

THE SOCIAL COSTS OF AIR TRAFFIC DELAYS

Part One: A Survey

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Executive Summary

In this report we do a thorough review of theoretical and empirical studies on the estimation of costs of air traffic delays and on the literature about value of time. In general empirical estimations do not address the issues discussed by the theoretical models. The latter mainly focuses on the modeling of queues at congested airports while the former are based on previous studies of the value of time and the question of a proper definition of delays.

Besides methodological problems, empirical analysis provides very different estimates of delays and costs of delays. On the other side, theoretical models provide some insights although they are difficult to test and apply in practice. Nevertheless they show the crucial role of a right definition of delays.

The relevant delay must take into account the fact that airlines add some extra time to their schedules in addition to what it is technically required to avoid partial congestion. This is the so-called buffer delay. Airlines obtain different benefits from this practice. For instance it helps them building their reputation in terms of reliability, i.e., in their capacity to avoid cascading delays. The buffer delay is defined as the number of minutes the airlines should add to the schedule so that marginal cost of this buffer is equaled to the expected benefit. Similarly, for the society as a whole, minutes of congestion should be added up to the point where benefits equal costs. Therefore, *when we study delay costs, we should not consider the whole delay. We should be able to distinguish what it is the optimal delay, in the total amount of delays.* Only the difference between observed and optimal delays could be harmful for the consumers of air traffic and airport services.

1. Introduction

“Placing users at the heart of the transport policy” is one of the key actions proposed by the White Paper of the Commission of the European Community. Particularly, “Specific new measures are needed on user's rights in all modes of transport so that, regardless of the mode of transport used, users can both know their rights and enforce them.” (See European Commission, 2001.) “The Commission's aim over the next ten years is to develop and define the rights of users”. In the short term, the Commission intended to:

- Increase air passengers existing rights through new proposals concerning in particular, denied boarding due to overbooking, delays and flight cancellations.
- Put forward a regulation concerning requirements to air transport contracts.

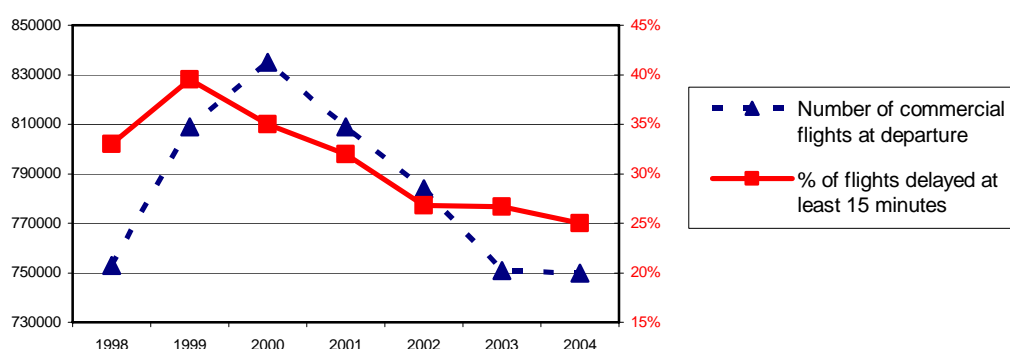
In fact, new regulation about passenger rights has entered into force in February 2005. The EU acted in 1991 to strengthen passenger's rights, particularly for the cases of overbooking. The new law has extended these rights to all kind of flights from and/or with destination the EU. It also has increased the monetary compensations in case of denied boarding; it includes compensations for some type of cancellations and cover also long delays.¹ However, no economic explanation has been presented to defend these new measures and airlines argue that it will immediately suppose an increase in cost that will be translated to prices.

For France, congestion and delays have become a common operational characteristic. According to l'observatoire des retards aeriens, the proportion of flights delayed more than 15 minutes was 25% with an average delay of 43 minutes. The delays in Europe were affected by the attacks of 2001 as well as they were disturbed by the war during 1999 in Yugoslavia. The most remarkable feature is the decline produced in the 2001 due to the terrorist attack of September 11th which undermined the confidence on passengers.

France presents quite high average delays however we have to take into account that this can be increased due to the central geographic position of France in Europe (more than one of each four flights in Europe cross the French airspace).

¹ See European Commission (2005).

Figure 1: Commercial flights and delays at the 15 biggest airports in France



Source: L'observatoire des retards du transport aerien

Even if airlines through Europe have signed voluntary agreements (not binding) to deliver defined standards of service to air travelers (such as the 2002 undertaking by the major players in the sector), in the absence of Community legislation, passengers are confronted with an increasing level of delays and with a set of national rules to protect them which are largely ineffective.

These figures raise several questions. First of all, one may question their relevance as the measures of delays could be based on imperfect or even incorrect definitions. Second, one may wonder whether the impact of delays on social welfare is significant.

In order to provide a set of objective replies, the Direction General de l'Aviation Civile (DGAC) of the French Ministry of Transport has commanded to the Institut D'Economie Industrielle (IDEI) a study of costs that passengers in French airports incur due to delays.

This interim report is aimed at providing a review of the state of literature on this topic. It accounts for with applied studies as well as theoretical and methodological considerations.

In a second section, we review the few applied studies that reckon costs of delays. The three main studies mainly deal with the cost for operators, and to a much smaller extent with the cost for users. They use a rough methodological framework based on a unique value of time, just taking into account the travel time spent and reckoning the gaps between the scheduled and the real times.

These studies contrast with the complexity and sophistication of theoretical models, which are discussed in the third section. Here we mainly focus on the question of a proper modeling of the congestion phenomenon. Theoretical models take into account

different dimensions of time delay like the gap between desired and actual arrival time, different values of time or the existence of buffer delay. To our knowledge however, none of these approaches has been used to reckon the cost of delays or even to set up a definition of delays and how to implement measures of delays according to these definitions.

The last section summarizes these results and draws some directions for future research or study.

2. Applied Studies on Cost of Delays

Three applied studies are reviewed:

- The ITA study “Costs of Air Transport Delay in Europe” (2000).
- The study “Evaluation of Congestion Costs for Madrid Airport” made in the framework of the UNITE research program by Nombela, de Rus and Betancor (2002).
- The report by the University of Westminster (2004).

These three studies deal with the cost of delays for the operator; the cost of delays for the users is just considered by the first two ones.

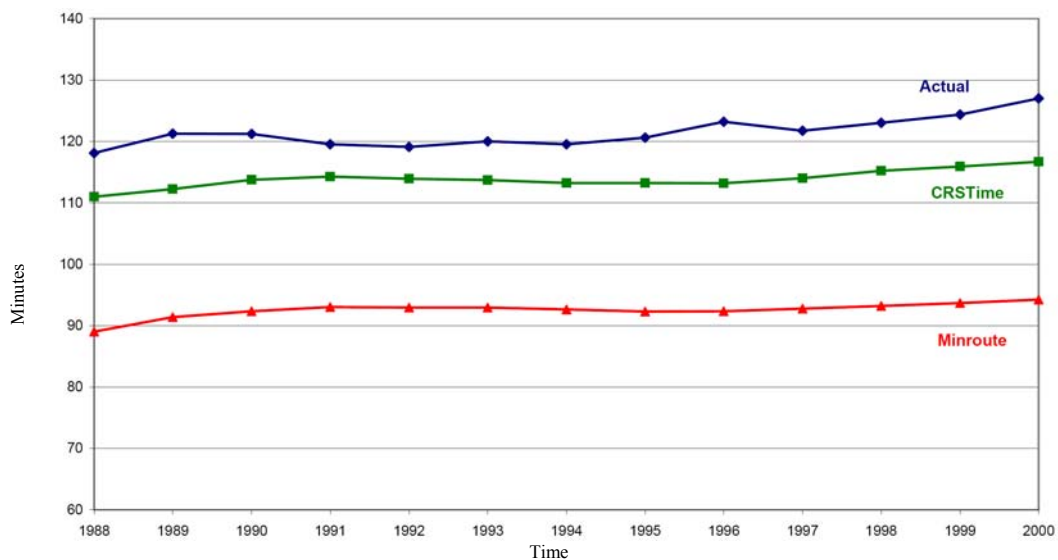
It must be mentioned that these studies use a crude definition of delays: the discrepancy between the scheduled arrival time and the real arrival time. However the Westminster study makes a distinction between arrival delays and departure delays.

For airlines, an important point noted in several studies is the difference between scheduled and what could be considered an optimal or minimum time for a trip. Due to the high cost that delays can represent for airlines, it is quite common to schedule a longer time for the trip than what could be gained without any kind of congestion. This difference is known as “buffer”, and is specially used by hub-airlines, which want to ensure the connections for all their passengers, and by low-cost companies that want to build a reputation of on-time flights. Airlines use buffers to recover from delay by “padding” the schedule so that they can improve the predictability of rotations and also improve their punctuality performance with respect to published schedules. This distinction between schedule and buffer is mentioned in the ITA study, but no reckoning is made (it is clear that the optimal travel time and the buffer time are not published, we know just their sum which is the scheduled time).

A few studies attempted to estimate this buffer time, or at least to approximate it since even the airlines only know an average estimate. (See, for instance, Morrison, Winston, Bailey and Khan, 1989.) They have been unsuccessful so that the most usual approach to estimate buffer time is to just compare the schedule time with the minimum travel time for each route that was obtained on the studied period. However this measure is imprecise as it could be affected by very favorable weather conditions. In that case it would be more adequate to consider some percentile of the distribution of buffer times. However nobody has even proposed such a measure.

If one sticks with a measure based on the minimum travel time, the delay time is 32 minutes in 2000 in the U.S. according to Mayer and Sinai (2003). These authors compare the values they obtain for minimum travel time with the scheduled and with actual travel time as we can see in figure 1.

Figure 2: Minimum, Scheduled and Actual Travel Times



Source: Mayer and Sinai (2003).

While actual and scheduled time increase at similar rates (around 10% in the whole period), minimum travel time just increased by five minutes, to 94 from 89, this means that airlines have increased scheduled travel time by about two-thirds of the growth in

average travel time.² A similar evolution can be expected in Europe due to the new regulation of the Commission for passenger rights.³

2.1. Costs of Air Transport Delay in Europe

The Institut du Transport Aérien (2000) estimates the delay costs for airlines and passengers in Europe. It provides an estimation of costs of delays based on the cost for passengers and airlines drawn from previous studies about the value of time. Note that they consider two concepts of delay that they denote as “operated flights versus schedule” and “schedule versus optimum” that we are going to denote as schedule delays and buffer delays respectively.

As we have previously explained buffer delays make reference to the extra time that airlines add to the schedule of a city pair, with respect to what is technically needed while schedule delays refer to the observed difference between announced arrival/departure time and the real one.

For the latter, the authors just add the different estimations of costs. They use data coming from IATA and ATA completed with data from EUROCONTROL in order to differentiate between primary and reactionary delays. They study just the delays due to Air Traffic Flow Management (ATFM).^{4,5}

The number of passengers affected by ATFM delays is calculated from an estimated number of delayed flights, and an average aircraft capacity and load factor. Passengers are distinguished between business, personal convenience and tourism travelers. Two scenarios for the value of time of the different categories, high and low, are considered. The values of time comes just from “a conservative range” that moves between 34 and 44 euros taken from values of time offered by previous studies.

² These figures have been obtained from a sample that includes 66.4 million flights at the top 27 US airports. The data set covers all airlines with at least one percent of all domestic traffic and only routes where flights are observed in each month of the entire sample period.

³ In fact, there is already evidence for the scheduled buffers. According to Eurocontrol, 13 percent of flights in 2004 departed before scheduled time and 34 percent of flights arrived before their scheduled time.

⁴ According to IATA, the study of IATA produces a very rough evaluation of the direct operating costs; the study of ATA covers only the U.S. and provides a valuation of 34.1\$ per minute without providing information on the procedure followed to obtain this value.

⁵ ATFM delay is defined as "duration between the last take-off time requested by the aircraft operator and the take-off slot given by the central flow management unit".

For buffer delay costs, the same values assumed for schedule delay are used for the passengers and a value of €45 per minute is assumed for airlines. The authors do not estimate what is the “optimum” time, but just consider two scenarios, increased flight time of either 5 or 10%.

They get a final estimation of 1.6 to 2.3 billion of euros for schedule delays costs for airlines in 1999, which rises to a ranging between 3 and 5.1 billion of euros if we consider also non-optimal scheduling. Costs of scheduled delays for passengers would be between 2.13 and 2.74 billion of euros and between 1.42 and 2.85 billion of euros for buffer delays (total costs range between 3.55 and 5.59 billions of euros).

The estimations offered by this study are quite rough as the authors recognize it for two reasons. First, the authors use a vague estimation of the cost per minute for both airlines and passengers for the two kinds of delays considered. The same value of time is applied for buffer and schedule delays. For airlines, as we will see in the study by the University of Westminster, these values are different; in particular, their approximation for the value of buffer time is on average 70% smaller than for the cost of schedule delay while here, the average cost for buffer is even a bit bigger than the cost for schedule delay, which makes unreasonable the existence of buffers. For passengers, is difficult to justify such a high cost for buffer time since passengers probably does not take the buffer time into their expectations, but rather look to the scheduled time.

Second, the ATFM delays do not include reactionary delays and does not take into account possible differences between the slot take-off time and the actual departure time caused by airport operations or aircraft operator operations.

2.2. Evaluation of Congestion Costs at Madrid Airport

Nombela, de Rus and Betancor (2002) study the congestion costs for Madrid airport in the period 1997-2000. They consider both airlines and passengers delay costs. The authors define the delay as the schedule delay, i.e., the difference between scheduled and actual arrival (and departure) times. They discuss how studies must be cautious when studying a network to avoid double-counting effect of delays, and the difficulty that presents the system to determine who causes the delays. In their case, as they want to study the congestion costs at Madrid airport regardless of who has caused the congestion, they just add up all experimented schedule delays.

The main contribution of their study is a small step they are performing toward the use of economic theory to define delay costs. Nonetheless, as for the Westminster report, their cost estimations are based on an accounting approach.

The authors develop a simple model to identify what are the basic variables that should be included to study congestion costs and explain why total congestion costs can be evaluated by adding the cost borne by passengers and by airlines separately.⁶

As in the preceding study, the final cost estimation is based on previous estimations of the value of time for the passengers and previous studies of the direct and indirect cost of delays for airlines. Values for passengers are estimated based on assuming hourly rates of 15.9 € and are presented on Table 1. The average cost is decreasing for both kinds of delays probably due to the increase in the capacity of the airport. However, the total cost is increasing because of the growth of total number of passengers.

Table 1: Total and Average Passenger Congestion Costs

	July 1997	July 1998	July 1999	July 2000
Monthly costs (million €)				
Arrivals	6.42	7.39	8.71	8.84
Departures	7.94	7.64	8.60	7.34
Average costs (in €/passenger)				
Arrivals	4.04	4.36	4.51	4.01
Departures	4.91	5.89	5.95	4.53

Source: Nombela, De Rus and Betancor (2002).

Table 2: Airlines' Congestion Costs (monthly costs, million euros)

	July 1997	July 1998	July 1999	July 2000
Arrivals	14.7	16.7	20.3	22.0
Departures	18.0	17.6	20.4	17.2

Source: Nombela, De Rus and Betancor (2002).

⁶ A presentation of their model is provided in Appendix 1.

Table 2 presents the results for airlines, which are obtained from an hourly cost of 5,000 €. Costs have increased for arrival delays while the evolution is not so clear for departures.

Considering July-2000 as a representative month of the activity at Madrid airport, and using the previous information the authors provide an annual estimated total costs for congestion around 665 million €. Table 3 shows the marginal congestion costs generated by each flight for both passengers and airlines, which seem to have improved after the enlargement of airport capacity.

Table 3: Marginal Congestion Costs Generated by a Flight (euros)

	July 1997	July 1998	July 1999	July 2000
Arrivals	6590	9250	7880	7070
Departures	6760	8720	8340	6710

Source: Nombela, De Rus and Betancor (2002).

In addition to these results, the main findings according to the authors are that, first, arrival and departure delays are highly correlated and second, spillover effects between one-hour intervals are present. Both results were foreseeable in both cases since 36% of flights in Madrid airport correspond to airlines that use Madrid as a hub.

The estimated values can be highly debatable. The value of time for passengers proceed from values estimated for Germany and Switzerland within the UNITE program and they apply the same value to all kind of passengers. The value of delayed time for airlines is just an average coming from private airlines studies - which could introduce an overestimation bias - and they assume that the representative aircraft at Madrid airport is a 135 seats plane whatever the airlines.

Moreover, the same values of time are applied for both arrival and departure delays, while, as we explain later in the section on value of time, this approach does not seem correct according to the current literature.

Total costs can be overestimated from the selection of the month. July is usually the busiest month of the year. For the studied period, July was the busiest month in 1997, 1998 and 2000, and the third busiest month in 1999.

Part of this bias could be corrected because the authors do not take into account the utilization of buffers, which could underestimate total costs. Nonetheless, we believe that the cost of delays is overestimated since the authors ignore the fact that some congestion is present at the optimal equilibrium that is explained in the next section.

2.3. Evaluating the True Cost of Delays for Airlines

On behalf of the Performance Review Commission, the University of Westminster prepared one of the latest studies about air congestion. The study focuses on the costs of delays due to air traffic management and born by air traffic carriers. It does not study the delay cost for passengers.

Two types of delays for airlines that we have previously discussed are considered here: *buffer delays* and *schedule delays*. Schedule delays are defined as the off-block/on block time of an aircraft relative to the operator's published schedule. Buffers refer in general to the extra time that airlines add to the schedule of a city pair, with respect to what is technically needed.⁷

Schedule delays

The study presents a thorough analysis of all possible costs that an airline face during a delay at the different phases of a flight (in airborne, on the ground, at the gate, during taxiing). It provides estimates for the cost that an airline faces for 15 and 65 minutes delays, typifying 'short' and 'long' delays. For each delay's duration they estimated a 'low', 'base' and 'high' cost scenarios.⁸

The study reports airborne and ground costs starting from each cost elements; i.e., crew costs, handling costs, fuel, maintenance costs, airport charges, passengers compensations for each kind of plane – the study considers twelve models of planes – and destination. The data are obtained through interviews conducted with airlines, handling agents, aircraft operating lessors and other parties.

⁷ For example, if an airline observed that a particular flight is usually late, it can set the return of the airplane at a later time in order to accommodate this regular delay. This is called by the authors "turnaround buffers." The airline can also add extra time to the scheduled time of the original flight. This is called "scheduled buffers."

⁸ For example, 'high' cost scenarios include a higher probability of missing connections, the highest load factor, the highest weight payload factor and so on.

The study includes a detailed description about depreciation, financing, valuation of the planes, accounting practices. It includes also the cost of reactionary delays thanks to a ‘scale up’ of the gate-to-gate costs. Since a delay at a given moment of time can produce cascading delays, the authors of the report scale up the cost of a delay by a factor that comes from a previous study on American Airlines by Beatty, Hsu, Berry and Rome (1998). This scale up varies on different categories of costs.

Buffer delay

The authors estimate the cost of incorporating one minute of buffer into the schedule in three cases: When airlines do not use it; when airlines use it exactly; when airlines use it plus some extra time. They follow the same analysis, studying element by element the possible cost for each kind of scenario, aircrafts, routes and the like. So finally they have estimations for airborne buffer costs, “at gate” buffer costs and taxi buffer costs.

In theory, *minutes of strategic buffer should be added to the airline schedule up to the point at which the cost of doing this equals the expected cost of the delays they are designed to absorb, possibly with some extra margin for uncertainty.* The costs can come from a decrease in the rotations of the plane on a day or from the increase in costs that suppose to register higher gate-to-gate times on the computers reservation system.

However, it is not possible to know the phase of a flight to which the extra-buffer should be associated because even the airlines do not know it. This problem forces the report’s authors not to include this cost into the global estimations, but to consider the hypothetical benefits of the reduction of buffers.⁹

The final estimation for the total cost of delays falls in the range of 800-1200 million euros, which represents an average cost of around 72 euros per minute of delay.

One of the main problems of their analysis is that the Westminster report estimates the cost just for two measures of delay, 15 and 65 minutes of delay, typifying ‘short’ and ‘long’ delay. The authors attribute the cost of the long delay to all the delays longer than 15 minutes. This produces an overestimation of the costs.¹⁰ In fact, approximately 99.9 percent of the total estimated cost comes from delays over 15 minutes. *Average cost per delay minute is around 1 euro for delays up to 15 minutes and around 84 euros for delays over 15 minutes.*

⁹ Through a “rudimentary estimation” they obtain a benefit between 5 and 40 euros per minute.

¹⁰ In 2004, almost 70% of flights delayed by more than 15 minutes were delayed by less than 30 minutes. For 2002, this number is around 65 percent. See Eurocontrol, 2003 and 2005.

Also, the cost of passenger delays for airlines comes from studies done by two airlines, Austrian Airlines and another carrier that wants to keep the confidentiality of its research. The problem is that for these private firms, the loss of a passenger represents a large cost because it represents a decrease of its market size while, when considering the whole air sector, it is expected that the passenger that quits an airline will move to another airline, and not just disappear. Hence the report probably overestimates the delay costs.

Finally, and most important, as we expect to show in our theoretical model (Part Two), we believe that their cost value is overestimated because the Westminster report captures too much congestion which differs from the optimal level of congestion.

As the report mentions, minutes of strategic buffer should be added to the airline schedule up to the point at which the cost of doing this equals the expected benefit. Similarly, for the society as a whole, minutes of congestion should be added up to the point where benefits equal costs. Therefore, *when we study delay costs, we should not consider the whole delay, but just the difference with respect to what could be considered an optimal delay, for both airlines and passengers.*

This critic is also relevant in the other studies we discuss above. In this sense both would be biased and present too high values for their estimation of delay costs.

2.4. Summary on Applied Studies

The main results from the previous studies are summarized on the following table:

Table 4: Summary on Studies of Costs of Air Traffic Delays

	ITA study	Madrid airport	Westminster Study
Market and time coverage	Europe 1999	Madrid airport July 1997-2000	Europe 2004
Costs estimated	Airlines and Passengers	Airlines and Passengers	Airlines
Kind of delays	Schedule and Buffer	Schedule	Schedule and Buffer ¹¹
Estimated costs for	Airlines	39.4-48.6 €/ min	83.3 €/ min
	Passengers	72 €/ min	0.74-1 €/min
		0.26 €/ min	

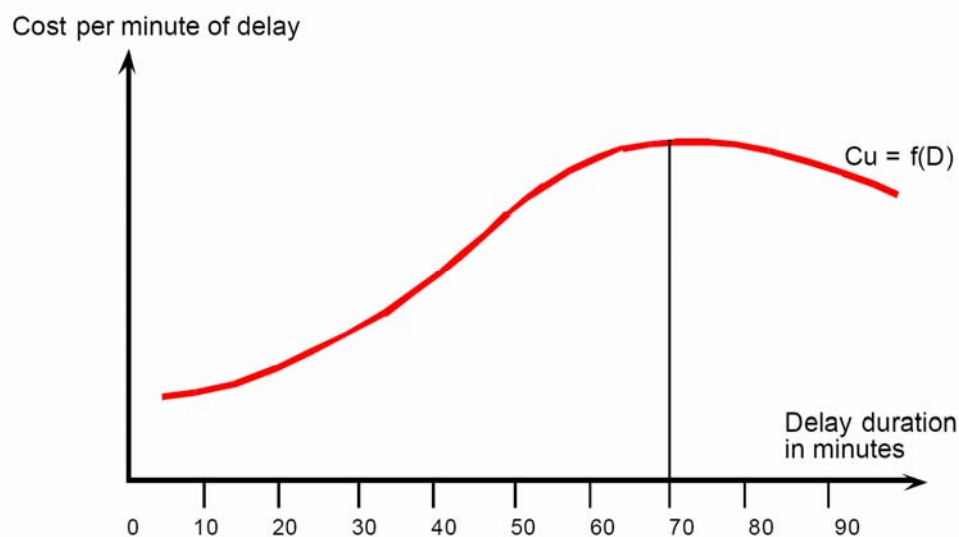
¹¹ Buffer delays are estimated in a theoretical way but not included on the final estimation of costs.

As we can see, the estimated values for both airlines and passengers costs, are relatively heterogeneous. The low value presented by the ITA study for airlines is especially noticeable, even if it is the only study that is taking into account buffer delays, which, in principle, should drive the estimates towards higher values. Probably this low value comes from the controversial definition they use for the delays and from a low estimation for the cost of a minute of delay.

Even if they do not apply it into their analysis, Nombela, De Rus and Betancor (2002) suggests that the impact of the delay depend on the duration of delay of a specific delay, on the nature of the airline and also on the interaction of delays for many flights (specially if hubbing is important).

Moreover, in the literature, the approach to delay cost estimation is based on strong assumptions, e.g., the cost of delay is an additive function of the cost of individual delay and the cost of each delay event is a linear function of the duration of the delay. However it is fairly reasonable to think that the delay is non-linearly related to duration and follows a distribution as depicted in Figure 2.

Figure 3: Cost of delay per minute distribution



Besides this problem and the lack of precision of estimates of delay costs, this review of applied studies shows several features that prove their poor methodological basis:

- A rough definition of delays, despite the fact that the policy of airlines shows that delays are not so simple to define. Indeed airlines include buffer times in the scheduled travel time in order to cope with delays. Some hints of these policies tend to the fact, which we will address more extensively in the forthcoming report (Part Two), that the optimal management of the time-tables implies some delays.
- A tough appraisal of values of time: A unique value of time is usually applied, while many research studies in the field of air transport as well as in the field of other modes show that there is a variety of values of time.
- No use of reliability per se: The fact that reliability, not just the travel time, is valued is not taken into account.
- No consideration of the fact that the users have more or less information on the probability of delays.
- A poor consideration of the situation of delays at connecting airports.

3. Theoretical Analysis on Congestion

The literature about delays and air traffic congestion can be classified into different categories:

- First, we find conceptual models that aim to explain the passengers' behavior, and result essentially in the definition and estimation of the values of time.
- Second, models that are more connected with reality that attempt to model congestion.

3.1. The conceptual models and the value of time

The oldest conception of costs related to travel time includes the trip time only, distinguishing if necessary the durations of different elements of the complete transport chain: Time spent on the way to the public transport systems or to the parking lots; possible waiting time in the case of public transport; trip time, differentiated according

to the transport mode (each mode presents disadvantages and advantages like comfort); then, again terminal time.¹²

A more complex and realistic element has been added when one introduces the concepts of desired arrival time and mismatch with respect to this desired arrival time. Usually this idea is formulated by considering the so-called disutility of transport (or the cost related to the transport), $U(t_h)$. Following Small (1992), this function is written as

$$U(t_h) = \alpha T + \beta SDE + \gamma SDL + \theta D_L.$$

It is a function of the departure time, t_h . It also involves:

- T which represents the trip time, and α , the value of this trip time;
- SDE or schedule delay-early, which is the gap of early arrival, and β the cost that represents to the traveler arriving before the preferred arrival time, that from now on we denote as PAT;
- SDL or schedule delay-late, is the gap of late arrival or delay, and γ the cost that arriving late stand for the passenger;
- DL is a dummy equal to 1 in the case of late arrival, and θ the fix cost associated.

The introduction of uncertainty is an additional level of complexity. Usually uncertainty is introduced through the expected utility of a risk-averse agent, which is equal to the expected total time \bar{T} , increased by a component proportional to the variance of the travel time σ , specifically:

$$EU^* = \alpha \bar{T} + v_r \sigma.$$

This functional form is usually chosen to econometrically estimate the coefficients α and v_r based on a behavioral analysis of trips with random duration. Note that v_r is considered as the unit value of reliability.

If we restrict ourselves to the original model, the literature considers that the value of the reliability v_r is perfectly measured by the costs of early and late arrival, β and γ ,

¹² This introduction is based on Bates, Polack, Jones and Cook (2001) and by Noland and Polack (2002).

which can be estimated through studies of declared or revealed preferences even without the presence of uncertainty on the travel time (see Arnott, de Palma and Lindsey, 1993), and therefore without the need to observe the behavior of agents in case of uncertainty. The model by Small has the advantage of introducing an asymmetry between the early and late schedule delay, which was not considered in the traditional model that includes only two parameters of the distribution function of trip times, namely the mean and the standard deviation.

Nevertheless this assimilation can present some problems. In fact, within the models that follow the idea of Small like the one of Arnott, de Palma and Lindsey, the agent knows perfectly the traffic situation, the late or early arrival that he will face. Therefore he can perfectly plan his agenda; for example, he can carry a book if he knows he will be in advance or announce his delay to a meeting if he knows he will be late. In the case of uncertainty, the agent cannot do it. From this point of view, if we assume to simplify the same value of time for the value of delay, β , and the value of early arrival, γ , a known delay of one hour is not equivalent to a situation where you have one hour of delay with a 50 percent probability or an early arrival of one hour with the same probability, in opposition to what is assumed by the model of Small. The difference between the two comes from the information that the agent has on the first case (the decisions are taken after receiving the information) and has not on the second case (decision of the utilization of time under uncertainty). *This could be interpreted as an option value* that would decrease the cost of mismatch in the schedule in the case of uncertainty over the arrival time. This difference is not taken into account by the literature; Specific surveys, probably based on declared preferences, allow us to measure the importance of this phenomenon. It can be particularly important in public transport, in which two types of schedule cost coexist: An expected schedule cost, based on the discreteness of timetables of transport services; and a random schedule cost that depends on unexpected early or late arrivals.

The case of public transport includes some other specific aspects that accentuate the considerations that we have presented above. This is extended in the Appendix 2, and show how the models, initially developed for the case of roads, where the departing time is chosen within a continuum set, could be applied to the case of public transport- in particular the case of air transport- where the departing times are discrete. In this case, most of the authors introduce new elements to the model like the fulfillment of the

departure time and the difference of the departure time with respect to the announced departure time; however there is almost no measure of the value associated to these elements.

3.2. The values of time

From the previous analysis we can conclude that it is required to consider several values for time:

- The value of trip time, which could be differentiated according to the degree of comfort;
- The value for the waiting time;
- The value of the time for the cases of early or late arrival;
- The value of the reliability, that it is linked to the value of early or late arrival in models that follow the idea of Small.
- Finally, in some models, the value of respecting schedules, for the case of public transport.

Several studies have been devoted to the value of travel time, mainly for surface transportation, i.e., road, rail, cars and bus. The studies on the value of travel time are not so common for the case of air transport. With respect to the studies addressing the value of the other components of trip duration, they are much less numerous and mainly focused on other means of transport; there is almost nothing for air transport.

The estimation methods of these values start from simple principles, based on the fact that all these values operate as parameters that explain the behavior of travelers facing the choice between different transport means or between itineraries characterized by different costs and by different trip durations.

We can estimate these parameters:

- Either by methods of revealed preferences, through traffic models that attempt to explain the choice and therefore the value of time is one of the parameters,
- Either by methods of declared preferences, by questionnaires where we propose to the agent to choose among several hypothetical choices between different

transport modes, different routes characterized by longer or shorter time and between larger or smaller prices.

3.2.1. The value of travel time

Several studies provide values of travel time. A non exhaustive list comprises: Étude MVA Consultancy (1987), Merlin (1991), Hague Consulting Group (1994), EURET (1996), Hague Consulting Group (1996), SNRA (1996), Hensher (1997), Morellet (1997), Small (1997), Wardman (1998), Boiteux (2000), Lam and Small (2001) and finally Quinet and Vickerman (2004).

The general results from these studies bear on the determinants of the change in the value of time:

- According to the trip motives. The value of travel time for business purposes is around (and usually under) the labor cost; it is higher than the value of travel time for commuting reasons, which itself is higher than the value of leisure time;
- According to the transport modes (keeping in mind that the modal choice results from income effect and the purpose of the trip): The value of time with air transportation is higher than the mean value of first class passengers in train, which is itself higher than the mean value of time for second class passengers in train, that is higher than the value of time by car;
 - The value of time increases with income, although at a slower rate (elasticity from 0.5 to 1);
 - The value of time for urban trips is smaller than the value of time for inter-city travels;
 - The value of time increase with the duration of the travel.

Several studies are devoted to urban transport, less numerous for inter-city travels, and really few for the air transport.¹³ Wardman (1998), in his review about estimated values of time for the United Kingdom, finds values for air transport that are almost double than the value of travel time by cars in inter-city travels, and are around 40 euros per hour.

¹³ In fact some studies have been done by airlines. They are not published due for confidentiality reasons.

Morellet (1997) estimates the values of time for all transport modes with the model MATISSE. These values vary as a function of several parameters, like income, size of the group, the purpose for the trip, its length, but also as a function of the modal competition. In general, the higher the competition on a particular route, the higher is the value of time. The mean results for France (length ≥ 80 km, year 1990) are on the following table:

Table 5: Value of time for France in 1990

Transport Mean	Value of time in euros
Train 2 nd class	9.5
Car	10.5
Train 1 st class	30.0
Airplane	47.0

Table 6: Time values in Inter-city travels (1998) per passenger

Mean	For distances smaller than		For the distances d included between 50 km or 150 km and 400 km	Stabilization for the distances superiors to 400 km
	50 km	150 km		
Road	8,4 €	-	50 km < d VDT= (d/10+50).1/6,56	13,7 €
Train 2° Cl.	-	10,7 €	150 km < dVDT=1/7(3d/10+445) .1/6,56	12,3 €
Train 1° Cl.	-	27,4 €	150 km < dVDT=1/7(9d/10+1125) .1/6,56	32,3 €
Airplane	-	-	45,7 €	45,7 €

Source: Rapport Boiteux (2000)

Table 7: Values of time

Relevant VOT studies	HCG 1994	HCG 1998	HCG 1998	SNRA 1997	EUNET 1998	UNITE Values Euro 1998
Transport Segment	Euro 1998					Euro 1998
Inflation to 1998						Normal
Transfer to Euro						travel
Passenger transport – VOT per person-hour						
<i>Car / motorcycle</i>		6.70		9.31		
Business	21.23	21.00		11.95		21.00
Commuting / private	5.53	6.37		3.91		6.00
Leisure / holiday	3.79	5.08		3.10		4.00
<i>Coach (Inter-urban)</i>						
Business	21.23					21.00
Commuting / private	5.95			5.40		6.00
Leisure / holiday	3.08			4.37		4.00
<i>Urban bus / tramway</i>						
Business	21.23					21.00
Commuting / private	5.95			4.94		6.00
Leisure / holiday	3.08			3.22		4.00
<i>Inter-urban rail</i>		4.97		8.50		
Business		18.43		11.95		21.00
Commuting / private		6.48		6.21		6.40
Leisure / holiday		4.41		4.94		4.70
<i>Air traffic</i>					40.60	
Business				16.20		28.50
Commuting / private				10.11		10.00
Leisure / holiday				10.11		10.00
Freight VOT						
<i>Road Transport</i>						
LGV	39.68	30.75	40.76			40.00
HGV	39.68	30.75	43.47			43.00
<i>Rail Transport</i>						
Full trainload		645.37	725.45			725.00
Wagon load		26.16	28.98			30.00
Average per tone			0.76			0.76
<i>Inland Navigation</i>						
Full ship load		178.55	201.06			200.00
Average per tone			0.18			0.18
<i>Maritime shipping</i>						
Full ship load		178.55	201.06			200.00
Average per tone			0.18			0.18
<i>Air transport</i>						
Average per tone						4.00

Source: Quinet and Vickerman, 2004.

Boiteux (2000), in a report for the French Government about the values of time to be used in investment economic appraisal, recommends the values of time for intercity travels presented in Table 6. These values are based on a critical review of the previously quoted studies.

Quinet and Vickerman (2004) collects the following values from the report UNITE in Table 7.

Finally we mention the values kept by Boiteux (2000) for urban transport, which results from a cautious synthesis of studies on the subject. They indirectly concern air transport, in the sense that trip by air starts and ends with an urban connection. (See Table 8.)

Table 8: Values of time in Euro/hour

Purpose	As a percent of wage	As a percent of gross wage	France Euro per hour	Ile de France Euro per hour
Business	61%	85%	10,5	13
Commuter	55%	77%	9,5	11,6
Others	30%	42%	5,2	6,4
Mean	42%	59%	7,2	8,8

Source: rapport Boiteux (2000)

3.2.2. The value of waiting time

- The value of waiting time is listed by the same studies that listed the value of trip time. Wardman (1998) finds that the values of waiting time and walking time are 1.6 higher than the value of a trip time. These values are smaller than the ones presented by Small (1992) and Merlin (1991) that find coefficients on the range 2 to 3.
- Note the results published by the consulting firm MVA for the RATP. These results are presented on Table 9 and expressed as a proportion of the waiting time.

Table 9: Values for different waiting times

Waiting	Seated trip	Stand-up trip	Stand-up and squeezed trip
100%	50%	65%	95%

Source: MVA (1987)

3.2.3. The value of time for early and late arrival

These values have been listed by several studies as de Palma and Rochat (1996), Noland and Polack (2002), and Bates, Polack, Jones and Cook (2001). De Palma and Rochat lean their study on their own research and on previous studies. The summary of these results is presented in Table 10.

Table 10: Value of time for early and late arrival

Author	Country or city	Ratio to the cost of travel time	
		Of the cost of arrival in advance	Of the cost of delayed arrival
Small (1992)	USA	0.64	2.39
Khattak; Schoffer and Koppelman (1995)	Brussels	0.38	1.03
De Palma and Rochat (1996)	Geneva	0.327	2.69

Bates, Polack, Jones and Cook (2001) determine the cost of early and late arrival from questionnaires of declared preferences for trips by train. They use a model based on Noland and Small that allows them to evaluate the coefficients of early and late arrival and to compare them to the coefficient of the mean value of trip time. The ratios are collected in Table 11.

Table 11: Value of time schedule expressed as a proportion of value of travel time

Average delay	Early arrival	Late arrival
1	0,5	1

3.2.4. Value of reliability

Within the framework of the Small's model, the valuation of reliability comes directly from the estimation of the value of time of early and late arrival. We have already discussed some elements of the value of reliability

However more direct estimations have been made. They attempts to resolve a previous problem: How do you measure reliability physically? The literature on the topic presents two measures: The standard deviation of the trip time and the interval between the quantile 50% and the quantile 90% of the travel time distribution. The use of this last measure is justified by the dissymmetry of consequences of the delays and by the dissymmetry of the distribution of travel times that spreads out for the delays.

Brownstone and Small (2005) review the values of reliability relative to road transport from some recent studies based on revealed and stated preferences (RP and SP respectively). The values are presented in Table 12.

Table 12: Comparison of selected model results

	Data Sources	Cc	Median VOT (\$/hour)	VOR median	
				Male	Female
<i>State Route 91</i>					
Lam-Small : route only	RP	RP	24	\$12/hr	\$30/hr
Lam-Small : route, mode, transponder	RP	RP	23	\$15/hr	\$32/hr
Small, Winston and Yan	RP/SP	RP/SP	9-25 ^a	\$20/hr (\$4/incident)	
<i>Interstate 15</i>					
Brownstone and alii. (wave 3)	RP	RP	30		NA
Steimetz-Brownstone (wave 5)	RP	RP	22-45 ^a		NR
Ghosh : route, mode, transponder	RP/SP	RP/SP	13-40 ^a		NA
<i>Other</i>					
Calfee-Winston-Stempski	SP	SP	4		NA

Legend: Cc= Coefficients used for computing; VOT= Value of time; VOR = Value of Reliability.

Notes:

NA: not applicable (variable not included in model) NR: not reported (variable included but resulting distribution not calculated).

^a Values obtained for different specifications of the cost function

Noland and Polack (2002) shows values obtained from questionnaires of declared preferences. They review three studies, and the table summarizing their results (table 13) deserves some comments, directly drawn from the quoted text.

Table 13: Comparison of Alternative Empirical Estimates of Travel Time Variability

	Black and Towriss (1993)			Noland and <i>al.</i> (1998)				Smaller and <i>al.</i> (1999)		
	All data	Cars, only commute trips	With standard deviation	With standard deviation and scheduling costs	With coefficient of variation, without lateness probability	With coefficient of variation	Without measure for variability	With standard deviation	With standard deviation and scheduling costs	Without standard deviation
Standard deviation	-0.0353	-0.0352	-0.1263	0.151	-	-	-	-0.274	0.0665 ^(a)	-
Coefficient of variation	-	-	-	-	-0.667	-0.346 ^(a)	-	-	-	-
Mean travel time	-0.0507	-0.0635	-0.0996	-0.0556	-0.129	-0.105	-0.098	-0.085	-0.0480	-0.0578
Cost	-0.0107	-0.0082	-	-	-	-	-	-1.304	-0.906	-1.0256
$E(SDE)$	-	-	-	-0.131	-0.097	-0.093	-0.095	-	0.0398 ^(a)	0.0236 ^(a)
$E[(SDE)^2]$	-	-	-	-	-	-	-	-	-0.00605	-0.00517
$E(SDL)$	-	-	-	-0.304	-0.281	-0.130	-0.128	-	-0.0403	-0.3181
Lateness probability	-	-	-	-2.564	-	-1.347	-1.529	-	-2.126	-1.849
Probability of extra late arrival	-	-	-	-	-	-	-	-	-1.661	-1.003
Log-likelihood	n.a.	n.a.	-2826.5	-2747.3	-2766.5	-2759.6	-2760.6	-3252.0	-3161.8	-3156.0
Reliability ratio	0.70	0.55	1.27	-	-	-	-	3.22	-	-
$\beta \ln \left(1 + \frac{\gamma}{\beta} \right)$	-	-	-	-0.157	-0.132	-0.081	-0.081	-	-	-

^(a) coefficient not significant at 95% level.

Black and Towris (1993) reckon a reliability coefficient, defined as the ratio between the coefficient of the standard deviation and the coefficient of the mean travel time.

Noland *et al.* (1998) and Small *et al.* (1999) made statistical estimations according both to the “standard deviation/mean” model (in that case Noland uses not the standard deviation, but the coefficient of variation which is the ratio between dispersion and mean) and to the Small (1992) model. In some models they include a probability of late arrival and also a probability of extra-late arrival, taking into account the flexibility of arrival at work. Small and alii introduce a quadratic term for the late arrival, taking into account the non-linearity of reactions of the users to the late arrival delay.

The MVA study already quoted and relative to the RATP gives as well estimations of the reliability for the case of urban public transport; the ratio between the value of the standard deviation and the value of travel time is 0.2, much smaller than the previous results; on the other hand the reliability of waiting times is higher: the correspondent ratio becomes equal to 0.5.

Bates *et al.* study the reliability of train services based on the results of the review of revealed preferences for the determination of the value of early and late arrival times. They use these values to model the departure time choice of a user as a function of the distribution of the travel time. The choice of the service depends not only on the mean delay but also on its dispersion, and the disutility linked to the non-reliability is weak when the dispersion of delays is small, and increases rapidly with this dispersion.

This example shows that the effect of delays depends on the knowledge that the users have. Bates *et al.* shows how an erroneous perception of the distribution of probabilities of delays could involve a loss for the user due to the wrong decision with respect to his departing time.

As we can see from this short review of the estimations for reliability, there are really few statistical results of the value of reliability in collective transport; this fact is similar to what happens for the value of travel time. In particular, we have not found any evaluation of the value of reliability in the area of air transport. As previously noted, it's probable that the air carriers have their own studies, but they are not published due to their confidentiality.

Besides, the studies present widespread results. Nevertheless the results are sufficiently significant to consider that reliability is an important element in the cost of transport, too often neglected.

However little attention is paid to the fact that the cost of reliability -essentially the delays with respect to the desired schedule- is or is not known in advance by the user. However the users seem to be sensible to the information level as shows the study done by inquiry to the users by Eurocontrol, and explained in the Appendix 3.

3.3. Theoretical Models of Congestion

Most of theoretical models on congestion that we present here focus on modeling the queues that the congestion can create, rather than on the reasons that create congestion. In general, congestion is assumed to be generated by some random process and the authors focus their attention on how, once congestion appears, it changes over time. It seems quite logical that, as congestion is present on a daily basis at most airports in the world, airlines can anticipate congestion and adapt their behavior accordingly.

As we have previously explained, this is captured by the buffer delays that airlines introduce into their schedule to control a part of the randomness of day-to-day operations.

The important question that we consider is why airlines do not account for a larger percentage of delays. In other words, what is the optimal level of average delays for airlines? Is there an optimal social average delay different from zero? We believe that the answer to these questions is positive. In the literature, only one paper addresses in part this issue.

We now summarize the models of congested transportation systems, which, according to Daniel (1995), can fall into three categories.

Econometric models

Econometric models estimate time-varying demand and delay functions and calculate equilibrium congestion fees. They use nonstructural specifications of delay functions and ignore intertemporal traffic adjustment in response to congestion fees. In this category we find for example Carlin and Park (1970), Park (1971) and Morrison and Winston (1989). Carlin and Park attempt to estimate the effects of using marginal cost

pricing at the Airport of LaGuardia with a simple model. They do not try to estimate total time and costs of delays, but to estimate the marginal delay cost that an additional operation can create; for this, they just assume different values of one minute of delay for passengers and airlines.

Morrison and Winston represent air traveler choice of air carrier and routing in the U.S. by a multinomial logit which includes, as explanatory variables among others, the average fare for the chosen fare class (fare class is assumed to be exogenous), the travel time, the schedule delay,¹⁴ transfer time, the percentage of flights on time (flights arriving within fifteen minutes of the scheduled arrival delay) and frequent flier miles awarded times the number of cities served by the carrier. They find that a 1 percentage change on the on-time performance records is valued at 1.21\$ per round trip. From this they get the value of time estimations gathered in Table 14.¹⁵

Table 14: Estimated time values

	1983 dollars per hour	Fraction of wage
Value of travel time	34.04	1.70
Value of transfer time	73.96	3.70
Value of schedule delay	2.98	0.15

Source: Morrison and Winston (1989)

From this point they study what would be the effects over congestion of using marginal cost pricing at airports. To study congestion they define delays as the sum of schedule and buffer delays, in particular the authors say that “scheduled flight time includes some delay” and therefore they estimate technologically feasible flight times. However it “led to unsatisfactory time predictions for many routes”. They assumed undelayed flight time for a route to be the minimum flight time achieved by any flight in the data set on the route during all the studied period.

They study what the social optimal for airports (for pricing and investment) is assuming that airlines operate into six mutually exclusive user classes (international, cargo, commuter, majors and nationals, other commercial and general aviation). They

¹⁴ "Schedule delay" refers in this case to the difference between the traveler's desired departure time and the closest available departure time.

¹⁵ The data comes from 5 randomly selected markets in the US in the third quarter of 1983: Allentown-Atlanta, Burbank-San Jose, Philadelphia-Orlando, San Francisco-Portland, and Dayton-New York (La Guardia).

get the expected result that optimal and arrival departures tolls should equal the extra cost that operation imposes on other users and on the airport authority. They also estimate the optimal capacity that is reached when the savings in delay costs from adding runway capacity equal the extra cost of that capacity.

According to their model, the increase in tolls that should be applied to improve the situation would make some users to be “tolled off” the airports. However the improvement in the financial health of these airports, plus the decrease on delays would more than offset these losses.

Also the study is conservative in this aspect since it is not considering possible substitutions. For example it assumes that the demand for using the airport in a given hour is a function of the price for that hour only. So the model does not capture “peak spreading”.

Also they study the effects of the deregulation of 1978 over air safety and the effects of mergers; in particular they study the case of the six mergers that took place on 1986-87.

By means of a simple model, Park (1971) studies the theoretical effects of imposing a congestion toll to the airlines or to the passengers in a context of flexible or fixed ticket price (perfect competition or competition just in schedules). This model is difficult to be applied in practice since for example delays are expressed just as an unknown increasing function on the number of flights and the total value of transportation is also an unknown function which increases with the number of passengers and decreases with delays. The model uses quite simplifying assumptions. It assumes for example that there exists only one destination and that traffic is homogeneous. It focuses its attention on the passenger loads which according to the model should be increased to achieve efficiency.

Bottleneck models

Bottleneck models generate equilibrium fees with intertemporal traffic adjustment, but generally employ simple deterministic queuing processes (see e.g., Vickrey (1969), Arnott, de Palma, and Lindsey (1990b, 1993) and Lindsey and Verhoef (2000)). They are focused on road transportation, which presents important differences with respect to air transport. For example, airport’s infrastructure is used by a relatively small number of agents whose decision of entry is not random but scheduled. They end up proposing

similar solutions as for roads: (1) enlarge capacity; (2) manage demand by peak-load pricing.

Queuing-theoretic models

Queuing-theoretic models capture the effects of stochastic arrivals on the evolution of queues, but assume exogenous arrival rates and do not calculate equilibrium congestion fees (see, e.g. Koopman (1972) and Menhdiratta and Kiefer (2000) and Janic and Stough (2003)). Most of this kind of models assumes a constant arrival rate. Such systems have steady-state solutions and do not adequately model airport queues resulting from rapid fluctuations of traffic rates

The model by Daniel (1995) is in between: it develops a stochastic queuing model implanted into a bottleneck model. His model is especially applicable to hubs, which experience rapid fluctuations and severe peaking of traffic rates and queue lengths.

Hub-and-spoke networks enable airlines to reduce their aircraft-operating cost and passenger schedule-delay (defined as the time between the most preferred travel time of a passenger and the closest available flight) by achieving higher load factors on larger aircraft with greater service frequency. To minimize costs, hub and spoke networks schedule arrivals and departures at hubs in “banks” of flights.

The model study arrival and departure queues and layover costs (time aircraft spend at hubs after exiting arrival queues and before entering departure queues) and interchange-encroachment costs (costs from the risk of passengers missing connections flights due to inadequate layover times). The model accounts for the effects of overlapping traffic and residual delays from one arrival or departure bank to the next.

With respect to his definition of delay, Daniel takes into account the buffers. According to his model the social planner can implement the optimal arrival schedule by imposing a congestion fee equal to the increase in costs imposed by the n th aircraft on all other aircraft. Because the airport authority cannot observe directly schedule times, aircraft operators would have incentives to misrepresent their schedule times if these were the basis of the toll assessments so the fee should be contingent on the actual arrival time.

He estimates the equilibrium of the model for five cases: no congestion fee and competition in the market, no-fee with a Nash dominant firm, no-fee with a Stackelberg

dominant firm, no-fee with joint-cost-minimizing airlines and the case of congestion fee and perfect competition.

Using Data from the Minneapolis-St. Paul Airport during a week of May of 1990, and according to his model, the atomistic model fits better the actual traffic patterns. Therefore it studies the theoretical effects that congestion pricing would have over this equilibrium.

The part of his model most open to criticism is that he assumes that there are two independent queuing systems at the airport, one for landings and one for takeoffs. Also, arrival and departure distribution is Poisson with time dependent rates and service times are deterministic and occur at equally spaced intervals. The study does not compute congestion costs.

Any of the studies we have discussed so far has studied why congestion appears and if congestion should disappear totally or if there is congestion on the social optimum. The model developed by Mayer and Sinai (2003) explains that delays appear largely due to network benefits from hubbing and to congestion externalities.

As the authors mention in general, it is believed that congestion is an externality, and agents do not take into account the externality that they create for others. This can explain why airports without a single dominant carrier should have high delays. However it is not valid to explain the persistence of congestion at airports with a dominant large carrier.

Brueckner (2002) follows this argument and shows that, when a monopolist dominates an airport, congestion is fully internalized. Under a Cournot oligopoly, however, carriers are shown to internalize only the congestion they impose on themselves. In this case a toll should be equal to the congestion cost from an extra flight times one minus the carrier's flight share.

There are two problems with this idea: First, even if overall the data presented by the author about congestion in US airports presents evidence against his theory, it is true that some of the airports with high levels of delays present a high degree of concentration on the airline market. Second, it is difficult to link congestion fees and market power, given the actual difficulty for new airlines to enter into most of the big European airports.

Mayer and Sinai (2003) suggest that air traffic congestion exists at some level due to the network benefits associated with the hub and spoke system. The authors construct a

measure of delay that is unaffected by airline scheduling, actual travel time minus minimum feasible travel time. According to their model, longer delays at hub airports are the efficient equilibrium outcome of a hub airline equating high marginal benefits from hubbing (one new round-trip flight from a hub connected to n cities will create $2n$ additional connecting routes) with the marginal costs of delay.¹⁶

The authors test their model with data from the US Department of Transportation which covers all airlines with at least one percent of all domestic traffic and 27 top U.S. airports from 1988 to 2000.

In this direction, according to the Logistics Management Institute, between April 1993 and April 1997 scheduled block time for flights among the 29 hub airports in the U.S. increased by 1.25 minutes over this four year period. This increases to 1.61 if the spacious new airport of Denver is excluded. Looking to individual airports the average increase ups to 3.28 minutes at Atlanta and to 4.71 minutes at Dallas-Ft. Worth (DFW). These number seems to be small but, just looking at DFW, that had around 450.000 scheduled departures in 1997 and assuming a cost of 40\$/block minute implies a cost increase of about \$85 million over the four years in direct costs for airlines alone.¹⁷

Also, following the idea of social gains coming from congestion, we could cite the paper of Betancor and Nombela (2002) explaining how an increase in the frequency of a service can increase the welfare of all travelers. So not only new destinations, but also increases in the frequency of old ones can present benefits for the society even if congestion is already present.

4. Towards an Objective Definition of Delays

As we have seen, the proper definition of delays is a complex problem. Most of the literature considers only the observed delays to study the cost of these delays. However several authors have observed that companies include buffer times in the scheduled travel time in order to cope with delays. This comes from the fact that delays produce also profits and not only costs and it can produce profits for both airlines and passengers. In hubs, the creation of a new route represents a big increase on the

¹⁶ A more detailed description of the model can be found on Appendix 4.

¹⁷ Source, Air Traffic Services Performance Focus Group: Airline metric concepts for evaluating air traffic service performance.

possibilities of combinations for all the users of the route and for other airports the passengers can benefit from an increase on the frequency of flights on a given route which would represent a diminution of the difference between their departing times and their desired departing time. These benefits must be confronted with the cost that the introduction of this new flight can represent over congestion.

These facts lead us to believe that to compute the cost of delays, we should measured not the observed delays but the difference between these and what we define as optimal delays, that are the delays that we will try to identify in Part Two of the report, and that result from equaling the cost and benefit of congestion.

Even if this is the most important problem of the literature, is not the only one. Specially, the effect of uncertainty over the cost that transport can represent for passengers has been obviated and the fact that passengers can adapt their behavior to the extent they know the existence of these delays has been understated.

The next part of the report will have as objective to model these phenomena to deduct a coherent economic definition of the delays for the user, and a formula that allows to calculate them. The model will therefore modelize the optimal decision for airlines and society in order to compare it to the actual situation.

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List of Acronyms

Abbreviations	Full term
AEA	Association of European Airlines
ATA	US Air Transport Association
ATFM	Air Traffic Flow Management
DGAC	Direction General de l'Aviation Civile
eCODA	Enhanced Central Office for Delays Analysis
EUNET	EUropean NETwork
EURET	European Research Programme for Transport
IDEI	Institut D'Economie Industrielle
ITA	Institut du Transport Aerien
IATA	International Air Traffic Association
PAT	Preferred Arrival Time
RATP	Régie Autonome des Transports Parisiens
RP	Revealed Preferences
SDE	Schedule Delay-Early
SDL	Schedule Delay-Late
SNRA	Swedish National Road Administration
SP	Stated Preferences
UNITE	Unification of accounts and marginal costs for Transport Efficiency
VOR	Value of Reliability
VOT	Value of Time

Appendix 1: A Model of Congestion Costs

This model is proposed by Nombela, De Rus and Betancor. Users' generalized cost of travel is expressed as:

$$g = p + v_t t + \gamma, \quad (\text{A1.1})$$

where p is the airfare; t is travel time (waiting and in-flight time); v_t is the value of time; and γ is some measure of quality (user's perception related to reliability, comfort and safety).

Producer surplus can be expressed as:

$$PS = pq - (c + \theta)q, \quad (\text{A1.2})$$

where q is the number of passengers, c is the marginal cost per passenger and θ is the compensation (assumed constant) paid by the airline to each passenger.

The change in users' surplus is then:

$$\Delta US = -(\Delta p + v_t \Delta t + \Delta \gamma - \theta)q, \quad (\text{A1.3})$$

where Δ represents the change in each variable between period 1 and period 0. Demand q is assumed to be constant, and also the utility obtained by travelers from used services, even though price and other components of generalized costs may vary between the two periods. The negative sign of expression (A1.3) indicates that the variation of user's surplus is simply equal to the change of total generalized costs.

The change in producer surplus when there is a system overload is:

$$\Delta PS = (\Delta p - \Delta c - \theta)q. \quad (\text{A1.4})$$

Change in social surplus (welfare), ΔW , is obtained as the sum of ΔUS and ΔPS . Without any modification of fares, change in total social welfare would be equal to (minus) total congestion costs:

$$\Delta W = - (vt\Delta t + \Delta c + \Delta \gamma)q. \quad (A1.5)$$

When airlines are able to pass their additional congestion costs to passengers through prices, $\Delta p = \Delta c + \theta$. We could then evaluate total congestion costs simply as the effects borne by passengers. In that case, congestion costs, CC , measured as the reduction of social surplus, are:

$$CC = (\Delta p - \theta + vt\Delta t + \Delta \gamma)q. \quad (A1.6)$$

Thus, congestion costs could be theoretically evaluated by computing the change in fares (induced by extra costs in airlines), the value of extra time spent by travelers and the loss of quality that they suffer, and deducting the monetary compensation θ received from airlines. However, as the authors assert, it is unrealistic to consider that this expression could be applied in practice to evaluate congestion costs. First, there is a large number of elements, such as changes in price of petrol or in the competitive environment that are likely to have a more important effect on costs than congestion. Second, quality changes for passengers (mainly derived from uncertainty related to flight unreliability) are probably as important as difficult to measure.

If we assume that quality effects are reflected in a higher valuation of time for passengers, total congestion costs can be evaluated as:

$$TCC = vt\Delta tq + \Delta cq, \quad (A1.7)$$

Therefore total congestion costs can be estimated from two separate parts: cost borne by passengers in term of extra time spent at airports, and extra costs assumed by airlines. As the increase in airline's costs per delayed passenger is difficult to estimate, they approximate it by evaluating the extra costs for airlines per hour of delay. They use estimations for both values of time (passenger and airlines) coming from previous studies).

Appendix 2: A theoretical decomposition of time value.

Consider the model by Small, which expresses the utility of a user as a function of the trip time, and the mismatch with respect to the desired arrival time:

$$U(t_h) = \alpha T + \beta SDE + \gamma SDL + \theta D_L. \quad (\text{A2.1})$$

In this expression, $U(t_h)$ represents the disutility (or the general cost of the transport) linked to the depart at time t_h ; T represents the trip time, and α the value of this trip time; SDE or schedule delay-early, is the gap of early arrival, and β the cost that represents to the traveler arriving before the preferred arrival time, that from now on we refer to as PAT; SDL or schedule delay-late, is the gap of late arrival or delay, and γ the cost that arriving late stand for the passenger; D_L is a dummy equal to 1 in the case of late arrival, and θ the fix cost associated.

The way we account for uncertainty is different whether we consider the case of road transport, where the departing time vary continuously, or we consider the case of a public transport, where the user can choose only between departures at discrete times and where, beside the reliability of arrival time, the reliability of departure time is added. Note that, in the case of car, the departing time is chosen by the user, while in the case of public transport, it is subject to uncertainty.

The departure time varies continuously

It is the case for road traffic. Then uncertainty comes from the trip time that becomes a random variable written:

$$T_h = T_f + T_x(t_h) + T_r(t_h). \quad (\text{A2.2})$$

In this expression, T_f is the trip time in the case of fluid conditions of traffic, $T_x(t_h)$ is the extra time due to congestion, which depends on the departing time, $T_r(t_h)$ is the random term that, as usual, depends of t_h , and whose probability density function is given by $f(T_r)$.

The user chooses his departing time for maximizing

$$E[U(t_h)] = \int U(t_h) f(T_r) dT_r \quad (\text{A2.3})$$

With the previously defined utility function, this expression can be written as:

$$E[U(t_h)] = \alpha E(T_h) + \beta E[SDE(t_h)] + \gamma E[SDL(t_h)] + \theta p_L(t_h) \quad (\text{A2.4})$$

Choosing the distribution of the random variable T_r we can calculate the expression for $E[U(t_h)]$ and determine the value of t_h that minimizes it. The mathematic expression can be simplified for some specific density functions. With an exponential distribution (defined by $f(u) = (1/b) \exp(-u/b)$, and where the average and the standard deviation are equal to b , the expression becomes:

$$EU^* = \alpha (T_f + T_x + b) + \theta p_L^* + b \left\{ \beta \ln \left[\frac{\theta + b(\beta + \gamma)}{b(\beta - \alpha \Delta)} \right] - \frac{b(\beta - \alpha \Delta)}{\theta + b(\beta + \gamma)} - \alpha \Delta \right\} \quad (\text{A2.5})$$

where

$$p_L^* = \frac{\theta(\beta - \alpha \Delta)}{\theta + b(\beta + \gamma)} \quad (\text{A2.6})$$

and : $\Delta = -T'_x / (1 + T'_x)$.

The optimal departing time is given by:

$$t_h^* = PAT - T_f - T_x - b \ln \left[\frac{\theta + b(\beta + \gamma)}{b(\beta - \alpha \Delta)} \right] \quad (\text{A2.7})$$

The previous expression can be rewritten as:

$$EU^* = \alpha (T_f + T_x + b) + \theta p_L^* + bH(\alpha, \beta, \gamma, b, \Delta) \quad (\text{A2.8})$$

With some additional hypothesis, $\theta = \Delta = 0$, and in the case of a logarithmic distribution of risks, it becomes:

$$EU^* = \alpha(T_f + T_x + b) + b\beta(1 + \gamma / \beta) \quad (A2.9)$$

We find therefore the actual expression for the expected value of a risk averse agent, equal to the expected total times plus a term that is proportional to the variance of trip time:

$$EU^* = \alpha\bar{T} + v_r\sigma \quad (A2.10)$$

This is the functional form usually employed in econometrics works, which try to estimate the two coefficients α and v_r from the analysis of the behavior in cases of aleatory trip time, and where v_r is considered the unit value of the reliability.

If we restrict ourselves to the original model, the literature considers that the value of the reliability v_r is perfectly measured by the costs of early and late arrival, β and γ , which can be estimated through studies of declared or revealed preferences even without the presence of uncertainty on the travel time (see the work by Arnott, de Palma and Lindsey), and therefore without the need to observe the behavior of the agents in case of uncertainty.

Discrete choice of the departing time

This is the case for the collective transports, in particular the case of air transport. The analysis is therefore quite more complex and is difficult to get general conclusions as we were able to do in the continuum case, because the choice of the user is not done within a continuum set but within discrete values, the departing times of the services. Besides, in the collective transport, several factors can be introduced in the formulation for the utility of the user for better explaining his choice:

- the waiting time (that is valued differently from the trip time, and that includes a security margin depending on the uncertainty over the schedule),
- the eventual correspondances,
- the fulfillment of the schedule (we can assume that the users attach a significative importance to the fulfillment of the timetable), as well at the depart than at the arrival,

- finally the information of the users about the probabilities. These probabilities are more numerous since, apart from the uncertainty about the trip time, there is also uncertainty about the arrival times.

But with the modeling of Small for the utility of the users, is remarkable that the assessment of the reliability will pass exclusively by the early and late arrival costs, besides the transport time costs.

Surely the final results in terms of cost of reliability combine this valuations with the probability laws of the different times (trip, arrival), but the diversity of cases and the discrete character of the choices prevent from succeeding on simple analytical formalizations like is the case for the road traffic.

The road modeling remains however an estimate of the isolated case of the services, all the more suitable as the intervals between services are reduced.

Bates *et. al* (2001) have simulated situations relative to the choice of passengers faced to railroad services. They first determine the optimal choice without uncertainty over the schedule and trip times. Then they look at the consequences of uncertainty over these schedules and trip times over the optimal decision, and the economic loss that results with respect to the situation without uncertainty. The results depend heavily on the particular situation studied. Although they can appear to be unexpected, they are very logical: The economic loss does not depend on the mean delay but also on its dispersion. On the one hand, a constant average delay would not imply any economic loss, but would drive to changes in the users' choices; on the other hand the dispersion of delays would drive to high economic losses. These results are obviously linked to the structure of the information of the users. Here the authors have assumed that the user knows perfectly the density functions over the transport offer.

Appendix 3: Perception of Passengers Delays: An explanatory study

The study run by Eurocontrol (2002) analyzes the differences between aircraft or flight-based performance indicators and actual passengers perception. It was elaborated through interviews to passengers during a three week summer period in 2001 at the airport of Barcelona. The most striking result is that, on a scale of 10, perception on delays averages 7.7; it is increased to 8.6 if the passenger already knows that she/he is delayed. Also when a passenger arrives on a delayed flight, he/she demands a thirty five percent faster airport processing time than when he/she arrives on time.

Other important findings from the study are:

- Arrival delays are quoted to be twenty percent more important to the passenger than departure delays;
- Passengers' expectations are independent of the trip chain organization. Passengers do not care about the sources of delays or how production takes place.;
- Expectations seem to be established rationally and formed in relation to passenger needs. For example, passengers flying through a hub airport and passengers with non domestic destinations are more concerned about arrival delays than passengers on a one-leg trip and passengers on a domestic flight respectively.;
- Eurocontrol data on delays provide a non accurate estimation of overall delays perceived by passengers while aircraft-based delay indicators are sufficient and not biased to capture them;
- Passengers do not seem to have full information of the trip chain events. Passengers are quite demanding on airport processes.

Appendix 4: A Model on Network Benefits and Flight Delays

This model is proposed by Mayer and Sinai (2003). Consider two profit maximizing airlines operating at a single airport: A hub airline connected to N cities and serving N^2 markets; an atomistic non-hub airline offering only point-to-point services. The airport operates continuously in discrete periods of time that they think of as being about 40 minutes long. Connecting passengers must wait at least one period and they can connect with any flights departing one or two periods after their arrival (they assume passengers are unwilling at any fare to wait more than two periods to connect). Hub airlines choose their number of arrivals (A_t) and departures (D_t) for each period to maximize the following value function:

$$V_h = p \left[\sum_t (A_t + D_t) \right] + q \left[\sum_t f_t A_t D_{t+1} + (1-b)(1-f_t) A_t D_{t+2} \right] - \sum_t (A_t + D_t)(A_t + D_t + N_t) \quad (\text{A4.1})$$

subject to:

1. $A_t \geq 0, D_t > 0$
2. All aircraft that arrive at an airport must eventually leave, so that total arrivals must equal total departures.

$$\sum_{t=0}^2 A_t = \sum_{t=0}^2 D_t$$

3. $D_t = (1 - f_{t-2}) A_{t-2} + f_{t-1} A_{t-1}$ and $A_t = f_t D_{t+1} + (1 - f_t) D_{t+2}$

Marginal benefit is represented by p , and it incorporates the revenue from fares charged for point-to-point service minus the marginal costs of serving the route segment, including items such as fuel, labor, and the rental costs of aircraft.

The second term in brackets describes the benefits of hubbing and generates increasing returns to scale based on the number of possible connection for each passenger. The additional net revenue obtainable by a hub airline, q , represents the price that connecting passengers are willing to pay in excess of the additional resource costs

times the number of destinations any passenger can feasibly connect to. On any plane arriving in period t , some fractions of passengers, f_t , connect to a departing plane in the next period and $(1-f_t) \cdot A_t$ can connect to D_{t+2} destinations in two periods. The discount factor, $0 < b < 1$, reflects the relative reduction in net revenues associated with two period connections compared to one-period connections.

The last term describes the congestion costs. Congestion increases linearly in the total number of flights in a period $(A_t + D_t + N_t)$ where N_t is the total number of arrivals and departures scheduled by the non-hub carrier in period t . They assume that congestion does not spill across periods. The hub carrier only cares about the congestion on it's own flights.

To model the flight choices of the non-hub carrier they assume that he operates atomistically, choosing a constant, nonzero number of flights each period.

To solve the maximization problem they assume that the number of passengers arriving at t that will connect with flights of the following periods depends on the relative shares of the departing flights in the subsequent two periods:

$$f_t = \frac{D_{t+1}}{(D_{t+1} + D_{t+2})} \quad (\text{A4.2})$$

The hub airline's problem is stationary under these assumptions and the problem reduces to repeated identical banks of three periods each, therefore the maximization problem becomes:

$$\begin{aligned} v_h = & p \left[\sum_{t=0}^2 (A_t + D_t) \right] + q \left[\sum_{t=0}^2 A_t \frac{D_{(t+1) \bmod 3}^2}{D_{(t+1) \bmod 3} + D_{(t+2) \bmod 3}} + (1-b) A_t \frac{D_{(t+2) \bmod 3}^2}{D_{(t+1) \bmod 3} + D_{(t+2) \bmod 3}} \right] \\ & - \sum_{t=0}^2 (A_t + D_t)(A_t + D_t + N_t) \\ \text{s.t. } D_t = & \frac{A_{(t-1) \bmod 3} D_t}{D_t + D_{(t+1) \bmod 3}} + \frac{A_{(t-2) \bmod 3} D_t}{D_{(t-1) \bmod 3} + D_t} \quad \forall t = 0, 1, 2; \quad \sum_{t=0}^2 A_t = \sum_{t=0}^2 D_t \end{aligned} \quad (\text{A4.3})$$

Maximizing equation A4.3 we find that the total hubs flight equal:

$$F_h = \sum_{t=0}^2 A_t^* + D_t^* = 16 \frac{p - N}{12 - 8q + 4qb + q^2 b^2} \quad (\text{A4.4})$$

That has positive derivative with respect to p and q .

In addition to selecting the total number of flights, the hub airlines must also decide how to schedule their flights during the three periods. At the optimum the hub carrier always schedules arrivals in two consecutive periods and clusters all departures in the third period.

The percentage of arrivals scheduled in the first period versus the second depends on the trade-off between the discount associated with two period connections and congestion costs from clustering arrivals in a single period. In particular, the share of total arrivals evaluated at the maximum of A4.3 is equal to:

$$\frac{A_t^*}{A_t^* + A_{t+1}^*} = \frac{1}{2} - \frac{q.b}{4} \quad (\text{A4.5})$$