictive power.

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Appendix

• **Proof of Proposition 1:** Constraint (1) is necessarily binding at a best-response by principal 1. Inserting the value of the corresponding optimal transfer t_1 into principal 1's objective function, and optimizing with respect to output yields the first-order condition satisfied by the equilibrium output that principal 1 would like to induce:

$$S_1'(q^*(\theta)) + t_2'(q^*(\theta)) = \theta. \tag{A1}$$

A similar condition holds also for principal 2 at a best-response:

$$S_2'(q^*(\theta)) + t_1'(q^*(\theta)) = \theta. \tag{A2}$$

Moreover, the agent chooses optimally the level of output and thus the following first-order condition holds:

$$\sum_{i=1}^{2} t_i'(q^*(\theta)) = \theta. \tag{A3}$$

Summing (A1) and (A2) and using (A3) yields efficiency of public good provision.

Using (A1) and (A3) yields also the local truthfulness condition (2)– principals' marginal contributions at the equilibrium points are equal to their marginal valuations.

The fact that there is full extraction of the agent's rent is proven in the following separate lemma.

Lemma 1 *At any equilibrium with admissible schedules, there is full extraction of the agent's rent.*

Proof: Consider the fictitious cooperative game that is induced by the principals' equilibrium contribution schedules and the characteristic function that is simply the agent's payoff from contracting with any subset of principals. Specifically, define the characteristic function as $\tilde{W}_K(\theta) = \max_{q\geq 0}\left\{\sum_{i\in K} t_i(q) - \theta q\right\}$ for $K \in \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ and where $\tilde{W}_\emptyset(\theta) = 0$. Observe that $\tilde{W}_K(\theta) \geq 0$ for any K since contributions are non-negative. We first show that this cooperative game is superadditive, which implies convexity in this setting.[27] To this end, suppose that $q_1 \in \arg\max_{q\geq 0}\{t_1(q) - \theta q\}$ and $q_2 \in \arg\max_{q\geq 0}\{t_2(q) - \theta q\}$, and without loss of generality, suppose that $q_1 \geq q_2 \geq 0$. Then, we have

$$\tilde{W}_{12}(\theta) \equiv \max_{q\geq 0}\{t_1(q) + t_2(q) - \theta q\} \geq t_1(q_1) + t_2(q_1) - \theta q_1 \geq \tilde{W}_1(\theta) + \max_{q\geq 0}\{t_2(q) - \theta q\}.$$

[27] Shapley's [14] definition of convexity is the requirement that $\tilde{W}_K + \tilde{W}_{K'} \leq \tilde{W}_{K\cup K'} + \tilde{W}_{K\cap K'}$. In the present case, it is sufficient to show superadditivity of the characteristic function to prove that the game is convex: i.e., $\tilde{W}_{12}(\theta) \geq \tilde{W}_1(\theta) + \tilde{W}_2(\theta)$.

The first inequality follows from maximization; the second inequality follows from the transfer functions being nondecreasing so $t_2(q_1) \geq t_2(q_2) - \theta q_2$. Hence,

$$\tilde{W}_{12}(\theta) \geq \sum_{i=1}^{2} \tilde{W}_i(\theta). \tag{A4}$$

From the fact that (1) is binding at a best-response by either principal, we have $\tilde{W}_{12}(\theta) = \tilde{W}_i(\theta)$ for $i = 1, 2$. (A4) yields thus that $\tilde{W}_{12}(\theta) \leq 0$. Since we have also $0 \leq \tilde{W}_{12}(\theta)$, we finally obtain $\tilde{W}_{12}(\theta) = \tilde{W}_i(\theta) = 0$. The agent gets no rent. ■

• **Proof of Proposition 2:** Observe that efficiency of the equilibrium output and full extraction of the agent's rent at the equilibrium imply:

$$W_{12}(\theta) - \sum_{i=1}^{2} V_i = 0 = \max_q \{t_j(q) - \theta q\} \text{ for } j = 1, 2. \tag{A5}$$

Moreover, the fact that $t_{-i}(q)$ is non-negative implies

$$\max_q S_i(q) + t_{-i}(q) - \theta q \geq W_i(\theta) = \max_q S_i(q) - \theta q. \tag{A6}$$

Since there is full extraction of the agent's rent, $t_i(q^*(\theta)) + t_{-i}(q^*(\theta)) - \theta q^*(\theta) = 0$ and the left-hand side of (A6) is also principal P_i's equilibrium payoff $V_i = S_i(q^*(\theta)) - t_i(q^*(\theta))$. Therefore, (4) immediately follows.

This immediately yields that any equilibrium payoffs vector (V_1, V_2) belongs necessarily to $\Gamma(\theta)$ and $\Sigma(\theta) \subseteq \Gamma(\theta)$. ■

• **Proof of Proposition 3:** Clearly, the inclusion $\Sigma^T(\theta) \subseteq \Sigma(\theta)$ holds. Reciprocally, fix any payoff vector $(V_1, V_2) \in \Gamma(\theta)$. The truthful schedules $t_i(q) = \max\{0, S_i(q) - V_i\}$ $(i = 1, 2)$ are best-responses to each other and yields those payoffs (V_1, V_2). Hence, we have also $\Gamma(\theta) \subseteq \Sigma^T(\theta) \subseteq \Sigma(\theta)$.

We just check that $\Gamma(\theta)$ is non-empty.

Lemma 2 *The cooperative game with characteristic function $W_K(\theta)$ for $K \in \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ is convex and thus $\Gamma(\theta)$ is non-empty.*

Proof: The proof of convexity of the game is similar to that of Lemma 1 with $S_i(\cdot)$ being used instead of $t_i(\cdot)$. ■ ■

• **Proof of Proposition 4:** We solve $(P_1)^{AS}$ in the case of small uncertainty, i.e., $\bar{\theta} - \underline{\theta}$ small enough.

Lemma 3 *$U_2(\cdot)$ is non-increasing in θ. Denote $q_2(\theta) = \arg\max_{q\geq 0}\{t_2(q) - \theta q\}$, then:*

$$\dot{U}_2(\theta) = -q_2(\theta), \tag{A7}$$

$$q_2(\theta) \text{ non-increasing, } \dot{q}_2(\theta) \leq 0. \tag{A8}$$

Proof: Standard revealed preferences arguments show that

$$t_2(q_2(\theta)) - \theta q_2(\theta) \geq t_2(q_2(\theta')) - \theta q_2(\theta') \text{ and } t_2(q_2(\theta')) - \theta' q_2(\theta') \geq t_2(q_2(\theta)) - \theta' q_2(\theta)$$

for any pair (θ, θ) with $\theta > \theta'$. Summing yields $(\theta - \theta')(q_2(\theta) - q_2(\theta')) \leq 0$. Hence, $q_2(\theta)$ is non-increasing, and thus almost everywhere differentiable. (A7) follows immediately. That $U_2(\theta)$ is non-increasing in θ is immediate. ■

Note that, if $t_2''(\cdot) < 0$ (a property which holds in equilibrium for equilibrium outputs and which can be imposed on the extension for any non-negative off equilibrium outputs as long as the contribution is positive), $q_2(\theta)$ is given either by the first-order condition:

$$t_2'(q_2(\theta)) = \theta, \tag{A9}$$

or by the corner solution $q_2(\theta) = 0$.

Now, let us turn to the properties of the rent profile $U(\theta)$.

Lemma 4 *$U(\cdot)$ is non-increasing in θ with*

$$\dot{U}(\theta) = -q(\theta),$$
$$q(\theta) \text{ non-increasing, } \dot{q}(\theta) \leq 0.$$

Proof: The proof is similar to that of Lemma 3 and is thus omitted. ■

Because the agent with type θ finds it optimal to choose output $q(\theta)$, we have:

$$\sum_{i=1}^{2} t_i'(q(\theta)) = \theta. \tag{A10}$$

Comparing (A9) and (A10), we observe that $q(\theta) > q_2(\theta)$ if $t_1'(\cdot) > 0$ for equilibrium outputs. This monotonicity property of $t_1(\cdot)$ will be checked on the equilibrium schedules found below when $\bar{\theta} - \underline{\theta}$ is small enough.

Since $q(\theta) > q_2(\theta)$, the participation constraint (12) binds at $\bar{\theta}$ only. We can write

$$U(\theta) = \int_{\theta}^{\bar{\theta}} q(x)dx + U(\bar{\theta}), \tag{A11}$$

where

$$U(\bar{\theta}) = \max_{q \geq 0} \left\{ \sum_{i=1}^{2} t_i(q) - \bar{\theta} q \right\} = U_2(\bar{\theta}). \tag{A12}$$

A similar condition would be obtained by looking at principal 2's best response.

The equilibrium output in (15) is then derived using the argument in the text.

The first-order condition (15) is also sufficient because principal 2's objective is concave. Indeed $S_1''(q) + t_2''(q) < 0$ when $t_2'(q) = S_2'(q) - \theta^c(q) + \underline{\theta}$ for any equilibrium output q in the range of $q^c(\cdot)$. This concavity property also holds for off the equilibrium outputs given that $t_2(q)$ is extended conveniently on its positive domain. ■

Full extraction of the $\bar{\theta}$-agent's rent: We begin the analysis of the set of equilibrium payoffs with a Lemma which will be useful in the proof of Proposition 6 below.

Consider the equilibrium strategies defined in (20) and their admissible extensions for any out of equilibrium output $q \leq q^c(\bar{\theta})$. We have:

Lemma 5 *The constants (c_1, c_2) defined in (20) satisfy the system of linear equations*

$$\frac{1}{3}\left(\sum_{i=1}^{2} S_i(q^c(\bar{\theta})) + 2\underline{\theta} q^c(\bar{\theta}) \right) - \bar{\theta} q^c(\bar{\theta}) - \sum_{i=1}^{2} c_i = \max_i \left\{ \max_{0 \leq q \leq q^c(\bar{\theta})} \{t_i(q) - \bar{\theta} q\} \right\}. \tag{A13}$$

Proof: Equation (A13) is obtained using (A12) for principal 1 and a similar condition for principal 2. We have by definition:

$$U(\bar{\theta}) = \max_{q \geq 0} \left\{ \sum_{i=1}^{2} t_i(q) - \bar{\theta} q \right\} = \sum_{i=1}^{2} t_i(q^c(\bar{\theta})) - \bar{\theta} q^c(\bar{\theta}).$$

Using the expressions of the equilibrium contributions at $q^c(\bar{\theta})$ from (19) yields:

$$U(\bar{\theta}) = \frac{1}{3}\left(\sum_{i=1}^{2} S_i(q^c(\bar{\theta})) + 2\underline{\theta} q^c(\bar{\theta}) \right) - \bar{\theta} q^c(\bar{\theta}) - \sum_{i=1}^{2} c_i. \tag{A14}$$

When computing $U_i(\bar{\theta})$, we have:

$$U_i(\bar{\theta}) = \max_{0 \leq q} \{t_i(q) - \bar{\theta} q\} = \max_{0 \leq q \leq q^c(\bar{\theta})} \{t_i(q) - \bar{\theta} q\} \tag{A15}$$

since for outputs greater than $q^c(\bar{\theta})$, we have $t_i'(q) < t_i'(q^c(\bar{\theta})) = \bar{\theta} - t_{-i}'(q^c(\bar{\theta})) < \bar{\theta}$ where the first inequality follows from $t_i(\cdot)$ being concave and the second from the definition of $q^c(\bar{\theta})$ and the fact that $t_{-i}'(q^c(\bar{\theta})) > 0$ when $\bar{\theta}$ is close enough to $\underline{\theta}$. The equality (A15) holds for any extension of the equilibrium tariff $t_i(q)$ for any out of equilibrium output $q \leq q^c(\bar{\theta})$. This ends the proof of Lemma 5. ■

Lemma 6 *At any equilibrium with admissible schedules, there is full extraction of the $\bar{\theta}$-agent's rent:*

$$U(\bar{\theta}) = \frac{1}{3}\left(\sum_{i=1}^{2} S_i(q^c(\bar{\theta})) + 2\underline{\theta} q^c(\bar{\theta})\right) - \bar{\theta} q^c(\bar{\theta}) - \sum_{i=1}^{2} c_i = 0. \tag{A16}$$

Proof: Consider the fictitious cooperative game with the characteristic function $\tilde{W}_K(\bar{\theta}) = \max_{q\geq 0}\left\{\sum_{i\in K} t_i(q) - \bar{\theta} q\right\}$ for $K \in \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ with $\tilde{W}_\emptyset(\bar{\theta}) = 0$. Note that only the $\bar{\theta}$ agent's participation constraint is binding at a best-response for both principals and that $\tilde{W}_K(\bar{\theta})$ is superadditive. The rest of the proof follows then exactly that of Lemma 1 and is thus omitted. ■

This ends the proof of Proposition 4. ■

• **Proof of Proposition 5:** First, denote principal i's equilibrium payoff under asymmetric information when dealing with the least efficient agent as

$$V_i^{AS}(\bar{\theta}|\bar{\theta}) = S_i(q^c(\bar{\theta})) - t_i(q^c(\bar{\theta})|\bar{\theta}). \tag{A17}$$

As we will see below, this payoff is necessarily positive when $\bar{\theta} - \underline{\theta}$ is small enough, justifying therefore the fact that principal i finds it worth contracting even with the worst type of the agent (see Footnote 16).

Second, observe that principal i's expected payoff under asymmetric information and a uniform distribution on $[\underline{\theta}, \bar{\theta}]$ can be defined as:

$$V_i^{AS}(\bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} (S_i(q^c(\theta)) - t_i(q^c(\theta)|\bar{\theta}))\frac{d\theta}{\bar{\theta} - \underline{\theta}}. \tag{A18}$$

Using the fact that there is full extraction at $\bar{\theta}$ only, $V_i^{AS}(\bar{\theta})$ can be rewritten as

$$V_i^{AS}(\bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} (S_i(q^c(\theta)) + t_{-i}(q^c(\theta)|\bar{\theta}) - (2\theta - \underline{\theta})q^c(\theta))\frac{d\theta}{\bar{\theta} - \underline{\theta}}. \tag{A19}$$

Lower bound on equilibrium payoffs: We first determine a lower bound on principal i's payoff in any equilibrium. To do so, suppose that this principal contracts alone with the agent. Under this exclusive arrangement, principal i gets an expected payoff which can be defined as:

$$V_i^{ASE}(\bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} (S_i(q_i^E(\theta)) - t_i^E(q_i^E(\theta)|\bar{\theta}))\frac{d\theta}{\bar{\theta}-\underline{\theta}} \tag{A20}$$

where the optimal output that this principal would implement under asymmetric information solves

$$S_i'(q_i^E(\theta)) = 2\theta - \underline{\theta}.$$

Denote $\Theta_i^E(q)$ the inverse of the monotonically strictly decreasing function $q_i^E(\theta)$. The corresponding nonlinear schedule taking into account that the agent with type $\bar{\theta}$ gets zero rent is such that:

$$t_i^E(q|\bar{\theta}) = \Theta_i^E(q)q + \int_{\Theta_i^E(q)}^{\bar{\theta}} q_i^E(x)dx. \tag{A21}$$

Proceeding as above and using (A20) and (A21) yields a first expression of $V_i^{ASE}(\bar{\theta})$ as

$$V_i^{ASE}(\bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} (S_i(q_i^E(\theta)) - (2\theta-\underline{\theta})q_i^E(\theta))\frac{d\theta}{\bar{\theta}-\underline{\theta}}. \tag{A22}$$

Using (A19), (A22) and the fact that $t_{-i}(q|\bar{\theta})$ is non-negative gives us:

$$V_i^{ASE}(\bar{\theta}) \leq \int_{\underline{\theta}}^{\bar{\theta}} (S_i(q_i^E(\theta)) + t_{-i}(q_i^E(\theta)|\bar{\theta}) - (2\theta-\underline{\theta})q_i^E(\theta))\frac{d\theta}{\bar{\theta}-\underline{\theta}} \leq V_i^{AS}(\bar{\theta}). \tag{A23}$$

This gives a lower bound on principal i's payoff in the asymmetric information game.

Convergence as $\bar{\theta} \to \underline{\theta}$: Integrating by parts the right-hand side of (A18), we obtain:

$$V_i^{AS}(\bar{\theta}) = V_i^{AS}(\bar{\theta}|\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} (S_i'(q^c(\theta)) - t_i'(q^c(\theta)|\bar{\theta}))\dot{q}^c(\theta)\left(\frac{\theta-\underline{\theta}}{\bar{\theta}-\underline{\theta}}\right)d\theta.$$

Taking into account (18), we get:

$$V_i^{AS}(\bar{\theta}) = V_i^{AS}(\bar{\theta}|\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} \dot{q}^c(\theta)(\theta-\underline{\theta})^2\frac{d\theta}{\bar{\theta}-\underline{\theta}}. \tag{A24}$$

Similarly, we have:

$$V_i^{ASE}(\bar{\theta}) = V_i^{ASE}(\bar{\theta}|\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} \dot{q}_i^E(\theta)(\theta-\underline{\theta})^2\frac{d\theta}{\bar{\theta}-\underline{\theta}}. \tag{A25}$$

Since we are interested in the limit of the set of equilibrium payoffs as $\bar{\theta}$ converges towards $\underline{\theta}$, note that the second terms on the right-hand sides of both (A24) and (A25) go to zero.

Turning now to the determination of $V_i^{AS}(\bar{\theta}|\bar{\theta})$ on the right-hand side of (A24), and using full extraction at $\bar{\theta}$, (A13) can be rewritten as:

$$\sum_{i=1}^{2} S_i(q^c(\bar{\theta})) - \bar{\theta}q^c(\bar{\theta}) - \sum_{i=1}^{2} V_i^{AS}(\bar{\theta}|\bar{\theta}) = 0. \tag{A26}$$

Also, using (A23) and (A24), we get

$$V_i^{AS}(\bar{\theta}|\bar{\theta}) \geq V_i^{ASE}(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \dot{q}^c(\theta)(\theta - \underline{\theta})^2 \frac{d\theta}{\bar{\theta} - \underline{\theta}}. \tag{A27}$$

As $\bar{\theta}$ converges towards $\underline{\theta}$, $q^c(\bar{\theta})$ converges towards $q^*(\underline{\theta})$, $q_i^E(\bar{\theta})$ converges towards $q_i^*(\underline{\theta})$ and $V_i^{ASE}(\bar{\theta})$ converges thus towards $W_i(\underline{\theta})$. Therefore, the set of payoffs $(V_1^{AS}(\bar{\theta}|\bar{\theta}), V_2^{AS}(\bar{\theta}|\bar{\theta}))$ satisfying (A26) and (A27) converges towards the set of payoffs (V_1, V_2) satisfying (3) and (4). Hence, we have:

$$lim_{\bar{\theta}\to\underline{\theta}} \Sigma(\bar{\theta}, \underline{\theta}) \subseteq \Gamma(\underline{\theta}).$$

Reciprocally, fix a payoff vector $(V_1, V_2) \in \Gamma(\underline{\theta})$. We construct equilibrium schedules of the common agency game under asymmetric information with payoffs for the principals which converge towards (V_1, V_2) as limits. We distinguish between the cases where (V_1, V_2) belongs to the interior of $\Gamma(\underline{\theta})$ ((4) is strict for all i) and where it lies on the boundary of that set. We treat each case in turn.

(V_1, V_2) belongs to the interior of $\Gamma(\underline{\theta})$: Consider the admissible schedules (differentiable at $q^c(\bar{\theta})$):

$$t_i(q|\bar{\theta}) = \begin{cases} S_i(q) - \int_{q^c(\bar{\theta})}^{q} (\theta^c(x) - \underline{\theta})dx - V_i^{AS}(\bar{\theta}|\bar{\theta}) \text{ if } q \geq q^c(\bar{\theta}) \text{ with } \theta^c(q) = \underline{\theta} \text{ for } q \geq q^c(\underline{\theta}), \\ \max\{0, S_i(q) - (\bar{\theta} - \underline{\theta})(q - q^c(\bar{\theta})) - V_i^{AS}(\bar{\theta}|\bar{\theta})\} \text{ if } q \leq q^c(\bar{\theta}). \end{cases} \tag{A28}$$

An agent with type $\bar{\theta}$ chooses $q^c(\bar{\theta})$ when taking both contracts and principal i gets a payoff $V_i^{AS}(\bar{\theta}|\bar{\theta})$ when contracting with this agent. These schedules also form a Nash equilibrium of the common agency game under asymmetric information (when $\bar{\theta} - \underline{\theta}$ is small enough) if there is full extraction of the type $\bar{\theta}$ agent's surplus. This implies first that (A26) holds. The pair $(V_1^{AS}(\bar{\theta}|\bar{\theta}), V_2^{AS}(\bar{\theta}|\bar{\theta}))$ such that

$$V_i^{AS}(\bar{\theta}|\bar{\theta}) = V_i + \frac{1}{2}\left(\sum_{i=1}^{2} S_i(q^c(\bar{\theta})) - \bar{\theta}q^c(\bar{\theta}) - W_{12}(\underline{\theta})\right) \tag{A29}$$

satisfies this property. Full extraction implies also that the agent must get a negative payoff when taking only one contract which implies that $V_i^{AS}(\bar{\theta}|\bar{\theta})$ must also satisfy:

$$V_i^{AS}(\bar{\theta}|\bar{\theta}) \geq \max_{q \leq q^c(\bar{\theta})} \{S_i(q) - (\bar{\theta} - \underline{\theta})(q - q^c(\bar{\theta})) - \bar{\theta}q\}. \tag{A30}$$

Using (A29), this means that one must have

$$V_i \geq \frac{1}{2}\left(W_{12}(\underline{\theta}) - \left(\sum_{i=1}^{2} S_i(q^c(\bar{\theta})) - \bar{\theta}q^c(\bar{\theta})\right)\right) + \max_{q \leq q^c(\bar{\theta})} \{S_i(q) - (\bar{\theta} - \underline{\theta})(q - q^c(\bar{\theta})) - \bar{\theta}q\}. \tag{A31}$$

This condition holds for $\bar{\theta} - \underline{\theta}$ small enough when $V_i > W_i(\underline{\theta})$ because the right-hand side of (A31) converges towards $W_i(\underline{\theta})$. Turning to (A27), this condition is also satisfied when $V_i^{AS}(\bar{\theta}|\bar{\theta})$ is defined by (A29) as $\bar{\theta}$ converges towards $\underline{\theta}$.

(V_1, V_2) *on the boundary of* $\Gamma(\underline{\theta})$*:* Consider now the case where $V_i = W_i(\underline{\theta})$ for one i (and thus $V_{-i} = W_{12}(\underline{\theta}) - W_i(\underline{\theta}) > W_{-i}(\underline{\theta})$). Consider the following values of $(V_1^{AS}(\bar{\theta}|\bar{\theta}), V_2^{AS}(\bar{\theta}|\bar{\theta}))$:

$$V_i^{AS}(\bar{\theta}|\bar{\theta}) = \max\left\{V_i^{ASE}(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \dot{q}^c(\theta)(\theta - \underline{\theta})^2 \frac{d\theta}{\bar{\theta} - \underline{\theta}}; \max_{q \leq q^c(\bar{\theta})} \{S_i(q) - (\bar{\theta} - \underline{\theta})(q - q^c(\bar{\theta})) - \bar{\theta}q\}\right\}$$

and

$$V_{-i}^{AS}(\bar{\theta}|\bar{\theta}) = -V_i^{AS}(\bar{\theta}|\bar{\theta}) + \sum_{i=1}^{2} S_i(q^c(\bar{\theta})) - \bar{\theta}q^c(\bar{\theta}).$$

Inserting these values into (A28) gives us a pair of admissible contributions which are best-responses to each other.

Note that $(V_i^{AS}(\bar{\theta}|\bar{\theta}), V_{-i}^{AS}(\bar{\theta}|\bar{\theta}))$ converges towards $(W_i(\underline{\theta}), W_{12}(\underline{\theta}) - W_i(\underline{\theta}))$ when $\bar{\theta}$ converges towards $\underline{\theta}$. By definition of $(V_1^{AS}(\bar{\theta}|\bar{\theta}), V_2^{AS}(\bar{\theta}|\bar{\theta}))$ above, conditions (A30) and (A27) both hold for i by construction and hold for $-i$ when $\bar{\theta} - \underline{\theta}$ is small enough also.

This shows that there exists a sequence of equilibria of the asymmetric information game whose payoffs for the principals converge again towards (V_1, V_2).

We finally have always:

$$\Gamma(\underline{\theta}) \subseteq lim_{\bar{\theta} \to \underline{\theta}} \Sigma(\bar{\theta}, \underline{\theta}).$$

■

• **Proof of Proposition 6:** We start by characterizing payoffs in natural-extension equilibria of the asymmetric information game, assuming that $\bar{\theta} - \underline{\theta}$ is small enough and θ is uniformly distributed on $[\underline{\theta}, \bar{\theta}]$. Given the definition of natural-extension schedules in (21), next lemma provides conditions that the constants (c_1, c_2) of those schedules must satisfy in equilibrium.

Lemma 7 *The constants* (c_1, c_2) *satisfy the following linear constraints:*

$$\sum_{i=1}^{2} c_i = \max_{q \geq 0}\left\{\frac{1}{3}\left(\sum_{i=1}^{2} S_i(q) + 2\underline{\theta}q\right) - \bar{\theta}q\right\}, \tag{A32}$$

$$\max_{q\geq 0}\left\{\frac{1}{3}\left(2S_i(q)-S_{-i}(q)+\underline{\theta}q\right)-\bar{\theta}q\right\}\leq c_i \tag{A33}$$

Proof: From (17), we have $t_1'(q|\bar{\theta}) = S_1'(q)+\underline{\theta}-\theta^c(q)$, for any positive output q at a natural-extension equilibrium. Thus $t_1'(q|\bar{\theta})$ remains positive when $\bar{\theta}-\underline{\theta}$ is small enough. A similar argument holds also for $t_2'(q|\bar{\theta})$. Because $t_i'(q|\bar{\theta})\geq 0$ for $i=1,2$, we can use Lemmas 5 and 6 to get:

$$\frac{1}{3}\left(\sum_{i=1}^{2}S_i(q^c(\bar{\theta}))+2\underline{\theta}q^c(\bar{\theta})\right)-\bar{\theta}q^c(\bar{\theta})-\sum_{i=1}^{2}c_i$$
$$=0\geq\max_i\left\{\max_{0\leq q}\left\{\frac{1}{3}(2S_i(q)-S_{-i}(q)+\underline{\theta}q)-c_i-\bar{\theta}q\right\}\right\} \tag{A34}$$

which yields (A32) and (A33). ■

Note first that, when $\bar{\theta}$ converges to $\underline{\theta}$, (A32) becomes

$$\sum_{i=1}^{2}c_i=\frac{1}{3}\max_{q\geq 0}\left\{\sum_{i=1}^{2}S_i(q)-\underline{\theta}q\right\}=\frac{1}{3}W_{12}(\underline{\theta}). \tag{A35}$$

Observing that the principals' payoffs in the complete information game can be written as $V_i=c_i+\frac{1}{3}W_{12}(\underline{\theta})$, (A35) becomes:

$$\sum_{i=1}^{2}V_i=W_{12}(\underline{\theta}). \tag{A36}$$

Let us turn to (A33) when $\bar{\theta}$ converges to $\underline{\theta}$. Using (A35), it becomes:

$$c_i\leq\frac{1}{3}W_{12}(\underline{\theta})-\frac{1}{3}\max_{q\geq 0}\{2S_{-i}(q)-S_i(q)-2\underline{\theta}q\}.$$

Thus, we get

$$V_i\leq\frac{2}{3}W_{12}(\underline{\theta})-\frac{1}{3}\max_{q\geq 0}\{2S_{-i}(q)-S_i(q)-2\underline{\theta}q\}. \tag{A37}$$

The claim in Proposition 6 is true whenever the right-hand side of (A37) is strictly less than $W_{12}(\underline{\theta})-W_{-i}(\underline{\theta})$, i.e.:

$$W_{12}(\theta)>-\max_{q\geq 0}\{2S_{-i}(q)-S_i(q)-2\underline{\theta}q\}+3\max_{q\geq 0}\{S_{-i}(q)-\underline{\theta}q\},\ \text{for } i=1,2. \tag{A38}$$

Take for instance $i=1$ and note that:

$$\sum_{i=1}^{2}S_i(q)-\underline{\theta}q+2S_2(q)-S_1(q)-2\underline{\theta}q=3(S_2(q)-\underline{\theta}q),\ \text{for all } q\geq 0.$$

Hence

$$W_{12}(\theta) + 2S_2(q) - S_1(q) - 2\underline{\theta}q > 3(S_2(q) - \underline{\theta}q), \text{ for all } q \geq 0, \ q \neq q^*(\theta).$$

In particular, since $q_2^*(\theta) \neq q^*(\theta)$ when $S_1' > 0$, we get

$$W_{12}(\theta) + 2S_2(q_2^*(\theta)) - S_1(q_2^*(\theta)) - 2\underline{\theta}q_2^*(\theta) > 3(S_2(q_2^*(\theta)) - \underline{\theta}q_2^*(\theta))$$

which implies (A38). ■

• **Proof of Proposition 7:** With an exponential distribution on an infinite support, principal 1's problem under asymmetric information becomes:

$$(P_1)^{AS}: \max_{\{q(\cdot),U(\cdot)\}} \int_{\underline{\theta}}^{+\infty} [S(q(\theta)) + t_2(q(\theta)) - \theta q(\theta) - U(\theta)] \lambda \exp(-\lambda(\theta - \underline{\theta}))d\theta$$

subject to (9)-(11) and (12)

where we assume that this principal finds it optimal to contribute whatever the agent's type (we show optimality of this strategy below).

Our first step is to replace the participation constraint (12) by a boundary condition.

Lemma 8 *When principals offer admissible schedules, the boundary condition*

$$lim_{\theta\to+\infty}U(\theta) = lim_{\theta\to+\infty}U_2(\theta) \tag{A39}$$

is equivalent (12).

Proof: $U(\cdot)$ and $U_2(\cdot)$ are both monotonically decreasing in θ and non-negative, so that they have a limit as θ converges towards $+\infty$. Passing to the limit, (12) implies:

$$lim_{\theta\to+\infty}U(\theta) \geq lim_{\theta\to+\infty}U_2(\theta).$$

Suppose that the inequality is strict, then principal 1 can improve his payoff by reducing all transfers uniformly by some strictly positive amount ϵ, a contradiction with optimality. Hence, (A39) holds.

Reciprocally, suppose (A39) holds. Because principals offer admissible schedules such that $t_i'(q) \geq 0$ for any $q \geq 0$, we have also $\dot{U}(\theta) = -q(\theta) \leq \dot{U}(\theta) = -q_2(\theta)$ and (A39) implies (12). ■

We will thus replace accordingly (12) by (A39) into $(P_1)^{AS}$ to get a new infinite horizon problem $(P_1)^{AS'}$. An admissible pair $(U(\theta), q(\theta))$ is one that satisfies (9)-(11) and (A39). As usual, we neglect the monotonicity condition (11) that will be checked ex post on the equilibrium output schedule.

Denote $\alpha(\theta)$ the co-state variable for (9). The Hamiltonian of this problem is:

$$H(\alpha, U, q, \theta) = [S(q) + t_2(q) - \theta q - U]\,\lambda \exp(-\lambda(\theta - \underline{\theta})) - \alpha q$$

and its Lagrangian, which is strictly concave in (U, q) as

$$L(\mu, \alpha, U, q, \theta) = H(\alpha, U, q, \theta) + \mu(U - lim_{\theta\to+\infty} U_2(\theta)).$$

We follow Seierstad and Sydsaeter (1987, Chapter 3, Theorem 12) in writing the necessary Halkin conditions for optimality as:

$$S_1'(q(\theta)) + t_2'(q(\theta)) = \theta + \frac{\alpha(\theta)}{\lambda}\exp(\lambda(\theta - \underline{\theta})), \tag{A40}$$

$$\dot{\alpha}(\theta) = \lambda \exp(-\lambda(\theta - \underline{\theta})) \text{ with } \alpha(\underline{\theta}) = 0. \tag{A41}$$

Denote $(\tilde{U}(\theta), \tilde{q}(\theta))$ the optimal solution satisfying (9), (A39), (A40) and (A41). At the equilibrium, we will have of course $(\tilde{U}(\theta), \tilde{q}(\theta)) = (U_\lambda^c(\theta), q_\lambda^c(\theta))$.

From (A41), we also get

$$\alpha(\theta) = 1 - \exp(-\lambda(\theta - \underline{\theta})). \tag{A42}$$

From Seierstad and Sydsaeter [13, Chapter 3, Theorem 13], the necessary conditions are also sufficient (given the strict concavity of the Lagrangian in (U, q)) if

$$\underline{lim}_{\theta\to+\infty}\alpha(\theta)(\tilde{U}(\theta) - U(\theta)) \geq 0 \tag{A43}$$

for any admissible profile $U(\theta)$.

Since we have $lim_{\theta\to+\infty}U(\theta) = lim_{\theta\to+\infty}\tilde{U}(\theta) = lim_{\theta\to+\infty}U_2(\theta)$ and this limit is finite in equilibrium (see below the condition $\tilde{U}(\theta) = U_\lambda^c(\theta) = \int_{\underline{\theta}}^{+\infty} q_\lambda^c(x)dx < +\infty$), for any admissible profile $U(\theta)$ and $\alpha(\theta) < 1$, (A43) holds.

Inserting (A42) into (A40) yields:

$$S'(q(\theta)) + t_2'(q(\theta)) = \theta + \frac{1}{\lambda}(\exp(\lambda(\theta - \underline{\theta})) - 1), \tag{A44}$$

Adding up (A44) and a similar equation obtained by looking at principal 2's best-response yields (23). Note that $lim_{\theta\to+\infty}q_\lambda^c(\theta) = 0$.

Still using (A44), we finally get the symmetric expression of the marginal contributions of both principals as:

$$t_i'(q) = t'(q) = S'(q) - \frac{1}{\lambda}(\exp(\lambda(\theta_\lambda^c(q) - \underline{\theta})) - 1).$$

Denote the equilibrium schedules as $t_i(q) = S(q) - \int_0^q \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx - V_i$. Using (A39) at the equilibrium, we find:

$$lim_{\theta\to+\infty}\left\{\max_{q\geq 0} 2S(q) - \theta q - 2\int_0^q \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx - \sum_{i=1}^{2} V_i\right\} = 0$$
$$\geq lim_{\theta\to+\infty}\left\{\max_{q\geq 0} S(q) - \theta q - \int_0^q \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx - V_i\right\} \quad \text{for } i = 1, 2. \tag{A45}$$

Note that $\max_{q\geq 0} 2S(q) - \theta q - 2\int_0^q \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx = 2S(q^c_\lambda(\theta)) - \theta q^c_\lambda(\theta) - 2\int_0^{q^c_\lambda(\theta)} \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx$ and denote $\varphi(\theta)$ this expression. We have $\dot{\varphi}(\theta) = -q^c_\lambda(\theta) < 0$ and $\varphi(\theta) \geq 0$ so that $lim_{\theta\to+\infty}\varphi(\theta)$ exists. Given that $lim_{\theta\to+\infty}q^c_\lambda(\theta) = 0$ and $S(0) = 0$ by assumption, this limit is necessarily zero. Moreover the max on the right-hand side of (A45) is achieved for a positive output $q^{'}_\lambda(\theta)$ defined as $S'(q^{'}_\lambda(\theta)) = \theta + \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(q^{'}_\lambda(\theta)) - \underline{\theta}))$. Denote $\varphi'(\theta) = \max_{q\geq 0} S(q) - \theta q - \int_0^q \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx = S(q^{'}_\lambda(\theta)) - \theta q^{'}_\lambda(\theta) - \int_0^{q^{'}_\lambda(\theta)} \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx$. We have $\dot{\varphi}'(\theta) = -q^{'}_\lambda(\theta) < 0$ and $\varphi'(\theta) \geq 0$ so that $lim_{\theta\to+\infty}\varphi'(\theta)$ exists. Given that $lim_{\theta\to+\infty}q^{'}_\lambda(\theta) = 0$ and $S(0) = 0$ by assumption, this limit is necessarily again zero. Inserting those findings into (A45) yields immediately $V_1 = V_2 = 0$.

Moreover, since by assumption $\int_{\underline{\theta}}^{+\infty} q^c_\lambda(x)dx < +\infty$, the agent's rent can be written as $U^c_\lambda(\theta) = \int_\theta^{+\infty} q^c_\lambda(x)dx$. Then, note that the net surplus that each principal withdraws from contracting with any type θ is $\phi(\theta) = 2S(q^c_\lambda(\theta)) - \theta q^c_\lambda(\theta) - \int_0^{q^c_\lambda(\theta)} \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx - \int_\theta^{+\infty} q^c_\lambda(x)dx$ with $\dot{\phi}(\theta) = \frac{\dot{q}^c_\lambda(\theta)}{\lambda}(\exp(\lambda(\theta - \underline{\theta})) - 1) < 0$ so that $\phi(\theta)$ is decreasing and $\phi(\theta) = \varphi(\theta) + \int_0^{q^c_\lambda(\theta)} \frac{1}{\lambda}(\exp(\lambda(\theta^c_\lambda(x) - \underline{\theta})) - 1)dx - \int_\theta^{+\infty} q^c_\lambda(x)dx$. Since $lim_{\theta\to+\infty}q^c_\lambda(\theta) = lim_{\theta\to+\infty}\varphi(\theta) = lim_{\theta\to+\infty}\int_\theta^{+\infty} q^c_\lambda(x)dx = 0$, we have $lim_{\theta\to+\infty}\phi(\theta) = 0$. Hence, $\phi(\theta)$ is positive and it is worth for each principal to contract with any type. ■

• **Proof of Proposition 8:** Fix any $q \geq 0$. First observe that $t'_\lambda(q) = \frac{\theta^c_\lambda(q)}{2}$ and

$$\theta^c_\lambda(q) - \underline{\theta} = \frac{1}{\lambda}log(2S'(q) - \theta^c_\lambda(q)) + \frac{log(\lambda)}{\lambda} \approx_{\lambda\to+\infty} \frac{log(\lambda)}{\lambda}.$$

Hence,

$$lim_{\lambda\to+\infty}\theta^c_\lambda(q) = \underline{\theta} = 2S'(q^*(\underline{\theta})).$$

This yields the result in the text. ■

• **Proof of Proposition 9:** By deviating and inducing another output $q \neq \hat{q}$, the best that principal i can do is to offer a contract which extracts the agent's rent and makes him produce $q^*_i(\theta) = \arg\max_{q\geq 0} S_i(q) - \theta q$. Principal i gets $W_i(\theta)$ with such a deviation. The agent prefers to take this new offer and getting nothing from principal $-i$ than producing $\hat{q}$ since $0 \geq \hat{t}_{-i} - \theta\hat{q}$ when (27) holds. Hence, the deviation is non-profitable when:

$$V_i \geq W_i(\theta)$$

from which we derive immediately that principal i's payoff is non-negative. From (27), the principals' payoffs (V_1, V_2) must also satisfy:

$$\sum_{i=1}^{2} S_i(\hat{q}) - \theta\hat{q} - \sum_{i=1}^{2} V_i = 0 \geq \max_i \{S_i(\hat{q}) - \theta\hat{q} - V_i\}.$$

This completes the characterization of payoffs with forcing contracts under complete information given in (29) and (30). ■

• **Proof of Proposition 10:** Suppose that principal 2 offers a forcing contract $\hat{t}_2(q) = \begin{cases} \hat{t}_2 \geq 0 & \text{for } q = \hat{q} \\ 0 & \text{for } q \neq \hat{q} \end{cases}$. We want to prove that a best-response to that forcing contract is also a forcing contract $\hat{t}_1(q) = \begin{cases} \hat{t}_1 \geq 0 & \text{for } q = \hat{q} \\ 0 & \text{for } q \neq \hat{q} \end{cases}$ even under asymmetric information, when $\bar{\theta}$ is close enough to $\underline{\theta}$.

Consider a deviation by principal 1 to a schedule $t_1(q)$ that would implement an output $q(\theta) \leq \hat{q}$ for all θ under asymmetric information. The best of such deviations solves $(P_1)^{AS}$ with the participation constraint (12) being now expressed as:

$$U(\theta) \geq U_2(\theta) = \max\{0, \hat{t}_2 - \theta\hat{q}\}, \quad \forall \theta \in \Theta. \tag{A46}$$

Assuming that $U_2(\theta) = 0$ for all θ (we come back on this assertion below), the solution is straightforward. It entails a monotonically decreasing output $q_1^E(\theta)$ such that $S_1'(q_1^E(\theta)) = 2\theta - \underline{\theta}$ and $q_1^E(\theta) \leq q_1^*(\theta) < \hat{q}$ by assumption. Principal 1's expected payoff for that deviation is $V_1^{ASE}(\bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} \left(S_1(q_1^E(\theta)) - (2\theta - \underline{\theta})\, q_1^E(\theta)\right) \frac{d\theta}{\bar{\theta} - \underline{\theta}}$ which converges towards $W_1(\underline{\theta})$ from below when $\bar{\theta}$ converges towards $\underline{\theta}$.

Consider now a deviation by principal 1 to a schedule that would induce an output $q(\theta)$ such that $q(\theta^*) \geq \hat{q}$ for some θ^*. Because $q(\theta)$ must be non-increasing, this deviation satisfies $q(\theta) \geq \hat{q}$ for all $\theta \leq \theta^*$ and we may define θ^* such that $q(\theta) \leq \hat{q}$ for $\theta \in [\theta^*, \bar{\theta}]$. Since $q_1^E(\theta) \leq q_1^*(\theta) \leq q_1^*(\underline{\theta}) < \hat{q}$ by assumption, since principal 1's objective function under asymmetric information is concave and maximized pointwise at $q_1^E(\theta)$, and since $\hat{t}_2 \geq 0$ the best of such deviations by principal 1 has necessarily $q(\theta) = \hat{q}$ for all $\theta \leq \theta^*$. Principal 2's forcing contract is also taken by the agent with type $\theta \leq \theta^*$. Denote $\hat{t}_1$ the corresponding fixed payment on that interval. Principal 1 gets payoff $\int_{\underline{\theta}}^{\theta^*} (S_1(\hat{q}) - \hat{t}_1) \frac{d\theta}{\bar{\theta} - \underline{\theta}}$ on that interval. Types $\theta \in [\theta^*, \bar{\theta}]$ do not take principal 2's contract for the best of such deviations and they receive the best exclusive option that principal 1 can offer, i.e., a contract that implements $q_1^E(\theta)$ and yields to that principal an expected profit $\int_{\theta^*}^{\bar{\theta}} \left(S_1(q_1^E(\theta)) - (2\theta - \underline{\theta})\, q_1^E(\theta)\right) \frac{d\theta}{\bar{\theta} - \underline{\theta}}$ on that interval. Moreover, the agent's rent is continuous at type θ^* so that:

$$\hat{t}_1 + \hat{t}_2 - \theta^*\hat{q} = \int_{\theta^*}^{\bar{\theta}} q^E(x)dx.$$

Taking into account this expression, the optimal cut-off θ^* chosen at a best-response by principal 1 is defined as the solution to the following optimization problem

$$\max_{\theta^*\in\Theta} \int_{\underline{\theta}}^{\theta^*} \left(S_1(\hat{q}) + \hat{t}_2 - \theta^*\hat{q} - \int_{\theta^*}^{\bar{\theta}} q^E(x)dx \right) \frac{d\theta}{\bar{\theta}-\underline{\theta}} + \int_{\theta^*}^{\bar{\theta}} \left(S_1(q_1^E(\theta)) - (2\theta - \underline{\theta})\, q_1^E(\theta) \right) \frac{d\theta}{\bar{\theta}-\underline{\theta}}.$$

The derivative of that maximand is positive at $\theta^* = \bar{\theta}$, i.e., all types choose the contract $(\hat{t}_1, \hat{q})$, when:

$$S_1(\hat{q}) + \hat{t}_2 \geq (2\bar{\theta} - \underline{\theta})\hat{q} + S_1(q_1^E(\bar{\theta})) - (3\bar{\theta} - 2\underline{\theta})q_1^E(\bar{\theta}). \tag{A47}$$

Note that this optimality condition implies also that principal 1's payoff is greater when inducing all types to choose a forcing contract than to deviate and deal exclusively with him (choosing $\theta^* = \underline{\theta}$), i.e., $S_1(\hat{q}) - \hat{t}_1 \geq V_1^{ASE}(\bar{\theta})$. A condition similar to (A47) follows from computing principal 2's best-response:

$$S_2(\hat{q}) + \hat{t}_1 \geq (2\bar{\theta} - \underline{\theta})\hat{q} + S_2(q_2^E(\bar{\theta})) - (3\bar{\theta} - 2\underline{\theta})q_2^E(\bar{\theta}). \tag{A48}$$

To induce all types to choose the forcing contracts $(\hat{t}_1, \hat{q})$ and $(\hat{t}_2, \hat{q})$, equilibrium transfers must finally satisfy:

$$\hat{t}_1 + \hat{t}_2 = \bar{\theta}\hat{q}. \tag{A49}$$

Using equations (A47), (A48) and (A49), an equilibrium with forcing contracts inducing production $\hat{q}$ by all types is such that:

$$\sum_{i=1}^{2} S_i(\hat{q}) - \bar{\theta}\hat{q} \geq 2(\bar{\theta} - \underline{\theta})\hat{q} + \sum_{i=1}^{2} S_i(q_i^E(\bar{\theta})) - (3\bar{\theta} - 2\underline{\theta})q_i^E(\bar{\theta}). \tag{A50}$$

When $\bar{\theta}$ converges towards $\underline{\theta}$, the left-hand side converges towards $\sum_{i=1}^{2} S_i(\hat{q}) - \underline{\theta}\hat{q}$ whereas the right-hand side converges towards $\sum_{i=1}^{2} W_i(\underline{\theta})$. Therefore, (A50) holds when $\bar{\theta}$ is close enough to $\underline{\theta}$ since we have assumed that (28) is satisfied. Hence, for $\bar{\theta} - \underline{\theta}$ close enough to zero, there exists $(\hat{t}_1, \hat{t}_2)$ that satisfy conditions (A47) to (A49) and hold the pair of forcing contracts as an equilibrium of the common agency game under asymmetric information. In particular, using (A48) and (A49) we get, for any θ, $\hat{t}_2 - \theta\hat{q} \leq \hat{t}_2 - \underline{\theta}\hat{q} \leq S_2(\hat{q}) - \bar{\theta}\hat{q} - (S_2(q_2^E(\bar{\theta})) - (3\bar{\theta} - 2\underline{\theta})q_2^E(\bar{\theta}))$ which is negative for $\bar{\theta} - \underline{\theta}$ small enough since it converges towards $S_2(\hat{q}) - \underline{\theta}\hat{q} - W_2(\underline{\theta}) < 0$. This validates the way we solved $(P_1)^{AS}$ above since $U_2(\theta) = 0$ for all θ when $\bar{\theta} - \underline{\theta}$ is small enough.

Rewriting equations (A47), (A48) and (A49) in terms of the principals' payoffs $(V_1(\bar{\theta}), V_2(\bar{\theta}))$ in that forcing equilibrium under asymmetric information, we obtain respectively:

$$V_i(\bar{\theta}) = S_i(\hat{q}) - \hat{t}_i \geq (\bar{\theta} - \underline{\theta})\hat{q} + S_i(q_i^E(\bar{\theta})) - (3\bar{\theta} - 2\underline{\theta})q_i^E(\bar{\theta}), \quad i = 1, 2, \tag{A51}$$

$$V_1(\bar{\theta}) + V_2(\bar{\theta}) = \sum_{i=1}^{2} S_i(\hat{q}) - \bar{\theta}\hat{q}. \tag{A52}$$

Finally, the set $\Sigma^{nd}(\bar{\theta}, \underline{\theta}, \hat{q})$ of payoffs of the asymmetric information game $(V_1(\bar{\theta}), V_2(\bar{\theta}))$ satisfying (A51) and (A52) is such that:

$$lim_{\bar{\theta}\to\underline{\theta}}\Sigma^{nd}(\bar{\theta}, \underline{\theta}, \hat{q}) \subseteq \Sigma^{nd}(\underline{\theta}, \hat{q}).$$

Reciprocally, consider a pair of forcing contracts $\{\hat{t}_1(q), \hat{t}_2(q)\}$ as in (26) that is an equilibrium under complete information at $\theta = \underline{\theta}$ and yields payoffs (V_1, V_2) in $\Sigma^{nd}(\underline{\theta}, \hat{q})$. Suppose first that $V_i > W_i(\underline{\theta})$ for $i = 1, 2$. Define the following payoffs

$$V_i(\bar{\theta}) = V_i - \frac{1}{2}(\bar{\theta} - \underline{\theta})\hat{q}.$$

These are equilibrium payoffs with forcing contracts under asymmetric information when (A51) and (A52) hold. The corresponding transfers $\hat{t}_i(\bar{\theta}) = S_i(\hat{q}) - V_i(\bar{\theta})$ satisfy then (A47), (A48) and (A49). Note that, by construction, $(V_1(\bar{\theta}), V_2(\bar{\theta}))$ satisfy (A52). Moreover, since $V_i > W_i(\underline{\theta})$, since $V_i(\bar{\theta})$ converges towards V_i as $\bar{\theta}$ goes to $\underline{\theta}$ and since the right-hand side of (A51) converges towards $W_i(\underline{\theta})$, $V_i(\bar{\theta})$ satisfies (A51) when $\bar{\theta} - \underline{\theta}$ is small enough.

Consider now the case where $V_i = W_i(\underline{\theta})$ for either $i = 1$ or 2. Then, by construction, $V_{-i} = \sum_{i=1}^{2} S_i(\hat{q}) - \underline{\theta}\hat{q} - W_i(\underline{\theta})$. Let $V_i(\bar{\theta}) = (\bar{\theta} - \underline{\theta})\hat{q} + S_i(q_i^E(\bar{\theta})) - (3\bar{\theta} - 2\underline{\theta})q_i^E(\bar{\theta})$ and $V_{-i}(\bar{\theta}) = \sum_{i=1}^{2} S_i(\hat{q}) - \bar{\theta}\hat{q} - V_i(\bar{\theta})$. $(V_1(\bar{\theta}), V_2(\bar{\theta}))$ converges towards (V_1, V_2) as $\bar{\theta} - \underline{\theta}$ goes to zero. By construction, $V_i(\bar{\theta})$ satisfies (A51) and $V_{-i}(\bar{\theta})$ satisfies a similar inequality for $\bar{\theta} - \underline{\theta}$ small enough. Indeed, the right-hand side of (A51) for subscript $-i$ converges towards $W_{-i}(\underline{\theta})$ and $V_{-i}(\bar{\theta})$ converges towards $\sum_{i=1}^{2} S_i(\hat{q}) - \underline{\theta}\hat{q} - W_i(\underline{\theta}) > W_{-i}(\underline{\theta})$ when (28) holds at $\underline{\theta}$. Hence, $(V_1(\bar{\theta}), V_2(\bar{\theta}))$ is a vector of principals' payoffs in a forcing equilibrium of the asymmetric information game and we have:

$$\Sigma^{nd}(\underline{\theta}, \hat{q}) \subseteq lim_{\bar{\theta}\to\underline{\theta}}\Sigma^{nd}(\bar{\theta}, \underline{\theta}, \hat{q}).$$

■

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