Comment on The Identification Power in Games by A. Aradillas-Lopez and E. Tamer:

Thierry Magnac, Toulouse School of Economics, magnac@cict.fr

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The paper by Andres Aradillas-Lopez and Elie Tamer clarifies how identification can be obtained in games and provides techniques to do so. In other words, it offers constructive ways of analyzing either point or set identification of parameters governing the preferences and beliefs of players in different games. In this short comment, I am trying to highlight the key issues that are brought out by the paper.

First, as sometimes alluded to by the authors, one way of reading their results is to recognize that concepts of Nash equilibrium or Bayesian Nash equilibrium lead to fixed point equations. It is then a quite natural consequence to use iterations to arrive at equilibrium since fixed point problems are usually solved by an appeal to a contraction argument and by iterating an infinite number of times the fixed point equation (Rust, Traub and Wozniakowski, 2002). This iteration technique sets two kinds of questions. It poses the question of the interpretation of the iterations and also the question of the uniqueness of solutions.

Iterations may have structural interpretation. For instance in dynamic choice models, they are viewed as the steps in a real or mental process of backward induction by a decision maker and, in this paper, a nice interpretation of "levels of rationality" can be attached to the iterations. Iterations need not have structural interpretation. The convergence to a stable equilibrium in a simple demand and supply set up, using a cobweb iteration technique, might not be structural. A standing question remains about the gain of having a structural interpretation for iterations.

The question of uniqueness of equilibrium is very well described by the three examples worked out in the paper. The first example is the simplest because we arrive at equilibrium after one step. The second example is more sophisticated because we need an infinite number of steps before converging to the equilibrium if the equilibrium is unique. If it is not unique, the limit set is larger than the set of equilibria. The last example in first-price auctions is more involved since we cannot have convergence to the equilibrium by following Battigalli & Siniscalchi in their construction of iterations. Multiplicity of solutions is thus a more severe issue than what is obtained under equilibrium assumptions and the approximation of the strength that an equilibrium hypothesis is providing, by a set of iterations, is less clear.

A second way of reading the paper is to look at the relationship, investigated by the authors, between the set of rationalizable strategies and the set of structural parameters (a set which can be a singleton). The tools that the authors are using, are coming from the literature dealing with set identification, to which Elie Tamer has contributed a lot (e.g. Tamer, 2003 and Honoré and Tamer, 2006). The formal exercise of writing down the transformation between the space of strategies and the space of parameters is not unified in the paper yet. To clarify the issue (without taking much care with what these spaces are), I am trying to discuss it more formally now.

Let us first look at the example of a single decision maker. A standard structural model would say that the decision, say y, is a function of observed and exogenous characteristics, say x, of unobserved characteristics, ε and of a finite dimensional parameter β:

$$y = d(x, \varepsilon; \beta). \tag{1}$$

Semiparametric identification consists in characterizing the binary relationship between the reduced form given by the probability distribution, $P(y \mid x)$, and the structural form, given by the parameter $\theta = (\beta, F_\varepsilon(. \mid x))$ where $F_\varepsilon(. \mid x)$ is the distribution of ε. The former characterization of the data, $P(y \mid x) = P_{\theta,d}(y \mid x)$, is induced by the latter structural parameter θ **and** the structural equation (1) . The reduced form may respectively have none, one or many images in the space of structural parameters, in which case we say that parameters respectively are overidentified, point-identified or set-identified. Examples of the last instance are not frequent albeit not uncommon. Multicolinearity in linear models or measurement error models (Leamer, 1987) provide such instances.

When there are two decision makers, such as in a game, the previous setting applies without modification only if the economic model $d(.)$ in equation (1) remains a function. It suffices to reinterpret y as applying to decisions for both, or all, players. What changes in the games studied by Aradillas Lopez and Tamer is that the structural model delivers structural *correspondences* instead of functions. We now have that:

$$Y = d(x, \varepsilon; \beta). \tag{2}$$

where Y is a set. For instance, in Example 1 using Nash equilibrium in mixed strategies, the "central square" of the $(t_1, t_2)-$space (the authors' ε) yields all solutions, $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$. Other examples that the authors study, can be framed like that, either using an equilibrium concept or a rationalizability one.

The formal discussion may help to derive some general results. For instance, the structural assumption of level k rationality is weaker than the corresponding one at level $k+1$ so that correspondences at levels k and $k+1$ satisfy inclusion properties $Y_k \supseteq Y_{k+1}$. *As a consequence*, identified sets under successive levels of rationality might be shown to satisfy the same relationship.

Besides, defining the structural model as a correspondence shows that an aspect of what the authors are proposing is not specific to games. Return to the single decision maker case by considering identification in dynamic discrete choice models (Magnac and Thesmar, 2002) where we might have a correspondence instead of a function. Probabilities of future events, anticipated by the decision maker, could be ambiguous, for instance, because they satisfy inequality restrictions instead of being point identified. The imaginative setting of Aradillas Lopez and Tamer can be extended to these general frameworks to analyze set identification in these models. Furthermore, dealing with equation (2) where Y is a set, is the objective of a recent thoughtful paper by Galichon and Henry (2006) whose results could then be used to establish asymptotic properties of estimators.

Nevertheless, the additional twist with games is related to the standing question in the literature about dynamic choices about the separate identification of preferences and expectations (Manski, 2004). What is interesting in games is that beliefs are a function of preferences and beliefs held by the other agents so that they are intertwined. It is this property that is used in the paper since rationalizability delivers constraints on beliefs. It would not be the case with single decision maker models.

Another useful question for applied researchers is to clarify which normalizations of the parameters are needed and what is the space of parameters where we can hope to get identification. The difference between a normalization and an identifying restriction might not be clear at first sight. A possible definition would be that normalizations do not affect the family of probability distributions in spite of a reduction in the parameter space. As before, let $\theta \in \Theta$, the parameter space. A normalization would be defined formally as: For any $\theta \in \Theta$, there exists $\theta_0 \in \Theta_0$ where $\Theta_0 \subset \Theta$, the inclusion being strict, such that $Pr(y \mid x, \theta) = \Pr(y \mid x, \theta_0)$ almost everywhere

(y, x).

For instance, in the simple game with complete information, is the assumption about the discrete nature of the distribution function of (t_1, t_2) a normalization? and how should this function be normalized? This case is simple because we can summarize the problem by the probability weights of the nine regions of the parameter space which are defined as lying below or above, 0 or the parameters α. We thus need only 8 numbers to describe the problem so that, as α_1 and α_2 are the structural parameters of interest, we just need 6 probabilities. This is obtained by normalization, that is, there exists a transformation of the parameter space in a way that can be written as a function of 6 numbers and α_1, α_2. I believe that it is a question to be solved before running the linear program (3.1) that is proposed by the authors, lest complications might arise. The same remark applies to the other examples in the paper where normalization and identification assumptions might be distinguished more clearly.

There are also more specific questions. Some index restrictions are used although we do not understand immediately their necessity. More deeply, the rôle of unobserved heterogeneity in beliefs and the common prior assumption is clearly high on the research agenda and the paper makes useful steps towards stating sufficient conditions.

In conclusion, this is a nice paper because it sets up more questions than it solves although it solves many. It is certainly a very nice research agenda that the authors are proposing to anybody working in this field.

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