The Growth-Environment Trade-off: Horizontal vs Vertical Innovations*

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Abstract

This paper explores the trade-off between economic growth and environmental quality along two paradigms of endogenous growth theory: variety expansion (HIP) and quality improvements (VIP). We compare the policies that match the decentralized economies' paths with the optimal "strong sustainable" growth path, characterized by growth in consumption and improvements in environmental quality. Three policy tools are employed: subsidies to monopolists and R&D, and taxes on emissions. The latter is increasing at the optimum, to keep the weight of tax revenues over output constant. All policy tools equal, the growth rate is higher in the VIP than in the HIP. The optimal subsidy to R&D is therefore greater and the cumulative loss in output smaller under HIP than the VIP.

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1 Introduction

The recent environmental policy agenda has focused on the need to control emissions of green-house gases. Prospects for sustained economic growth are challenged by these environmental constraints. Technologies have to be adjusted substantially to make environmental and economic growth targets compatible.

Economic theory has characterized the case of global pollution by the two-fold role of the environment[1]. On the one hand, environmental quality affects utility of the representative agent as a non-rival good. On the other hand, production exploits rival services from the environment (sink for waste and emissions). In this respect it represents a renewable resource, that assimilates waste according to its own laws, not subject to human manipulation. The public nature of the environment justifies public intervention either through regulation of technological standards, or through some pricing mechanism for emissions (taxes or tradeable permits). This paper explores the issue within the two main paradigms of growth theory that endogenize technological change through research and development activities.

Stokey (1998) proposes a useful framework of analysis, where emissions are an input to production. Within this framework, an AK economy cannot develop along a trajectory of sustainable growth. This is because as environmental standards are improved, the marginal product of capital is reduced. Whereas if labor productivity grows at an exogenous rate, the marginal product of capital may be kept high enough to induce continuous investment. In this case, the economy may follow a path of "strong sustainable" growth, that is one where consumption and environmental quality increase simultaneously. Building on Stokey's framework, Aghion and Howitt (1998) endogenize the rate of productivity growth in an aggregate version of the schumpeterian models of growth, to show that with this approach "strong sustainable" growth might be optimal. This model is reviewed in section 2.

In this paper we present two different approaches to decentralize the Aghion-Howitt economy. Section 3 introduces the disaggregated structure of these economies. The first one refers to the theory of endogenous growth through horizontal innovations, introduced by Romer (1990). The second one builds on the theory of endogenous growth through vertical innovations, developed by Grossman and Helpman (1991) and Aghion and Howitt (1992). In sections 4 and 5 we analyze the decentralized versions of the models. Three policy tools are introduced into the economy, in order to correct for the distortions present in these economies: a subsidy to monopolies to correct for

[1]See Smulders (1995) for an excellent survey of the literature.

market power; a subsidy to R&D to match social and private returns to this activity; and a tax on emissions of pollutants to price this input.[2] One virtue of the schumpeterian approach is that it allows us to highlight the channels through which environmental policy affects medium to long term growth. This analysis is undertaken in section 6, where we compare the equilibrium outcomes in the two paradigms of growth, and present the trade-off between economic growth and environmental quality. In section 7 we characterize the optimal policy that matches the decentralized paths of growth with the optimal sustainable path. Conclusions are contained in the last section.

A recent strand of literature has explored environmental policy in endogenous growth models with horizontal innovations. Elbasha and Roe (1996), Hung, Chang and Blackburn (1993), and Verdier (1993) study the case where the flow of emissions affects utility, and intermediary inputs are differentiated according to their intensity in pollution. They all analyze market equilibria and compare them with social optima. Hung, Chang and Blackburn (1993) and Verdier (1993) find that at equilibrium the economy might even growth slower that at the optimum. However none of these works implements explicitly the optimal policy.

2 Optimal Sustainable Growth

Aghion and Howitt (1998) analyze the pollution-production relationship in an aggregate economy along the following lines.

Final output is produced according to :

$$Y_t = z_t \left[D_t(1-n_t)\right]^{1-\alpha} K_t^{\alpha} \tag{1}$$

With $\alpha \in (0,1)$, and K_t, D_t being the stock of capital and the level of technology (i.e. labor productivity); labor, available in a fixed amount (normalized to unity) is shared between the production of the final good, in share $(1-n)$, and research and development (R&D) activities, in share n. Finally z is a technological parameter measuring the "pollution intensity" of output. It is constrained by :

$$z_t \in (0,1] \tag{2}$$

So, when the technology standard is not affected by environmental considerations, then $z = 1$ and production is maximum. However as environmental technological standards are restricted, then $z < 1$ and output is lower than its potential.

[2] In the framework we have adopted to control for pollution with taxes or with tradeable pollution permits is equivalent. See the conclusion for an explanation.

Emissions of pollutants result from production :

$$P_t = z_t^{\gamma} Y_t \tag{3}$$

with $\gamma > 0$. Combining equations (1) and (3), we can write emissions directly as inputs in the production function :

$$Y_t = P_t^{\frac{1}{1+\gamma}} \left([D_t(1-n_t)]^{1-\alpha} K_t^{\alpha} \right)^{\frac{\gamma}{1+\gamma}}$$

characterized by unitary elasticity of substitution between emissions and "usual" output. Constraint (2) implies that there is an upper bound on emissions inputs, beyond which the marginal productivity of emissions falls to zero.

The stock of pollution, S, accumulates according to the law :[3]

$$\dot{S}_t = P_t - \theta S_t \tag{4}$$

where $\theta \in (0,1)$ is the natural rate of assimilation. It is further assumed that the environment has a carrying capacity, i.e. there exists an upper bound on the stock of pollution beyond which, for simplicity, production becomes impossible. Therefore :

$$S_t \in [0, \bar{S}) \tag{5}$$

Finally accumulation of capital and knowledge (labor productivity) proceed according to :

$$\dot{K}_t = Y_t - c_t \tag{6}$$

$$\dot{D}_t = \delta D_t n_t \tag{7}$$

where $\delta > 0$, and c denotes aggregate consumption. Constant returns with respect to D in (7) are necessary for endogenous growth, while constant returns with respect to n are assumed for simplicity but are not necessary.

The utility function of the infinitely lived representative agent depends on the level of consumption and the stock of pollution. It is assumed to be separable and globally concave. The social planner program is:

$$\max \int_0^{\infty} e^{-\rho t} \left(\frac{c_t^{1-\varepsilon}}{1-\varepsilon} - \frac{S_t^{1+\omega}}{1+\omega} \right) dt \tag{8}$$

[3]We acknowledge that this reproduction function is by no means neutral. Farmer (1998) shows that if the environment regenerates according to a logistic function (a more realistic assumption), sustainable growth paths, in a weak sense of stability of the stock of pollution, exist only under very restrictive circumstances.

subject to (1), (2), (3), (4), (5), (6), (7), with $\varepsilon, \omega > 0$. The problem has three control variables (c, z, n), three states (K, D, $-S$) and current value costate variables (ν, μ, ζ, respectively). The necessary and sufficient conditions are presented in appendix I.

Proposition 1 *(Aghion-Howitt, 1998) There exists an optimal sustainable growth path if:*
(a) the discount rate is lower than the potential rate of growth of knowledge:

$$\delta > \rho$$

(b) the intertemporal elasticity of substitution is lower than unity:

$$\frac{1}{\varepsilon} < 1$$

(c) the natural rate of pollution assimilation is high enough:

$$(\varepsilon - 1)(\delta - \rho) < \theta \left[(1+\omega)\varepsilon + \frac{\varepsilon + \omega}{\gamma(1-\alpha)} \right]$$

Along the sustainable balanced growth path consumption grows without bound and pollution is reduces indefinitely. It is characterized by:[4]

$$g_c^o = g_Y^o = g_K^o = (\delta - \rho)\frac{(1+\omega)\gamma(1-\alpha)}{\varepsilon + \omega + (1+\omega)\gamma(1-\alpha)\varepsilon}$$

$$g_D^o = \frac{\varepsilon + \omega + (1+\omega)\gamma(1-\alpha)}{(1+\omega)\gamma(1-\alpha)} g_c^o > g_c^o$$

$$g_z^o = -\frac{\varepsilon + \omega}{(1+\omega)\gamma} g_c^o < 0 \tag{9}$$

$$g_S^o = \frac{1-\varepsilon}{1+\omega} g_c^o < 0$$

$$n^o = \frac{\delta - \rho}{\delta}\left(\frac{\varepsilon + \omega + (1+\omega)\gamma(1-\alpha)}{\varepsilon + \omega + (1+\omega)\gamma(1-\alpha)\varepsilon} \right) \in (0,1) \tag{10}$$

Proof. The proof consists in showing that for any initial level of pollution S_0 there exists a couple of capital and knowledge stocks, (K_0, D_0), such that the economy is initially on this balanced growth path. Then to check that all necessary and sufficient conditions are satisfied. For details see appendix 1 to chapter 5 in Aghion and Howitt (1998). ■

[4] $g_x = \frac{\dot{x}}{x}$ is the instantaneous growth rate. Superscript o stands for "optimum".

Assumption (a) is usual for endogenous growth to be optimal. We will basically restrict the analysis to the case of assumption (b), although it is quite restrictive. Its role is crucial and this will become clear when we will study the decentralized equilibria in section 5. Assumption (c) is more technical, and is necessary to ensure that $P/S = \theta + g_S > 0$ holds.

Few comments are worthwhile. First knowledge is accumulated at a faster rate than physical capital ($g_D^o > g_K^o$), because knowledge accumulation has to counter both decreasing returns to investment (which require at least $g_D = g_K$), and the depressing effect of environmental restrictions ($g_z^o < 0$) on the productivity of capital. Second, the lower the intertemporal elasticity of substitution, the smaller the optimal growth rate, thus the lower the opportunity cost of environmental policy, or equivalently the greater the social return to investment in environmental quality ($-S$). This suggests that the scope for optimality of the sustainable path is enlarged when consumption and environmental quality are complements in the utility function, and *vice versa* is reduced when they are substitutes (see Michel and Rotillon, 1996). Third, along this sustainable growth path the stock of pollution tends asymptotically towards zero. Michel (1993) calls this state the "ecological paradise". Michel (1993) and Xepapadeas (1994) find this result when abatement activities are characterized by increasing returns. In the Aghion-Howitt model abatement expenditure is defined only implicitly, and is characterized by decreasing returns.[5]

3 The Disaggregated Economies

This section introduces two modified models of endogenous growth theory. The first one extends the model economy of Paul Romer (1990), where R&D fosters growth by increasing the variety of intermediate goods, to include pollution inputs. The second builds on the models of Grossman and Helpman (1991) and Aghion and Howitt (1992), where R&D targets directly productivity improvements over a given space of intermediate goods.

The goods of the two economies are the same as in section 2. However the "level of technology", D, is interpreted differently in the two models. It will be shown that at the optimum the two economies are described by the aggregate economy presented in the previous section.

[5]Implicit abatement expenditure is $\Lambda = (1-z)[D(1-n)]^{1-\alpha}K^{\alpha} = (1-z)y$, and pollution $P = (y-\Lambda)^{1+\gamma}\, y^{-\gamma}$. Thus, at any level of potential output, y, we have $(\partial P)/(\partial \Lambda) < 0$ and $(\partial^2 P)/(\partial \Lambda^2) > 0$, as long as $\gamma > 0$. We thank Sjak Smulders for his comments on this point.

3.1 The Horizontal Innovation Paradigm

In the horizontal innovations paradigm (hereafter HIP) the production function of the representative final sector firm is :

$$Y_t = z_t(1 - n_t)^{1-\alpha} \int_0^{B_t} x_{jt}^{\alpha} dj \tag{11}$$

where B is the mass of intermediate goods and represents a proxy for the state of technological knowledge.

R&D employs labor, in quantity n, and non-rival knowledge, B. Its output is the instantaneous flow of blueprints for new goods $\dot{B} = f(n, B)$. It is crucial to assume that $f(.)$ has constant returns with respect to B, because this allows for indefinite growth of B along a balanced growth path where n is constant. We also assume for simplicity (i.e. it's not crucial) and coherence with the previous model that constant returns hold with respect to labor inputs too. Therefore we have :

$$\dot{B}_t = \delta n_t B_t \tag{12}$$

where δ is the index of labor productivity in R&D.

Once a new good is invented, it is produced using capital alone, according to a one-to-one production function, thus $\forall\ j \in [0, B_t]$:

$$x_{jt} = K_{jt}$$

Intermediate goods are therefore services from capital goods. Aggregate capital is given by :

$$K_t = \int_0^{B_t} x_{jt} dj$$

3.2 The Vertical Innovation Paradigm

In the vertical innovation paradigm (hereafter VIP) the production function of the representative final sector firm is :

$$Y_t = z_t(1 - n_t)^{1-\alpha} \int_0^{1} A_{jt} x_{jt}^{\alpha} dj \tag{13}$$

Here A_{jt} is the productivity index for good j at date t and therefore *ex-ante* is a random variable. Let us also define the average productivity index as $A_t = \int_0^1 A_{jt} dj$.

R&D firms specialize on one good j and employ labor in a stochastic production function, governed by a Poisson process with instantaneous arrival parameter λ (equal across sectors). When an innovation is obtained the

highest productivity present on the market, denoted by $\bar{A}_t \equiv \max_j\{A_{jt}\}$, is applied to the new generation of good j. This feature captures the intersectoral spillover within the R&D sector.

The Poisson arrival rate in sector j at date t is assumed to be λn_{jt}, for simplicity and coherence with previous settings. Therefore the instantaneous flow of innovations in the economy as a whole is just λn_t. Moreover we assume an intertemporal spillover, such that the state of knowledge (proxied by $\bar{A}$) grows at a rate proportional to the flow of innovations, according to a spillover coefficient $\kappa > 0$:

$$\frac{\dot{\bar{A}}_t}{\bar{A}_t} = \kappa\lambda n_t \equiv \delta n_t \tag{14}$$

where we have defined $\delta \equiv \kappa\lambda$. Finally to obtain full compatibility between this disaggregated economy and the aggregate economy in section 2, we assume that R&D intensity (i.e. the n_{jt}'s) is uniform across sectors.[6]

The production function of the intermediate good $j \in [0,1]$ is now assumed to be :

$$x_{jt} = \frac{K_{jt}}{A_{jt}}$$

More productive goods are more capital intensive. Aggregate capital is :

$$K_t = \int_0^1 A_{jt} x_{jt} dj$$

3.3 Optimal aggregation

In both models the environment and preferences are modeled by (3), (4), (5), and (8) as in section 2.

Proposition 2 *At steady state these two disaggregated economies are compatible with the aggregate economy considered by Aghion and Howitt. That is, their optimal aggregate production functions coincide with (1), and, for n constant, the laws of motion of productivity indexes (B for the HIP and A for the VIP) are equal to (7). Therefore they share the same optimal balanced growth path.*

Proof. see appendix II. ■

[6]While this is an abrupt assumption for the disaggregated economy, we will see that it is the equilibrium outcome in the decentralized economy.

4 The decentralized economies

It is assumed that, in both economies, there are a competitive final sector, a competitive R&D sector, perfect markets for capital and labor, and monopolized markets for intermediate goods. The price of the final good is normalized to unity, w denotes the wage rate, r the rental rate of capital, and p_j the price of intermediate good j. To cope with the three distortions present in these economies, we introduce three policy tools :

- a tax h levied on the final sector to price pollution emissions inputs;
- a subsidy to capital income, τ, to correct for market power;
- a subsidy to R&D, σ, to match private and public returns to R&D.

4.1 The final sector

The profit function of the representative competitive final sector firm is :

$$\begin{aligned}\Psi_t &= z_t(1-n_t)^{1-\alpha}\int_0^{B_t} x_{jt}^{\alpha}dj - w_t(1-n_t) - \\ &\quad - \int_0^{B_t} p_{jt}x_{jt}dj - h_t z_t^{1+\gamma}(1-n_t)^{1-\alpha}\int_0^{B_t} x_{jt}^{\alpha}dj\end{aligned}$$

for the HIP economy, and :

$$\begin{aligned}\Psi_t &= z_t(1-n_\tau)^{1-\alpha}\int_0^{1} A_{jt}x_{jt}^{\alpha}dj - w_t(1-n_t) - \\ &\quad - \int_0^{1} p_{jt}x_{jt}dj - h_t z_t^{1+\gamma}(1-n_t)^{1-\alpha}\int_0^{1} A_{jt}x_{jt}^{\alpha}dj\end{aligned}$$

for the VIP economy.

Therefore, in both cases, the endogenous choice of z is :

$$z_t = \begin{cases} 1 & \text{if} \quad h_t \leq \frac{1}{(1+\gamma)} \quad \text{(Dirty Regime)} \\ ((1+\gamma)h_t)^{-\frac{1}{\gamma}} & \text{if} \quad h_t > \frac{1}{(1+\gamma)} \quad \text{(Green Regime)} \end{cases} \tag{15}$$

Thus pollution intensity of output is affected by green taxes only if the tax is larger than the elasticity of output with respect to emissions inputs (we call this case the green regime).

Instead the inverse demand functions for labor and intermediate inputs in the HIP case are :

$$w_t = (1-\alpha)z_t(1-h_t z_t^{\gamma})(1-n_t)^{-\alpha}\int_0^{B_t} x_{jt}^{\alpha}dj \tag{16}$$

and, $\forall j \in [0, B_t]$:

$$p_{jt} = \alpha z_t (1 - h_t z_t^{\gamma})(1 - n_t)^{1-\alpha} x_{jt}^{\alpha-1} \tag{17}$$

while in the VIP they are :

$$w_t = (1-\alpha) z_t (1 - h_t z_t^{\gamma})(1 - n_t)^{-\alpha} \int_0^1 A_{jt} x_{jt}^{\alpha} dj \tag{18}$$

and, $\forall j \in [0, 1]$:

$$p_{jt} = A_{jt} \alpha z_t (1 - h_t z_t^{\gamma})(1 - n_t)^{1-\alpha} x_{jt}^{\alpha-1} \tag{19}$$

Notice that green taxes, h, depress demand for all inputs in both models. However, while demand is uniform across sectors in the HIP, it is skewed towards the most productive intermediate good in the VIP.

4.2 The intermediate goods sector

The cost of capital net of subsidy for the monopolist is $(1-\tau)r$. From the production functions of intermediate goods, we know that the instantaneous profits of monopolist j are $\Pi_{jt} = (p_{jt} - (1-\tau_t)r_t)x_{jt}$ in the HIP, and $\Pi_{jt} = (p_{jt} - A_{jt}(1-\tau_t)r_t)x_{jt}$ in the VIP. Profit maximization, taking into account the demand functions (17) and (19), gives equilibrium prices, sales, and profits. In the HIP they are given by :

$$\hat{p}_{j,t} = \hat{p}_t = \frac{(1-\tau_t)r_t}{\alpha}$$

$$x_{jt} = \hat{x}_t = \left(\frac{\alpha^2 z_t (1 - h_t z_t^{\gamma})}{(1-\tau_t)r_t} \right)^{\frac{1}{1-\alpha}} (1 - n_t) \tag{20}$$

$$\Pi_{jt} = \Pi_t = \frac{1-\alpha}{\alpha} (1-\tau_t) r_t \hat{x}_t \tag{21}$$

and in the VIP case, by :

$$\hat{p}_{j,t} = A_{jt} \frac{(1-\tau_t)r_t}{\alpha}$$

$$x_{jt} = \hat{x}_t = \left(\frac{\alpha^2 z_t (1 - h_t z_t^{\gamma})}{(1-\tau_t)r_t} \right)^{\frac{1}{1-\alpha}} (1 - n_t) \tag{22}$$

$$\Pi_{jt} = A_{jt} \frac{1-\alpha}{\alpha} (1-\tau_t) r_t \hat{x}_t \tag{23}$$

While quantities sold are equal for all sectors in both models, profits are proportional to the productivity index in the VIP, and they are uniform across sectors in the HIP.

4.3 The R&D sector

Let us now turn to the R&D sector. In the HIP, $1/\delta B_t$ units of labor are required to produce one marginal blueprint, since knowledge accumulation reduces the labor intensity of R&D. Labor cost in the R&D sector equals the equilibrium wage net of a government financed subsidy, that is $(1-\sigma_t)w_t$. Since the R&D sector is competitive, the equilibrium level of activity equates the cost and the value of a marginal innovation. The value of innovation is the present value of the stream of profits accruing to the patent holder, V_t. Then the R&D arbitrage condition is :

$$n_t \in (0,1) \qquad \Rightarrow \qquad \frac{(1-\sigma_t)w_t}{\delta B_t} = V_t \tag{24}$$

where :

$$V_t = \int_t^\infty e^{-\int_t^s r_u du} \Pi_s ds \tag{25}$$

In the VIP, if research takes place at all, then its equilibrium level is such that the marginal cost of labor in R&D (the wage net of subsidy) equals the marginal expected return, that is the Poisson arrival parameter times the value of an innovation, V_t. This is summarized by the following arbitrage condition :

$$n_t \in (0,1) \qquad \Rightarrow \qquad (1-\sigma_t)w_t = \lambda V_t \tag{26}$$

where now we have :

$$V_t = \int_t^\infty e^{-\int_t^s r_u du} e^{-\lambda \int_t^s n_u du} \Pi_s(\bar{A}_t) ds \tag{27}$$

with $\Pi_s(\bar{A}_t) = \bar{A}_t \frac{1-\alpha}{\alpha}(1-\tau_s) r_s \hat{x}_s$. The stream of profits is discounted by the rate of return on alternative placements but also by the probability of survival, because the arrival of an innovation on the same sector makes the incumbent patent obsolete and its value nil. Profits of an innovator depend on the productivity applied to the innovation, and this is $\bar{A}_t$ independently of the sector j where the innovation is implemented. Since also the cost of labor is common to all sectors, the equilibrium level of R&D will be uniform across sectors.

4.4 Consumers and the government

Consumers maximize their discounted flow of utility, given by (8), subject to their intertemporal budget constraint. The latter can be summarized by the instantaneous law of accumulation of assets, W :

$$\dot{W}_t = w_t + r_t W_t - c_t - T_t$$

(where T are lump-sum taxes) together with the no-Ponzi game condition :

$$\lim_{t\to\infty} e^{-\int_0^t r_s ds} W_t = 0$$

Since pollution is a non-rival "bad" and it is assumed that consumers have no direct instruments to control it, the representative consumer problem is solved by the usual Ramsey rule :

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\varepsilon} \tag{28}$$

Finally the government must respect its own intertemporal budget constraint. To be simple, but without loss of generality, we assume that the government keeps the budget balanced through lump-sum taxes (or transfers) to households, T. Then the government budget constraint is represented by the equality between tax revenue and subsidy expenditure :

$$T_t + h_t P_t = \tau_t r_t K_t + \sigma_t w_t n_t \tag{29}$$

5 Green Regime Balanced Growth

Along a balanced growth path all variables grow at constant rates and r, n, τ and σ are constant.

5.1 The Horizontal Innovations Paradigm

The value of an innovation arrived at date t can be computed using (20) and (21) to substitute for Π in the integral of (25), and is :

$$V_t = \frac{\frac{1-\alpha}{\alpha}\left(\alpha^2\Gamma\right)^{\frac{1}{1-\alpha}}\left[(1-\tau)r\right]^{\frac{-\alpha}{1-\alpha}} h_t^{\frac{-1}{\gamma(1-\alpha)}}(1-n)}{r + \frac{1}{\gamma(1-\alpha)} g_h} \tag{30}$$

where $\Gamma = \gamma\left(1+\gamma\right)^{\frac{-(1+\gamma)}{\gamma}} < 1$ is obtained by substituting z from condition (15) under the green regime. The numerator is the initial (at date t) instantaneous profit, and is decreasing in the level of the green tax and the net cost of capital. The denominator includes the discount rate r, and the rate of growth of green taxes, because these crowd out profits by depressing demand for intermediate goods. We call this effect the green crowding-out.

For a balanced growth path to exist it is necessary that :

$$g_Y = g_B - \frac{g_h}{\gamma(1-\alpha)} \tag{31}$$

This is the dynamic version of the R&D arbitrage condition, which must hold for n to be constant[7]. It states that whenever the green tax is increasing, knowledge accumulation must proceed faster than capital accumulation (i.e. same rate as output growth along a BGP). This is because $g_h > 0$ implies by (15) a reduction in z, and thus a faster decline in the productivity of capital than otherwise. As a result in order to keep the marginal product of capital constant, knowledge accumulation must compensate for both decreasing returns to capital and the adoption of increasingly cleaner but less productive techniques.

To determine the equilibrium interest rate in the green regime we have to simplify the R&D arbitrage condition. Write initial profits as function of initial sales, x_t, then substitute labor demand from the final sector with (16) and z by (15), finally substitute for $\hat{x}$ from (20) and simplify the R&D arbitrage condition to obtain :

$$r + \frac{g_h}{\gamma(1-\alpha)} = \delta\alpha\frac{(1-n)}{(1-\sigma)}$$

then substitute for n using (12), (28) and (31): $n = g_B/\delta = (r-\rho)/\delta\varepsilon + g_h/\delta\gamma(1-\alpha)$. Gather all terms in r to obtain :

$$r^e = \frac{\varepsilon}{\varepsilon(1-\sigma)+\alpha}\left[\alpha\delta + \frac{\alpha}{\varepsilon}\rho - [\alpha + (1-\sigma)]\frac{g_h}{\gamma(1-\alpha)}\right] \tag{32}$$

The first term in the denominator is generated by the discount rate, the second by the initial level of profits. In brackets on the numerator we have the first three terms which reflect initial profits, and the last term generated by the green crowding-out effect.

The equilibrium level of R&D employment can be directly derived using again (12), (28) and (31), and is :

$$n^e = \frac{1}{\varepsilon(1-\sigma)+\alpha}\left[\alpha - (1-\sigma)\frac{\rho}{\delta} + (\varepsilon-1)(1-\sigma)\frac{g_h/\delta}{\gamma(1-\alpha)}\right] \tag{33}$$

Notice that for $\varepsilon > 1$ the equilibrium level of R&D is increasing in the rate of growth of green taxes. To interpret this result remember that in the overlapping generations model of growth when $\varepsilon > 1$ a reduction in the rate of interest increases the saving rate because the income effect dominates the

[7]Since σ is constant, the left-hand side of the arbitrage condition (24) grows with wages and decreases with B. Wages in turn are proportional to output, since $w_t = (1-\alpha)Y_t/(1-n)$ and n is constant along a BGP. Then the right-hand side is decreased by the green crowding-out effect, as results from equation (30).

substitution effect. In our model, n represents a saving rate too, because it increases future output through knowledge accumulation by foregoing present consumption. Then an increasing path of green taxes here reduces the equilibrium return to savings from (32) and therefore fosters investment when the intertemporal elasticity of substitution is smaller than unity ($\varepsilon > 1$).

Proposition 3 *When $\sigma = g_h = 0$, an admissible solution exists if :*

$$\rho/\delta < \alpha$$

(this implies $n^e \in (0,1)$ and $r^e > 0$).
Environmental policies compatible with the existence of solutions with positive growth are restricted. The most restrictive constraint applies when $\sigma = 0$, and is :

$$g_h < \bar{g}_h^{HIP} \equiv \frac{\gamma(1-\alpha)}{1+\alpha}(\alpha\delta - \rho)$$

where $\bar{g}_h^{HIP} > 0 \Leftrightarrow \rho/\delta < \alpha$.

Proof. From equations (32) and (33), it is immediate that $r^e > 0$ and $n^e < 1$ when $\sigma = g_h = 0$. In this case $n^e > 0$ iff $\rho/\delta < \alpha$.
For $g_h > 0$ and $\sigma > 0$, we have than $g_Y^{HIP} > 0 \Leftrightarrow r^e > \rho$, implies $g_h < \bar{g}_h = \frac{\gamma(1-\alpha)}{\alpha+(1-\sigma)}[\alpha\delta - (1-\sigma)\rho]$. This upper bound is increasing in σ if $\sigma < 1$, and is the most restrictive at $\sigma = 0$, which is given by $\bar{g}_h^{HIP}$.
Then $n^e > 0$, implies $g_h > \frac{\gamma(1-\alpha)}{\varepsilon-1}[\rho - \alpha\delta/(1-\sigma)]$ if $\varepsilon > 1$. Notice however that this lower bound is negative $\forall\sigma \in [0,1)$ if $\rho/\delta < \alpha$. Instead if $\varepsilon < 1$, the condition is $g_h < \hat{g}_h = \frac{\gamma(1-\alpha)}{1-\varepsilon}[\alpha\delta/(1-\sigma) - \rho]$. However, $\bar{g}_h < \hat{g}_h \ \forall\sigma \in [0,1)$.
Finally, $n^e < 1$, implies $g_h < \tilde{g}_h = \frac{\gamma(1-\alpha)}{\varepsilon-1}[\rho + \varepsilon\delta]$, but $\bar{g}_h < \tilde{g}_h \ \forall\sigma \in [0,1)$. Instead if $\varepsilon < 1$, $n^e < 1$ is trivially satisfied. ■

Remark: it is worth mentioning that, if $\varepsilon > 1$ and $\alpha < \rho/\delta < \alpha\epsilon$, there exists an environmental policy leading to negative growth with positive levels of R&D activity. Indeed a policy satisfying :

$$\frac{\rho - \alpha\delta}{\varepsilon - 1} < \frac{g_h}{\gamma(1-\alpha)} < \frac{\alpha}{1+\alpha}(\delta + \rho/\varepsilon)$$

implies $n^e \in (0,1)$ and $0 < r^e < \rho$.

5.2 The Vertical Innovations Paradigm

Along a balanced growth path the cross sectoral distribution of relative (to the leading edge) productivity parameters is constant. This is an important

property since it allows us to compute average and aggregate values of variables by switching the space of integration from that of nominal sectors (j is the name of a sector) to that of relative productivity (see appendix II for details). We can evaluate the average productivity A_t and show that it is proportional to the leading edge productivity :

$$A_t = \frac{\bar{A}_t}{1+\kappa}$$

so that $g_A = g_{\bar{A}}$.

Since equilibrium production of intermediate goods, $\hat{x}$, is uniform across sectors, using the definition of aggregate capital we can write :

$$\hat{x}_t = \frac{K_t}{A_t}$$

Then aggregate output can be computed as:

$$Y_t = z_t[A_t(1-n_t)]^{1-\alpha}K_t^{\alpha}$$

Using equations (22) and (23) to substitute $\Pi_s(\bar{A}_t)$ into (27), we obtain :

$$V_t = \frac{\bar{A}_t \frac{1-\alpha}{\alpha} (\alpha^2\Gamma)^{\frac{1}{1-\alpha}} [(1-\tau)r]^{\frac{-\alpha}{1-\alpha}} h_t^{\frac{-1}{\gamma(1-\alpha)}}(1-n)}{r+\lambda n + \frac{1}{\gamma(1-\alpha)}g_h} \tag{34}$$

The numerator is the initial instantaneous profit of an innovator, and is increasing in the level of general knowledge and decreasing in the level of green taxes and the net cost of capital. The denominator shows as before the discount and the green crowding-out rates, plus a new effect called creative destruction, specific to the vertical innovation paradigm.

Along a balanced growth path for the dynamic version of the R&D arbitrage condition to hold it is necessary that :

$$g_Y = g_{\bar{A}} - \frac{g_h}{\gamma(1-\alpha)}$$

since wages grow at the same pace of output. Notice that, although the arbitrage conditions differ in the two models, this dynamic constraint for balanced growth is equivalent to condition (31) for the HIP.

The equilibrium interest rate is determined using the same procedure as before, and is[8] :

$$r^e = \frac{\varepsilon}{\varepsilon(1-\sigma)+\alpha\frac{1+\kappa}{\kappa}+(1-\sigma)/\kappa}\left[\left(\alpha\delta+\frac{\alpha}{\varepsilon}\rho\right)\frac{1+\kappa}{\kappa}- \right. \tag{35}$$

[8]Here we also make use of $\lambda/\delta = 1/\kappa$ and $\bar{A}/A = 1+\kappa$.

$$-\left(\alpha\frac{1+\kappa}{\kappa}+(1-\sigma)\right)\frac{g_h}{\gamma(1-\alpha)}+\frac{(1-\sigma)}{\kappa}\left(\frac{\rho}{\varepsilon}-\frac{g_h}{\gamma(1-\alpha)}\right)\Big]$$

There are two kinds of differences with respect to the HIP. First, all terms derived from the initial level of profits (i.e. the second term in the denominator and the first three terms in brackets) are now compounded by the factor $(1+\kappa)/\kappa > 1$. This is because while in the HIP profits are uniform across sectors, in this setting demand is skewed towards the most productive good and therefore the initial profit of innovators is greater. Second, three new terms generated by the creative destruction effect appear: the third term in the denominator and the last two terms in brackets. Notice that a positive g_h now lowers the equilibrium interest rate through this additional channel. The equilibrium level of R&D activity can be derived as in the HIP case, and is :

$$n^e=\frac{\left[\alpha\frac{1+\kappa}{\kappa}-(1-\sigma)\frac{\rho}{\delta}+(\varepsilon-1)(1-\sigma)\frac{g_h/\delta}{\gamma(1-\alpha)}\right]}{\varepsilon(1-\sigma)+\alpha\frac{1+\kappa}{\kappa}+(1-\sigma)/\kappa} \tag{36}$$

The only changes with respect to the HIP here consist of the stronger importance of the level of initial rents to innovators (both on the numerator and the denominator) and the creative destruction effect appearing as the third term in the denominator, which lowers *ceteris paribus* the equilibrium level of R&D activity.

Proposition 4 *When $\sigma = g_h = 0$, an admissible solution exists if :*

$$\rho/\delta < \alpha\frac{1+\kappa}{\kappa}$$

(this implies $n^e \in (0,1)$ and $r^e > 0$).
Environmental policies compatible with the existence of solutions with positive growth are restricted. The most restrictive constraint applies when $\sigma = 0$, and is :

$$g_h < \bar{g}_h^{VIP} \equiv \frac{\gamma(1-\alpha)}{1+\alpha}(\alpha\delta-\frac{\kappa}{1+\kappa}\rho)$$

where $\bar{g}_h^{VIP} > 0 \Leftrightarrow \rho/\delta < \alpha\frac{1+\kappa}{\kappa}$.

Proof. From equations (35) and (36), it is immediate that $r^e > 0$ and $n^e < 1$ when $\sigma = g_h = 0$. In this case $n^e > 0$ iff $\rho/\delta < \frac{1+\kappa}{\kappa}\alpha$.
For $g_h > 0$ and $\sigma > 0$, we have than $g_Y^{VIP} > 0 \Leftrightarrow r^e > \rho$, implies $g_h < \bar{g}_h = \frac{\gamma(1-\alpha)}{\alpha+(1-\sigma)}[\alpha\delta - \frac{\kappa}{1+\kappa}(1-\sigma)\rho]$. This upper bound is increasing in σ if $\sigma < 1$, and is the most restrictive at $\sigma = 0$, which is given by $\bar{g}_h^{VIP}$.
Then $n^e > 0$, implies $g_h > \frac{\gamma(1-\alpha)}{\varepsilon-1}\left[\rho - \frac{1+\kappa}{\kappa}\frac{\alpha\delta}{(1-\sigma)}\right]$ if $\varepsilon > 1$. But this lower bound

is negative $\forall\sigma \in [0,1)$ if $\rho/\delta < \frac{1+\kappa}{\kappa}\alpha$. Instead if $\varepsilon < 1$, the condition is $g_h < \hat{g}_h = \frac{\gamma(1-\alpha)}{1-\varepsilon}\left[\frac{1+\kappa}{\kappa}\frac{\alpha\delta}{(1-\sigma)} - \rho\right]$. However, $\bar{g}_h < \hat{g}_h \ \forall\sigma \in [0,1)$.
Finally, $n^e < 1$, implies $g_h < \tilde{g}_h = \frac{\gamma(1-\alpha)}{\varepsilon-1}[\rho + \frac{1+\varepsilon\kappa}{\kappa}\delta]$, but $\bar{g}_h < \tilde{g}_h \ \forall\sigma \in [0,1)$. Instead if $\varepsilon < 1$, $n^e < 1$ is trivially satisfied. ■

As in the HIP there can be a configuration of parameters such that environmental policy can lead to negative growth, although the economy is engaged in R&D activity.

It is peculiar of this paradigm that growth may be positive at equilibrium, even when this is not optimal. Indeed, the condition for equilibrium growth, $\rho/\delta < \alpha\frac{1+\kappa}{\kappa}$ might be satisfied even when the one for optimality is not, $\rho/\delta > 1$. To understand this possibility consider the limit case of a spillover coefficient, κ, close to zero. Then R&D would be ineffective in increasing productivity. Yet the decentralized agents would still engage in R&D activity to "steal" monopoly rents (if $\lambda > 0$). Socially, this would be a wasteful allocation of resources.

6 Growth versus Environment : Comparisons

To summarize we restate that the two economies evolve along a green BGP described by :

$$g_Y^e = g_K^e = g_c^e = \frac{r^e - \rho}{\varepsilon}$$

$$g_Y^e = \delta n^e - \frac{g_h}{\gamma(1-\alpha)}$$

$$g_{\hat{x}}^e = \frac{-g_h}{\gamma(1-\alpha)}$$

$$g_z^e = -\frac{g_h}{\gamma}$$

$$g_S^e = g_P^e = g_Y^e - g_h$$

with the equilibrium return to savings, r^e, and R&D activity, n^e, given by (32) and (33) for the HIP economy, and by (35) and (36) for the VIP. Therefore :

$$g_Y^{HIP} = \frac{1}{\alpha + (1-\sigma)\varepsilon}\left[\alpha\delta - (1-\sigma)\rho - \frac{[\alpha + (1-\sigma)]}{\gamma(1-\alpha)}g_h\right] \tag{37}$$

$$g_S^{HIP} = \frac{\alpha\delta - (1-\sigma)\rho}{\alpha + (1-\sigma)\varepsilon} - \left[1 + \frac{1}{\gamma(1-\alpha)}\frac{[\alpha + (1-\sigma)]}{[\alpha + (1-\sigma)\varepsilon]}\right]g_h \tag{38}$$

$$g_Y^{VIP} = \frac{1}{\alpha + (1-\sigma)\frac{1+\varepsilon\kappa}{1+\kappa}} \left[\alpha\delta - \frac{\kappa}{1+\kappa}(1-\sigma)\rho - \frac{[\alpha + (1-\sigma)]}{\gamma(1-\alpha)} g_h \right] \tag{39}$$

$$g_S^{VIP} = \frac{\alpha\delta - \frac{\kappa}{1+\kappa}(1-\sigma)\rho}{\alpha + (1-\sigma)\frac{1+\varepsilon\kappa}{1+\kappa}} - \left[1 + \frac{1}{\gamma(1-\alpha)} \frac{[\alpha + (1-\sigma)]}{[\alpha + (1-\sigma)\frac{1+\varepsilon\kappa}{1+\kappa}]} \right] g_h \tag{40}$$

6.1 Channels of transmission of environmental policy

¿From equations (37) to (40) it appears that the rate of growth of green taxes depresses the equilibrium growth rate of output, and reduces the rate of accumulation of the pollution stock. The level of the green tax rate, h, instead has an impact on the level of aggregate output and emissions, but not on their rates of growth. The analysis of the decentralized economies allows us to understand through which channels g_h affects equilibrium growth. Let us therefore have a closer look.

Along the green regime balanced growth path, g_h induces the final sector to reduce progressively the emissions intensity of output (see (15)). This depresses output growth directly because one of the inputs (emissions) is reduced. However, it also implies that the productivity of all other inputs are reduced. Thus demands for labor and for intermediate inputs from the final sector decrease with g_h (see (16) to (19)). Lower demand for intermediate goods depresses monopolists' profits through the green crowding-out effect (see (20) to (23)) and *ceteris paribus* decreases incentives to engage in research and development activities.

So far for the direct consequences, let us now turn to general equilibrium effects. First, lower demand for labor from the final sector tends to lower the equilibrium wage, and reduce through this channel the cost of R&D. Also the depressive effect of g_h on the equilibrium interest rate tends to increase the present value of the stream of profits accruing to patent holders. These two indirect channels increase incentives to R&D activity. It turns out that for $\varepsilon > 1$ these last general equilibrium effects dominate, and equilibrium R&D investment increases. The opposite is true if $\varepsilon < 1$. The only difference between the two models, concerning the channels of transmission, runs through this general equilibrium outcome. Indeed only in the VIP, the impact of g_h on n^e, further affects incentives to R&D through the creative destruction channel. However, even if R&D increases in equilibrium the rate of growth of output decreases. Therefore, the greater the rate of growth of green taxes, the higher the cumulative loss of output.

6.2 Comparing the growth-environment trade-offs

We have seen that the incentives to engage in R&D in the two economies differ in two main respects. In the VIP with respect to the HIP, the initial profit of innovators is greater because demand is skewed towards most recent goods. Yet the expected lifetime of monopoly power is shorter. These two features act in opposite directions, and it is not clear *a priori* which one dominates. We have the following result.

Proposition 5 *All policy tools equal, growth is stronger under the VIP than the HIP, at least until it is positive. That is,* $\forall \tau, \sigma, h$ *equal in the two economies and* $\forall g_h$ *such that* $g_Y > 0$ *:*

$$g_Y^{VIP} > g_Y^{HIP}$$

Furthermore the parameters restriction for the existence of a solution is less strict in the VIP than in then HIP.

Proof. We first show that at $g_h = 0$, $g_Y^{VIP} > g_Y^{HIP}$ if $r^e > g_Y^e$, which is always the case in equilibrium. Then we show that the g_h such that $g_Y = 0$ is higher in the VIP than in the HIP.
For $g_h = 0$, using equations (37) and (39), we find that $g_Y^{VIP} > g_Y^{HIP}$ iff :

$$\varepsilon > 1 - \left[1 + \frac{1-\sigma}{\alpha}\right] \frac{\rho}{\delta}$$

Comparing the equilibrium interest rates to the growth rates, this condition implies $r^e > g_Y^e$. But this is necessary for the no-Ponzi game condition, which we have imposed on households, to hold since assets, W, grow at the same rate of income along a BGP[9].
To prove the second part of the proposition we generalize results obtained in propositions 3 and 4 to the case $\sigma \in (0, 1)$. Define $\bar{g}_h$ such that $g_Y = 0$. Then we have :

$$\bar{g}_h^{VIP} = \frac{\gamma(1-\alpha)}{\alpha + (1-\sigma)}[\alpha\delta - \frac{\kappa}{1+\kappa}(1-\sigma)\rho] > \frac{\gamma(1-\alpha)}{\alpha + (1-\sigma)}[\alpha\delta - (1-\sigma)\rho] = \bar{g}_h^{HIP}$$

The last part of the proposition follows directly from propositions 3 and 4, indeed for $\alpha < \rho/\delta < \alpha\frac{1+\kappa}{\kappa}$, equilibrium growth might result in the VIP while it cannot in the HIP. ■

This means that the effect due to higher initial profits dominates the one generated by the shorter lifetime of monopolies, and incentives to engage in

[9]The result can also be derived from the transversality condition of the representative consumer's maximization problem.

R&D are greater in the vertical innovation economy than in the horizontal innovations one.

The following thought experiment might improve the understanding of the result. The property holds because at equilibrium the rate of interest is greater than the growth rate. Consider an exogenous increase in ε holding n (hence g_Y) constant. This is equivalent to an increase in r, since the spread is $r - g_Y = \rho + (\varepsilon - 1)g_Y$. What are the consequences on the R&D arbitrage conditions? The reduction in wages and average profits is the same in the two models. Then the only difference between the two arbitrage conditions, runs through the impact on the denominator of the value of innovations (equations (30) and (34)). The discount factor is more sensitive to changes in r in the HIP case than in the VIP. Thus incentives to R&D are affected relatively more in the HIP than in the VIP.

Proposition 6 *The equilibrium growth rates of output and stock pollution react more strongly to the growth rate of green taxes in the VIP than in the HIP if $\varepsilon > 1$, and vice versa :*

$$\begin{array}{rcccccc} \varepsilon > 1 & \Longleftrightarrow & \left(\frac{\partial g_Y}{\partial g_h}\right)^{VIP} & < & \left(\frac{\partial g_Y}{\partial g_h}\right)^{HIP} & < & 0 \\ & \textit{and} & \left(\frac{\partial g_S}{\partial g_h}\right)^{VIP} & < & \left(\frac{\partial g_S}{\partial g_h}\right)^{HIP} & < & 0 \\ \varepsilon < 1 & \Longleftrightarrow & \left(\frac{\partial g_Y}{\partial g_h}\right)^{HIP} & < & \left(\frac{\partial g_Y}{\partial g_h}\right)^{VIP} & < & 0 \\ & \textit{and} & \left(\frac{\partial g_S}{\partial g_h}\right)^{HIP} & < & \left(\frac{\partial g_S}{\partial g_h}\right)^{VIP} & < & 0 \end{array}$$

Furthermore the equilibrium level of R&D is more sensitive to the rate of growth of green taxes in the HIP than in the VIP :

$$\begin{array}{rcccccc} \varepsilon > 1 & \Longleftrightarrow & 0 & < & \left(\frac{\partial n^e}{\partial g_h}\right)^{VIP} & < & \left(\frac{\partial n^e}{\partial g_h}\right)^{HIP} \\ \varepsilon < 1 & \Longleftrightarrow & \left(\frac{\partial n^e}{\partial g_h}\right)^{HIP} & < & \left(\frac{\partial n^e}{\partial g_h}\right)^{VIP} & < & 0 \end{array}$$

Proof. The first result is immediate since $\varepsilon > 1 \Leftrightarrow \varepsilon > \frac{1+\varepsilon\kappa}{1+\kappa}$ and vice versa. Then, taking derivatives of (36) with respect to g_h :

$$\left(\frac{\partial n^e}{\partial g_h}\right)^{VIP} = \frac{\frac{(\varepsilon-1)(1-\sigma)}{\delta\gamma(1-\alpha)}}{\varepsilon(1-\sigma) + \alpha\frac{1+\kappa}{\kappa} + (1-\sigma)/\kappa}$$

and from (33) :

$$\left(\frac{\partial n^e}{\partial g_h}\right)^{HIP} = \frac{\frac{(\varepsilon-1)(1-\sigma)}{\delta\gamma(1-\alpha)}}{\varepsilon(1-\sigma) + \alpha}$$

The first expression is always smaller than the second in absolute value. The sign of change is positive if $\varepsilon > 1$ and negative if $\varepsilon < 1$. ■

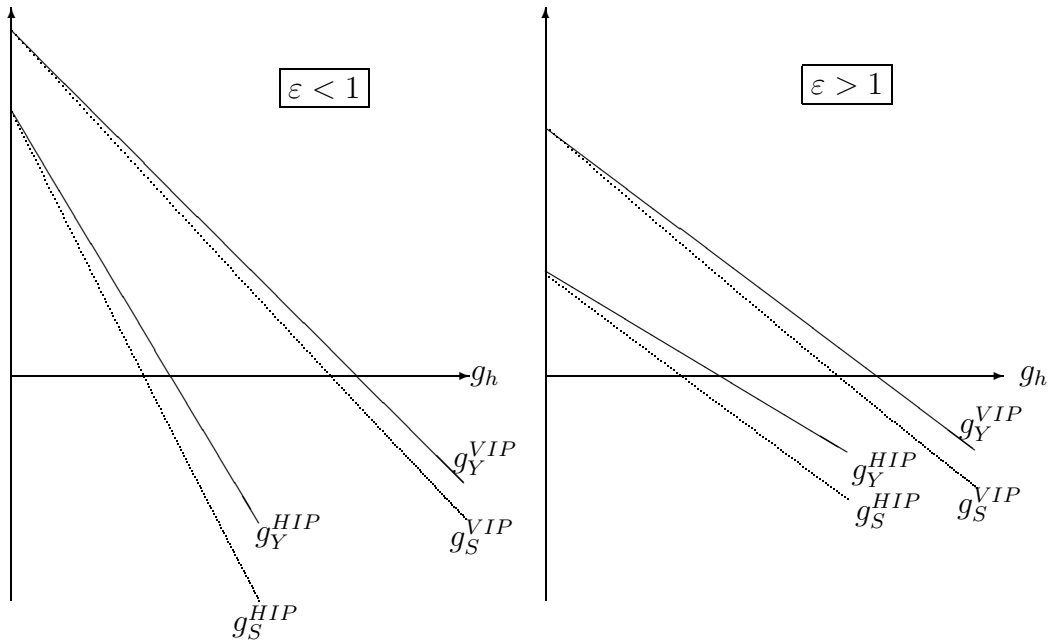

Figure 1 : Equilibrium configurations.

Proposition 7 *The rate of growth of green taxes beyond which environmental quality improves is greater in the VIP than in the HIP. That is, defining* $\hat{g}_h$ *such that* $g_S^e = 0$*, we have :*

$$\hat{g}_h^{VIP} > \hat{g}_h^{HIP}$$

Proof. Define $\hat{g}_h$ such that $g_S^e = 0$. From (40) we have that :

$$\hat{g}_h^{VIP} = \frac{\gamma(1-\alpha)[\alpha\delta - \frac{\kappa}{1+\kappa}(1-\sigma)\rho]}{[1+\gamma(1-\alpha)]\alpha + [1+\gamma(1-\alpha)\frac{1+\varepsilon\kappa}{1+\kappa}](1-\sigma)}$$

and from (38) :

$$\hat{g}_h^{HIP} = \frac{\gamma(1-\alpha)[\alpha\delta - (1-\sigma)\rho]}{[1+\gamma(1-\alpha)]\alpha + [1+\gamma(1-\alpha)\varepsilon](1-\sigma)}$$

then with some algebra, we obtain that $\hat{g}_h^{VIP} > \hat{g}_h^{HIP}$ implies :

$$\varepsilon > 1 - \left[1 + \frac{(1-\sigma)}{\alpha}\right] \frac{[1+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \frac{\rho}{\delta}$$

but this lower bound is always smaller than the one ensuring $r^e > g_Y^e$, which we have seen is true at equilibrium (see proof of proposition 5). ■

The last two results imply that when $\varepsilon > 1$, the percentage loss in the growth rate of output necessary to stabilize the stock of pollution, is larger in the VIP than in the HIP[10]. The opposite is true for $\varepsilon < 1$.

The results are summarized in figure 1. Functions (37) to (40) can be drawn on the (g_h, g_Y) and (g_h, g_S) planes. As shown the g_Y^{VIP} schedule lies everywhere above g_Y^{HIP}. Notice that both schedules have steeper slopes in the VIP case when $\varepsilon > 1$, and vice versa.

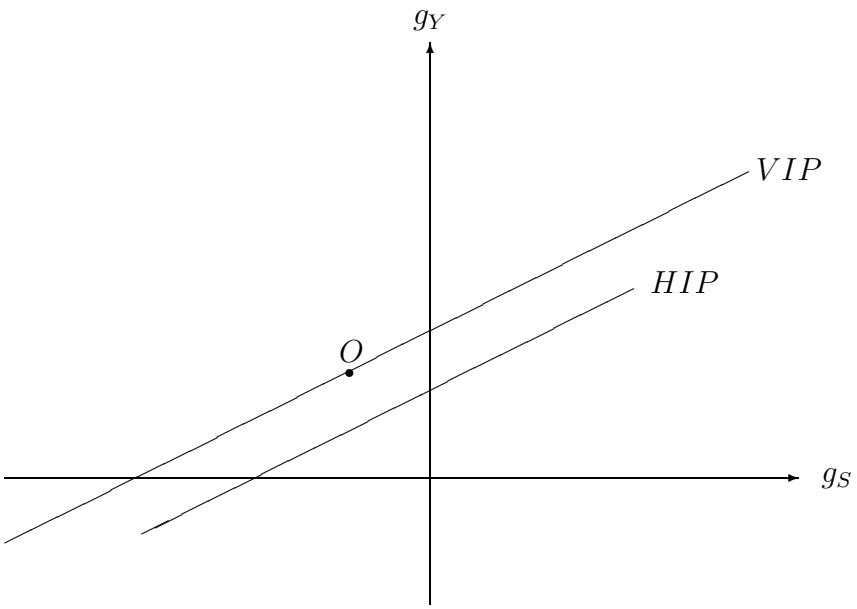

Figure 2 : The growth-environment trade-off.

Figure 2 shows the locus of balanced growth paths in the (g_S, g_Y) space for the two paradigms. It is upwards slopping, and this highlights the trade-off between economic growth and environmental protection. For any chosen level of pollution growth, equilibrium output growth is greater in the VIP than in the HIP. Of course, the carrying capacity constraint (5) rules out the region $g_h > 0$ as unsustainable. Finally the loci shift upwards as R&D subsidies are increased.

7 Optimal Policy

We have three instruments to correct for three distortions.

[10]The percentage loss in the growth rate of output is $(\bar{g}_Y - \hat{g}_Y)/\bar{g}_Y$, where $\bar{g}_Y$ is such that $g_S = \bar{g}_Y$, and $\hat{g}_Y$ is such that $g_S = 0$.

First the capital income subsidy is used to correct for market power in the intermediate goods sector. To set the level of capital (or output of the intermediate sector) at its socially optimal level, the market price has to match the social marginal cost. The common element of marginal cost is r which will be targeted using the R&D subsidy. Therefore we only have to impose $\hat{p} = (1-\tau)r/\alpha = r$ in the HIP and $\hat{p}_j = (1-\tau)A_j r/\alpha = A_j r$ in the VIP, which imply $\tau^*_{HIP} = \tau^*_{VIP} = 1-\alpha$.

The task of the green tax is to tackle the optimal level and growth rate of the technological parameter z. As to the optimal growth rate of the green tax we have the following result.

Proposition 8 *If $\varepsilon > 1$, the optimal tax on emissions is increasing at a constant rate :*

$$g^*_h = \frac{(\varepsilon+\omega)\gamma(1-\alpha)}{\varepsilon+\omega+(1+\omega)\gamma(1-\alpha)}\delta n^o \tag{41}$$

where n^o is given by (10).

Proof. We have seen that the decentralized choice of z is such that $g^e_z = -g_h/\gamma$ in the green regime. The optimal rate of reduction of z is $g^o_z = g^o_Y(\varepsilon+\omega)/\gamma(1+\omega)$ by (9). For the same relationship to hold in the decentralized economy the growth rate of green taxes should be $g_h = g^e_Y(\varepsilon+\omega)/(1+\omega)$. Substituting (31) for g^e_Y, and using (12) (or (14)) we obtain (41). ■

As to the matter of the optimal level of green taxes, we know that at date $\hat{t}$ when z is optimally set to unity $h_{\hat{t}} = 1/(1+\gamma)$. Date $\hat{t}$ is defined by the ratio of the shadow prices of environmental quality and capital $\zeta_{\hat{t}}/\nu_{\hat{t}} = 1/(1+\gamma)$ (see appendix I). The shadow prices in turn depend on the levels of the stock variables.

Let us now turn to the rate of subsidy to R&D.

Proposition 9 *If $\varepsilon > 1$, the optimal subsidy to R&D is greater under the HIP than under VIP. That is $\sigma^*_{HIP} > \sigma^*_{VIP}$.*

Proof. Consider first the HIP. Substituting (23) into (25) to evaluate the price of patents, and replacing the result into the arbitrage condition (24) together with (16) for w, (15) for z, and (20) for $\hat{x}$, we obtain :

$$(1-\sigma)\left[r + \frac{1}{\gamma(1-\alpha)}g_h\right] = \delta\alpha(1-n)$$

Using (28) and (31), substitute for $r = \rho + \varepsilon g^e_Y = \rho + \varepsilon g_B - \varepsilon g_h/\gamma(1-\alpha)$ to get :

$$(1-\sigma)\left[\rho + \varepsilon\delta n + (1-\varepsilon)\frac{g_h}{\gamma(1-\alpha)}\right] = \delta\alpha(1-n)$$

Then substitute g_h by its optimal value (41) :

$$(1-\sigma)\left[\rho+\frac{\varepsilon+\omega+(1+\omega)\gamma(1-\alpha)\varepsilon}{\varepsilon+\omega+(1+\omega)\gamma(1-\alpha)}\delta n\right]=\delta\alpha(1-n)$$

Finally, if σ is optimal then n is at its optimal level, given by (10). In this case the expression reduces to :

$$(1-\sigma^*)^{HIP}=\alpha(1-n^o)$$

Applying the same procedure to the VIP, we obtain:

$$(1-\sigma^*)^{VIP}=\alpha\frac{1+\kappa}{\kappa}\frac{(1-n^o)}{(1+n^o/\kappa)} \tag{42}$$

There two kinds of differences. First the factor $(1+\kappa)/\kappa$ due to the greater initial level of profits under VIP than HIP. Second the denominator which results from the creative destruction effect and tends to increase the optimal subsidy, because it reduces incentives to engage in research. Then $[(1+\kappa)/\kappa]/[1+n^o/\kappa]=(1+\kappa)/(\kappa+n^o)>1$, since $n^o<1$, so that $(1-\sigma^*)^{VIP}>(1-\sigma^*)^{HIP}$. It follows that $\sigma^*_{HIP}>\sigma^*_{VIP}$. ■

Point O in figure 2 represents the optimal couple (g^o_Y, g^o_S). Since the equilibrium configurations of the VIP lie always above those of the HIP, and the schedule shifts upwards with σ, it is clear that to attain point O in the HIP it is necessary a greater subsidy than in the VIP.

It is interesting to notice that under the VIP there exists a theoretical possibility that the optimal subsidy to R&D be negative (i.e. a tax). Indeed (42) is always positive but there might be configurations of parameters such that $(1-\sigma)>1$. To be in a "normal" situation ($\sigma>0$) the spillover effect of R&D on general knowledge must be strong "enough". [11]

As a conclusion we can state that the optimal policy is more expensive under the HIP than under the VIP. Indeed we have seen that h, P, r, τ, K, w and n are identical in the two cases, yet $\sigma^*_{HIP}>\sigma^*_{VIP}$ and from the budget constraint (29), we know that lump-sum taxes will be higher (or transfers lower) in the HIP than in the VIP. The cost in terms of foregone growth in output is, however, larger in the vertical innovations economy.

[11] $\sigma<0$ implies $\alpha\frac{1+\kappa}{\kappa+n^o}(1-n^o)>1$, that is: $(\varepsilon+\omega)\left[\left(\alpha\frac{1+\kappa}{\kappa}+1\right)\frac{\rho}{\delta}-2\right]+(1+\omega)\gamma(1-\alpha)\left[\left(\alpha\frac{1+\kappa}{\kappa}+1\right)\left(\frac{\rho}{\delta}-1\right)+\left(\alpha\frac{1+\kappa}{\kappa}-1\right)\varepsilon\right]>0$. This inequality is satisfied when $1<\frac{\rho}{\delta}<\alpha\frac{1+\kappa}{\kappa}$. This is the case when the spillover coefficient, κ, is very small relative to the productivity of labor in R&D, λ, and growth is positive in *laisser faire* although it is not optimal. In this case it is optimal to tax R&D activity because it is not effective in fostering average productivity growth.

Finally, substituting for z in (3) using (15) we can compute the share of green tax revenue over output in the green regime as :

$$\frac{h_t^* P_t^o}{Y_t^o} = \frac{1}{1+\gamma}$$

Which represents a very simple rule for optimal environmental policy: once the optimal emissions intensity of output (P^o/Y^o) has been tackled, the government only has to keep real green tax revenue at a level equal to the elasticity of output to emissions inputs. Along the optimal sustainable path of growth, emissions decrease continuously as firms adjust their technology, and the green tax is increased to offset the depressing effect on green tax revenue.

8 Conclusion

In this paper, we have studied and compared two main paradigms of endogenous growth theory in the particular context of sustainable growth when pollution is an implicit input to production. First, we have chosen two disaggregated economies, with horizontal and vertical differentiation respectively, and we have shown that they are compatible with the aggregate economy studied by Aghion and Howitt (1998). Then, we have characterized the decentralized equilibria of the two economies, introducing three policy tools: subsidies to monopolists and to R&D, and taxes on emissions of pollutants. Along a balanced growth path, subsidies are constant and green taxes increase at a constant rate.

For each model we obtain a continuum of balanced growth paths, highlighting a trade-off between the rate of growth of output and that of the pollution stock. That is, the faster the green taxes grow, the slower is equilibrium growth of output and consumption, but the slower (faster) the rate at which environmental quality deteriorates (improves). We have identified the channels through which environmental policy affects output growth. Some of these channels are direct. First, the policy reduces the growth rate of output by increasing the cost of pollution inputs. Second, it tends to reduce returns to R&D by depressing the demand for intermediate inputs, thus crowding out innovators' rents. The other channels are indirect, but influence the general equilibrium outcome. According to the value of the intertemporal elasticity of substitution, they can dominate the first direct effect, and increase incentives to engage in R&D activity.

Comparing the growth-environment trade-off in the two model economies, we have found that all policy tools equal, growth is higher in the vertical

innovations economy than in the horizontal innovations one. This result goes beyond environmental issues, and holds because demand is skewed towards innovations in the vertical innovations paradigm, while it is uniform across sectors in the horizontal case.

Finally, we have obtained the policy tools that allow the social planner to implement the optimal "strong sustainable" growth path. The optimal tax on emissions increases over time as firms adopt cleaner technologies. The optimal subsidy to R&D is greater in the HIP than in the VIP, though the cumulative loss in output growth is more important in the VIP.

The same results hold in the case of tradeable pollution permits (see Grimaud, 1999). Indeed, only the final sector employes emissions of pollutants as an input in these models. Therefore, a policy that controls the quantity (pollution permits) is equivalent to one controlling the price (green taxes). The equivalence is obtained when the quantity of pollution allowed is such that the equilibrium price of pollution permits on the secondary market matches the green tax that we have analyzed.

To conclude, we shall signal that we are aware of some caveats and limits of our analysis. First of all, we have not analyzed the case of high intertemporal elasticity of substitution ($\varepsilon < 1$), the transition towards the green-regime, and how the policy maker can bring the economy on the sustainable growth path. Then there are two theoretical issues at stake. We have relied on a setting where emissions enter as inputs in the aggregate production function with unitary elasticity of substitution. As shown by Dasgupta and Heal (1974) this is necessary for sustainable growth to be feasible in the presence of essential non-renewable resources. Yet it is not a particularly plausible assumption for corner situations (when one of the inputs is close to zero). We also assumed that R&D employs only labor and is not polluting, while letting capital enter the R&D production function would be natural and important[12]. However, this would limit the scope for sustainable growth because emissions would enter indirectly as inputs of R&D activity, the engine of growth.

[12]Aghion and Howitt (1998, ch 3) show that letting capital enter the R&D production function, brings about complementarities between capital accumulation and innovation, breaking the neoclassical dichotomy.

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Appendix I

The current value Hamiltonian is :

$$H = U(c,S)+\nu[z[D(1-n)]^{1-\alpha}K^{\alpha}-c]+\mu\delta nD-\zeta[z^{1+\gamma}[D(1-n)]^{1-\alpha}K^{\alpha}-\theta S]$$

The first order conditions with respect to the three controls, c, z and n, and the Euler equations with respect to the state variables, K, D, and $(-S)$ are :

$$U_c = c_t^{-\epsilon} = \nu_t$$

$$(1+\gamma)z_t^{\gamma}\zeta_t = \nu_t$$

Which, taking into account the constraint $z \in (0,1]$ gives :

$$z_t = \begin{cases} 1 & \text{if } \frac{\zeta_t}{\nu_t} \leq \frac{1}{(1+\gamma)} \quad D.R. \\ (\frac{\nu_t}{(1+\gamma)\zeta_t})^{\frac{1}{\gamma}} & \text{if } \frac{\zeta_t}{\nu_t} > \frac{1}{(1+\gamma)} \quad G.R. \end{cases}$$

$$\mu_t\delta D_t = \begin{cases} (1-\alpha)(1-\frac{\zeta_t}{\nu_t})\frac{\nu_t Y_t}{(1-n)} & D.R. \\ (1-\alpha)\frac{\gamma}{1+\gamma}\frac{\nu_t Y_t}{(1-n)} & G.R. \end{cases}$$

$$\frac{\dot{\nu}_t}{\nu_t} = \begin{cases} \rho-\alpha(1-\zeta_t/\nu_t)\frac{Y_t}{K_t} & D.R. \\ \rho-\alpha\frac{\gamma}{1+\gamma}\frac{Y_t}{K_t} & G.R. \end{cases}$$

$$\frac{\dot{\mu}_t}{\mu_t} = \begin{cases} \rho-\delta n-(1-\alpha)(1-\zeta_t/\nu_t)\frac{Y_t}{D_t}\frac{\nu_t}{\mu_t} & D.R. \\ \rho-\delta n-(1-\alpha)\frac{\gamma}{(1+\gamma)}\frac{Y_t}{D_t}\frac{\nu_t}{\mu_t} & G.R. \end{cases}$$

$$\frac{\dot{\zeta}_t}{\zeta_t} = \rho+\theta-\frac{S_t^{\omega}}{\zeta_t}$$

Finally the transversality conditions are :

$$\lim_{t\to\infty} e^{-\rho t}\nu_t K_t = 0$$

$$\lim_{t\to\infty} e^{-\rho t}\mu_t D_t = 0$$

$$\lim_{t\to\infty} e^{-\rho t}\zeta_t S_t = 0$$

Appendix II

Proof of proposition 2.

For n and z given, the social planner chooses each x_j to minimize aggregate capital K necessary to produce a given level of output $\bar{Y}$. In the HIP the problem is :

$$x_j, j \in [0, B_t] \min \int_0^{B_t} x_j dj$$

subject to :

$$\bar{Y} = z_t (1 - n_t)^{1-\alpha} \int_0^{B_t} x_j^{\alpha} dj$$

the first order conditions ensure that production is uniform across sectors, that is $x_j = x^{\circ}\ \forall j \in [0, B_t]$. Then aggregate capital is $K_t = \int_0^{B_t} x_j dj = B_t x^{\circ}$. Substituting for x in the aggregate production function we obtain :

$$Y_t = z_t (1 - n_t)^{1-\alpha} \int_0^{B_t} \left(\frac{K_t}{B_t}\right)^{\alpha} dj = z_t \left[(1 - n_t) B_t\right]^{1-\alpha} K_t^{\alpha}$$

The law of motion of productivity is given by (12), which is equivalent to (7).

In the VIP the problem is :

$$x_j, j \in [0, 1] \min \int_0^{1} A_j x_j dj$$

subject to :

$$\bar{Y} = z_t (1 - n_t)^{1-\alpha} \int_0^{1} A_j x_j^{\alpha} dj$$

the first order conditions ensure that production is uniform across sectors, that is $x_j = x^{\circ}\ \forall j \in [0, 1]$. Then aggregate capital is given by $K_t = \int_0^1 A_j x_j dj = A_t x^{\circ}$, where A_t is the average productivity index. Substituting in the aggregate production function we have :

$$Y_t = z_t (1 - n_t)^{1-\alpha} \int_0^{1} A_{jt} \left(\frac{K_t}{A_t}\right)^{\alpha} dj = z_t \left[(1 - n_t) A_t\right]^{1-\alpha} K_t^{\alpha}$$

Finally, if R&D intensity is uniform across sectors, i.e. $n_j = n\ \forall j \in [0, 1]$, and constant, then average productivity is proportional to the leading edge productivity index $\bar{A}$, and therefore they grow at the same rate, that is :

$$\frac{\dot{A}_t}{A_t} = \kappa \lambda n$$

which is equivalent to (7). The proof of this property is given by Aghion and Howitt (1998, ch. 3, app. 1) and is reported in what follows.

Consider the cumulative distribution $F(.,t)$ of the productivity parameters A_j's over the space of intermediate goods $[0,1]$ at the steady state. Denote with A_o the leading edge technology at date t_0. Then we have :

$$F(A_o, t_0) = 1$$

$$\frac{dF(A_o,t)}{dt} = -\lambda n F(A_o, t)$$

At date t_0 the parameter A_o is at the top and then it slides down the ladder as innovations arrive in other intermediate goods. The second equation states that at time t of the $F(A_o, t)$ goods of lower quality than A_o some (of mass λn) overcome technology A_o.

The solution to the system above gives $\forall t \geq t_0$:

$$F(A_o, t) = e^{-\lambda n(t-t_0)}$$

But from (14) we know that :

$$\dot{\bar{A}}_t = \kappa \lambda n \bar{A}_t$$

$$\bar{A}_{t_0} = A_o$$

Therefore :

$$\bar{A}_t = A_o e^{\kappa \lambda n (t-t_o)}$$

Which rearranged and together the previous results gives :

$$F(A_o, t) = \left(\frac{A_o}{\bar{A}_t}\right)^{\frac{1}{\kappa}}$$

Define $a_{jt} = \frac{A_j}{\bar{A}_t} \in (0,1]$ as the relative productivity parameter or, in other words, the age of each technology. In the long run, when the memory of date t_0 fades away, all relative productivity parameters are generated by the following Cdf :

$$H(a) = a^{\frac{1}{\kappa}}$$

which is constant over time. The density being $h(a) = a^{\frac{1}{\kappa}-1}/\kappa$.

We can apply this result immediately to derive :

$$\begin{aligned} A_t = \int_0^1 A_{jt} dj &= \bar{A}_t \int_0^1 a_{jt} dj = \\ &= \bar{A}_t \int_0^1 a h(a) da = \\ &= \bar{A}_t \int_0^1 \frac{a^{\frac{1}{\kappa}}}{\kappa} da = \frac{\bar{A}_t}{1+\kappa} \end{aligned}$$