Screening Risk-Averse Agents Under Moral Hazard[1]

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August 2001

[1]The authors thank P.A. Chiappori, D. de Meza, R. Myerson, C. Prendergast, S. Rosen and D. Webb for helpful discussions and comments, as well as seminar participants in Berkeley, Chicago, Montréal, Northwestern, Rome, Stanford and Wisconsin. Bruno Jullien gratefully acknowledges financial support from the Fédération Francaise des Sociétés d'Assurance; Bernard Salanié thanks the University of Chicago for its hospitality.

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Abstract

Principal-agent models of moral hazard have been developed under the assumption that the principal knows the agent's risk-aversion. This paper extends the moral hazard model to the case when the agent's risk-aversion is his private information, so that the model also exhibits adverse selection. We characterize the optimal menu of contracts; while its detailed properties depend on the setting, we show that some of them must hold for all environments. In particular, the power of incentives always decreases with risk-aversion. We also characterize the relationship between the outside option and the optimal contracts. We then apply our results to insurance, managerial incentive pay, and corporate governance.

Introduction

The traditional literature on moral hazard emphasizes the trade-off between risk-sharing and the provision of incentives. It derives optimal contracts whose shape depends closely on the agent's risk-aversion. One may then wonder how the principal knows so precisely the agent's preferences; and it seems likely that the agent will try to manipulate the principal's perception of his risk-aversion. We here extend the moral hazard model to the case when there is adverse selection on the agent's risk-aversion. While such an extension is of theoretical interest, we argue that it may also help solve some empirical puzzles that have been uncovered in recent years. Several papers (e.g. Chiappori-Salanié (2000)) indeed have shown that the standard models of insurance under asymmetric information predict correlations that cannot be found in the data. Thus it seems that we need a richer model to account for the empirical evidence.

We analyze here a two-outcome/two-types model of moral hazard with adverse selection on the agent's risk-aversion. We first show that when the agent's risk-aversion is public information (the public risk-aversion model), very little can be said on the link between risk-aversion and the optimal contract. In particular, intuition suggests that more risk-averse agents should face lower-powered incentives. In fact, we find that the power of incentives may be a non-monotonic function of risk-aversion.

Surprisingly, this anomaly disappears when the agent's risk-aversion is his private information (the private risk-aversion model). We indeed show that in the contract space, the traditional Spence-Mirrlees condition is verified under weak regularity conditions. This implies that adverse selection imposes more structure on the design of optimal contracts: more risk-averse agents must opt for contracts with lower-powered incentives, as intuition suggests.

While the single-crossing property greatly simplifies the analysis of the private risk-aversion model, it still turns out to allow for a variety of configurations. In the "regular" case in which more risk-averse agents face lower-powered incentives in the public risk-aversion model, the private risk-aversion optimum always separates types. But while more risk averse agents always face lower-powered incentives, they may well provide more effort since this reduces the risk they face. *Hence the relationship between the incentives and the observed probability distribution of outcomes is ambiguous.* As announced above, this important result allows for a better understanding of the links between incentives, performance and risk, and may be of interest in many economic activities (insurance, but also managerial pay, corporate governance…).

In the "non-responsive" case in which more risk-averse agents face higher-powered incentives in the public risk-aversion model, the private risk-aversion optimum may involve separation or bunching, depending on the parameters.

It also turns out that the power of the incentives implicit in the outside option plays a crucial role in the analysis. In an insurance model, on the one hand, the power of incentives is maximal for the outside option of no-insurance. In labor economics models, on the other hand, the power of incentives is typically minimal for the outside option. We find that whether there is overprovision or underprovision of incentives relative to the public risk-aversion model, and which type of agent benefits from an informational rent, depends in a monotonic way on the characteristics of the outside option. Moreover, a novel feature of the private risk-aversion model is that the power of incentives increases (weakly) with that of the outside option, contrary to the public risk-aversion model in which it is independent of that of the outside option. This implies that observed probabilities of success should be higher for agents who face more powerful incentives in their outside options.

We apply our analysis to three situations of interest: insurance, managerial incentive pay, and the financing of a project. In an insurance context, our results may explain the empirical findings by Chiappori and Salanié (2000) who find no correlation between risk and coverage for automobile insurance. When applied to executive compensation, our results imply that there is underprovision of incentives when risk-aversion is private and that less risk-averse executives should have more performance-sensitive pay. Finally we develop an example of corporate governance to show that our model predicts that the economic performance of a project should be positively related to the degree of self-financing, but not necessarily to the debt-equity ratio.

On a theoretical level, our paper's contribution is both to introduce heterogeneous risk-aversions and to solve a model with both moral hazard and adverse selection (what Myerson (1982) calls a generalized agency model). Heterogeneity in risk aversion by itself doesn't create in general acute problems for studying adverse selection, as shown by the papers of Salanié (1990) for vertical contracting, Landsberger and Meilijson (1994) for insurance and Laffont-Rochet (1998) for the regulation of firms. However, it creates new difficulties when there is moral hazard, as the degree of risk aversion affects the agent's behavior and thus the principal's expected utility from a given contract. The existing papers on the generalized agency model (Laffont and

Tirole (1986), McAfee and McMillan (1987), Baron and Besanko (1987), Faynzilberg and Kumar (1997, 2000)) have focussed on the case where the agent's private characteristic affects the technology. In that case, effort is a monotonic function of type under some regularity conditions. Our model is much richer in that higher risk-aversion leads to higher effort (in the regular case) but also to lower-powered incentives (and thus to lower effort), so that the relationship between effort and type is fundamentally ambiguous[1].

To the best of our knowledge, very little is known on optimal contracting when the agent's risk-aversion is his private information. We should mention, however, related work by de Meza and Webb (2000) and by Pauly (1974), Villeneuve (2000) and Wambach (1997). Our paper is also related to the work of Chassagnon and Chiappori (1997) and Stewart (1994) on insurance with adverse selection and moral hazard.

Section 1 of the paper analyses the agent's choice of effort. We state and prove the Spence-Mirrlees condition in Section 2. We then specialize the model to CARA utility functions, and study the public (resp. private) risk-aversion model in Section 3 (resp. Section 4). Finally, Section 5 applies our results to the three situations listed above.

1 The Choice of Effort

Consider a risk-averse agent facing a binomial risk: success is worth W, while failure yields a lower revenue $(W - \Delta)$. The agent can exert a costly effort $e \in [0, \bar{e}]$ to reduce the probability $p(e) \in]0, 1[$ of failure. We shall refer to $C = (W, \Delta)$ as a contract, and with a slight abuse of interpretation to Δ as the power of incentives. We shall also use the inverse function $c(p)$ whenever convenient; by definition, $c(p(e)) = e$. Assume decreasing returns, and the usual Inada conditions:

Assumption 1 *The function $p(e)$ is decreasing convex on $[0, \bar{e}]$, with $p'(0) = -\infty$, $p'(\bar{e}) = 0$.*

This section is devoted to the comparative statics governing the effort choice. Keep in mind that we mainly want to discuss the effects of a change

[1] Hemenway (1990, 1992) identified the first effect in the insurance context and called it "propitious selection".

in risk-aversion on effort. It is therefore important to ensure that whatever their risk-aversion, all types of agents have access to the same technology, as defined by the function $p(e)$. In other words, the marginal rate of substitution between effort and wealth should not depend on the agent's risk-aversion. This is only possible with the so-called "monetary cost" formulation of effort[2]. Consequently we assume that the agent has a concave Von Neumann-Morgenstern utility function u, and that effort and wealth are perfect substitutes. We define the agent's problem as

$$\mathcal{U}(W, \Delta) \equiv \max_p \, (1-p)u(W - c(p)) + pu(W - \Delta - c(p)) \tag{1}$$

and we denote by $p(W, \Delta)$ a solution to this program.

It is well-known (see Ehrlich-Becker (1972), Dionne-Eeckhoudt (1985), and Jullien-Salanié-Salanié (1999)) that this apparently simple problem displays complex properties. In particular, the effect of a change in risk-aversion is ambiguous: even when the Arrow-Pratt index of risk-aversion is a constant (the CARA case), the probability of failure $p(W, \Delta)$ may be non-monotonic with respect to risk-aversion. Hence, and contrary to what intuition suggests, more risk-averse agents do not necessarily exert more effort. The reason is that effort reduces the income in case of failure, so that a more risk-averse agent facing a high probability of failure may opt for increasing his worst-case income instead of reducing the probability of failure[3]. As a consequence, note that the effect of varying W also is ambiguous if wealth effects are present.

One may also wonder whether stronger incentives (a higher Δ) indeed reduce the probability of failure. Once more, this is not generally true, because of wealth effects. Conversely, in the absence of wealth effects we prove in the Appendix that

Lemma 1 *Under CARA utilities, the probability of failure $p(W, \Delta)$ is decreasing with respect to Δ, and is independent of W.*

[2]Note that the usual "non-monetary cost" formulation defines the agent's payoff as $(u(R) - e)$. Therefore the MRS is $u'(R)$. Such a formulation does not allow to properly study the impact of risk-aversion, since modifying u into v more risk-averse than u also modifies the agent's preferences under certainty.

[3]Jullien-Salanié-Salanié (1999) shows that for a given incentive contract (W, Δ) and a given technology c, the optimal probability of failure (locally) decreases in risk aversion if and only if this probability is small enough.

2 Regularity and single-crossing

As we have just seen, there are not many general results for the solution of program (1). Nevertheless this section proves an important result that will make tractable the analysis of contracts under adverse selection.

The value $\mathcal{U}(W, \Delta)$ of the agent's program (1) defines some indifference curves in the space (W, Δ) of contracts. The Spence-Mirrlees property then says that more risk-averse agents should be compensated more for an increase in incentives; or equivalently the slope $\left(\frac{\partial W}{\partial \Delta}\right)_{\mathcal{U}}$ must be higher for a more risk-averse agent. This single crossing property would imply that, *ceteris paribus*, more risk-averse agents prefer lower-powered schedules. This is what intuition suggests; but as shown by the fact that more risk-averse agents may expend less effort, intuition may be a poor guide in these models. The difficulty, of course, is that p is endogenous, and varies when risk-aversion is changed.

In this section we offer two distinct sets of sufficient conditions for the single-crossing property to hold. Our first condition is the following:

Property (Q): *consider two contracts $C_1 = (W_1, \Delta_1)$ and $C_2 = (W_2, \Delta_2)$ such that $\Delta_2 > \Delta_1$. Suppose that an agent is indifferent between both contracts: $\mathcal{U}(W_1, \Delta_1) = \mathcal{U}(W_2, \Delta_2)$. Denote e_k an optimal choice of effort under C_k. Then*

$$W_2 - \Delta_2 - e_2 < W_1 - \Delta_1 - e_1 < W_1 - e_1 < W_2 - e_2.$$

This property is easily understood. Since $\Delta_1 < \Delta_2$ and the agent is indifferent between both contracts, then it must be that $W_1 < W_2$. But this in turn implies that $W_1 - \Delta_1 > W_2 - \Delta_2$, because otherwise contract C_2 would dominate contract C_1. Summarizing:

$$W_2 - \Delta_2 < W_1 - \Delta_1 < W_1 < W_2.$$

In other words, the second contract is riskier in the sense that the range of possible wealth levels is enlarged. Property (Q) requires that this ordering of contracts remains valid in utility space, that is once the agent has adjusted his choice of effort. Clearly (Q) holds if effort is set at the same level for both contracts, and it also holds if effort does not vary too much with incentives. It is fairly natural in an insurance context, for instance: if an agent is indifferent between two contracts, then (Q) means that the contract with a higher deductible should have more extreme outcomes in utility space.

We are now ready to state our first result, whose proof is in the Appendix:

Proposition 1 *If (Q) holds, then the Spence-Mirrlees condition holds: more risk-averse agents prefer lower-powered incentive contracts.*

While (Q) seems fairly natural, it would be nice to have assumptions on primitives that imply it. To do this, we now define:

Property (P): $pc'(p)$ is increasing in p.

Property (P) is satisfied by several simple classes of technologies, for instance

$$p(e) = \frac{p_0}{(1+ke)^a} \text{ for all positive } a \text{ and } k.$$

It is also easy to see that it is a natural condition to guarantee the quasi-concavity of the agent's program (1), under the CARA assumption. Moreover, consider the expected wealth of the agent $ER = W - p\Delta$. For a risk-neutral agent, p minimizes $(p\Delta + c(p))$ and thus $\Delta = -c'(p)$. Property (P) then is equivalent to ER being an increasing function of p and thus a decreasing function of Δ. To rephrase this, property (P) must hold if an increase in Δ reduces expected wealth for a risk-neutral agent, which seems natural.

It turns out that with CARA utility functions, property (P) implies property (Q). Thus the justifications for (P) extend to (Q).

Lemma 2 *Suppose that the utility function is CARA and that property (P) holds. Then property (Q) also holds.*

One can also get rid of the CARA assumption by assuming that agents can be ordered in the Ross (1981) sense[4]. Then the Spence-Mirrlees property holds under a slightly weaker version of property (P):

Property (WP): $pc'(p)$ is non-decreasing in p.

In particular, (WP) (but not (P)) holds for $p(e) = \exp(-\lambda e)$. We prove in the Appendix that:

[4]That is, v is Ross-more risk-averse than u if and only if for all x and y, $-v''(x)/v'(x) \geq -u''(y)/u'(y)$. See also Gollier(2001).

Proposition 2 *Suppose that agent H is more risk-averse than agent L, in the sense of Ross. Then (WP) implies that the single-crossing condition holds: agent H prefers lower-powered incentives than agent L.*

3 The Optimal Contract with Public Risk-aversion

Let us now introduce a risk-neutral principal, in charge of selecting an incentive contract $(W,\,\Delta)$. Denote S the wealth created in case of success, and $(S-D)$ the wealth created in case of failure. Assume that the agent has a reservation option given by an incentive scheme (W_0, Δ_0). In an insurance model, this option would be the no-insurance situation: an agent of initial wealth $W_0 = S$ faces a potential damage $\Delta_0 = D$. In a labor context, the reservation option is unemployment, with $\Delta_0 = 0$ and wealth W_0.

Because wealth effects make the analysis extremely cumbersome, we shall focus from now on on the case of $CARA$ utility functions:

Assumption 2 *The agent's utility function is CARA: $u_\sigma(x) = -\exp(-\sigma x)$, where σ is the risk-aversion index; and (WP) holds.*

The assumption on (WP) is here only to ensure that (by Proposition 2) the Spence-Mirrlees single crossing condition holds.

In the CARA case, the level of effort only depends on the power of incentives Δ and not on the level of W. We thus denote it $p_\sigma(\Delta)$. It is more convenient to work with certainty equivalents than with expected utility. For a given contract (W, Δ) and utility function u_σ, the certainty equivalent of the agent is given by

$$CE_\sigma(W,\Delta) = W - c(p_\sigma(\Delta)) - \frac{1}{\sigma}\log(1 - p_\sigma(\Delta) + p_\sigma(\Delta)e^{\sigma\Delta}).$$

Given that the agent would exert no effort under a sure wealth, it can be interpreted as the level of sure wealth that the agent would exchange for the contract (W, Δ). An important feature of this utility function is that it is additively separable in W. This is also the case for the principal's profit, which writes

$$\begin{aligned}\pi_\sigma(W,\Delta) &= (1 - p_\sigma(\Delta))(S - W) + p_\sigma(\Delta)(S - D - W + \Delta)\\ &= S - W - p_\sigma(\Delta)(D - \Delta).\end{aligned}$$

Assume that S is high enough:

Assumption 3 *$\pi_\sigma(W_0, \Delta_0) \geq 0$ for all relevant σ.*

This ensures that, without loss of generality, we can restrict attention (both in the cases without and with adverse selection) to situations where the principal contracts with all types (see Jullien (2000)). In the case of insurance, $\pi_\sigma(W_0, \Delta_0) \equiv 0$: contracting on (W_0, Δ_0) is formally equivalent to no trade. In the case of a labor contract the assumption means that the firm makes no loss, even if the agent is not paid and exerts no effort.

We can finally define the total surplus as $B_\sigma(\Delta) = \pi_\sigma(W, \Delta) + CE_\sigma(W, \Delta)$ or

$$B_\sigma(\Delta) = S - p_\sigma(\Delta)(D - \Delta) - c(p_\sigma(\Delta)) - \frac{1}{\sigma}\log(1 - p_\sigma(\Delta) + p_\sigma(\Delta)e^{\sigma\Delta}).$$

Let us analyze the optimal contract when the principal knows the agent's risk-aversion. The situation thus only involves moral hazard. Maximizing $\pi_\sigma(W, \Delta)$ under

$$CE_\sigma(W, \Delta) \geq CE_\sigma(W_0, \Delta_0)$$

boils down, since both objective functions are linear in W, to maximizing over Δ the total surplus $B_\sigma(\Delta)$ and adjusting W to bring the agent on his reservation utility curve.

We will denote $(W_\sigma^1, \Delta_\sigma^1)$ the resulting contract for agent σ. Note that unlike W_σ^1, Δ_σ^1 does not depend on the reservation option (W_0, Δ_0). The following result is proved in the Appendix:

Lemma 3 *Under Assumptions 1 and 2, one has $0 < \Delta_\sigma^1 < D$. Moreover, Δ_σ^1 increases with respect to D.*

Thus, the increase in surplus from failure to success is shared between the principal and the agent.

One would expect that Δ_σ^1 decreases in σ, since more risk-averse agents prefer lower-powered incentive schedules. However, the derivative of Δ_σ^1 in σ depends *inter alia* on the cross derivative of $p_\sigma(\Delta)$ in (σ, Δ), on which little is known in general. It is in fact possible to find examples in which Δ_σ^1 is non-monotonic in σ.

Example 1: Assume that the utility function is $CARA(\sigma)$ and that the technology is given by

$$p(e) = p_0 \exp(-\lambda e)$$

where λ is a positive constant. Equivalently,

$$c(p) = -\frac{1}{\lambda} \log \frac{p}{p_0}$$

This technology satisfies (WP) ($pc'(p)$ is constant). Then easy calculations show that the optimum of the agent's program is equal to p_0 if $\sigma > \lambda$ and to

$$\min \left(p_0, \frac{\sigma}{\lambda - \sigma} \frac{1}{e^{\sigma \Delta} - 1} \right) \tag{2}$$

otherwise. As we have a closed form for $p_\sigma(\Delta)$, it is easy to maximize B_σ numerically and to see how its maximizer Δ_σ^1 depends on σ. We ran such a simulation for $D = \lambda = 10$ and $p_0 = 0.1$. We found that while Δ_σ^1 decreases in σ for $\sigma < 8.4$, it starts increasing afterwards. The behavior of p_σ^1 is even more complicated, as it increases with σ until we reach $\sigma = 0.3$, then decreases, and starts increasing again after $\sigma = 8$. This counterintuitive behavior takes place without the constraint $p \leq p_0$ becoming binding in this region.

It is possible to strengthen our assumptions so as to find a set of sufficient conditions on the technology that ensure that the problem is well-behaved[5]. These assumptions hold in Example 1; however, they are not very illuminating and we shall not pursue the matter further. To simplify the exposition, we assume from now on:

Assumption 4 $B_\sigma(\Delta)$ *is strictly quasi-concave.*

This is all the information we shall need about total surplus.

4 The Optimal Contract with Private Risk-aversion

We now introduce adverse selection on the agent's risk-aversion. We assume that there are two types of agents, the high risk-aversion type H and the low risk-aversion type L. As the properties of the optimal contract with adverse

[5] If $pc'(p)$ is concave, then $B_\sigma(\Delta)$ is strictly quasi-concave. Under a slightly stronger assumption, Δ_σ^1 cannot increase in σ in regions where, as one normally expects, $p_\sigma(\Delta)$ is decreasing in σ.

selection depend on the comparison between Δ_H^1 and Δ_L^1, we shall distinguish between the "regular" case when $\Delta_H^1 < \Delta_L^1$ and the "non-responsive"[6] case when $\Delta_H^1 > \Delta_L^1$.

The prior probability of the high risk-aversion type H is denoted μ. The principal's problem then is to choose a pair of contracts (W_H, Δ_H) and (W_L, Δ_L) to maximize his expected profit

$$\mu\pi_H(W_H, \Delta_H) + (1-\mu)\pi_L(W_L, \Delta_L)$$

or, using $\pi_\sigma(W, \Delta) = B_\sigma(\Delta) - CE_\sigma(W, \Delta)$,

$$\mu(B_H(\Delta_H) - CE_H(W_H, \Delta_H)) + (1-\mu)(B_L(\Delta_L) - CE_L(W_L, \Delta_L))$$

given the two incentive constraints

$$\begin{cases} CE_L(W_L, \Delta_L) \geq CE_L(W_H, \Delta_H) \\ CE_H(W_H, \Delta_H) \geq CE_H(W_L, \Delta_L) \end{cases}$$

and the two participation constraints

$$\begin{cases} CE_L(W_L, \Delta_L) \geq CE_L(W_0, \Delta_0) \\ CE_H(W_H, \Delta_H) \geq CE_H(W_0, \Delta_0) \end{cases}$$

This problem is best studied on figures. We consider from now on the (Δ, W) plane, and we denote O the point that corresponds to the reservation contract (Δ_0, W_0). From the single-crossing condition, indifference curves for the H type are steeper than for the L type wherever they cross. As a consequence, any incentive compatible contract must verify:

$$\Delta_H \leq \Delta_L.$$

4.1 The Regular Case

In the regular case with $\Delta_H^1 < \Delta_L^1$, the optimal contract still depends on where the reservation incentives Δ_0 lie. We first prove that if $\Delta_H^1 < \Delta_0 < \Delta_L^1$, then private information on risk-aversions does not matter. Then we look at two quite different cases:

[6]We borrow the term "non-responsive" from Guesnerie-Laffont (1984) and the survey by Caillaud-Guesnerie-Rey-Tirole (1988).

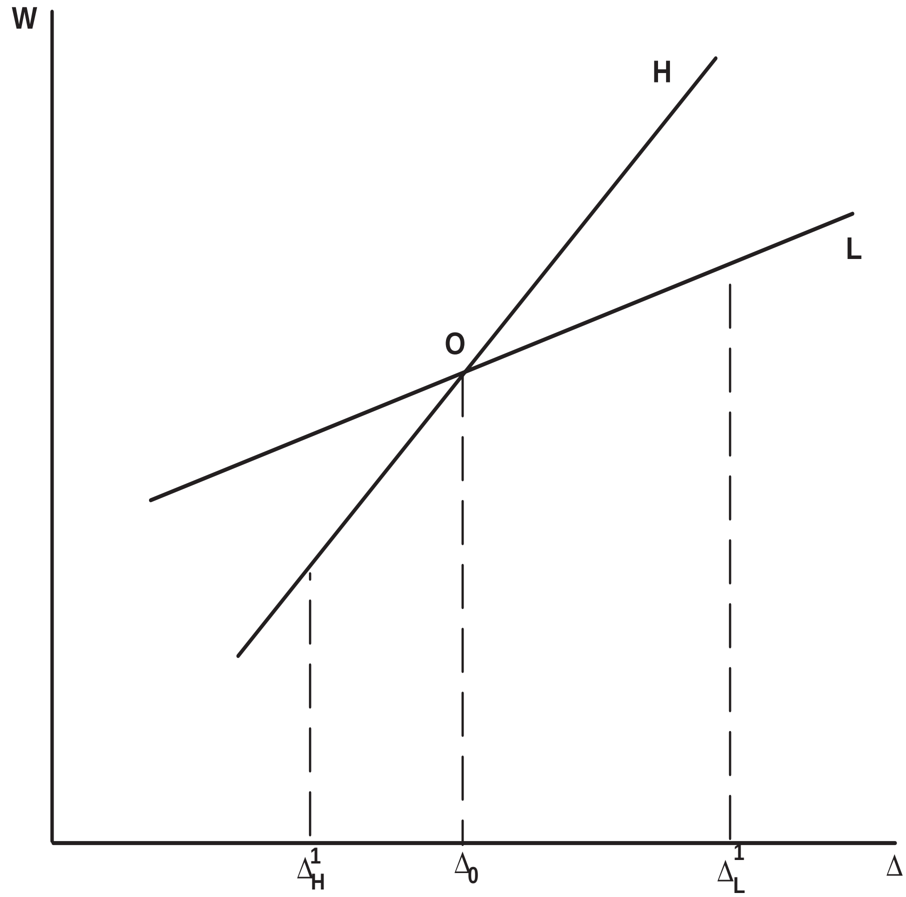

Figure 1: When Private Risk-aversion Does Not Matter

- for $\Delta_H^1 < \Delta_L^1 < \Delta_0$, we have the "insurance case", which includes in particular the standard insurance model for which $\Delta_0 = D$ (recall that Lemma 3 has shown that $\Delta_\sigma^1 < D$ for all σ);
- for $\Delta_0 < \Delta_H^1 < \Delta_L^1$, we study the "labor economics case", with a low-powered outside option (for example unemployment) (Lemma 3 has shown that $\Delta_\sigma^1 > 0$ for all σ).

4.1.1 When Private Risk-aversion Does Not Matter

Assume that $\Delta_H^1 \leq \Delta_0 \leq \Delta_L^1$. Then the pair of contracts depicted on Figure 1 is incentive compatible and it coincides with the public risk-aversion pair of contracts. This gives us a first result:

Theorem 1 *Assume Assumptions 1 to 4. When $\Delta_H^1 \leq \Delta_0 \leq \Delta_L^1$, it does not matter whether risk-aversion is publicly observed or not: the incentive constraints do not bind and no agent gets an informational rent.*

It is easy to check by playing with the figures that it is only in this case that the public risk-aversion pair of contracts is incentive compatible. Note also that this efficiency result does not extend to more than two types.

4.1.2 The Insurance Model

Let us now assume that $\Delta_L^1 < \Delta_0$, as in the insurance model. Then all action takes place below point O, as we will see. To simplify the exposition, we start from the relaxed maximization program obtained by neglecting the incentive constraint for type L. We will show later that the solution to the relaxed program satisfies this constraint.

Note that given CARA utility functions, $(CE_L(W, \Delta) - CE_H(W, \Delta))$ is a function of Δ_L only, that we denote $\phi(\Delta)$. Moreover, the Spence-Mirrlees condition exactly says that

$$\frac{\partial CE_L}{\partial \Delta} \geq \frac{\partial CE_H}{\partial \Delta}$$

and therefore that the function ϕ is increasing. Now the three constraints to be considered are

$$\left\{ \begin{array}{c} CE_H(W_H, \Delta_H) \geq CE_H(W_L, \Delta_L) \\ CE_H(W_H, \Delta_H) \geq CE_H(W_0, \Delta_0) \\ CE_L(W_L, \Delta_L) \geq CE_L(W_0, \Delta_0) \end{array} \right.$$

First note that the third constraint (the participation constraint for type L) must be binding, as it is always possible to reduce W_L without affecting the other two constraints, and this increases profits.

Secondly, assume that $\Delta_L > \Delta_0$ at the optimum. Now replace the contract for type L with the no-trade contract O. Clearly, all three constraints are still satisfied. Moreover, $B_L(\Delta)$ is strictly quasi-concave and is highest below O (as $\Delta_L^1 < \Delta_0$), so that bringing the contract for L on O on the same indifference curve increases B_L and therefore profits π_L. Thus we are led to a contradiction, and we get that $\Delta_L \leq \Delta_0$.

Then using the function ϕ, our constraints imply:

$$\begin{cases} CE_H(W_H, \Delta_H) \geq CE_L(W_0, \Delta_0) - \phi(\Delta_L) \\ CE_H(W_H, \Delta_H) \geq CE_L(W_0, \Delta_0) - \phi(\Delta_0) \\ CE_L(W_L, \Delta_L) = CE_L(W_0, \Delta_0) \\ \Delta_L \leq \Delta_0 \end{cases}$$

Since ϕ is increasing, the second constraint disappears. Then it is optimal to reduce W_H so as to make the first constraint bind. Finally the relaxed program reduces to maximizing

$$\mu B_H(\Delta_H) + (1-\mu) B_L(\Delta_L) + \mu\phi(\Delta_L)$$

under the constraint $\Delta_L \leq \Delta_0$.

Obviously, the principal's profit on type H is maximal when the power of the incentive scheme is exactly at its first-best value Δ_H^1. Concerning Δ_L, let us define

Definition 1 Δ_L^2 *is a solution of* $\max_{\Delta \leq \Delta_0} \left(B_L(\Delta) + \frac{\mu}{1-\mu}\phi(\Delta) \right)$

Note that it is possible that this maximum be reached at $\Delta_L^2 = \Delta_0$. Given that $(B_L(\Delta) + \frac{\mu}{1-\mu}\phi(\Delta))$ is independent of Δ_0 and ϕ is increasing, this occurs for values of Δ_0 close to Δ_L^1 and/or for μ close to one. In this case the less risk averse agent receives the contract (W_0, Δ_0) while the high type also receives his reservation utility. In any case, since ϕ is increasing, it must be the case that $\Delta_L^2 > \Delta_L^1$: the L type faces higher-powered incentives than in the public risk-aversion model. As we are in the regular case $\Delta_H^1 < \Delta_L^1$, it follows that $\Delta_H^1 < \Delta_L^2$.

This pair of contracts is depicted on Figure 2 for $\Delta_L^2 < \Delta_0$. As appears clearly from the figure, it satisfies the incentive constraint for type L, so that it is the optimal menu of contracts.

We summarize here the results:

Proposition 3 *Assume Assumptions 1 to 4. Assume that* $\Delta_H^1 \leq \Delta_L^1 < \Delta_0$, *then the optimal contract is such that* $\Delta_H = \Delta_H^1$ *and* $\Delta_L = \Delta_L^2$. *Type L*

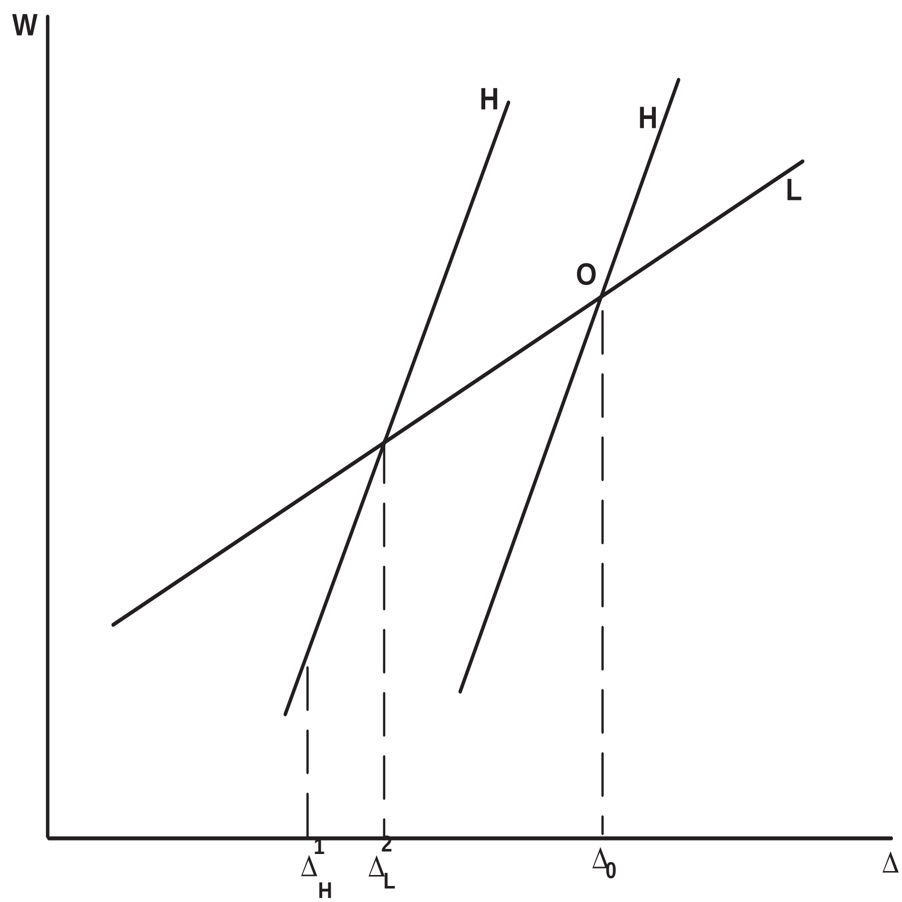

Figure 2: The Private Risk-aversion Optimum for the Insurance Model in the Regular Case

receives his reservation utility and faces higher-powered incentives than in the public risk-aversion model, while type H benefits from an informational rent $(\phi(\Delta_0) - \phi(\Delta_L^2))$.

While the characterization of the optimal menu of contracts is more involved here than in the usual adverse selection model, the intuition is quite simple. In an insurance model, the more risk-averse type has a higher willingness to pay for insurance and therefore he is more valuable to the insurer. Thus he can be identified to the "good type". Proposition 2 then restates the standard analysis in terms of our model: the good type gets the efficient contract, his incentive constraint is binding, and he gets an informational rent; while the bad type gets an inefficient contract and no informational rent.

4.1.3 The Labor Economics Case

The analysis of the case where $\Delta_0 < \Delta_H^1$ is exactly symmetric to that of the insurance model, so that we will be more sketchy here. We now define the relaxed program by neglecting the incentive constraint for type H. It is easy to see that the participation constraint for type H is binding, so that the three constraints of the relaxed program become

$$\begin{cases} CE_L(W_L, \Delta_L) \geq CE_H(W_0, \Delta_0) + \phi(\Delta_0) \\ CE_L(W_L, \Delta_L) \geq CE_H(W_0, \Delta_0) + \phi(\Delta_H) \\ CE_H(W_H, \Delta_H) = CE_H(W_0, \Delta_0) \end{cases}$$

Then we prove $\Delta_H \geq \Delta_0$ by a similar argument to the one we used to prove $\Delta_L \leq \Delta_0$ for the insurance case, so that the second constraint is binding and the first one can be replaced by $\Delta_0 \leq \Delta_H$.

Now the objective for the relaxed program can be reduced to maximizing

$$\mu B_H(\Delta_H) + (1-\mu) B_L(\Delta_L) - (1-\mu)\phi(\Delta_H)$$

subject to $\Delta_0 \leq \Delta_H$.

Let us define $\Delta_H^2 < \Delta_H^1$ as

Definition 2 Δ_H^2 *is a solution of* $\max_{\Delta \geq \Delta_0} \left(B_H(\Delta) - \frac{1-\mu}{\mu}\phi(\Delta) \right)$.

The very same reasoning then shows that

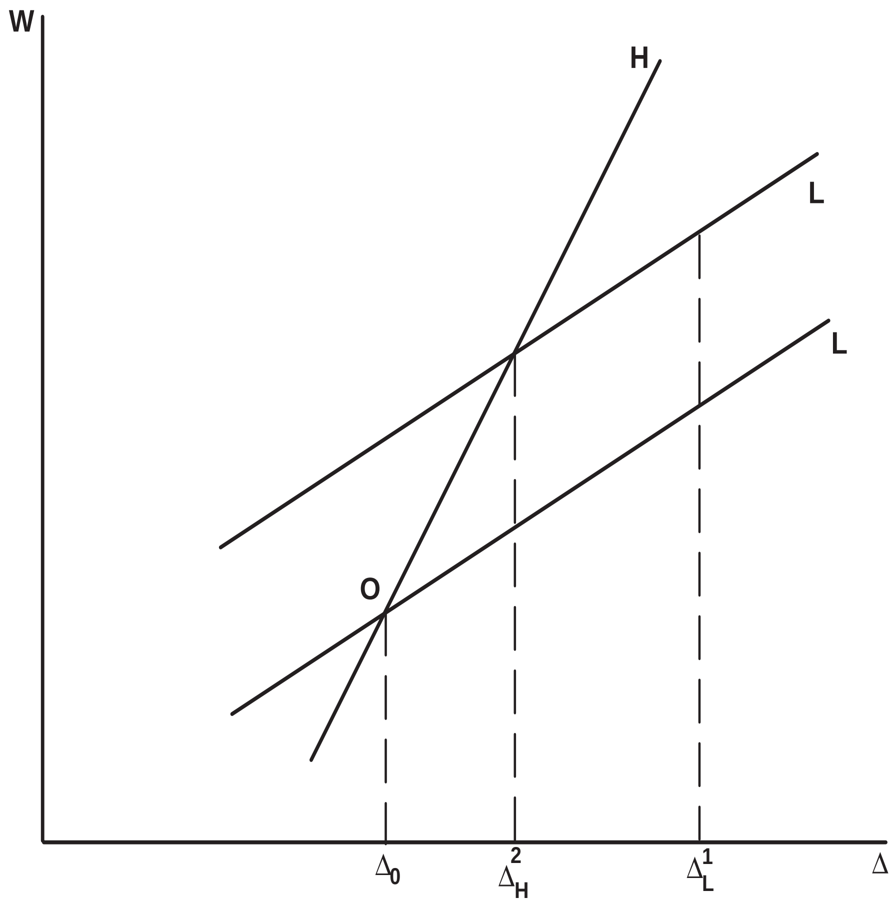

Figure 3: The Private Risk-aversion Optimum for the Labor Economics Model in the Regular Case

Proposition 4 *Assume Assumptions 1 to 4. Assume that* $\Delta_0 < \Delta_H^1 \leq \Delta_L^1$*, then the optimal contract is such that* $\Delta_H = \Delta_H^2$ *and* $\Delta_L = \Delta_L^1$*. Type H receives his reservation utility and faces lower-powered incentives than in the public risk-aversion model, while type L benefits from an informational rent* $(\phi(\Delta_H^2) - \phi(\Delta_0))$.

Here again it is possible that both types receive their reservation utility, which occurs when μ is small enough and/or Δ_H^1 is close to Δ_0, so that $\Delta_H^2 = \Delta_0$. Then the contract for the H type is in O. Figure 3 graphs the optimal contracts when $\Delta_H^2 > \Delta_0$. Once again, the incentive constraint for L is clearly satisfied, so that the solution of the relaxed program indeed is the private risk-aversion optimum.

Note that the intuition here is that the low risk-aversion individual is more valuable to the employer, as the employer himself is risk-neutral and he wants to align the agent's incentives on his own. Thus here the good type is type L, and all results are reversed with respect to the insurance case.

4.1.4 A Summary for the Regular Case

We summarize the results on Figure 4, which plots the power of incentives as a function of Δ_0. As Δ_0 increases from very small to very large, we start from the labor economics model, for which the incentives are $(\Delta_L^1, \Delta_H^2 \geq \Delta_0)$. When Δ_0 is small enough for that model, Δ_H^2 is higher than Δ_0; then it becomes equal to it. When Δ_0 increases again, we reach the range $\Delta_H^1 < \Delta_0 < \Delta_L^1$, for which we know that the optimal contract is (Δ_L^1, Δ_H^1). Then when Δ_0 overtakes Δ_L^1, we enter the realm of the insurance model, where the optimal contract is $(\Delta_L^2 \leq \Delta_0, \Delta_H^1)$. For that model, Δ_L^2 coincides with Δ_0 when the latter is small enough. Also note that the only regions where one type of agent gets a positive informational rent are when Δ_0 is very small (and L gets a rent) and when Δ_0 is very large (and H gets a rent).

Thus as we move from the labor economics model to the insurance model, the downward distortion on the type H is reduced (weakly) while the upward distortion on the type L increases. The rent of type L decreases and the rent of type H increases. Moreover, the power of incentives increases for each type as Δ_0 increases, a feature not present in the public risk-aversion case:

Proposition 5 *Assume Assumptions 1 to 4. When* Δ_0 *increases in the regular case, the incentives* Δ_H *and* Δ_L *faced by each type increase (weakly).*

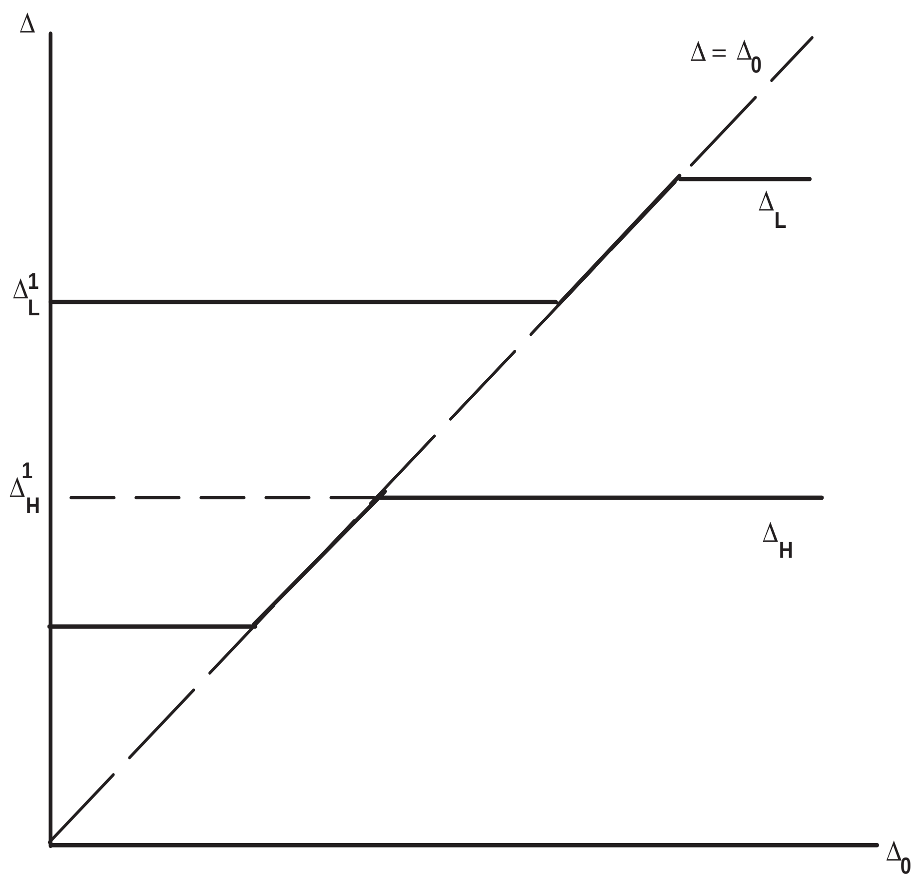

Figure 4: A Summary for the Regular Case

As a consequence, the probabilities of success also (weakly) increase with Δ_0. Finally, these probabilities also increase with D.

The last part of the Proposition is a straightforward consequence of Lemma 3, which shows that Δ_σ^1 increases with D. From Definitions 1 and 2, this property extends to Δ_σ^2. Hence the result obtained under pure moral hazard that incentives are more powerful when the stake D is higher is robust to the introduction of adverse selection. In particular, it applies in the insurance model, for which $\Delta_0 = D$.

4.2 The Non-responsive Case

Now assume that $\Delta_H^1 > \Delta_L^1$. The key difference with the regular case is that it is now possible that $\Delta_L^2 < \Delta_H^1$, or that $\Delta_H^2 > \Delta_L^1$, which would violate incentive compatibility[7].

If the parameters of the problem are such that in the insurance model, $\Delta_L^2 > \Delta_H^1$, then nothing in the argument is changed and Proposition 3 immediately applies. If in the labor economics model the parameters imply that $\Delta_H^2 < \Delta_L^1$, then Proposition 4 remains true. The intuition is fairly simple. Take for instance the insurance model. In the regular case, type H gets a (possibly zero) informational rent. The inequality $\Delta_L^2 > \Delta_H^1$ holds when μ (the prior probability of type H) is high enough. In that case it becomes important for the principal to reduce informational rents, which can only be done by distorting the incentives for type L away from Δ_L^1, so much indeed that the inequality $\Delta_L^2 > \Delta_H^1$ holds even though $\Delta_L^1 < \Delta_H^1$. The symmetric intuition holds for the labor economics model.

Thus for some parameter ranges, we obtain in the non-responsive case exactly the same separating contracts as in the regular case. However, the optimal contract may be bunching in other parameter ranges. To see this, take for instance the insurance model and assume that μ is small. Then Δ_L^2 is close to Δ_L^1 and is therefore smaller than Δ_H^1, so that the pair of contracts (Δ_H^1, Δ_L^2) is not incentive compatible. Now assume that the optimum (Δ_H, Δ_L) still is separating. Then one of the incentive constraints cannot be binding (otherwise we would have $\Delta_H = \Delta_L$, as is easily seen by adding both constraints).

[7]In such cases one may want to slightly modify definitions 1 and 2: simply select the highest solution for Δ_L^2, and the lowest for Δ_H^2.

- assume the incentive constraint for L does not bind. Then we can go through the analysis of the relaxed program in Section 4.1.2: (Δ_H, Δ_L) must be a solution of that program, that is a maximizer of

$$\mu B_H(\Delta_H) + (1-\mu) B_L(\Delta_L) + \mu\phi(\Delta_L)$$

under the constraint $\Delta_L \leq \Delta_0$. But if we take μ small enough, that objective is strictly quasi-concave, and its unique maximizer is close to (Δ_H^1, Δ_L^2), which we know is not incentive compatible. Thus we are led to a contradiction.

- now assume that the incentive constraint for H does not bind. Then we can go through the analysis of the relaxed program in Section 4.1.3: (Δ_H, Δ_L) must be a solution of that program, that is a maximizer of

$$\mu B_H(\Delta_H) + (1-\mu) B_L(\Delta_L) - (1-\mu)\phi(\Delta_H)$$

under the constraint $\Delta_H \geq \Delta_0$. If μ again is small enough, the solution must be close to (Δ_H^2, Δ_L^1); but this is not incentive compatible and we have a contradiction.

Therefore we have proved that for μ small enough in the insurance model, the optimum must be bunching. Symmetrically, one can prove that when μ is large enough, there is bunching in the labor economics model. In both cases the optimal bunching contract (W, Δ) is obtained by maximizing

$$\mu(B_H(\Delta) - CE_H(W, \Delta)) + (1-\mu)(B_L(\Delta) - CE_L(W, \Delta))$$

given the two participation constraints

$$\begin{cases} CE_H(W, \Delta) \geq CE_H(W_0, \Delta_0) \\ CE_L(W, \Delta) \geq CE_L(W_0, \Delta_0) \end{cases}$$

Which constraint is binding depends on Δ_0.

5 Applications

5.1 Insurance

The standard insurance model considers a monetary loss D, which is insured with a deductible F against a premium P. It fits within our general model, with

$$\begin{cases} \Delta_0 = D \\ W = W_0 - P \\ \Delta = F \end{cases}$$

In this case the optimal menu of contracts is (Δ_H^1, Δ_L^2) when it is separating, which corresponds to decreasing levels of deductible. Thus low risk-aversion insurees have a lower coverage. As we saw, the optimal contract can also be bunching in the non-responsive case.

Chiappori-Salanié (2000) have recently given evidence of an empirical puzzle. In the Rothschild-Stiglitz (1976) model of competitive insurance markets, equilibrium, when it exists, has higher-risk insurees getting better coverage. Thus their model predicts a positive correlation between risk and coverage. Using data on the French car insurance market, Chiappori-Salanié find that the correlation of risk and coverage is in fact close to zero[8]. They suggest that combining moral hazard and screening on risk-aversions may explain this finding: if more risk-averse insurees both buy better coverage and drive more cautiously, then taking the correlation of risk and coverage without controlling for risk-aversion will yield a negative correlation. This is indeed one of the motivations behind our paper.

In a recent paper, de Meza-Webb (2000) investigate the competitive equilibrium in a model with moral hazard and screening on risk-aversions, with a different focus (on public policy). Their model has some simplifying assumptions: the L type is risk-neutral, he does not make any effort, and more risk-averse agents make more effort. Moreover, they introduce an administrative cost of insurance which underlies the benefits of public intervention. Finally, it is in a competitive setting, while we consider only one principal and thus a model of the insurer as a monopoly[9]. Both their work and ours show the possibility of bunching. Empirically, this may be interpreted as no correlation between risk and coverage. However, in their model it is due to an explicit assumption that the single-crossing condition does not hold[10], whereas it does in our model.

[8]Cawley-Philipson (2000) also find no evidence for asymmetric information, using health insurance data.

[9]Thus our model is closer to that of Stiglitz (1977). It should be noted that Stiglitz's model, like Rothschild-Stiglitz's, implies a positive correlation between risk and coverage.

[10]This is due to their non-monetary cost formulation of effort (see footnote 2).

It turns out that our model is rich enough that depending on the parameters, it may imply positive, negative, or (approximately) zero correlation between risk and coverage, even under separation. The point is that while less risk averse agents choose higher deductible levels, they could take less precautions, so that they may well end up with a higher probability of accident. Indeed this is just what basic intuition suggests.

To see this, consider the most plausible scenario: the regular case where $\Delta_H^1 > \Delta_L^1$. Denote

$$p_\sigma^1 = p_\sigma(\Delta_\sigma^1)$$

the risk of agent σ without adverse selection and

$$p_\sigma^2 = p_\sigma(\Delta_\sigma^2)$$

his risk with adverse selection. We already know that $\Delta_L^2 > \Delta_H^1$; therefore positive correlation between risk and coverage is equivalent to $p_H^1 = p_H^2 > p_L^2$. Now let μ be close to one. Then Δ_L^2 is close to $\Delta_0 = D$ and p_2^L is close to $p_L(D)$. But the latter can be larger or smaller than p_1^H, depending on the primitives of the model. Therefore even in the most regular case, the correlation of risk and coverage can take either sign, or even be close to zero as found by Chiappori-Salanié[11].

We should emphasize two points here. First, this explanation assumes that one cannot control fully for risk-aversion, which seems reasonable enough. Second, it requires the existence of some market power. It is indeed possible to show that incentive-compatibility and the zero-profit conditions (in the absence of transaction costs or loading factors) are sufficient to get a positive correlation[12]. However actuarial pricing is problematic in our context, even under free entry (see Chassagnon-Chiappori (1997)).

If the correlation between risk and coverage doesn't provide a good test for adverse selection when it is driven by risk aversion, there may be ways out. One alternative would be to exploit the fact that risk-aversion is a fundamental characteristic of the individuals. This means that more risk-averse agents should choose lower deductibles on all their insurance contracts, which would not occur if private information bears on risk as in the Rothschild-Stiglitz model, or if there is only pure moral hazard as there is no reason to think that there is a positive correlation between the levels of different risks (say

[11] While we have illustrated this with μ close to one, it is easily seen that since p_L^2 decreases in μ, the correlation must be negative for μ small enough.

[12] This is proved in Chiappori et al (2000).

housing, health and car insurance). Combining data on insurance purchases and portfolio choices would also help, as in Guiso-Jappelli (1996).

5.2 Executive Compensation

One of the best-known applications of the labor economics model is on how firms should pay their workers, and in particular their managers. Here the reservation contract is typically taken to be the null contract.[13] Our results then show that the firm should offer a menu of wages W and bonuses Δ, with again higher risk-aversion employees choosing lower bonuses. This time the employees who get an informational rent are those who have low risk-aversion. Notice that we don't make the claim here that productivity doesn't matter. The key question in determining the most relevant adverse selection variable is to identify the dimension that is the most likely to be private information of the agent. Indeed productivity may vary across individuals but (in particular in the case of a top executive) the firm may have access to substantial information on productivity based on past records.[14] The key dimension may then be moral hazard and risk aversion.

With moral hazard, managers' pay should be sensitive to their firms' performance. The "pay-performance sensitivity" has been estimated by many papers (see Murphy (1999)). The seminal contribution is that of Jensen-Murphy (1990); using data on CEOs of US firms from 1969 to 1983, they obtained what seemed to be very low estimates of the elasticity of executive compensation to firm performance. More recent estimates (such as Hall-Liebman (1998)) point to much higher elasticities. In this empirical literature, the pay-performance sensitivity corresponds to the Δ in our model. A first prediction of our model thus is that the pay-performance sensitivity should be lower than what is implied by the standard moral hazard model. Second, we saw that for the insurance model, our model predicts that the correlation between risk and coverage may take either sign. It is easy to see

[13] The outside option depends on the nature and the tightness of the labor market. In our model what really matters is $\phi(\Delta_0)$, which is the difference between the utility levels obtained by the two types when not employed by the firm. Assuming $\Delta_0 = 0$ or low seems reasonable for instance when considering the process of hiring a CEO whose best other available alternative is a top executive position at a lower rung, as there is a large difference between the incentives provided to CEOs and those provided at lower hierarchical levels.

[14] On the other side, at lower levels, employees may have few information on their true productivity for the job.

that in the field of managerial incentives, the analog result is that the correlation between firm performance and managerial incentives may go either way. Thus consider a regression of the form

$$P_i = \alpha + \beta \Delta_i + u_i$$

of a performance measure P_i for firm i on, say, the bonus rate of the CEO of firm i. Then if one does not control adequately for the CEO's risk-aversion in this regression, the estimated β may take either sign and in any case it will be a biased estimate of the effect of incentive-based pay on firm performance. This seems to raise a difficulty for papers such as Abowd (1990), which estimates such a regression to find evidence that higher bonus rates improve performance.

One interesting extension in the context of labor contracts would be to allow for different positions within the firm. If higher level positions correspond also to positions where moral hazard considerations are the most acute, our analysis suggests that (at equal individual productivity levels) the firm should select less risk-averse agents at the higher position and induce highly risk averse agents to stay in lower positions. This means that we should find less and less risk-averse individuals as we move upward in the internal hierarchy. This may provide a rationale for the use of an internal tournament because since a tournament is risky, less risk averse agents will be more willing to engage in the tournament generating thus more self-selection.

5.3 An Application to Corporate Finance

Investment funds are typically involved in financing activities that are both risky and subject to moral hazard problems. Indeed the whole design of the financial contract is shaped by the problem of providing correct incentives (see Tirole (2000)). To draw the implications of the analysis for such a case, consider the following example.

An entrepreneur can invest up to $I = I_1$ in a project. The project succeeds with probability $p(e)$, in which case the return is RI. In case of failure the return is rI, with $r < R$. We assume that the entrepreneur has personal resources $I_0 < I_1$, but must borrow $(I_1 - I_0)$ from a monopoly lender to invest more. Moreover we assume that it is always efficient to invest the full amount I_1. The contract consists of the entrepreneur borrowing $(I_1 - I_0)$, repaying d and giving the lender a share α of profits. The no-trade contract has no borrowing and $d = \alpha = 0$. In this context we have

$$\begin{cases} S = RI_1 \\ D = (R-r)I_1 \\ W_0 = RI_0 \\ \Delta_0 = (R-r)I_0 \\ W = (RI_1 - d)(1-\alpha) \\ \Delta = (R-r)I_1(1-\alpha) \end{cases}$$

The contract can be seen as a combination of debt and equity. The share α of the firm owned by the external financier is $(1 - \frac{\Delta}{D})$ while the face value of the debt d is $(RI_1 - \frac{WD}{\Delta})$. Our model then predicts that more risk averse entrepreneurs should have lower-powered incentives, which here can be interpreted as the lender receiving more equity and less debt. This is due to optimal risk-sharing, and this holds also under public risk-aversion.

Notice that high levels of Δ_0 correspond to cases where the entrepreneur already has a high capital. As the power of optimal incentives increases with Δ_0, this means that the entrepreneur should give away less equity and the firm should perform better when the internal capital I_0 is high. As a consequence the model predicts a positive correlation between the degree of self-financing and the performance, at equal investment levels. On the other hand, as for the analysis of the correlation between risk and coverage in the insurance case, we cannot infer an a priori correlation between the performance and the level of the debt, or the composition of the external financing. These two points are reminiscent of the findings of Leland-Pyle (1977), even though the underlying mechanism in their paper is quite different, as it relies on the fact that more self-financing signals a more profitable project, while it signals lower risk-aversion in our model.

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Appendix: proofs.

Proof of Lemma 1: In the CARA case $u(x) = -\exp(-\sigma x)$, with $\sigma > 0$. The agent's program (1) thus is

$$\max_p \ (-p\exp(-\sigma(W - \Delta - c(p)) - (1-p)\exp(-\sigma(W - c(p)))\,.$$

Denote $\gamma = \exp(\sigma\Delta) - 1 > 0$. Then the program becomes

$$\min_p \ (\sigma c(p) + \log(1 + \gamma p) - \sigma W)\,. \tag{3}$$

The derivative with respect to p is

$$\sigma c'(p) + \frac{\gamma}{1 + \gamma p} \tag{4}$$

which is an increasing function of γ and thus of Δ. Therefore by the implicit function theorem, the optimal p is a decreasing function of Δ.

Proof of Proposition 1: Assume that agents are ordered in increasing risk-aversion: if $\tau > t$, then u_τ is more risk-averse than u_t. Suppose that agent t is indifferent between contract C_1 with optimal effort e_1 and contract C_2 with optimal effort e_2. Assume that $\Delta_1 < \Delta_2$. For an agent type $\tau > t$, let e_2^τ be an optimal effort level for agent τ under C_2. Let τ approach t from above. As effort is upper hemi-continuous in risk-aversion, e_2^τ must approach some effort level that is optimal for t under C_2; choose e_2 to be that effort level. Property (Q) applied at (C_1, e_1) and (C_2, e_2) implies that for τ close enough to t,

$$W_2 - \Delta_2 - e_2^\tau < W_1 - \Delta_1 - e_1 < W_1 - e_1 < W_2 - e_2^\tau.$$

Now let F_1 (resp. F_2) be the cumulative distribution function of outcomes under (C_1, e_1) (resp. under (C_2, e_2^τ)):

- for $R < W_2 - \Delta_2 - e_2^\tau$, $F_1(R) = F_2(R) = 0$;
- on $[W_2 - \Delta_2 - e_2^\tau, W_1 - \Delta_1 - e_1)$, $F_1 = 0$ and $F_2 = p(e_2^\tau)$;
- on $[W_1 - \Delta_1 - e_1, W_1 - e_1)$, $F_1 = p(e_1)$ and $F_2 = p(e_2^\tau)$;

- on $[W_1 - e_1, W_2 - e_2^\tau)$, $F_1 = 1$ and $F_2 = p(e_2^\tau)$;
- for $R \geq W_2 - e_2^\tau$, $F_1 = F_2 = 1$.

Hence $(F_2 - F_1)$ is positive then negative. A result of Jewitt (1989) (see also Jullien-Salanié-Salanié(1999)) then implies that the more risk-averse agent τ must prefer (C_1, e_1) to (C_2, e_2^τ). A fortiori, τ must prefer C_1 to C_2. Since $\Delta_1 < \Delta_2$ and t is indifferent between C_1 and C_2, this completes the proof.

Proof of lemma 2: we know that $W_2 - \Delta_2 < W_1 - \Delta_1 < W_1 < W_2$. Given that $e_1 < e_2$ with CARA utility functions, we get $W_2 - \Delta_2 - e_2 < W_1 - \Delta_1 - e_1$.

The proof of $W_1 - e_1 < W_2 - e_2$ requires some computations. Under CARA, (4) can be used to get

$$1 + \gamma p = \frac{1}{1 + \sigma p c'(p)} > 0$$

so that from (3) the utility for agent σ of contract (W, Δ) is equal to

$$\sigma(c(p) - W) - \log(1 + \sigma p c'(p)).$$

Besides the agent is indifferent between C_1 and C_2; therefore

$$\sigma(c(p_1) - W_1) - \log(1 + \sigma p_1 c'(p_1)) = \sigma(c(p_2) - W_2) - \log(1 + \sigma p_2 c'(p_2)).$$

Finally $e_1 < e_2$, or equivalently $p_1 > p_2$. The strict version of (P) then yields the result.

Proof of Proposition 2: Denote p_H (resp. p_L) the optimal choice of agent H (resp. L) for a given contract (W, Δ). We have

$$\left(\frac{\partial W}{\partial \Delta}\right)_{\mathcal{U}} = \frac{p u'(W - \Delta - c(p))}{p u'(W - \Delta - c(p)) + (1 - p) u'(W - c(p))}$$

and therefore we want to prove that

$$A_L \equiv \frac{1 - p_L}{p_L} \frac{u_L'(W - c(p_L))}{u_L'(W - \Delta - c(p_L))} \geq A_H \equiv \frac{1 - p_H}{p_H} \frac{u_H'(W - c(p_H))}{u_H'(W - \Delta - c(p_H))}.$$

Suppose first that $p_H \geq p_L$. Then

$$\frac{1-p_L}{p_L} \geq \frac{1-p_H}{p_H}$$

Now note that under the Ross ordering, the function

$$w_H(x) = u_H(x - c(p_H))$$

is more concave than the function

$$w_L(x) = u_L(x - c(p_L))$$

In particular, the ratio

$$\frac{w'_H(x)}{w'_L(x)}$$

must be decreasing in x. By comparing the values of this ratio in $x_0 = W$ and $x_1 = W - \Delta$, it follows that

$$\frac{u'_L(W - c(p_L))}{u'_L(W - \Delta - c(p_L))} \geq \frac{u'_H(W - c(p_H))}{u'_H(W - \Delta - c(p_H))}$$

which allows us to conclude.

Suppose now that $0 < p_H < p_L$. Then p_H must be an interior solution to (1), so that

$$\begin{aligned} &u_H(W - \Delta - c(p_H)) - u_H(W - c(p_H)) \\ &= c'(p_H)\left(p_H u'_H(W - \Delta - c(p_H)) + (1-p_H)u'_H(W - c(p_H))\right) \end{aligned}$$

This yields

$$A_H = -\frac{1}{p_H c'(p_H)} \frac{w_H(x_0) - w_H(x_1)}{w'_H(x_1)} - 1$$

Similarly for agent L, for whom we only know that the derivative at p_L is non-negative (as p_L may equal p_0):

$$u_L(W - \Delta - c(p_L)) - u_L(W - c(p_L))$$

$$\geq c'(p_L)\left(p_L u'_L(W - \Delta - c(p_L)) + (1-p_L)u'_L(W - c(p_L))\right)$$

This yields

$$A_L \geq -\frac{1}{p_L c'(p_L)} \frac{w_L(x_0) - w_L(x_1)}{w'_L(x_1)} - 1$$

Therefore it is sufficient to prove that

$$\frac{1}{-p_L c'(p_L)} \frac{w_L(x_0) - w_L(x_1)}{w'_L(x_1)}$$

is greater than

$$\frac{1}{-p_H c'(p_H)} \frac{w_H(x_0) - w_H(x_1)}{w'_H(x_1)}$$

Under Assumption 1 and $p_H < p_L$, we have $p_H c'(p_H) \leq p_L c'(p_L)$. Besides, defining a function k by $w_H = k \circ w_L$, we claim that

$$w_H(x_1) \leq w_H(x_0) + (w_L(x_1) - w_L(x_0)) \frac{w'_H(x_0)}{w'_L(x_0)}$$

This follows from the concavity of k (which is again a consequence of the Ross ordering) and from $k'(w_L(x)) = w'_H(x)/w'_L(x)$. This allows us to conclude.

Proof of Lemma 3: We have

$$B_\sigma(\Delta) = [S - p_\sigma(\Delta)(D - \Delta)] - \min_p \, [c(p) + \frac{1}{\sigma} \log(1 + \gamma p)].$$

For $\Delta \geq D$, the first term is decreasing since $p_\sigma(\Delta)$ is decreasing with respect to Δ, from Lemma 1. And the second term strictly decreases with γ, and thus with Δ. This shows that $\Delta^1_\sigma < D$.

For Δ small, (4) simplifies to

$$c'(p_\sigma(\Delta)) \simeq -\Delta$$

and under Assumption 1, it implies $\frac{\partial p}{\partial \Delta} < 0$ in $\Delta = 0$. Therefore

$$B'_\sigma(0) = -D \frac{\partial p}{\partial \Delta} + p - \frac{p(\gamma + 1)}{1 + \gamma p} = -D \frac{\partial p}{\partial \Delta} > 0$$

because $\gamma = \exp(\sigma \Delta) - 1$ is zero at $\Delta = 0$.

The second part of the Lemma follows directly from the fact that the derivative of B with respect to D is $-p_\sigma(\Delta)$, which is increasing with respect to Δ.