

Ideological Uncertainty and Lobbying Competition¹

David Martimort² and Aggey Semenov³

This version: 19th October 2007

Abstract: Polarized interest groups compete to influence a decision-maker through monetary contributions. The decision-maker chooses a one-dimensional policy and has private information about his ideal point. Competition between interest groups under asymmetric information yields a rich pattern of equilibrium strategies and payoffs. Policies are systematically biased towards the decision-maker's ideal point and it may sometimes lead to a “laissez-faire” equilibrium where the decision-maker is freed from any influence. Either the most extreme decision-makers or the most moderate ones may get information rent depending on their ideological bias. The market for influence may exhibit segmentation with interest groups keeping an unchallenged influence on ideologically close-by decision-makers. Interest groups refrain from contributing when there is too much uncertainty on the decision-maker's ideology and when the latter is ideologically too far away.

Keywords: Lobbying Competition, Common Agency, Asymmetric Information, Contributions.

JEL Classification : D72; D82.

¹We thank Parkash Chander, Jacques Drèze, Didier Laussel, Michel Le Breton, Jean-François Mertens, Humberto Moreira, Larry Samuelson, Lars Stole, Jean Tirole, Shlomo Weber and especially Shan Zhao for useful comments on an earlier version. We also thank two referees and the Editor Antonio Merlo whose comments have improved the paper.

²Toulouse School of Economics (IDEI-GREMAQ and EHESS), France.

³National University of Singapore, Singapore.

1 Introduction

The pluralistic view of politics promulgated by political scientists over the last few decades has highlighted the important role played by special interests in shaping political decision-making.¹ One major thrust of this literature is that competition between interest groups induces efficient and balanced policies, therefore, large political representation of private interests ensures implemented policies better represent social welfare. The current paradigm to model pluralistic politics, namely the “common agency model” of policy formation, provides theoretical support to this conclusion.² Under common agency, competing lobbying groups (the principals) non-cooperatively design contributions to influence a decision-maker (the agent). At equilibrium, all organized interest groups actively contribute irrespectively of their ideological distances to the decision-maker. This decision-maker chooses from the array of contributions to accept and the appropriate policy to implement. Under complete information, this decentralized political process is efficient, i.e., the aggregate payoff to the grand-coalition comprising all principals and their common agent is maximized.

However, many observers have forcefully argued that politics is plagued with transaction costs resulting both from asymmetric information and the limited ability to enforce contributions.³ Both casual and empirical evidence show that interest groups have limited knowledge about legislators’ preferences and that this imperfect knowledge determines whether groups contribute or not, and, if they do, the amount of their contributions. For instance, Kroszner and Stratman (1999) and Stratman (2005) pointed out that interest groups adopt different attitudes when dealing with young legislators whose preferences are concealed, compared to older legislators whose ideology has been better revealed by their past responses to earlier PACs contributions. Moreover, those contributions seem to increase over time as legislators clarify their ideologies. Among others, Kroszner and Stratman (1998) and Wright (1996) also provided strong evidence suggesting that the ideological distance between an interest group and a decision-maker is the key to assess the importance of contributions. Wright (1996), for example, noticed that the National Automobile Dealers Association (NADA) contributed far more heavily to the conservatives (78,1 percent) than to the liberals (about 12 percent) during the election cycles 1979-1980 and 1981-1982. He suggested that such pattern could be explained by the close ideological connection between members of NADA (who are generally pro-business) with the conservative politicians.

¹Dahl (1961), Lowi (1979), Moe (1981), Truman (1952) and Wilson (1973) among others.

²This model was cast earlier by Bernheim and Whinston (1986) in a complete information abstract framework and then adapted by Grossman and Helpman (1994) and others towards various political economy applications (international trade, tax policies, regulation, etc.).

³Dixit (1996).

In this paper, we revisit the common agency model of pluralistic politics. However, to account for reported evidence, we introduce asymmetric information between interest groups and decision-makers whose ideologies are privately known. Asymmetric information creates transaction costs in the relationships between interest groups and decision-makers. These costs are reflected by limited activism by some interest groups, segmentation of the market for influence, and weak contributions. Our theory thus provides a richer pattern of equilibrium behaviors than what has been predicted by complete information models; a pattern better suited to reconcile theory with evidence.

First, far from aggregating the preferences of interest groups efficiently, equilibrium policies might not be as responsive to private interests as in a complete information and frictionless world. This phenomenon can be so pronounced that a “laissez-faire” equilibrium might arise when the decision-maker, free from any influence, chooses his own ideal policy. Second, due to frictions caused by asymmetric information, interest groups may choose to target only decision-makers who are ideologically close and thus easier to influence. They eschew contributing to ideologically distant decision-makers. This feature explains the prevalence of one-lobby influence on ideologically adjacent decision-makers in environments where interest groups are sufficiently polarized.

To obtain these results, we model competition between two interest groups who want to influence a common decision-maker. For instance, these principals can be thought of as two legislative Committees ambitious to influence a regulatory agency, or as two lobbying groups dealing with an elected political decision-maker who chooses a one-dimensional policy on behalf of the society.⁴ The policy variable can be a regulated price, an import tariff, a wage level or a number of permits depending on the application. The interest groups and the decision-maker all have quadratic preferences with ideal points in that one-dimensional policy space, with the interest groups’ being located on opposite sides of the policy space. Groups thus favor policy distortions in opposite directions. This assumption allows us to parameterize equilibrium patterns, first, with respect to, the ideological distances between interest groups and the decision-maker; and second, with respect to the degree of polarization between groups. Trade policy gives an interesting example of such diametrically opposed interest groups. Typically, upstream producers prefer low tariffs for the downstream products, while downstream producers prefer high tariffs.⁵

The groups’ preferences are common knowledge. On the contrary, much ideological uncertainty surrounds the legislator’s ideological bias. Hence, groups non-cooperatively design contributions to influence the decision-maker’s choice as under complete informa-

⁴The observation that policies result from the influence of two (major) competing groups is common in the empirical literature. See for instance Kroszner and Stratman (1999) and Gawande et al. (2005).

⁵Gawande et al. (2005) provided an interesting empirical study along these lines.

tion and also to elicit revelation of his preferences.

The decision-maker trades off the contributions he receives from the interest groups against his own ideological bias. Stronger ideological biases are expected on issues of much relevance to the public at large. This is because the decision-maker's ideology plays an important role in macroeconomic issues such as unemployment, debt and inflation - if the decision-maker is elected. A weaker ideological bias is more likely on issues which appear to be too technical to the public. Regulatory and trade policies are two examples.

Since a given interest group does not internalize the impact of contribution modification by competing groups intended to extract the decision-maker's information rent, therefore in equilibrium there is excessive rent extraction. The interest groups' marginal contributions no longer reflect their marginal utility as under complete information; they are always too low or can be null. As a result, equilibrium policies are pushed towards the decision-maker's ideal point. Lobbying competition, far from reaching efficiency and balanced policies, shifts the balance of power significantly towards the decision-maker. The decision-maker thus may find it more attractive to refuse some contributions, especially when he is ideologically too distant from the contributing interest group.

Altogether, asymmetric information and interest groups' competition provide a less optimistic view of the political process than predicted by the complete information models of pluralistic politics. Equilibrium patterns under asymmetric information might significantly differ depending on the importance the decision-maker gives to his own ideology, the extent of ideological uncertainty, and the degree of polarization between competing interest groups. When the decision-maker's ideological bias is sufficiently large and groups are sufficiently polarized, there exists a unique equilibrium. This equilibrium is characterized by greater contributions and information rents for extreme decision-makers and also by segmented areas of influence for interest groups. When polarization between interest groups is small compared to ideological uncertainty, it is likely that interest groups end up being ideologically closer to each other than to the decision-maker. Their only concern is then to coordinate their respective contributions. Multiple equilibria may arise from miscoordination problems. Contributions overlap and the more moderate decision-makers secure greater contributions and more information rent. Counter-lobbying always arises in equilibrium.

Let us now review the relevant literature. Following Grossman and Helpman (1994), Dixit, Grossman and Helpman (1997) and others, the large extent of the common agency literature explaining interest groups' behavior has focused on complete information. Few contributions have explicitly analyzed the transaction costs of contracting due to asymmetric information between principals and agent and how the pattern of contributions and the political landscape is affected. Under moral hazard, i.e., when the decision-maker's

action (or effort) is nonverifiable, Dixit (1996) argued that a bureaucracy subject to the conflicting influences of various legislative committees/interest groups may end up having very low incentives for exerting effort, but because of an implicit focus on models of intrinsic common agency, groups never eschew contributions.

Le Breton and Salanié (2003) considered a decision-maker who has private information on the weight given to social welfare in his objective function. A binary political decision may be favored by some groups while others oppose to it. Groups only contribute to lobby for their most preferred option. In such contexts, an interest group is active only upon learning that it is not too costly to move the decision-maker away from social welfare maximization. Martimort and Semenov (2007a) generalizes this insight to the case of asymmetric lobbies and a more continuous policy.

Epstein and O'Halloran (2004) also studied a common agency model under asymmetric information with spatial preference similar to ours but with the decision-maker's ideal point taking only two possible values. Restricting the analysis to direct mechanisms, their main concerns is on the incentives of asymmetric interest groups to collude.

Restricted participation of interest groups and low equilibrium contributions have already found other rationales in the literature. Mitra (1999) (under complete information) and Martimort and Semenov (2007b) (under symmetric but incomplete information) investigated how the equilibrium payoffs from the common agency game where interest groups play with public officials determine whether interest groups find it worthwhile to enter the political arena when they face some exogenous fixed-cost of organization. Merging a common agency model of lobbying with legislative bargaining, Helpman and Persson (2001) demonstrated that equilibrium contributions may be quite small and still have a significant impact on policies. Lastly, Felli and Merlo (2006) argued that some interest groups may not participate in the lobbying process in a model with active voters and candidates choosing from which lobbies they want support. They also found some tendency towards moderate policies but for reasons different from ours.

Section 2 presents the model and the complete information benchmark. Section 3 analyzes a hypothetical benchmark where interest groups form a coalition and cooperatively design their contributions. Section 4 deals with the case of competing interest groups and characterize various equilibrium patterns. Section 5 summarizes our main results and proposes possible extensions. Proofs are relegated to the Appendix.

2 Model and Complete Information Benchmark

Preferences, information, contracts: Two polarized interest groups P_1 and P_2 (thereafter principals) simultaneously offer contributions to influence a political decision-maker (thereafter the common agent). Let $q \in \mathbb{R}$ be a one-dimensional policy parameter controlled by the decision-maker. Interest group P_i ($i = 1, 2$) has a quasi-linear utility function over policies and monetary contributions t_i which is given by:

$$V_i(q, t_i) = -\frac{1}{2}(q - a_i)^2 - t_i.$$

The parameter a_i is P_i 's ideal point in the one-dimensional policy space. The principals' ideal points are symmetrically located around the origin, and without loss of generality, we use the normalization $a_1 = -a_2 = 1$.

The decision-maker has similar quasi-linear preferences given by:

$$U\left(q, \sum_{i=1}^2 t_i, \theta\right) = -\frac{\beta}{2}(q - \theta)^2 + \sum_{i=1}^2 t_i,$$

where $\beta \geq 0$. The parameter β characterizes how the agent trades off contributions against his own ideological bias. As β increases, the agent puts more emphasis on ideology. This implies that principals have to contribute more to influence the agent.⁶

The decision-maker has private information on his ideal policy θ . This parameter is uniformly distributed on a set $\Theta = [-\delta, \delta]$ centered around zero with δ representing the degree of ideological uncertainty. The decision-maker has moderate views when his ideal point lies near the origin and more extreme otherwise.

As δ increases, polarization between the two organized groups on the particular policy decreases since it becomes more likely that the groups' ideal points are relatively closer to each other than to the decision-maker's.

Interest groups influence the decision-maker by credibly committing to offer non-negative contributions $t_i(q) \geq 0$, which specify group i 's monetary transfer as a function of the decision maker's policy choice q .⁷ With the agent's ideal point θ being privately known, contributions also serve (as usual in the screening literature) to elicit this parameter. The set of feasible transfer schedules consists of continuous, piece-wise differentiable functions of the policy variable q .

Timing: The timing of the game unfolds as follows:

⁶Goldberg and Maggi (1999) estimated β in the U.S. patterns of protection to be between 50 and 88. Bradford (2003) suggested a lower value of β but still higher than one.

⁷Campante and Ferreira (2007) considered the effect of imperfect commitment in a complete information framework.

- The decision-maker learns his ideal point θ ;
- Interest groups non-cooperatively offer contributions $\{t_1(q), t_2(q)\}$ to the decision-maker;
- The decision-maker decides whether to accept or refuse each of these offers. If he refuses all offers, he chooses his most preferred policy and obtains his status quo payoff of zero;
- Finally, if the decision-maker accepts the offer(s), he chooses the policy q and receives the corresponding payments.

Contracting takes place at the interim stage, i.e., once the decision-maker is already informed about his ideal point but lobbies do not have this information. Note that since contributions are non-negative, a weakly dominant strategy for the decision-maker is to always accept all offers. The agent's outside opportunity if he refuses all contributions is in fact his payoff if he chooses his own ideal policy. We refer to this setting as the “laissez-faire” outcome.

The equilibrium concept is subgame-perfect Nash equilibrium (thereafter SPNE or equilibrium in short) and we focus on pure strategy equilibria.

Common agency under complete information: Under complete information, there exist equilibria of the common agency game between lobbyists that achieve an efficient policy $q^{FB}(\theta)$ maximizing the aggregate payoff of the grand-coalition comprising both principals and their common agent:

$$q^{FB}(\theta) = \arg \max_{q \in \mathbb{R}} \left\{ \sum_{i=1}^2 V_i(q, t_i) + U \left(q, \sum_{i=1}^2 t_i, \theta \right) \right\} = \frac{\beta \theta}{\beta + 2}. \quad (1)$$

As the decision-maker's ideological bias is more pronounced (i.e., β increases), the optimal policy is shifted towards his own ideal point. Nevertheless, this policy always reflects both groups' preferences.

To achieve such efficient outcome, interest groups offer the truthful contributions (Bernheim and Whinston, 1986, Dixit, Grossman and Helpman, 1997):

$$t_i(q|\theta) = \max \left\{ 0, -\frac{1}{2}(q - a_i)^2 - C_i(\theta) \right\}, \text{ for all } q,$$

and for some constant $C_i(\theta)$ (that we leave unspecified for simplicity) where we make explicit the dependence of this schedule on θ since this parameter is common knowledge. The agent is always (at least weakly) better off accepting all such non-negative contributions, and with such payments, each interest group P_i makes the decision-maker residual

claimant for the payoff of the bilateral coalition. This ensures that preferences are aggregated efficiently.

Although strict concavity of the objective functions ensures uniqueness of the efficient policy, many possible distributions of equilibrium payoffs might be feasible. A payoff vector for the interest group corresponds to a pair $(C_1(\theta), C_2(\theta))$ which ensures that the agent is at least as well-off by taking both contracts than accepting only the truthful contribution of a single group.

Example: Consider the case of a moderate decision-maker whose ideology is known to be located at 0. The efficient policy is $q^{FB}(0) = 0$. Thus, there exists a unique equilibrium with truthful contributions given by:

$$t_i(q|0) = \max \left\{ 0, -\frac{1}{2}(q - a_i)^2 + \frac{\beta + 2}{2(\beta + 1)} \right\}, \text{ for all } q \text{ and } i = 1, 2.$$

It yields payoffs $U^{FB}(0) = \frac{1}{1+\beta}$ to the decision-maker and $V_i^{FB}(0) = -\frac{\beta+2}{2(\beta+1)}$ to each group.

3 Coalition of Interest Groups

As a benchmark, first consider the case where groups form a coalition (or merged principal) which cooperatively designs a contribution $t(q)$ to the decision-maker. Under asymmetric information, the decision-maker might get some information rent from privately knowing his ideal point and exaggerating his distance from that coalition's ideal point to raise more contributions. Of course, the optimal policy is no longer efficient as a result of a trade-off between rent extraction and allocative efficiency.

The merged entity now has an objective which can be written as:

$$V_M(q, t) = -\frac{1}{2}\{(q - 1)^2 + (q + 1)^2\} - t,$$

where t is now the groups' joint contribution. The merged entity's ideal point is located at zero. This principal gives more weight to ideology than each interest group taken separately.

For a given contribution $t(q)$,⁸ let us denote $U(\theta)$ as the agent's payoff when he accepts that contribution and $q(\theta)$ as the corresponding optimal policy. By definition, we have:

$$U(\theta) = \max_{q \in \mathbb{R}} \left\{ t(q) - \frac{\beta}{2}(q - \theta)^2 \right\} \text{ and } q(\theta) = \arg \max_{q \in \mathbb{R}} \left\{ t(q) - \frac{\beta}{2}(q - \theta)^2 \right\}.$$

⁸We know from the Taxation Principle (Rochet, 1985, for instance) that there is no loss of generality in considering that class of mechanisms in the case of monopolistic screening. Beyond the case of a merged principal, this Taxation Principle is also useful in non-cooperative screening environments. Peters (2001) and Martimort and Stole (2002) showed that there is no loss of generality in restricting the analysis to the case where principals compete through nonlinear contributions $t_i(q)$, $i = 1, 2$.

The following Lemma characterizes the implementable profiles $\{U(\theta), q(\theta)\}$.

Lemma 1 : $U(\theta)$ and $q(\theta)$ are almost everywhere differentiable and at any differentiability point

$$\dot{U}(\theta) = \beta(q(\theta) - \theta), \quad (2)$$

$$\dot{q}(\theta) \geq 0. \quad (3)$$

The set of incentive constraints in the decision-maker's problem is equivalent to conditions (2) and (3). Condition (2) is obtained from the first-order conditions of the agent's optimization problem (local optimality). Condition (3) guarantees the global optimality of the allocation. Together, these conditions fully describe the set of implementable allocations. Hence, under interim contracting, the merged entity solves the following problem:

$$(\mathcal{P}^M) : \quad \max_{\{q(\cdot), U(\cdot)\}} \int_{-\delta}^{\delta} \left(-\frac{1}{2}(q(\theta) - 1)^2 - \frac{1}{2}(q(\theta) + 1)^2 - \frac{\beta}{2}(q(\theta) - \theta)^2 - U(\theta) \right) \frac{d\theta}{2\delta},$$

subject to (2), (3) and

$$U(\theta) \geq 0, \quad \text{for all } \theta \in \Theta. \quad (4)$$

Since the decision-maker can only be moved away from his ideal point which gives him his reservation payoff if he receives a positive contribution, the non-negativity of contributions is equivalent to the interim participation constraint (4). In screening models, the subset of types where (4) binds plays an important role. Contrary to the standard screening models where agents have monotonic preferences in terms of the policy choice, the slope of the agent's rent does not necessarily keep a constant sign and the participation constraint may not necessarily bind at the end-points $\pm\delta$.⁹ To get a full description of the optimum and limit technicalities associated to the non-standard feature of the screening problem, we rely on the quadratic utility functions and the fact that the distribution of the agent's ideal point is uniform.

Proposition 1 : Assume that interest groups jointly design contributions. The optimal policy $q^M(\theta)$ and the decision-maker's information rent $U^M(\theta)$ both depend on the decision-maker's ideological bias.

Weak Ideological Bias, $0 < \beta < 2$.

- The optimal policy is inefficient and is distorted towards the decision-maker's ideal point: $q^M(\theta) = \frac{2\beta\theta}{\beta+2}$, for any $\theta \in \Theta$;

⁹This is reminiscent of the analysis of countervailing incentives explored by Lewis and Sappington (1989) and Maggi and Rodriguez-Clare (1995).

- Only moderate decision-makers get information rent, extreme ones do not. This information rent is non-negative, zero at both endpoints $\pm\delta$, and is strictly concave:

$$U^M(\theta) = \frac{\beta(2-\beta)}{2(\beta+2)}(\delta^2 - \theta^2), \text{ for any } \theta \in \Theta;$$

- The coalition of interest groups offers a positive and strictly concave contribution on its positive part which is maximized for the most moderate decision-maker:

$$t^M(q) = \max \left\{ 0, -\frac{(2-\beta)}{4}q^2 + \frac{\beta(2-\beta)}{2(\beta+2)}\delta^2 \right\}, \text{ for any } q.$$

Strong Ideological Bias, $\beta \geq 2$.

- The optimal policy always coincides with the decision-maker's ideal point: $q^M(\theta) = \theta$, for any $\theta \in \Theta$;
- The decision-maker's information rent is always zero: $U^M(\theta) = 0$, for any $\theta \in \Theta$;
- There is no contribution.

Under asymmetric information, policies are inefficient and are always closer to the agent's ideal policy than under complete information. The impact of asymmetric information under monopolistic screening is akin to an increase of the agent's bargaining weight within the grand-coalition, making his preferences more relevant to evaluate policy outcomes.

Intuitively, a decision-maker would like to pretend to be extreme because it would increase contributions aimed at shifting the policy towards the coalition's ideal point. These incentives are weakened when the policy is closer to the decision-maker's ideal point and the contribution designed for a moderate type is sufficiently large so that the contributions left to the extreme types are less attractive. By the same token, the decision-maker's rent decreases with his ideological distance with the merged principal. This is an important feature of optimal contracting under monopolistic screening.

More precisely, consider the hypothetical case where the merged entity still wants to reward the decision-maker for implementing an efficient policy $q^{FB}(\theta) = \frac{\beta\theta}{\beta+2}$, exactly as under complete information. To induce the decision-maker to reveal his ideal point, he must receive some information rent. The corresponding rent profile is rather concave with a steep increasing part on $[-\delta, 0]$ (with slope $\frac{2\beta\theta}{\beta+2}$) and a steep decreasing part on $[0, \delta]$ (with slope $-\frac{2\beta\theta}{\beta+2}$).¹⁰ This rent is costly from the point of view of the interest groups.

¹⁰The nonlinear contribution $t^{FB}(q) = \frac{\beta\delta^2}{\beta+2} - \beta q^2$ generates this profile and ensures that all types participate with the most extreme decision-makers being just indifferent between participating or not.

Reducing this rent is done by making it somewhat flatter, i.e., by better aligning the policy $q(\theta)$ with the decision-maker's ideal point so that the quantity $q(\theta) - \theta$ is reduced in (2). Shifting the optimal policy towards the agent's ideal point and making it more sensitive to his ideological preferences reduces the agent's rent.

When the decision-maker has a sufficiently strong ideological bias ($\beta \geq 2$), it becomes too costly for interest groups to move the decision-maker away from his ideal point. The merged entity prefers not contributing at all and let the agent choose his ideal point. Influence has to be easy to buy to allow the coalition of interest groups to overcome the cost of asymmetric information and have access to the decision-maker. Otherwise, transaction costs of asymmetric information keep the coalition of interest groups outside the political process. This effect will be significantly magnified when interest groups compete since it appears for lower values of β .

4 Competition Between Interest Groups

So far, our analysis has only emphasized informational asymmetries as the only potential source of rent for the decision-maker. Competition between interest groups and their conflicting desires to influence the decision-maker introduces another source of rent under competitive screening.

As before, we denote $U(\theta)$ as the decision-maker's payoff when he accepts both contributions $\{t_1(q), t_2(q)\}$ and let $q(\theta)$ be the corresponding optimal policy. The rent-policy profile $\{U(\theta), q(\theta)\}$ which is implemented by the pair of contributions $\{t_1(q), t_2(q)\}$ now satisfies:

$$U(\theta) = \max_{q \in \mathbb{R}} \left\{ \sum_{i=1}^2 t_i(q) - \frac{\beta}{2}(q - \theta)^2 \right\} \text{ and } q(\theta) = \arg \max_{q \in \mathbb{R}} \left\{ \sum_{i=1}^2 t_i(q) - \frac{\beta}{2}(q - \theta)^2 \right\}.$$

Lemma 1 again applies and conditions (2) and (3) characterize implementable profiles. Similarly, the rent-policy profile $\{U_i(\theta), q_i(\theta)\}$ that is implemented had the agent accepted only principal P_i 's contribution is defined as:

$$U_i(\theta) = \max_{q \in \mathbb{R}} \left\{ t_i(q) - \frac{\beta}{2}(q - \theta)^2 \right\} \text{ and } q_i(\theta) = \arg \max_{q \in \mathbb{R}} \left\{ t_i(q) - \frac{\beta}{2}(q - \theta)^2 \right\}.$$

For a given contribution $t_{-i}^*(q)$ offered by principal P_{-i} , principal P_i 's best-response now solves a relaxed problem (by neglecting the second-order condition (3)) which can be written as:

$$(\mathcal{P}_i^C) : \max_{\{q(\cdot), U(\cdot)\}} \int_{-\delta}^{\delta} \left(-\frac{1}{2}(q(\theta) - a_i)^2 - \frac{\beta}{2}(q(\theta) - \theta)^2 + t_{-i}^*(q(\theta)) - U(\theta) \right) \frac{d\theta}{2\delta},$$

subject to

$$\dot{U}(\theta) = \beta(q(\theta) - \theta); \quad (2)$$

$$U(\theta) \geq U_{-i}(\theta) \text{ for all } \theta \in \Theta. \quad (5)$$

The new participation constraint (5) ensures that the decision-maker prefers taking both contributions rather than accepting only P_{-i} 's contract, assuming that P_i 's contribution is non-negative. A simplifying consequence of this non-negativity is that the participation constraint (4) is always implied by (5) and can be omitted.

Definition 1 : *A subgame-perfect equilibrium of the common agency game under asymmetric information is a pair of contributions $\{t_1^*(\cdot), t_2^*(\cdot)\}$ which implement a rent-policy profile $\{U^*(\theta), q^*(\theta)\}$ solving both $(\mathcal{P}_1^C)$ and $(\mathcal{P}_2^C)$.*

To solve $(\mathcal{P}_i^C)$ for a given contribution schedule $t_{-i}^*(q)$ we define the respective Hamiltonian of $(\mathcal{P}_i^C)$ as

$$H_i(U, q, \lambda_i, \theta) = \left\{ -\frac{1}{2}(q - a_i)^2 - \frac{\beta}{2}(q - \theta)^2 + t_{-i}^*(q) - U \right\} \frac{1}{2\delta} + \lambda_i \beta (q - \theta),$$

and its Lagrangian as

$$L_i(U, q, \lambda_i, p_i, \theta) = H_i(U, q, \lambda_i, \theta) + p_i(U - U_{-i}),$$

where λ_i is the co-state variable associated to (2) and p is the multiplier of the participation constraint (5). Maximizing the Hamiltonian with respect to the control variable q leads to the following first-order condition

$$-(\beta + 1)q(\theta) + a_i + \beta\theta + t_{-i}'^*(q(\theta)) + 2\lambda_i(\theta)\beta\delta = 0, \quad (6)$$

whereas the co-state variable evolves according to

$$\dot{\lambda}_i(\theta) = -\frac{\partial L_i}{\partial U} = \frac{1}{2\delta} - p_i(\theta), \text{ for almost all } \theta \in \Theta. \quad (7)$$

Different kinds of differentiable equilibria may be sustained depending on the parameter values for β and δ . These equilibria are characterized by different areas where the participation constraint (5) binds and $p_i > 0$. To compute the interest groups' equilibrium payoffs, we need to determine the decision-maker's payoff when accepting only the contract from one principal. When accepting only one contribution, the agent may want to adopt more extreme approaches and significantly shift his policy choice beyond the range of possibilities obtained when taking both contracts. Therefore we need to extend contributions for off the equilibrium outputs.¹¹ Following Martimort and Stole (2007), our

¹¹This is a familiar argument from Bernheim and Whinston (1986), Klemperer and Meyer (1989) and Martimort and Stole (2003, 2007) who reduced the problem of the multiplicity of equilibria in various multiprincipal settings by putting restrictions on out-of-equilibrium strategies.

model with quadratic utility functions and uniform distribution of types allows us to focus on equilibria with *natural contributions* which are themselves piecewise quadratic and keep the same analytical expressions (6) for decisions q both on and off the equilibrium path.¹²

To be more explicit, let $q^*(\theta)$ be the equilibrium policy and suppose that it is monotonically increasing $\dot{q}^*(\theta) > 0$ ¹³ so that we can unambiguously define the inverse function $\theta^*(q)$. Inserting into (6) yields the expression of the marginal contribution that P_{-i} offers in equilibrium:

$$-(\beta + 1)q + a_i + \beta\theta + t_{-i}'(q) + 2\lambda_i(\theta^*(q))\beta\delta = 0. \quad (8)$$

For the different scenarios analyzed below, our assumption on quadratic preferences and uniform distribution ensures that $\lambda_i(\theta)$ and $q^*(\theta)$ are both linear in θ . Hence, $\lambda_i(\theta^*(q))$ can be linearly extended beyond the policy equilibrium range $[q^*(-\delta), q^*(\delta)]$. Integrating and keeping the positive part of those schedules leads us to the definition of (piecewise quadratic) *natural contributions* as:

Definition 2 *A natural contribution schedule of the common agency game under asymmetric information is defined as:*

$$t_i^N(q) = \max \left\{ (\beta + 1) \frac{q^2}{2} - a_i q - \beta \int_0^q (\theta^*(x) + 2\delta \lambda_i(\theta^*(x))) dx - C_i, 0 \right\} \quad \forall q, \quad (9)$$

for some constant C_i .

A subgame-perfect equilibrium is *natural* if the equilibrium contribution schedules $t_i^*(q)$ are themselves natural. In the rest of our analysis, equilibria should be understood as the natural equilibria.

4.1 The “Laissez-Faire” Equilibrium: No Influence

First, let us investigate whether there might exist an equilibrium such that none of the interest groups ever contributes. As a result, the decision-maker always chooses his ideal point: the “laissez-faire” outcome. In this case, and although they are both organized, both groups simultaneously eschew any contribution.

¹²One motivation for focusing on this particular extension is that the marginal contributions in those equilibria are kept unchanged as the support of the uniform distribution of types is slightly enlarged (i.e., as δ increases).

¹³This will be checked ex post for the various equilibrium patterns.

Proposition 2 : Strong Ideological Bias/Large Ideological Uncertainty. *Assume that $\beta \geq 1$ and that $1 \leq \delta$. There is no contribution by either interest group in the unique equilibrium. The decision-maker always chooses his ideal point $q^*(\theta) = \theta$ and gets his status quo payoff $U^*(\theta) = 0$ for all $\theta \in \Theta$.*

Lobbying competition under asymmetric information significantly erodes the interest groups' influence when the decision-maker puts sufficient weight on ideology and there is sufficient ideological uncertainty, i.e., when the interest groups' ideal points both lie within the interval defined by the most extreme views of the agent. As such, the situation can be viewed as though the decision-maker was free from any influence and could always choose his most preferred policy.

The same zero-contribution outcome already occurred had the coalition of interest groups formed when $\beta \geq 2$ (see Proposition 1). The point is that the non-cooperative behavior of principals exacerbates this effect which now also arises for lower levels of β as well, i.e., although the decision-maker gives only a moderate weight to his ideology. Note also the difference between the "laissez-faire" outcome above where competing groups do not contribute and the complete information equilibrium given at the end of Section 2. Indeed, and although the decision-maker eventually chooses his ideal point, this was due to the fierce competition between opposite groups which contribute significantly to avoid a policy shift towards the other end of the political spectrum.

To give some intuition to Proposition 2, let us think about the case with only one interest group, P_1 and let us look for the optimal contribution that P_1 offers in such a hypothetical monopolistic screening environment. Compared to the analysis performed in Section 3, this single principal is now biased on one side of the policy space: He prefers a higher policy than the decision-maker's average ideal point. Also, ideology matters less at the margin to this principal than for a coalition of interest groups since P_1 is not concerned about P_2 's utility. Even if both P_1 and the coalition has to give up the same information rent in order to implement a given policy profile and shift the optimal policy towards the decision-maker's ideal point by the same amount, P_1 suffers more from that increase than the coalition. This implicit increase in the weight given to the agent's ideology erodes all P_1 's bargaining power when there is sufficient ideological uncertainty. Considering the case of two groups, an equilibrium where both principals do not contribute arises.

4.2 Partial Influence: Non-Overlapping Areas

In the "laissez-faire" equilibrium, none of the interest groups secures any area of influence. Even when the ideological distance between the agent and an interest group is small, the principal cannot ensure that the decision-maker will only follow his own recommenda-

tion because there is too much uncertainty on the decision-maker's preferences which may be too distant from that of the group. We now investigate conditions under which such unchallenged influence occurs instead. The market for influence is then segmented with interest groups on both sides of the political spectrum being linked in exclusive relationships with decision-makers who are sufficiently close ideologically. Such a pattern of influence is called a *partition equilibrium of type 1*. From Proposition 2, the degree of polarization between interest groups (resp. the ideological uncertainty) must increase (resp. decrease) for such pattern to arise.

Definition 3 : *In a partition equilibrium of type 1, principal P_i offers a positive contribution only on a non-empty subset Ω_i of Θ . Moreover, the principals' areas of influence are disconnected, i.e., $\Omega_1 \cap \Omega_2 = \emptyset$. A partition equilibrium of type 1 is symmetric (SPE1) when there exists $\tau \in (0, \delta)$ such that $\Omega_2 = [-\delta, -\tau]$ and $\Omega_1 = [\tau, \delta]$. Let $\Omega_0 = [-\tau, \tau]$ be the area where none of the principals contribute.*

The conditions below ensure existence and uniqueness of a SPE1.

Proposition 3 : Strong Ideological Bias/Intermediate Ideological Uncertainty. *Assume that $\beta > 1$ and $\frac{1}{\beta} \leq \delta < 1$. The unique equilibrium of the common agency game is a SPE1. Let $\tau = \tau_1 = -\tau_2 = \frac{\beta\delta-1}{\beta-1}$:*

- *The equilibrium policy $q^*(\theta)$ reflects the preferences of the contributing interest group only for the most extreme realizations of θ and otherwise is equal to the decision-maker's ideal point*

$$q^*(\theta) = \begin{cases} \frac{2\beta\theta - (\beta-1)\tau_i}{\beta+1} & \text{if } \theta \in \Omega_i, \\ \theta & \text{if } \theta \in \Omega_0; \end{cases}$$

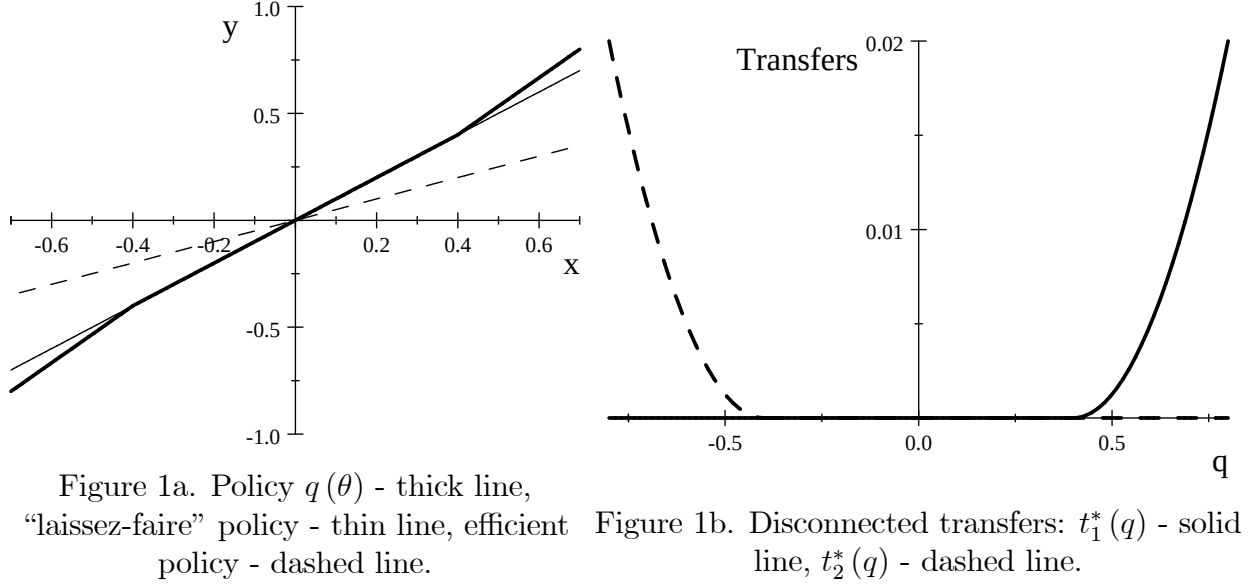
- *The decision-maker's information rent $U^*(\theta)$ is convex and equal to his status quo payoff only on Ω_0*

$$U^*(\theta) = \begin{cases} \frac{\beta(\beta-1)}{2(\beta+1)} (\theta - \tau_i)^2 & \text{if } \theta \in \Omega_i, \\ 0 & \text{if } \theta \in \Omega_0; \end{cases}$$

- *Contributions are piecewise continuously differentiable and convex*

$$t_1^*(q) = \begin{cases} \frac{(\beta-1)}{4} (q - \tau)^2 & \text{if } q \geq \tau, \\ 0 & \text{otherwise;} \end{cases} \quad \text{and } t_2^*(q) = t_1^*(-q).$$

On Figures 1a and 1b, we have represented the respective equilibrium policy and contributions with disconnected areas of influence for the parameter values $\beta = 2$ and $\delta = 0.7$.



A partition equilibrium shares some common features with the "laissez-faire" equilibrium. In both cases, the decision-maker might be freed from the principals' influence but now this occurs only when the decision-maker is sufficiently moderate. Interest groups are now able to secure unchallenged influence when their ideological distance with the agent is sufficiently small. The most extreme decision-makers are thus linked in exclusive relationships with nearby groups.

To understand the shape of contributions and the equilibrium pattern, it is important to first think about the case where one interest group, say P_1 , is alone and always has more extreme views than the agent ($1 \geq \delta$). When $\beta > 1$ influencing the agent's is very costly for that interest group. If there was complete information on the decision-maker's ideal point, the policy $q_1^*(\theta) = \frac{\beta\theta+1}{\beta+1}$ should be implemented and to do so, a transfer $t_1^*(\theta) = \frac{\beta(1-\theta)^2}{2(\beta+1)^2}$ would be required to compensate the decision-maker for not preferring his own ideal point. This transfer increases with the distance between the decision-maker and the group's ideal points, namely $1 - \theta$. Under asymmetric information, such high transfer becomes very attractive for the most extreme types close to P_1 's ideal point who want to look more moderate. To limit those incentives, the interest group reduces his contribution to the most moderate types and shifts the policy towards the decision-maker's ideal point (Figure 1). The optimal policy for the most close-by types is as found in Proposition 3 above, $q^*(\theta) = \frac{2\beta\theta-\beta\delta+1}{\beta+1}$. As the agent's ideal point comes closer to that of the group, this second-best policy does not need to be as distorted towards the agent's ideal point.

Formally, we have $\dot{q}^*(\theta) > 1$. As a result, the decision-maker's rent and the contribution he receives are both convex since:

$$\ddot{U}(\theta) = \beta(\dot{q}^*(\theta) - 1) > 0 \text{ and } t_1''(q^*(\theta)) = \frac{\ddot{U}(\theta)}{\dot{q}^*(\theta)}.$$

4.3 Partial Influence: Overlapping Areas

Suppose now that interest groups are more polarized, or ideological uncertainty decreases. Transaction costs of contracting under asymmetric information diminish and both interest groups suffer less from not knowing the decision-maker's preferences. Moderate decision-makers now receive positive contributions from both interest groups. We will call such a pattern of influence a *partition equilibrium of type 2*.

Definition 4 : *In a partition equilibrium of type 2, principal P_i offers a positive contribution only on a non-empty subset Ω_i of Θ . Moreover, the principals' areas of influence overlap, i.e., $\Omega_1 \cap \Omega_2 = \Omega_0 \neq \emptyset$ where Ω_0 is the area where both principals simultaneously contribute. A partition equilibrium of type 2 is symmetric (SPE2) when there exists $\tau \in (0, \delta)$ such that $\Omega_2 = [-\delta, \tau]$ and $\Omega_1 = [-\tau, \delta]$.*

We provide the conditions ensuring existence of a SPE2 below. As in the case of disconnected areas of influence this equilibrium is unique.¹⁴

Proposition 4 : Strong Ideological Bias/Smaller Ideological Uncertainty. *Assume that $\beta > 1$ and $\frac{\beta+2}{\beta(2\beta+1)} \leq \delta < \frac{1}{\beta}$. The unique equilibrium of the common agency game is a SPE2. Let $\gamma = 1 - \beta\delta$, and $\tau = \tau_1 = -\tau_2 = \frac{(\beta+2)\gamma}{\beta(\beta-1)}$:*

- *The equilibrium policy $q^*(\theta)$ reflects the preferences of both groups only for moderate decision-makers and is otherwise biased towards the preferences of the nearby group for more extreme ones*

$$q^*(\theta) = \begin{cases} \frac{2\beta\theta+\gamma}{\beta+1} & \text{if } \theta \in [\tau, \delta], \\ \frac{3\beta\theta}{\beta+2} & \text{if } \theta \in [-\tau, -\tau], \\ \frac{2\beta\theta-\gamma}{\beta+1} & \text{if } \theta \in [-\delta, -\tau]; \end{cases}$$

- *The decision-maker's information rent $U^*(\theta)$ is convex and minimized for the most moderate type with $U^*(0) = \frac{3\gamma^2}{\beta-1} > 0$;*

¹⁴Note that the set of parameter values corresponding to both types of equilibria do not overlap.

- Contributions are convex and continuously differentiable

$$t_1^*(q) = \begin{cases} 0 & \text{if } q \leq \frac{-3\gamma}{\beta-1}, \\ \frac{(\beta-1)}{6}q^2 + \gamma q + \frac{3\gamma^2}{2(\beta-1)} & \text{if } q \in [\frac{-3\gamma}{\beta-1}, \frac{3\gamma}{\beta-1}], \\ \frac{(\beta-1)}{4}q^2 + \frac{\gamma}{2}q + \frac{9\gamma^2}{4(\beta-1)} & \text{if } q \geq \frac{3\gamma}{\beta-1}. \end{cases} \text{ and } t_2^*(q) = t_1^*(-q),$$

On Figures 2a and 2b, we have represented the respective equilibrium policy and contributions with disconnected areas of influence for parameters $\beta = 2$ and $\delta = 0.47$.

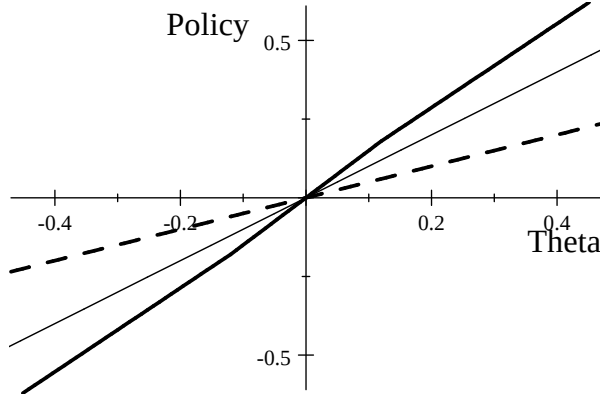


Figure 2a. Policy $q(\theta)$ - thick line, “laissez-faire” policy - thin line, efficient policy - dashed line.

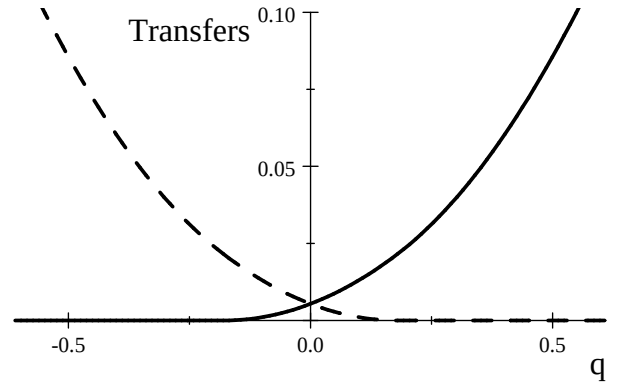


Figure 2b. Partially overlapping transfers: $t_1^*(q)$ - solid line, $t_2^*(q)$ - dashed line.

As the degree of polarization between groups increases, it is relatively easy for each interest group to influence nearby types of the decision-maker and they obtain a positive rent. For the rival interest group on the other side of the ideological space, the policies led by those distant types may be excessively biased towards opposite views. To counter this effect, the other principal must reward the agent himself although the latter is on the opposite side of the ideological spectrum. This strategy is valuable as long as the ideological distance with the agent is not too large. In such cases, areas of influence start to overlap for the most moderate decision-makers. This result is due to the fact that, under interim contracting, moving policy towards his own ideal point requires a principal to offer a positive contribution to the informed decision-maker. Under competition, contributions are piled up for the most moderate types to maintain their preferences for a balanced policy.

4.4 Fully Overlapping Areas of Influence

We now distinguish two cases depending on the decision-maker’s ideological concern in his utility function.

4.4.1 Strong Ideological Bias

Proposition 4 already showed that, as ideological uncertainty decreases, the interest groups' areas of influence start to overlap. When uncertainty is sufficiently small or alternatively when the degree of polarization is sufficiently large, both groups are always able to influence the decision-maker whatever his ideological location as shown in the next proposition.

Proposition 5 : Strong Ideological Bias/Small Ideological Uncertainty. *Assume that $\beta > 1$ and $\delta < \frac{\beta+2}{\beta(2\beta+1)}$. The interest groups' areas of influence fully overlap in the unique equilibrium. Still denoting $\gamma = 1 - \beta\delta$:*

- *The equilibrium policy $q^*(\theta)$ is more biased towards the decision-maker's ideal point than when groups cooperate*

$$q^*(\theta) = \frac{3\beta\theta}{\beta+2} \text{ with } |q^*(\theta) - \theta| \leq |q^M(\theta) - \theta|, \text{ for all } \theta \in \Theta; \quad (10)$$

- *The rent profile $U^*(\theta)$ is convex and strictly positive everywhere*

$$U^*(\theta) = \frac{\beta(\beta-1)}{\beta+2}\theta^2 - 3\left(\frac{\beta^2\delta^2}{\beta+2} - \frac{1}{2\beta+1}\right), \text{ for all } \theta \in \Theta;$$

- *Contributions are convex and positive everywhere*

$$t_1^*(q) = \max\left\{\frac{\beta-1}{6}q^2 + \gamma q - \frac{3}{2}\left(\frac{\beta^2\delta^2}{\beta+2} - \frac{1}{2\beta+1}\right), 0\right\}, \text{ and } t_2^*(q) = t_1^*(-q).$$

On Figures 3a and 3b, we have represented respectively the equilibrium policy and contributions with overlapping areas of influence for $\beta = 2$ and $\delta = 0.2$.

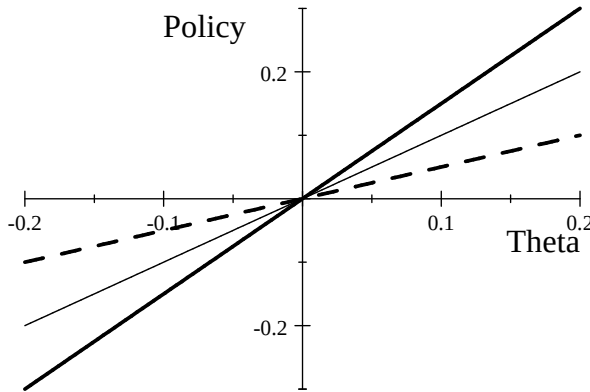


Figure 3a. Policy $q(\theta)$ - thick line, "laissez-faire" policy - thin line, efficient policy - dashed line.

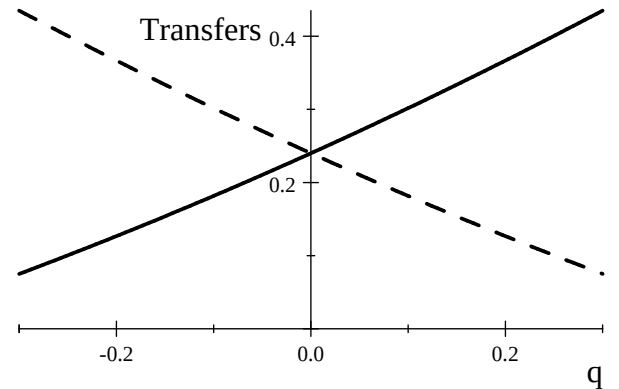


Figure 3b. Fully overlapping transfers: $t_1^*(q)$ - solid line, $t_2^*(q)$ - dashed line.

Altogether, Propositions 2, 3, 4 and 5 provide some interesting comparative statics on the role of ideological uncertainty. The decision-maker's rent is equal to his status quo payoff only when there is sufficient ideological uncertainty compared to the degree of polarization between groups (δ is large). As ideological uncertainty diminishes, each group secures some area of influence if the decision-maker is sufficiently close ideologically and let the other group enjoy unchallenged influence if he is more distant. A moderate decision-maker receives positive contributions only when ideological uncertainty is sufficiently small and groups find it worthwhile to compete head-to-head for his services, thereby raising contributions and giving up some positive rent. Fully overlapping areas of influence arise when ideological uncertainty is sufficiently small. Interest groups now compete for all types and, whatever his type, the decision-maker gets a positive rent.

This rent now has two sources. First, interest groups find it worthwhile bidding for the agent's services as his ideology is better known. But now, an extra source of information rent comes from the possibility for an extreme decision-maker to behave as being more moderate. In fact, by doing so he would increase the ideological distance with both principals, and grasp greater contributions.

It is interesting to look at the limit of the equilibrium above when δ converges to zero and the decision-maker is known for sure to be being located at the origin. In the limit, the decision $q^*(0) = 0$ is clearly efficient whereas the contributions become:

$$t_1^*(q) = \max \left\{ \frac{\beta - 1}{6} q^2 + q + \frac{3}{2(2\beta + 1)}, 0 \right\}, \text{ and } t_2^*(q) = t_1^*(-q).$$

This yields a payoff $U^*(0) = \frac{3}{2\beta+1}$ to the decision-maker and $V_i^*(0) = -\frac{1}{2} - \frac{3}{2(2\beta+1)}$ to either group. These schedules form an equilibrium of the complete information game although they are not truthful and do not induce the same payoff distributions between the groups and the decision-maker as exhibited at the end of Section 2. This points to the failure of the truthfulness criterion as a device to select equilibria in the complete information game.¹⁵

4.4.2 Weak Ideological Bias

When $\beta < 1$, the decision-maker's ideology becomes less concern and interest groups can now influence the decision-maker's policy choice with relative ease. Intuitively, one should expect that the resulting equilibrium policy is less sensitive to ideology than before. This means that as the decision-maker becomes more extreme, the difference between his ideal point and what the nearby interest group would like to implement decreases. Filling this

¹⁵Martimort and Stole (2007) question also the justification of the truthfulness criterion in a public good common agency model and show that truthful equilibria are not particularly attractive limits of equilibria of the game under asymmetric information.

gap requires an increasingly lower marginal contribution as policies become more extreme. The equilibrium contribution of a given group is now concave and increasing as his own ideal point comes closer to that of the decision-maker.

The next two propositions describe the different equilibrium patterns in distinguishing, as we usually do, between the cases of a large and a small ideological uncertainty. We comment below on the differences and similarities between those two cases.

Proposition 6 : Weak Ideological Bias/Large Ideological Uncertainty. *Assume that $\beta < 1$ and $\delta \geq \delta^* = \frac{1}{2\beta+1} \sqrt{\frac{3(\beta+2)}{\beta}}$. Then, there exists $\lambda_1^* \leq \frac{1}{2}$ such that, for each $\lambda_1 \in [-\frac{1}{2}, \lambda_1^*)$, with $1 < (2\beta + 1)\delta \sqrt{\frac{\beta}{3(\beta+2)}} - \beta\delta(1 + 2\lambda_1)$, there exists a continuum of equilibria with groups having fully overlapped influences.*

- The equilibrium policy $q^*(\theta) = \frac{3\beta\theta}{\beta+2}$ is again more biased towards the decision-maker's ideal point than when groups cooperate;
- The rent profile $U^*(\theta)$ is strictly concave, zero only at both endpoints $\pm\delta$ and always lower than when groups cooperate

$$U^*(\theta) = \frac{\beta(1-\beta)}{\beta+2}(\delta^2 - \theta^2) \leq U^M(\theta) \text{ for all } \theta \in \Theta;$$

- Contributions are strictly concave on their positive part

$$t_1^*(q) = \max \left\{ 0, -\frac{1-\beta}{6}q^2 + \gamma q - C_1 \right\}, \quad t_2^*(q) = \max \left\{ 0, -\frac{1-\beta}{6}q^2 - \gamma q - C_2 \right\}, \quad (11)$$

where $\gamma = 1 + 2\beta\delta\lambda_1$. There exists an interval of possible values for the constants (C_1, C_2) . Those constants satisfy the following linear constraints:

$$C_1 + C_2 = -\frac{\beta(1-\beta)}{\beta+2}\delta^2, \text{ and } C_i \geq \frac{3(\beta\delta + \gamma)^2}{2(2\beta+1)} - \frac{\beta\delta^2}{2}, \quad i = 1, 2.$$

Turning now to the case of a small ideological uncertainty, we again obtain the uniqueness of the equilibrium. When the ideological uncertainty is small the issue of competition between principals dominates coordination.

Proposition 7 : Weak Ideological Bias/Small Ideological Uncertainty. *Assume that $\beta < 1$ and $\delta < \delta^*$. The unique equilibrium has fully overlapping influences and entails:*

- An equilibrium policy $q^*(\theta) = \frac{3\beta\theta}{\beta+2}$;

- A strictly concave rent profile $U^*(\theta)$ which is strictly positive at both endpoints $\pm\delta$

$$U^*(\theta) = \frac{\beta(1-\beta)}{\beta+2}(\delta^2 - \theta^2) - 3\left(\frac{\beta^2\delta^2}{\beta+2} - \frac{1}{2\beta+1}\right) \text{ for all } \theta \in \Theta;$$

- Contributions are strictly concave on their positive part

$$t_1^*(q) = \max\left\{0, -\frac{1-\beta}{6}q^2 + \gamma q - \frac{3}{2}\left(\frac{\beta^2\delta^2}{\beta+2} - \frac{1}{2\beta+1}\right)\right\}, \quad t_2^*(q) = t_1^*(-q) \quad (12)$$

where $\gamma = 1 - \beta\delta$.

On Figure 4 we draw the equilibrium policy in the case of $\beta = \frac{1}{2}$ and $\delta = 1$.

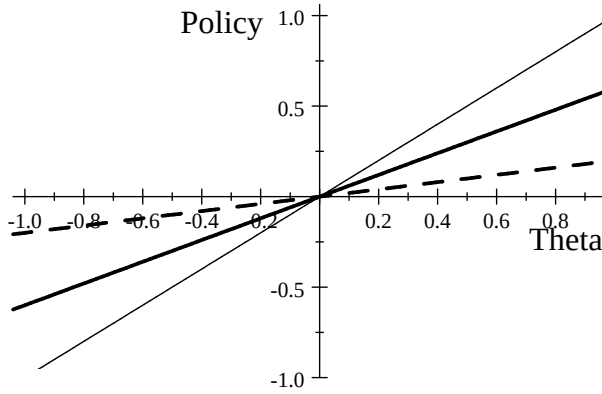


Figure 4. Policy $q(\theta)$ - thick line,
“laissez-faire” policy - thin line, efficient
policy - dashed line.

When $\beta < 1$, the equilibrium shares some common features with the optimal solution achieved had interest groups cooperated in designing contributions: The situation can be viewed as though the decision-maker’s ideology had a greater implicit weight in the policy process. However, this effect is magnified compared to the cooperative outcome. The equilibrium policy comes closer to the decision-maker’s ideal point. Because $|q^*(\theta) - \theta| \leq |q^M(\theta) - \theta|$, the rent profile under lobbying competition is now flatter. To reduce the agent’s information rent, each interest group needs to shift the policy towards the agent’s ideal point and, to do so, offers a relatively flat contribution. However, a given interest group does not take into account that his rival also offers such a flat contribution so that the equilibrium policy is already significantly shifted towards the decision-maker’s ideal point. The rent profile is excessively flat compared with merged contracting.¹⁶ This

¹⁶This revisits in the context of spatial preferences a result already found in other common agency games with public screening devices in case where principals have monotonic preferences (see for instance Martimort and Stole (2007)).

contractual externality between interest groups leads to an excessive bias towards the agent's ideal point compared to the case where contributions are jointly designed.

When ideology does not matter so much for the decision-maker, moderate types receive more rent than extreme ones. Exactly as when groups collude, those types may want to look more extreme than what they really are to raise contributions. Two cases may arise.

If polarization is strong or ideological uncertainty is small: Head-to-head competition between interest groups ensures that the equilibrium payoffs of all players are uniquely defined. Even the decision-makers with the most extreme types get a positive rent out of the groups' aggressive bidding for their services.

If polarization is weak or ideological uncertainty sufficiently large: Interest groups now become more congruent. They both want to extract as much rent as possible from the agent. Because a moderate decision-maker can look more extreme, he gets some rent. This congruence between competing interest groups creates a coordination problem leading to multiple equilibria. Multiplicity comes from the leeway in choosing both the levels of contributions and their margins. First, there are different ways of designing contributions so that interest groups collectively extract the rent of the most extreme types and prevent those types from offering services exclusively to the closer interest group. This coordination problem only affects the level of contributions. Second, for a given amount of ideological uncertainty, interest groups compete more fiercely for the services of the most extreme decision-maker; with the group the further away from an extreme decision-maker having to concede the most to get some influence. Indeed, an extreme agent finds it more attractive to take only the contribution of the nearby group. This hardens his participation constraint and makes him a tougher bargainer with the opposite interest group.¹⁷ As competition for the services of an extreme decision-maker becomes tougher, the screening possibilities of the interest group which is on the other side of the ideological space are more limited. This increases the marginal contribution of this group.

5 Conclusion and Directions for Future Research

Let us first briefly recapitulate the main results of our analysis. In passing, we suggest some testable implications that immediately follow from our work.

Inefficient Policies: Under asymmetric information, competition between interest groups leads to huge inefficiencies in policy choices. There always exists a strong bias towards the decision-maker's ideal point. If ideological uncertainty is sufficiently large, transaction

¹⁷There remains some freedom in choosing the corresponding multiplier λ_1 , with this multiplier being greater if the reservation payoff that the agent gets by taking only one contract is steeper.

costs also become large. Interest groups might refrain from stop contributing and leave the decision-maker free to pursue his own ideological views.

This result may explain the apparent near welfare-maximizing behavior of U.S. policy-makers in some areas, especially in trade policy. Using a complete information model, Gawande et al. (2005) for instance argued that the competition between interest groups (final producers and intermediate ones) whose impacts cancel out might explain why the estimated implicit weight given to contributions in the decision-maker's preferences is too low. Nevertheless they concluded their study by noticing that their estimated parameter is still excessively high. Introducing asymmetric information in such analysis would magnify the policy bias towards the agent's ideal point (who may have the social-welfare maximizing policy as an ideal point) and might help to solve this important empirical puzzle.

Contributions and Segmentation of the Market for Influence: When the decision-maker's ideological bias is strong and there is sufficient uncertainty, interest groups may not contribute to a decision-maker whose ideal point lies too far away from their own preferences. The market for influence is segmented with exclusive relationships between interest groups and decision-makers whose preferences are close in the political spectrum. This is likely to occur for general policies that have a broad appeal to the public and that decision-makers value a lot (maybe for electoral concerns), for young legislators who have not yet revealed much on their preferences through past voting behavior, and for those who have previously not shown any expertise or interest in a particular field.

As ideological uncertainty decreases, the areas of influence of competing groups begin to overlap. More extreme legislators continue on collecting most contributions though they may still receive contributions from opposite groups. Thus one should expect older decision-makers whose preferences are better known to gain more support from both sides of the political spectrum.

If the decision-maker's ideological bias is not too strong, possibly due to the fact that the policy at stake is sector specific and has little appeal for the general public, interest groups always contribute. However, the nature of competition is highly dependent on the amount of ideological uncertainty. Interest groups are more congruent when facing much ideological uncertainty since it becomes quite likely that both ideal policies are on the same side of the decision-maker's own ideal point. The main features of the pattern of contributions then seem as though groups had cooperated in designing contributions. Moderate legislators collect the bulk of contributions and rent. On the other hand, with less uncertainty, competition induces interest groups to raise contributions even for the most extreme decision-makers.

Extensions: Let us mention a few possible extensions of our framework.

First, our closed-form characterization of equilibria relies on our choice of quadratic utility functions and a uniform distribution for the decision-maker's location in the ideology space. Although the first assumption is by now standard in the political economy literature¹⁸ in tandem with the second it makes our analysis tractable and allows us to determine explicitly the areas where a given group stops contributing, if any. More general choices would not lead to such simple characterization.

Second, the symmetry of the interest groups' ideal points around the origin also eases the analysis. Most of our results, particularly the fact that equilibria with quadratic transfers can be explicitly derived, could nevertheless be generalized to the case of asymmetric principals at the cost of more cumbersome notations.

Third, readers may find troublesome the timing of the lobbying game which assumes that interest groups commit to contribution schedules even though those contracts cannot be enforced in practice. This full commitment assumption is borrowed from Grossman and Helpman (1994) and has been found as a useful short-cut to understand patterns of influence under complete information. In this paper, we have relaxed this last assumption and analyzed how asymmetric information may change contribution patterns by keeping in the background the usual motivation that repeated relationships may help enforce those contracts. Although the full commitment-common agency model has received some empirical support,¹⁹ which suggests functionality of the methodological short-cut, a full-fledged analysis would require to take into account the non-enforceability of contributions. This self-enforceability might be harder to obtain as there is more uncertainty on the decision-maker's ideal point. This should reinforce our previous findings.

Fourth, it would be interesting to investigate what would happen when there exist more than two interest groups.²⁰ The characterization of equilibria may become quite complex even though a general feature of our previous findings certainly continues to hold: A given group will refrain contributing to decision-makers who are ideologically too far away. This might lead to complex patterns of contributions which would mix what arises in the different equilibrium configurations arising with only two groups. For instance, groups on the same side of the political space may be rather congruent whereas they oppose more fiercely with groups on the other side of the political spectrum. In such environment, it might be worth investigating the incentives to collude for interest groups biased in similar directions.²¹

Fifth, our view of the political process has also been simplified by focusing on a

¹⁸Austen-Smith and Banks (2000).

¹⁹See Goldberg and Maggi (1999) and Bradford (2003).

²⁰Laussel and Le Breton (2001) have analyzed the structure of equilibria with more than two groups in complete information environments.

²¹See Epstein and O'Halloran (2004) for some steps in those directions.

one-dimensional policy space. More complex multi-dimensional policy spaces and spatial preferences could be investigated. Interest groups may tailor their contributions to the particular policy dimensions they are interested in or they may make contributions conditional on the whole array of policies. From the earlier common agency literature under asymmetric information,²² it is well-known that the pattern of policy distortions and rent distributions may depend on the interest groups' ability to contract or not on the whole array of policies. It would be worthwhile investigating whether the strong bias towards the agent's ideal point that we found also occurs in those more general environments.

Sixth, in ongoing relationships, interest groups learn about the decision-maker's preferences from the past history of decisions he makes. It would be interesting to extend our analysis to such dynamic settings. As time passes, uncertainty on the preferences of the decision-maker gets resolved. This certainly raises contributions so that old legislators whose preferences have already been revealed attract more contributions.

Finally, we have simplified our modelling of the political process assuming a unique decision-maker. A less abstract description of legislative organizations would require a more detailed modelling of the legislature taking into account the fact that decision result from interaction among legislators. Multiagent/multiprincipals models which are not yet developed may have some strong appeal in that respect.

All those are extensions that we hope to undertake in future studies.

References

- Austen-Smith, D. and J. Banks, (2000), *Positive Political Theory 1: Collective Preferences*, Michigan University Press.
- Bernheim, D. and M. Whinston, (1986), "Menu Auctions, Resource Allocations and Economic Influence", *Quarterly Journal of Economics*, 101: 1-31.
- Bradford, S. (2003), "Protection and Jobs: Explaining the Structure of Trade Barriers Across Industries", *The Journal of International Economics*, 61:19-39.
- Campante, F. and F. Ferreira, (2007), "Inefficient Lobbying, Populism and Oligarchy", *Journal of Public Economics*, 91: 993-1021.
- Dahl, R., (1961), *Who Governs? Democracy and Power in American City*, New Haven: Yale University Press.
- Dixit, A., (1996), *The Making of Economic Policy: A Transaction-Cost Politics Perspective*, MIT Press, Cambridge.

²²Martimort (1992) and Stole (1991).

- Dixit, A., Grossman, G. and E. Helpman, (1997), "Common Agency and Coordination", *Journal of Political Economy*, 105: 752-769.
- Epstein, D. and S. O'Halloran, (2004), "Modelling Madison: A Multiple Principals Model of Interest Group Competition", Mimeo Columbia University.
- Felli, L. and A. Merlo, (2006), "Endogenous Lobbying", *Journal of European Economic Association*, 4: 180-215.
- Gawande, K., (1997), "US Non-tariff Barriers as Privately Provided Public Goods", *Journal of Public Economics* 64: 61-81.
- Gawande, K., P. Krishna and M. Olarreaga, (2005), "Lobbying Competition Over Trade Policy", *NBER Working Paper* No 11371.
- Goldberg, P. and G. Maggi, (1999) "Protection for Sale: An Empirical Investigation," *American Economic Review* 89: 1135-1155.
- Grossman, G. and E. Helpman, (1994), "Protection for Sale", *American Economic Review*, 84: 833-850.
- Helpman, E. and T. Persson, (2001), "Lobbying and Legislative Bargaining", *Advances in Economic Analysis and Policy*, 1(1) Article 3, <http://www.bepress.com/beeap>.
- Klemperer, P. and M. Meyer, (1989), "Supply Function Equilibria in Oligopoly Under Uncertainty," *Econometrica* 57: 1243-1277.
- Kroszner, R. and T. Stratman, (1998), "Interest-Group Competition and the Organization of Congress: Theory and Evidence from Financial Services's Political Action Committee," *American Economic Review*, 88: 1163-1187.
- Kroszner, R. and T. Stratman, (1999), "What Drives Deregulation? Economics and Politics of the Relaxation of Bank Branching Restrictions," *Quarterly Journal of Economics*, 114: 1437-1467.
- Laffont, J.J. and D. Martimort, (2002), *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, Princeton.
- Laussel, D. and M. Le Breton, (2001) "Conflict and Cooperation: The Structure of Equilibrium Payoffs in Common Agency", *Journal of Economic Theory* 100:93-128.
- Le Breton, M. and F. Salanié, (2003), "Lobbying under Political Uncertainty", *Journal of Public Economics*, 87: 2589-2610.
- Lewis, T. and D. Sappington, (1989), "Countervailing Incentives in Agency Problem", *Journal of Economic Theory*, 49: 294-313.
- Lowi, T., (1979), *The End of Liberalism*, New-York: Norton.

- Maggi, G. and A. Rodriguez-Clare, (1995), "On Countervailing Incentives", *Journal of Economic Theory*, 66: 238-263.
- Martimort, D., (1992), "Multi-Principaux avec Anti-Selection", *Annales d'Economie et de Statistiques*, 28: 1-38.
- Martimort, D. and A. Semenov, (2007a), "Political Bias in Lobbying under Asymmetric Information", *Journal of European Economic Association Papers and Proceedings*, 5: 614-623.
- Martimort, D. and A. Semenov, (2007b), "The Pluralistic View of Politics: Asymmetric Lobbyists, Ideological Uncertainty and Political Entry", *Economics Letters*, 97:155-161.
- Martimort, D. and L. Stole, (2002), "The Revelation and Delegation Principles in Common Agency Games", *Econometrica*, 70: 1659-1674.
- Martimort, D. and L. Stole, (2003), "Contractual Externalities and Common Agency Equilibria", *Advances in Theoretical Economics*, 3(1) Article 4, <http://www.bepress.com/bejte>.
- Martimort, D. and L. Stole, (2007), "Selecting Equilibria in Common Agency Games", Mimeo IDEI and University of Chicago.
- Mezzetti, C., (1997), "Common Agency with Horizontally Differentiated Principals", *Rand Journal of Economics*, 28: 323-334.
- Mitra, D., (1999), "Endogenous Lobby Formation and Endogenous Protection: A Long-Run Model of Trade Policy Formation", *American Economic Review*, 89: 1116-1134.
- Moe, T., (1981), *The Organization of Interests*, Chicago: University of Chicago.
- Peters, M., (2001), "Common Agency and the Revelation Principle", *Econometrica*, 69: 1349-1372.
- Rochet, J-C., (1985), "The Taxation Principle and Multitime Hamilton-Jacobi Equations," *Journal of Mathematical Economics*, 14: 113-128.
- Seierstad, A. and K. Sydsaeter, (1987), *Optimal Control Theory with Economic Applications*, Amsterdam: North Holland.
- Stole, L., (1991), "Mechanism Design under Common Agency", Mimeo Chicago University.
- Stratman, T., (2005), "Some Talk: Money in Politics. A (Partial) Review of the Literature," *Public Choice*, 124: 135-156.
- Truman, D., (1952), *The Governmental Process*, New-York: Knopf.
- Wilson, J., (1973), *Political Organizations*, New-York: Basic Books.

Wright, J., (1996), *Interest Groups and Congress*, Boston, Allyn and Bacon.

Appendix

• **Proof of Lemma 1** The proof is standard and thus omitted. See Laffont and Martimort (2002) for instance. ■

• **Proof of Proposition 1:** The implementability conditions for a profile $\{U(\theta), q(\theta)\}$ are given by (2) and (3). We ignore the monotonicity constraint (3) in $(\mathcal{P}^M)$ which is checked ex post once the equilibrium is derived and consider the reduced problem $(\tilde{\mathcal{P}}^M)$ with the state variable $U(\theta)$ and control variable $q(\theta)$. The program of the merged entity is now:

$$(\tilde{\mathcal{P}}^M) : \max_{\{q(\cdot), U(\cdot)\}} \int_{-\delta}^{\delta} \left\{ -\frac{1}{2}(q(\theta) - 1)^2 - \frac{1}{2}(q(\theta) + 1)^2 - \frac{\beta}{2}(q(\theta) - \theta)^2 - U(\theta) \right\} \frac{d\theta}{2\delta},$$

subject to (2) and (4).

Denoting λ as the co-state variable for (2), the Hamiltonian of $(\tilde{\mathcal{P}}^M)$ can be written as:

$$H(U, q, \lambda, \theta) = \left\{ -\frac{1}{2}(q - 1)^2 - \frac{1}{2}(q + 1)^2 - \frac{\beta}{2}(q - \theta)^2 - U(\theta) \right\} \frac{1}{2\delta} + \lambda\beta(q - \theta).$$

Since $H^*(U, \lambda, \theta) = \max_{q \in \mathbb{R}} H(U, q, \lambda, \theta)$ and the state constraint (2) are both linear in U , the problem is concave in U . Therefore, the sufficient conditions for optimality with pure state constraints (see Seierstad and Sydsaeter (1987, Theorem 1, p. 317-319)) are also necessary. To write these conditions, consider the following Lagrangian:

$$L(U, q, \lambda, p, \theta) = H(U, q, \lambda, \theta) + pU,$$

where p is the Lagrange multiplier associated with the state constraint (4). Let $\{U(\theta), q(\theta)\}$ be an admissible pair which solves $(\tilde{\mathcal{P}}^M)$. The sufficient conditions for optimality are:

$$\frac{\partial H}{\partial q} = 0, \text{ for almost all } \theta; \tag{A1}$$

$$\dot{U}(\theta) = \beta(q(\theta) - \theta); \tag{A2}$$

$$\dot{\lambda}(\theta) = \frac{1}{2\delta} - p(\theta); \tag{A3}$$

$$p(\theta)U(\theta) = 0, p(\theta) \geq 0, U(\theta) \geq 0; \tag{A4}$$

$$\lambda(-\delta) = \lambda(\delta) = 0; \tag{A5}$$

where $p(\theta)$ and $\lambda(\theta)$ are piecewise continuous and $\lambda(\theta)$ may have jump discontinuities at τ_j , such that, $-\delta \leq \tau_j \leq \delta$ ($j = 1, \dots, n$), and for these jumps:

$$\lambda(\tau_j^-) - \lambda(\tau_j^+) = \epsilon_j, \epsilon_j \geq 0, \tag{A6}$$

with

$$\epsilon_j = 0 \text{ if } \begin{cases} \text{either} & a) U(\tau_j) > 0, \\ \text{or} & b) U(\tau_j) = 0 \text{ and } q(\theta) \text{ is discontinuous at } \tau_j. \end{cases} \quad (\text{A7})$$

From (A1), the optimal policy is

$$q(\theta) = \frac{\beta\theta + 2\beta\delta\lambda(\theta)}{\beta + 2}, \text{ for almost all } \theta. \quad (\text{A8})$$

This expression together with (A6) leads to the following useful Lemma (which proof is immediate):

Lemma 2 : *The equilibrium policy $q(\theta)$ and the co-state variables $\lambda(\theta)$ are both continuous at any interior point $\theta \in (-\delta, \delta)$. $\lambda(\theta)$ is continuous at $\theta = \pm\delta$ only if $q(\theta)$ is discontinuous at those end-points and $U(\pm\delta) = 0$.*

We are now ready to prove Proposition 1. Using the sufficient conditions for optimality given above, let us guess the form of the solution and check that it satisfies all the conditions for optimality (A1) to (A6). Two cases are possible:

Zero rent on all Θ : $p(\theta) > 0$ for all $\theta \in \Theta$ implies $U(\theta) = 0$. Then the quadruple $(U(\theta), q(\theta), \lambda(\theta), p(\theta)) = \left(0, \theta, \frac{\theta}{\beta\delta}, \frac{\beta-2}{2\beta\delta}\right)$ satisfies (A1) to (A6) if and only if $\beta \geq 2$. Note that $\lambda(\cdot)$ is discontinuous at both endpoints $\tau_1 = -\tau_2 = \delta$, with $\epsilon_1 = \epsilon_2 = \frac{1}{\beta} > 0$.

Zero rent only at endpoints: To find the solution when $\beta < 2$, let us set $p(\theta) = 0$ for all θ in Θ . The quadruple $(U(\theta), q(\theta), \lambda(\theta), p(\theta)) = \left(\frac{\beta(\beta-2)}{2(\beta+2)}(\theta^2 - \delta^2), \frac{2\beta\theta}{\beta+2}, \frac{\theta}{2\delta}, 0\right)$, satisfies (A1) to (A6) if and only if $0 \leq \beta \leq 2$. Note again that $\lambda(\cdot)$ is discontinuous at both endpoints $\tau_1 = -\tau_2 = \delta$, with $\epsilon_1 = \epsilon_2 = \frac{1}{2} > 0$. ■

• **Lobbying competition. Preliminaries:** To characterize equilibria under interim contracting, we first consider the reduced problem $(\tilde{\mathcal{P}}_i^C)$ where the monotonicity constraint (3) is ignored. This constraint will be checked ex post on the solution of the relaxed problem. This is now an optimal control problem with a unique pure state constraint whose Lagrangian can be written as:

$$L_i(U, q, \lambda_i, p_i, \theta) = H_i(U, q, \lambda_i, \theta) + p_i(U - U_{-i}(\theta)),$$

where λ_i is the co-state variable, p_i is the multiplier of the pure state constraint (5). The Hamiltonian of $(\tilde{\mathcal{P}}_i^C)$ is:

$$H_i(U, q, \lambda_i, \theta) = \left\{ -\frac{1}{2}(q - a_i)^2 - \frac{\beta}{2}(q - \theta)^2 + t_{-i}^*(q) - U \right\} \frac{1}{2\delta} + \lambda_i\beta(q - \theta).$$

To characterize the solution of $(\tilde{\mathcal{P}}_i^C)$, we shall apply the sufficient conditions for optimality (see Seierstad and Sydsaeter (1987, Chapter 5, Theorem 1, p. 317)) which hold when $\hat{H}_i(U, \lambda_i(\theta), \theta) = \max_q H_i(U, q, \lambda_i(\theta), \theta)$ is concave in U . First, we show:

Lemma 3 : *The pair $\{t_1^*(q), t_2^*\}$ is an equilibrium only if it generates a rent-policy profile $\{U^*(\theta), q^*(\theta)\}$ which solves problems $(\mathcal{P}_i^C)$ for $i = 1, 2$. Moreover, if $t_i^{*''}(q) < \beta + 1$ $i = 1, 2$, then $H_i(U, q, \lambda_i(\theta), \theta)$ is concave in q .*

Proof: If, for the transfers $t_i^*(q)$, the property stated in Lemma 3 is true, then the objective functions in problems $(\mathcal{P}_i^C)$ are concave in q . It can be verified ex post (i.e., once the equilibrium schedules $t_i^*(\cdot)$ are obtained) that the condition of Lemma 3 holds so that the Hamiltonian is indeed concave in q . ■

The sufficient conditions for optimality ensure that there exists a pair of piecewise continuous functions $\lambda_i(\theta)$ and $p_i(\theta)$, and constants $\epsilon_k \geq 0$, $k = 1, \dots, n$, such that:

$$\dot{U}(\theta) = \beta(q(\theta) - \theta), \quad (\text{A9})$$

$$\frac{\partial H_i}{\partial q}(U, q, \lambda_i, \theta) = 0, \text{ for almost all } \theta \in \Theta, \quad (\text{A10})$$

$$\dot{\lambda}_i(\theta) = \frac{1}{2\delta} - p_i(\theta), \text{ for almost all } \theta \in \Theta, \quad (\text{A11})$$

$$p_i(\theta)(U(\theta) - U_{-i}(\theta)) = 0, \quad p_i(\theta) \geq 0, \quad U(\theta) \geq U_{-i}(\theta), \quad (\text{A12})$$

$$\lambda_i(-\delta) = \lambda_i(\delta) = 0. \quad (\text{A13})$$

Moreover, $\lambda_i(\theta)$ may have jump discontinuities at points τ_k , for $-\delta \leq \tau_k \leq \delta$, ($k = 1, \dots, n$) such that

$$\lambda_i(\tau_k^-) - \lambda_i(\tau_k^+) = \epsilon_k \geq 0, \quad (\text{A14})$$

and

$$\epsilon_k = 0 \text{ if } \begin{cases} \text{either} & a) U(\tau_k) > U_{-i}(\tau_k), \\ \text{or} & b) U(\tau_k) = U_{-i}(\tau_k) \text{ and } q(\theta) \text{ is discontinuous at } \tau_k. \end{cases} \quad (\text{A15})$$

We use the above conditions to derive some properties of the equilibria.

Properties of the equilibrium policy: Assuming strict concavity of the agent's objective function (which turns out to hold once we have derived the equilibrium schedules below), the corresponding first-order condition for the agent's behavior is written as:

$$\sum_{i=1}^2 t_i'(q(\theta)) = \beta(q(\theta) - \theta). \quad (\text{A16})$$

From (A10), we obtain:

$$-(\beta + 1)q(\theta) + a_i + \beta\theta + t_{-i}^{*'}(q(\theta)) + 2\lambda_i(\theta)\beta\delta = 0. \quad (\text{A17})$$

Using (A17), a similar equation obtained by permuting indices and (A16), we get the following expression of an equilibrium policy:

$$q(\theta) = \frac{\beta\theta + 2(\lambda_1(\theta) + \lambda_2(\theta))\beta\delta}{\beta + 2}, \text{ for almost all } \theta \in \Theta. \quad (\text{A18})$$

Also, we can generalize Lemma 2 to the case of competing principals:

Lemma 4 : *The equilibrium policy $q(\theta)$ is everywhere continuous and $\lambda_i(\theta)$ is continuous at any interior point $\theta \in (-\delta, \delta)$. If $\lambda_i(\theta)$ is discontinuous at $\theta = \delta$ or at $\theta = -\delta$, then $q(\theta)$ is continuous and the state constraint is binding at this point.*

Proof: Suppose that at an interior point $\tau \in (-\delta, \delta)$ $\lambda_i(\cdot)$ is discontinuous with $\varepsilon_i = \lambda_i(\tau^-) - \lambda_i(\tau^+) > 0$. Then, from (A15), $q(\theta)$ must be continuous at τ . From (A18) it is possible only if $\lambda_{-i}(\cdot)$ is discontinuous at τ with $\lambda_{-i}(\tau^-) - \lambda_{-i}(\tau^+) = -\varepsilon_i$. But $-\varepsilon_i < 0$, a contradiction with (A14). ■

Properties of the rent profile: Let us now give a first property of the rent profile that helps limiting the investigation of the different equilibrium patterns.

Lemma 5 : *The shape of the agent's information rent depends on whether β is greater or less than one:*

- *If $\beta \leq 1$ (resp. < 1), the agent's information rent $U(\theta)$ is concave (resp. strictly concave) in θ .*
- *If $\beta > 1$, the information rent $U(\theta)$ is strictly convex on a non-empty interval $[\delta_1, \delta_2]$ if and only if $p_1(\theta) + p_2(\theta) < \frac{\beta-1}{\beta\delta}$ for all $\theta \in [\delta_1, \delta_2]$.*
- *If $\beta = 1$, the agent's information rent $U(\theta)$ is linear on a non-empty interval $[\delta_1, \delta_2]$ if and only if $p_i(\theta) = 0$ on this interval.*

Proof: Using (2) and differentiating w.r.t. θ yields: $\ddot{U}(\theta) = \beta(\dot{q}(\theta) - 1)$. From (A11) and (A18), we obtain

$$\dot{q}(\theta) = \frac{\beta + 2\beta\delta(\frac{1}{\delta} - p_1(\theta) - p_2(\theta))}{\beta + 2} = \frac{3\beta - 2\beta\delta(p_1(\theta) + p_2(\theta))}{\beta + 2}.$$

Thus, we have:

$$\ddot{U}(\theta) = \frac{2\beta(\beta - 1 - \beta\delta(p_1(\theta) + p_2(\theta)))}{\beta + 2}.$$

Since $p_i(\theta) \geq 0$, Lemma 5 is proved. ■

The positiveness of the Lagrange multipliers $p_i(\theta)$ ($i = 1, 2$) has an important impact on the properties of equilibria. If $p_i(\theta) > 0$ on non-degenerate interval, the corresponding state constraint $U(\theta) \geq U_{-i}(\theta)$ is binding on that interval, and, consequently, the transfer $t_i(q(\theta))$ is identically equal to zero there.

From (A17), we obtain an expression for $t'_i(\cdot)$ that does not depend on $t_{-i}(\cdot)$. For each configuration of parameters that we consider below, (A17) uniquely defines the derivative of the equilibrium schedule in the equilibrium range. This leads to the unique equilibrium up to some constants of integration depending of the different possible configurations for the sign of those Lagrange multipliers of the problems $(\mathcal{P}_i^C)$, $i = 1, 2$. This is what we will do in the proceeding proofs. ■

• **Proof of Proposition 2:** Suppose that $t_2^*(q) = 0$ for all q so that $U_2(\theta) = 0$ for all $\theta \in \Theta$. Then, from (A18), we get that P_1 's best-response is to induce $q(\cdot)$ such that:

$$-(\beta + 1)q(\theta) + 1 + \beta\theta + 2\lambda_1(\theta)\beta\delta = 0.$$

The “laissez-faire” policy $q(\theta) = \theta$ is optimal when $\lambda_1(\theta) = \frac{\theta-1}{2\beta\delta}$. From (A11), we get $p_1(\theta) = \frac{\beta-1}{2\beta\delta} \geq 0$ if and only if $\beta \geq 1$, so that the participation constraint is everywhere binding, $U(\theta) = U_2(\theta) = 0$ on all Θ .

Finally, we must check the transversality conditions (A13) with the possible discontinuities at end-points given by (A14). Note that

$$\lambda_1(\delta^-) - \lambda_1(\delta^+) = \frac{\delta - 1}{2\beta\delta} = \epsilon \geq 0 \text{ if and only if } \delta \geq 1,$$

$$\lambda_1(-\delta^-) - \lambda_1(-\delta^+) = \frac{\delta + 1}{2\beta\delta} = \epsilon' \geq 0.$$

Since $U(\theta) = U_2(\theta) = 0$ on all Θ , $t_1^*(q) = 0$ for all q . Proceeding similarly for P_2 ends the proof. ■

• **Proof of Proposition 3:** Denote by $\Omega_0 = [-\tau, \tau]$, the symmetric interval where both principals offer null contributions:²³ $U(\theta) = U_1(\theta) = U_2(\theta) = 0$ and thus $q(\theta) = \theta$ on this interval. Using (A17) for principal P_1 , we obtain:

$$\lambda_1(\theta) = \frac{\theta - 1}{2\beta\delta} \text{ for } \theta \in \Omega_0. \quad (\text{A19})$$

We have on $\Omega_1 = [\tau, \delta]$, $U(\theta) = U_1(\theta) > U_2(\theta)$. From that, we deduce that $p_1(\theta) = 0$ on Ω_1 . Using the transversality condition (A13) and integrating (A11) yields:

$$\lambda_1(\theta) = \frac{\theta - \delta}{2\delta} \text{ for } \theta \in \Omega_1. \quad (\text{A20})$$

²³As in the proof of Lemma 3 one can show that the interval Ω_0 is indeed symmetric.

By continuity of $\lambda_1(\cdot)$ at τ we obtain $\tau = \frac{\beta\delta-1}{\beta-1}$. Rewriting the condition that τ should belong to $[0, \delta]$ we obtain that $\delta < 1 < \beta\delta$. Finally inserting the expression of $\lambda_1(\cdot)$ found in (A20) into (A17) and using $t_2^{*'}(q(\theta)) = 0$ for all $\theta \in \Omega_1$, we have:

$$q(\theta) = \frac{2\beta\theta + 1 - \beta\delta}{\beta + 1} \text{ for } \theta \in \Omega_1. \quad (\text{A21})$$

A similar and symmetric expression is obtained for $\theta \in \Omega_2 = [-\delta, -\tau]$. To find the expressions for the equilibrium contributions, we first note that, on Ω_1 , we have for the information rent:

$$\dot{U}(\theta) = \beta(q(\theta) - \theta) = \frac{\beta((\beta - 1)\theta + 1 - \beta\delta)}{\beta + 1} \text{ for } \theta \in \Omega_1.$$

From this equation we determine $U(\theta)$ on Ω_1 (and similarly on Ω_2):

$$U(\theta) = \frac{\beta(\beta - 1)}{2(\beta + 1)}\theta^2 + \frac{\beta(1 - \beta\delta)}{\beta + 1}\theta + C_1,$$

where C_1 is the constant of integration which is determined by the condition $U(\tau) = 0$. This leads to $C_1 = \frac{\beta(\beta\delta-1)^2}{2(\beta-1)(\beta+1)}$. Manipulating yields the expression for $U(\theta)$ in the text. Contributions are determined by $t_1(q(\theta)) = U(\theta) + \frac{\beta}{2}(q(\theta) - \theta)^2$ on Ω_1 (and similarly for $t_2(q(\theta))$ on Ω_2). ■

• **Proof of Proposition 4:** Consider again $\Omega_0 = [-\tau, \tau]$, the symmetric interval where both principals offer non-negative contributions. On this interval, it must be that $U(\theta) \geq \max\{U_1(\theta), U_2(\theta)\}$. Moreover $U(\theta) = U_1(\theta)$ (resp. $U(\theta) = U_2(\theta)$) only at $-\tau$ (resp. τ).

We have $p_1(\theta) = 0$ on $(-\tau, \delta]$. Using the transversality condition (A13) and integrating (A11) implies:

$$\lambda_1(\theta) = \frac{\theta - \delta}{2\delta} \text{ for } \theta \in (-\tau, \delta]. \quad (\text{A22})$$

Similarly, we have $p_2(\theta) = 0$ on $[-\delta, \tau)$ and:

$$\lambda_2(\theta) = \frac{\theta + \delta}{2\delta} \text{ for } \theta \in [-\delta, \tau). \quad (\text{A23})$$

Using (A18) for principal P_1 and (A22), we have:

$$-(1 + \beta)q(\theta) + 1 + \beta\theta + t_2^{*'}(q(\theta)) + \beta(\theta - \delta) = 0 \text{ for } \theta \in (-\tau, \delta]. \quad (\text{A24})$$

Similarly, using (A18) for principal P_2 and (A23), we obtain:

$$-(1 + \beta)q(\theta) - 1 + \beta\theta + t_1^{*'}(q(\theta)) + \beta(\theta + \delta) = 0 \text{ for } \theta \in [-\delta, \tau). \quad (\text{A25})$$

Summing (A24) and (A25) and using the agent's first-order condition yields:

$$q(\theta) = \frac{3\beta\theta}{\beta + 2} \text{ for } \theta \in (-\tau, \tau). \quad (\text{A26})$$

Using (A24) and taking into account that $t_2^*(q(\theta)) = 0$ on Ω_1 gives the following expression of the policy on that interval:

$$q(\theta) = \frac{2\beta\theta + 1 - \beta\delta}{\beta + 1} \text{ for } \theta \in [\tau, \delta]. \quad (\text{A27})$$

Continuity of $\lambda_2(\cdot)$ at τ is ensured when: $\frac{2\beta\tau + 1 - \beta\delta}{\beta + 1} = \frac{3\beta\tau}{\beta + 2}$, or when

$$\tau = \frac{(2 + \beta)(1 - \beta\delta)}{\beta(\beta - 1)}. \quad (\text{A28})$$

If $\beta > 1$, τ belongs to $(0, \delta]$ when $\beta\delta < 1 \leq \frac{\beta(2\beta+1)}{(\beta+2)}\delta$.

To find the expression of the equilibrium contribution $t_1^*(q)$, note that using (A25) yields for $\theta \in \Omega_0$:

$$t_1^*(q) = \begin{cases} 0 & \text{if } q \leq \frac{-3\beta\tau}{2+\beta}, \\ \frac{\beta(\beta-1)}{\beta+2}\tau + \frac{(\beta-1)}{3}q & \text{if } q \in \left[\frac{-3\beta\tau}{2+\beta}, \frac{3\beta\tau}{2+\beta}\right], \\ \beta(\beta-1)\left(q + \frac{\tau}{\beta+2}\right) & \text{if } q \geq \frac{3\beta\tau}{2+\beta}. \end{cases}$$

Note that $t_1(q(-\tau)) = 0$ because $U(-\tau) = U_2(-\tau)$ by definition of Ω_2 . Integrating and taking into account that $t_1(\cdot) = 0$ is continuous at $q(\tau) = \frac{3\beta\tau}{2+\beta}$ yields thus the expression in the text. It is important to note that $U^*(\cdot)$ is convex and piecewise continuously differentiable profile with:

$$\dot{U}^*(\theta) = \begin{cases} \frac{\beta(\beta-1)}{\beta+1}\left(\theta - \frac{\tau\beta}{\beta+2}\right) & \text{if } \theta \leq -\tau, \\ \frac{2\beta(\beta-1)}{\beta+2}\theta & \text{if } \theta \in [-\tau, \tau], \\ \frac{\beta(\beta-1)}{\beta+1}\left(\theta - \frac{\tau\beta}{\beta+2}\right) & \text{if } \theta \geq \tau. \end{cases}$$

$U^*(\cdot)$ is thus minimum at zero with: $U^*(0) = \frac{3\gamma^2}{\beta-1} > 0$. ■

• **Proof of Proposition 5:** We are now looking for an equilibrium such that the participation constraint $U(\theta) \geq U_2(\theta)$ (resp. $U(\theta) \geq U_1(\theta)$) binds only at the end-point $-\delta$ (resp. δ). Thus we have $p_1(\theta) = 0$ on $(-\delta, \delta]$. Using the transversality condition (A13) and integrating (A11) yields: $\lambda_1(\theta) = \frac{\theta-\delta}{2\delta}$ for $\theta \in (-\delta, \delta]$. Similarly, we have $p_2(\theta) = 0$ on $[-\delta, \delta)$. Using the transversality condition (A13) and integrating (A11) implies: $\lambda_2(\theta) = \frac{\theta+\delta}{2\delta}$ for $\theta \in [-\delta, \delta)$. Using (A18) for principal P_1 we obtain:

$$-(1 + \beta)q(\theta) + 1 + \beta\theta + t_2^*(q(\theta)) + \beta(\theta - \delta) = 0 \text{ for } \theta \in (-\delta, \delta]. \quad (\text{A29})$$

Similarly, using (A18) for principal P_2 we have:

$$-(1 + \beta)q(\theta) - 1 + \beta\theta + t_1^*(q(\theta)) + \beta(\theta + \delta) = 0 \text{ for } \theta \in [-\delta, \delta). \quad (\text{A30})$$

Summing (A29) and (A30) and using the agent's first-order condition implies:

$$q^*(\theta) = \frac{3\beta\theta}{\beta+2} \text{ for } \theta \in (-\delta, \delta). \quad (\text{A31})$$

Hence, $\dot{U}^*(\theta) = \frac{2\beta(\beta-1)}{\beta+2}\theta$ for $\theta \in (-\delta, \delta)$. Integrating yields the expression of $U^*(\theta)$ in the text. Similarly using (A30) and (A31) yields

$$t_1^*(q) = \frac{(\beta-1)}{3}q + \gamma \text{ for } q \in \left(-\frac{3\beta\delta}{\beta+2}, \frac{3\beta\delta}{\beta+2}\right). \quad (\text{A32})$$

Integrating and keeping the non-negative part yields $t_1^*(q) = \frac{(\beta-1)}{6}q^2 + \gamma q - C_1$, where C_1 is the constant of integration. Similarly we obtain for contributions $t_2^*(q) = \frac{(\beta-1)}{6}q^2 - \gamma q - C_2$. Note that the condition $U^*(-\delta) = U_2^*(-\delta)$ can be rewritten as:²⁴

$$\frac{\beta(\beta-1)}{\beta+2}\delta^2 - C_1 - C_2 = \max \left\{ 0, -C_2 + \max_q \frac{(\beta-1)}{6}q^2 - \gamma q - \frac{\beta}{2}(q+\delta)^2 \right\}, \quad (\text{A33})$$

or

$$\frac{\beta(\beta-1)}{\beta+2}\delta^2 - C_1 - C_2 = \max \left\{ 0, -C_2 + \frac{3}{2(2\beta+1)} - \frac{\beta\delta^2}{2} \right\}. \quad (\text{A34})$$

But, because we must have $U^*(0) = -C_1 - C_2 \geq 0$, the only possibility to solve (A34) is:

$$C_1 = C_2 = \frac{3\beta^2\delta^2}{2(\beta+2)} - \frac{3}{2(1+2\beta)}, \quad (\text{A35})$$

which is a negative number (as requested by the condition $U^*(0) = -C_1 - C_2 \geq 0$) when $1 \geq \beta\delta\sqrt{\frac{2\beta+1}{\beta+2}}$ but this latter condition holds since $\frac{\beta+2}{(2\beta+1)\beta} > \delta$ and $\beta > 1$. $\blacksquare$

• **Proofs of Propositions 6 and 7:** We consider an equilibrium such that the participation constraints (5) are binding only at endpoints. For all $\theta \in (-\delta, \delta)$ we thus have $p_1(\theta) = p_2(\theta) = 0$. The co-state variables $\lambda_i(\theta)$ are continuous for all $\theta \in (-\delta, \delta)$ but may still have discontinuities at end-points. From (A11), by integrating we obtain: $\lambda_i(\theta) = \frac{\theta}{2\delta} + \lambda_i$, where λ_1 and λ_2 are some constants. Using (A18) we have

$$q(\theta) = \frac{3\beta\theta + 2(\lambda_1 + \lambda_2)\beta\delta}{\beta+2}. \quad (\text{A36})$$

Therefore, we get: $\dot{U}(\theta) = \frac{2\beta(\beta-1)}{\beta+2}\theta + \frac{2(\lambda_1+\lambda_2)\beta^2\delta}{\beta+2}$, and thus

$$U(\theta) = \frac{2\beta}{\beta+2} \left[\frac{(\beta-1)(\theta^2 - \delta^2)}{2} + \beta\delta(\lambda_1 + \lambda_2)\theta \right] - 2C, \quad (\text{A37})$$

where C is a constant of integration. Because of the symmetry of the model, we focus on symmetric rent profile such that

$$U(\delta) = U(-\delta). \quad (\text{A38})$$

²⁴ A similar condition is obtained by writing the boundary condition $U^*(\delta) = U_1^*(\delta)$.

Using (A37), and (A38) yields necessarily, $\lambda_1 + \lambda_2 = 0$. From (A36), we conclude that

$$q(\theta) = \frac{3\beta\theta}{\beta + 2}. \quad (\text{A39})$$

To satisfy the transversality conditions (A13) it must be that $\lambda_i \in [-\frac{1}{2}, \frac{1}{2}]$. Then, observe that both co-state variables may have jumps at endpoints. Those jumps corresponds to the binding participation constraints $U(\delta) = U_2(\delta) = U_1(\delta)$ and $U(-\delta) = U_2(-\delta) = U_1(-\delta)$. Denote $-C = U(\delta) = U(-\delta)$ as the common utility level at both endpoints $\pm\delta$. Using (A37), the agent's information rent becomes:

$$U(\theta) = \frac{\beta(1-\beta)}{\beta+2}(\delta^2 - \theta^2) - 2C.$$

It is non-negative and concave only if $\beta \leq 1$ and $C \leq 0$.

From (A17) and (A18), we get the expression of the marginal contributions offered by both principals at any equilibrium policy:

$$t'_1(q) = -\frac{1-\beta}{3}q + (2\beta\delta\lambda_1 + 1) \quad \text{and} \quad t'_2(q) = -\frac{1-\beta}{3}q - (2\beta\delta\lambda_1 + 1), \quad (\text{A40})$$

where $\lambda_1 \in [-\frac{1}{2}, \frac{1}{2}]$ is arbitrary. Let us denote $\gamma = 2\beta\delta\lambda_1 + 1$. Integrating (A40) yields the following expressions of contributions up to some constants (C_1, C_2) :

$$t_1(q) = -\frac{1-\beta}{6}q^2 + \gamma q - C_1, \quad \text{and} \quad t_2(q) = -\frac{1-\beta}{6}q^2 - \gamma q - C_2. \quad (\text{A41})$$

Those expressions are valid as long as contributions are positive. The corresponding schedules are thus defined as:

$$t_1(q) = \max \left\{ 0, -\frac{1-\beta}{6}q^2 + \gamma q - C_1 \right\} \quad \text{and} \quad t_2(q) = \max \left\{ 0, -\frac{1-\beta}{6}q^2 - \gamma q - C_2 \right\}. \quad (\text{A42})$$

From this we obtain the policies chosen by the agent when he contracts only with one of the principals and the corresponding contribution is positive. These policies are respectively given by:

$$q_1(\theta) = \frac{3}{2\beta+1}[\beta\theta + \gamma] \quad \text{and} \quad q_2(\theta) = \frac{3}{2\beta+1}[\beta\theta - \gamma].$$

Observe that $q_1(\theta) > q(\theta)$, and $q_2(\theta) < q(\theta)$.

Let us turn now to the characterization of the pairs (C_1, C_2) . Because (5) is binding at both endpoints, those constants must satisfy the condition

$$\max_q \left\{ -\frac{1-\beta}{3}q^2 - \frac{\beta}{2}(q \pm \delta)^2 \right\} - C_1 - C_2$$

$$= \max \left\{ 0, \max_q \left\{ -\frac{1-\beta}{6}q^2 + \gamma q - \frac{\beta}{2}(q-\delta)^2 - C_1 \right\}, \max_q \left\{ -\frac{1-\beta}{6}q^2 - \gamma q - \frac{\beta}{2}(q+\delta)^2 - C_2 \right\} \right\}. \quad (\text{A43})$$

Two cases should be considered depending on whether $2C = \frac{\beta(1-\beta)}{\beta+2}\delta^2 + C_1 + C_2$ is negative or null.

Zero rent at endpoints (Proposition 6), $C = 0$: Then (A43) can be rewritten as a pair of conditions. The first condition $U(\pm\delta) = 0$ becomes:

$$C_1 + C_2 = -\frac{\beta(1-\beta)}{\beta+2}\delta^2. \quad (\text{A44})$$

To satisfy the second condition $U(\delta) = U_1(\delta) = 0$ (resp. $U(-\delta) = U_2(-\delta) = 0$), we must also have:²⁵

$$C_i \geq \max_q \left\{ -\frac{1-\beta}{6}q^2 + \gamma q - \frac{\beta}{2}(q-\delta)^2 \right\} = \frac{3(\gamma + \beta\delta)^2}{2(2\beta+1)} - \frac{\beta\delta^2}{2}. \quad (\text{A45})$$

The linear constraints (A44) and (A45) are compatible when

$$1 \leq (2\beta+1)\delta \sqrt{\frac{\beta}{3(\beta+2)}} - \beta\delta(2\lambda_1+1),$$

which gives the upper bound in the text.

Positive rent at endpoints (Proposition 7), $C < 0$: The conditions $U(\delta) = U_1(\delta) = -C > 0$ and $U(-\delta) = U_2(-\delta) = -C > 0$ yield:

$$-\frac{\beta(1-\beta)}{\beta+2}\delta^2 - C_1 - C_2 = \frac{3(\beta\delta + \gamma)^2}{2(2\beta+1)} - \frac{\beta\delta^2}{2} - C_1 = \frac{3(\beta\delta + \gamma)^2}{2(2\beta+1)} - \frac{\beta\delta^2}{2} - C_2 > 0.$$

This can be rewritten as $C_i = \frac{3}{2} \left(\frac{\beta^2\delta^2}{\beta+1} - \frac{(\gamma+\beta\delta)^2}{2\beta+1} \right)$. Note that, because $U(\delta) = U_1(\delta) = -C > 0$ and $U_2(\delta) \geq 0$ we must have $\lambda_1(\delta) = 0$ which means that necessarily $\lambda_1 = -\frac{1}{2}$ and $\gamma = 1 - \beta\delta$. ■

²⁵One can check that $U(\delta) = U_1(\delta) = 0$ implies also $U(-\delta) = U_1(-\delta) = 0$ and $U(-\delta) = U_2(-\delta) = 0$ implies $U(\delta) = U_2(\delta) = 0$.