Non Convexities, Imperfect Competition and Growth

André Grimaud*

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*Université de Toulouse I (GREMAQ, IDEI and LEERNA), 21 allée de Brienne, 31000 Toulouse, France, and ESCT.

Abstract

This paper starts from the fact that, when knowledge is used as input, technology generally exhibits increasing returns to scale. We consider an equilibrium where patents are given to the new ideas, which are public goods, rather than to the intermediate goods in which they are embodied as in the standard literature. In order to avoid the problem of existence of a competitive equilibrium, we assume that there is imperfect competition in all the economic sectors that use knowledge. The methodology is illustrated in growth models with an expanding variety of products.

1 Introduction

Most economists agree with the fact that one of the main factors of long term growth is the continuous accumulation of knowledge. For instance, in the standard growth models, this accumulation takes the form of an increase in the number of intermediate goods (Romer (1990), Grossman-Helpman (1991)), or in their quality (Aghion-Howitt (1992)). One of the main difficulties to understand how the research activity is financed comes from the basic public good nature of this knowledge. This raises essentially two types of problems.

The first ones are standard in the public goods theory : they are linked to the fact that these goods are non rival (or non depletable : see Mas-Collell-Whinston-Green (1995)) and, in some cases, non excludable. Let us consider for instance an innovation that takes the form of a scientific report that describes the theory underlying the building and functionning of a new type of engin that can be used for cars, airplanes, boats, and other types of goods. Suppose that the inventor has a patent that protects its monopoly. Assume now that this inventor is able to use a first-degree price discrimination, that is to say to extract from each user its willingness to pay for the innovation. In this case, the personalized prices paid to the inventor are exactly the Lindahl prices : we know that they optimally finance research. There are at least two reasons that explain why this type of discrimination is unlikely to happen in practice. First, it requires that the inventor have the ability to exclude any potential user, and thus to verify whether the innovation is used. Second, it requires complete information about individual willingnesses to pay.

The second type of problems comes from the fact that knowledge is used as an input in production processes, which implies non convex technologies. It is worthwhile to note that this property has been pointed out prior to the development of endogenous growth theory. For instance, Manning-Markusen-Mc Millan (1985, p. 236) write that "With public intermediate goods there is a presumption that the production functions of the consumption-goods industries exhibit increasing returns to scale". More recently, several authors recall that non rivalry of knowledge implies increasing returns to scale. For instance, Jones (2001, p. 6) writes that " . . . economists have recognized that the non rivalry of knowledge implies that aggregate production is characterized by increasing returns to scale." The main consequence of this property is that if firms pay to use knowledge, for instance if they pay the Lindahl prices (or less than these prices), and if they operate in competitive markets, their profits are negative. For instance, Feehan (1989, p. 239) writes : "Constant returns to scale in the primary factors means that placing user charges on firms, e.g., Lindahl prices, is infeasible. By Euler's theorem, payment to the private factors fully exhaust revenue so firms are unable to pay user charges." Similarly, in the abstract of his paper, Romer (1990) claims that "Because of the non convexity introduced by a non rival good, price taking competition cannot be supported."

Given this fundamental problem existence of competitive equilibrium , different types of equilibria can be considered. The first one is a benchmark. It can be assumed that all markets are competitive and that the firms which use knowledge are subsidized to avoid negative profits. In this case, we know from the first theorem of welfare economics that if the prices paid for knowledge are the Lindhal prices, the first best optimum is implemented. This equilibrium is interesting from a theoretical point of view because it allows to characterize the optimal prices in a complete market framework. However it is not realistic, in particular because it assumes that all of the research is publicly financed.

The second type of equilibrium is the one exhibited in the standard literature starting with the seminal works of Romer, Grossman-Helpman and Aghion-Howitt. To each innovation is associated one particular intermediate good, and the patent given to the producer of this good allows him to benefit from monopoly profits. Let us return to the previous example of a new type of engin. The standard growth theory assumes that the inventor embodies his innovation in a new engin and that he monopolistically sells this engin to firms which produce cars, airplanes, boats, and other machines. Note that this equilibrium has incomplete markets. Indeed, knowledge is not directly priced, and each innovation is indirectly financed by the profits on the private intermediate good in which it is embodied. This explains why this type of equilibrium is not generally optimal. Moreover, since knowledge is not priced, it is possible to maintain the assumption of perfect competition in all markets, except in those of the intermediate goods. In the previous example, the firms which produce cars, airplanes, boats, etc, do not directly pay knowledge, their markets are competitive, but they buy engins from a monopolist.

The main objective of this paper is to study a third type of equilibrium in which knowledge is directly and privately financed. To avoid the existence problem due to the non convexity of technologies, we assume that there is imperfect competition in all markets where knowledge is used as an input. In contrast with the equilibria considered in the standard literature, the equilibrium studied here has complete markets since knowledge is now directly priced. For instance, in the previous example, we now assume that the inventor of the new theory is directly rewarded by the firms that use it to produce engins : the patent concerns the scientific report which is an indivisible public good, and not the engin which is a divisible and private one. Thus, since the sectors of cars, airplanes, boats, etc, directly pay knowledge, we need to assume that they are imperfectly competitive.

The paper is organized as follows. In section 2, we present a very simple growth model in which the main features of the analysis are presented. In the next two sections, we use the methodology to construct equilibria with complete markets and imperfect competition in two models considered in the literature. In section 3, we consider the Isaac Newton model of Jones (2001) and in section 4 the basic model of Romer (1990).

2 A simple model without intermediate goods

2.1 The model

There are three types of goods in the economy : a final good (Y), labor (L), and innovations (n). There are two sectors : the final good sector and the research sector.

An innovation is an indivisible, public, and infinitely durable good, simultaneously used by the research sector and by the final good [1]. Formally, it is a point j on the segment $[0, n_t]$, where n_t is the measure of the space of innovations at time t. Innovations are produced by H firms. Each firm h has a production function

$$\dot{n}_t^h = q^h(L_t^h, n_t),\ h = 1, \dots, H, \tag{1}$$

where L_t^h is the quantity of labor used at time t. The total flow of innovations produced at t in the whole economy is

$$\dot{n}_t = \sum_h \dot{n}_t^h = \sum_h q^h(n_t, L_t^h). \tag{2}$$

Once invented, an innovation can be reproduced at zero cost. The final good is produced by I firms, according to

$$Y_t^i = F^i(L_t^i, n_t),\ i = 1, \dots, I, \tag{3}$$

where L_t^i is the labor used at t.

There is a continuous mass L of identical individuals. Each individual is endowed with one unit of flow of labor, and his utility is $\int_0^\infty u(c_t)e^{-\rho t}dt$,

[1] Basically, one thinks of a report in which is explained a new theory, a new methodology ... as for instance a new type of engin in the above introduction.

where c_t is his consumption. In this model, the whole final good is used for consumption : $Lc_t = \sum_i Y_t^i = Y_t$. Finally, we have

$$L_t^Y + L_t^R = L, \tag{4}$$

$$\text{where} \qquad L_t^Y = \sum_{i=1}^{I} L_t^i \quad \text{and} \quad L_t^R = \sum_{h=1}^{H} L_t^h.$$

Along this section, we use the following specification : $\dot{n}_t^h = q^h(L_t^h, n_t) = \delta n_t L_t^h, \delta > 0$, that implies $\dot{n}_t = \delta n_t L_t^R; Y_t^i = F^i(L_t^i, n_t) = AL_t^i n_t^\beta$, $A > 0, \beta > 0$, that implies $Y_t = AL_t^Y n_t^\beta; u(c_t) = c_t^{1-\varepsilon}/(1-\varepsilon), \varepsilon > 0$. Note that the two technologies have constant-returns-to-scale with respect to labor, that is the only private input in this model. Thus there are increasing returns with respect to the two types of inputs : labor and knowledge [2].

2.2 The first best

In this sub-section, we consider an equilibrium in which all markets (final output, labor and financial market) are perfectly competitive, and where innovations are financed by Lindahl prices. Since we assume constant-returns-to-scale with respect to labor in the two sectors, final output and research, all profits are nil in these sectors once labor is paid. Therefore we have to assume that the Lindahl prices which are used to pay knowledge, and thus to finance research, are subsidized by the government.

The price of good Y is normalized to one, and we denote by w_t and r_t the wage and the interest rate. We denote by v_t^i and v_t^h the Lindahl prices of one innovation corresponding to the firms $i(i = 1, \dots, I)$ and $h(h = 1, \dots, H)$. At each time t, the value of an innovation is

$$V_t = \int_t^\infty v_s e^{-\int_t^s r u du} ds, \tag{5}$$

where $v_s = \sum_i v_s^i + \sum_h v_s^h$ is the sum of the Lindahl prices paid at s for this innovation. We assume that, once one innovation has occured, these Lindahl prices are paid to the inventor from the date of invention to infinity.

In the final good sector, each competitive firm i maximizes its profit $\pi_t^i = F^i(L_t^i, n_t) - w_t L_t^i$. The first-order condition is

$$F_L^i - w_t = 0. \tag{6}$$

[2]It could be possible to study a more general model with physical capital. Each individual production function would be for instance $Y_t^i = A(K_t^i)^\alpha (L_t^i)^{1-\alpha} n_t^\beta$. The main results would not be modified.

F_L^i is independent of i, and we note it F_L. Moreover, (6) implies

$$\frac{\dot{w}_t}{w_t} = \frac{\dot{F}_L}{F_L}. \tag{7}$$

The Lindahl price corresponding to the firm i, that is to say the marginal profitability of an innovation, is

$$v_t^i = \frac{\partial \pi_t^i}{\partial n_t} = F_n^i. \tag{8}$$

In the research sector, the profit of the firm h on innovations produced at t is $\pi_t^h = q^h(L_t^h, n_t)V_t - w_t L_t^h$ [3]. Maximizing with respect to L_t^h leads to

$$q_L^h V_t - w_t = 0. \tag{9}$$

q_L^h is independent of h, and we note it q_L. Moreover, (9) implies

$$\frac{\dot{w}_t}{w_t} = \frac{\dot{q}_L}{q_L} + \frac{\dot{V}_t}{V_t}. \tag{10}$$

The Lindahl price corresponding to the firm h is

$$v_t^h = \frac{\partial \pi_t^h}{\partial n_t} = q_n^h V_t. \tag{11}$$

We assume that the government finance the Lindahl prices by using a lump-sum tax T_t paid by the households, such that its budget constraint is $T_t = n_t v_t$.

Finally, the representative household maximizes his utility, that leads to the standard condition :

$$\rho - \frac{u'' \dot{c}_t}{u'} = r_t. \tag{12}$$

We are now able to give a condition which, after eliminating the prices, characterizes any equilibrium path. Differentiating (5) with respect to t gives $r_t = \dot{V}_t/V_t + v_t/V_t$. From (7) and (10), one gets $\dot{V}_t/V_t = \dot{F}_L/F_L - \dot{q}_L/q_L$. From (6) and (9), one gets $V_t = F_L/q_L$. From (8) and (11), we obtain the total Lindahl prices paid at t for one innovation : $v_t = \sum_i F_n^i + (F_L/q_L)\sum_h q_n^h$. Finally, using (12) we obtain the following basic condition :

$$\rho - \frac{u'' \dot{c}}{u'} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{q_L}{F_L}\left(\sum_i F_n^i + \frac{F_L}{q_L}\sum_h q_n^h\right) \tag{13}$$

[3] The methodology used here allows to shorten the calculations. A more complete analysis consists in maximizing $\int_0^\infty (v_t n_t^h - w_t L_t^h) e^{-\int_0^t r u du} dt$, subject to $\dot{n}_t^h = q^h(L_t^h, n_t)$. It leads to the same results : see appendix A.1.

This condition is close to the Ramsey-Keynes one, $\rho - u''\dot{c}/u' = F_K$, that is obtained in the standard neoclassical model where the technology is $Y = F(K, L)$. But, here, the right side is the marginal productivity of Y if this good is indirectly invested in the research sector in order to accumulate knowledge.

Let us now use the specification given in sub-section 2.1 above. Then we have $u''\dot{c}/u' = -\varepsilon g_Y$ (g_Y is the rate of growth of Y), $F_L^i = An^\beta, F_n^i = \beta AL^i n^{\beta-1}, q_L^h = \delta n$, and $q_n^h = \delta L^h$. (13) becomes

$$\rho + \varepsilon g_Y = \beta g_n - g_n + \frac{\delta n}{An^\beta}\left(\beta AL^Y n^{\beta-1} + \frac{An^\beta}{\delta n}\delta L^R\right).$$

Since $Y = AL^Y n^\beta$, we have $g_Y = \beta g_n$. Using the fact that $L^Y + L^R = L$ and $\delta L^R = g_n$, one gets the rate of growth of the final output :

$$g_Y = \frac{\delta\beta L - \rho}{\varepsilon}. \tag{14}$$

Then, it is easy to obtain the rates of growth, quantities and prices at equilibrium. We have $g_n = (\delta L - \rho/\beta)/\varepsilon, L^R = (L - \rho/\delta\beta)/\varepsilon, L^Y = (L(\varepsilon - 1) + \rho/\delta\beta)/\varepsilon, V = An^{\beta-1}/\delta, v^Y = \sum_i v^i = \beta AL^Y n^{\beta-1}, v^R = \sum_h v^h = AL^R n^{\beta-1}, v = v^Y + v^R = An^{\beta-1}(\beta L^Y + L^R)$. It can be noted that these results, in particular the total Lindahl prices (v^Y and v^R), do not depend on the number of firms (I and H) in the two sectors, final good and research.

Remark : it is easy to verify that this equilibrium is optimal. Indeed, the social planner maximizes the utility $\int_0^\infty u(c_t)e^{-\rho t}dt$ subject to the constraints $\sum_i F^i(L_t^i, n_t) - Lc_t = 0, \dot{n}_t - \sum_h q^h(L_t^h, n_t) = 0$, and $\sum_i L_t^i + \sum_h L_t^h - L = 0$. This maximization leads to the condition (13) above and all the results obtained at equilibrium : see appendix A-2. In fact, since all markets are competitive and innovations are financed by Lindahl prices, this result of optimality is a direct consequence of the first welfare theorem.

2.3 Partial financing of research

In the previous sub-section we have assumed that, at each time t, each innovator receives a payment equal to the sum of the willingnesses to pay of all users of his innovation. As it is explained in the introduction, this case can be interpreted as a benchmark in which a patent is given to each innovation, each innovator being able to use a first-degree price discrimination, that is to say to extract from each user its willingness to pay (recall that, for the moment, these payments are subsidized by the government). Now we assume that for any reason (information problems, difficulty to exclude), only a part of the Lindahl prices can be extracted. More precisely, we assume that the payment for one innovation, $v_t = v_t^Y + v_t^R$, is such that $v_t^Y = \eta \sum_i v_t^i = \eta \sum_i F_n^i$ and $v_t^R = \theta \sum_h v_t^h = \theta (F_L/q_L) \sum_h q_n^h$, with $0 \leq \eta \leq 1$ and $0 \leq \theta \leq 1$. Then, the basic condition (13) above becomes

$$\rho - \frac{u'' \dot{c}}{u'} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_n}{q_n} + \frac{q_L}{F_L} \left(\eta \sum_i F_n^i + \theta \frac{F_L}{q_L} \sum_h q_n^h \right).$$

Using the same particular specification, one gets the new rate of growth under imperfect discrimination :

$$g_Y = \frac{\eta \delta \beta L - \rho}{\varepsilon + \eta - 1 + (1-\theta)/\beta}. \tag{15}$$

It is easy to see that $\partial g_Y / \partial \theta$ is positive. Moreover, $\partial g_Y / \partial \eta$ has the same sign than $\delta L(1-\theta) + (\delta \beta L(\varepsilon - 1) + \rho)$, which is also positive : indeed, $\delta \beta L(\varepsilon - 1) + \rho$ is positive because $L^Y = (L(\varepsilon - 1) + \rho/\delta\beta)/\varepsilon$ is positive. In others words, according to the first intuition, this result shows that a partial financing of research leads to an insufficient growth.

2.4 Private financing of research and imperfect competition

We have already observed that the equilibrium studied above is only a benchmark. Clearly it is not realistic, essentially because it is assumed that the whole research sector is financed by the government. Our objective now is to construct an equilibrium in which the research activity is totally privately financed. As explained above, on account of the reasonable assumption of increasing returns to scale in the final sector and in the research sector, we cannot continue to assume that these two markets are perfectly competitive. On the contrary, in order to have non negative profits in these sectors, we assume here that there is imperfect competition. More precisely, using directly the specified model, we make the following hypothesis.

First, we assume that the (imperfect) competition in the final sector and in the research sector leads to nil profits for all firms in these sectors :

$$\pi_t^i = A L_t^i n_t^\beta - w_t L_t^i - v_t^i n_t = 0, \quad i = 1, \ldots, I \tag{16}$$

$$\text{and} \quad \pi_t^h = \delta L_t^h n_t V_t - w_t L_t^h - v_t^h n_t = 0, \quad h = 1, \ldots, H \tag{17}$$

Second, we assume that, in these sectors, all firms are price takers on the markets of labor and of innovations. Then, if each firm minimizes its cost, one gets

$$\frac{v_t^i}{w_t} = \frac{\beta L_t^i}{n_t}, \quad i = 1, \dots, I \tag{18}$$

$$\text{and} \quad \frac{v_t^h}{w_t} = \frac{L_t^h}{n_t}, \quad h = 1, \dots, H \tag{19}$$

Observe that we assume that a given firm i (or h) pays all the innovations it uses (in number n_t) at the same price v_t^i (or v_t^h). We think that this assumption can be justified by the symmetry of innovations in this model. However, we allow that two different firms pay different prices ; in others words, there is a possible price discrimination between firms. If it was not the case, firms would have incentives to cluster in order to reduce the total payment for knowledge.

Our main objective now is to calculate the growth rate of the output, and then all the variables of the model. As in the first best equilibrium, we always have

$$r_t = \rho + \varepsilon g_Y = \frac{\dot{V}_t}{V_t} + \frac{v_t}{V_t}, \tag{20}$$

where V_t is given by (5).

Using (16) and (18), we obtain $AL_t^i n_t^\beta - w_t L_t^i (1 + \beta) = 0$, that gives

$$w_t = \frac{An_t^\beta}{1 + \beta}. \tag{21}$$

In the same way, using (17) and (19), we have $\delta L_t^h n_t V_t - 2w_t L_t^h = 0$, that gives

$$w_t = \frac{\delta n_t V_t}{2}. \tag{22}$$

Note that, for a given level of knowledge n_t, imperfect competition leads to a decrease in w_t. Indeed, in the perfect competition case, we have $w_t = An_t^\beta$ in the final sector (see (6)) and $w_t = \delta n_t V_t$ in the research sector (see (9)). That is why it is now possible to have non negative profits.

From (21) and (22), we obtain the value of an innovation at t :

$$V_t = \frac{2A}{\delta(1 + \beta)} n_t^{\beta - 1}, \tag{23}$$

that implies $\dot{V}_t / V_t = (\beta - 1) g_n$.

From (18) and (22), we get $v_t^i = \delta\beta V_t L_t^i/2$ (the more a firm is large, the more it pays for one innovation), that gives the total payment for one innovation by the final sector : $v_t^Y = \delta\beta V_t L_t^Y/2$.

Similarly, from (19) and (22), one gets $v_t^h = \delta V_t L_t^h/2$, that gives the payment by the research sector : $v_t^R = \delta V_t L_t^R/2$. Then we obtain the total payment for one innovation

$$v_t = v_t^Y + v_t^R = \frac{\delta(\beta L_t^Y + L_t^R)V_t}{2}. \tag{24}$$

Finally, (20) can be written

$$\rho + \varepsilon g_Y = (\beta - 1)g_n + \frac{\delta(\beta L^Y + L^R)}{2}.$$

Since $g_Y = \beta g_n$, $L^Y = L - L^R$, and $\delta L^R = g_n$, we obtain the following growth rate

$$g_Y = \frac{\delta\beta L/2 - \rho}{\varepsilon + (1-\beta)/2\beta}, \tag{25}$$

that can be put closer the first best one given by (14). Using (25), all the other variables (growth rates, quantities, prices) can be calculated as previously in 2.2.

It would be rather long to give a complete comparison between the two rates of growth given by (25) (imperfect competition) and (14) (first best optimum). In fact, according to the values of parameters, imperfect competition can increase or decrease the rate of return in research given by $\dot{V}_t/V_t + v_t/V_t$. However, we can observe that if $\beta = 1$ (the marginal productivity of knowledge in the final sector is constant), imperfect competition depresses growth.

This example shows that it is possible to simply calculate the rates of growth, quantities and prices, in an equilibrium where innovations, which are public goods, are directly and privately financed. This equilibrium is very different from the standard one studied in the literature since Romer and Grossman-Helpman, where innovations are indirectly financed by the profits on the intermediate goods in which they are embodied. Here markets are complete, but the imperfect competition which prevails in the sectors which use knowledge as input (final sector and research) leads to a non-optimal equilibrium.

3 Imperfect competition in the Jones' Isaac Newton growth model

The purpose of sections 3 and 4 is to show how our methodology can be used to construct equilibria with imperfect competition in other models. The first one is the Isaac Newton model studied by Jones (2001). As in section 2, it is a model without intermediate goods, but it is a semi-endogenous growth model [4]. We use the same notations as in section 2. The technology of the final good sector is

$$Y_t = n_t^\sigma L_t^Y, \sigma > 0. \tag{26}$$

As previously, it can be desagregated in I firms : $Y_t^i = n_t^\sigma L_t^i, i = 1, \dots, I$. In the research sector, the technology is

$$\dot{n}_t = \delta L_t^R, \delta > 0. \tag{27}$$

Here also, it can be desagregated : $\dot{n}_t^h = \delta L_t^h, h = 1, \dots, H$. We always have $L_t^Y + L_t^R = L_t$ (see (4)), but now the population grows at constant rate :

$$\frac{\dot{L}_t}{L_t} = \nu. \tag{28}$$

Jones assumes that a constant fraction s of the labor force works as researchers, so that $L_t^R = sL_t$ and $L_t^Y = (1-s)L_t$. Then he characterizes the steady state growth path. He obtains $g_y = \sigma\nu$, where $y_t = Y_t/L_t, L_t^R/n_t = \nu/\delta, L_t^Y/n_t = (1-s)\nu/s\delta$, and $L_t/n_t = \nu/s\delta$. Here we consider the same model than Jones, given by (26)-(27)-(28), but without the assumption $L_t^R = sL_t$.

As in more standard presentations (see for instance Barro-Sala-I-Martin (1995)), we assume that there are M households, and that the utility per household is

$$\int_0^\infty \frac{u(c_t)}{M} e^{(\nu-\rho)t} dt, \tag{29}$$

where $u(c_t) = (c_t^{1-\varepsilon})/(1-\varepsilon)$ and $c_t = Y_t/L_t$ is the individual consumption. This assumption allows to make a welfare analysis and to construct other types of equilibria.

[4] In the same paper, Jones studies a more complete model where the population growth rate is endogenous. The same methodology could be applied to this model.

3.1 The first best

In any steady state we have, as Jones, $g_{L^R} = g_{L^Y} = g_L = \nu$. From (26), one gets $g_Y = \sigma g_n + g_{L^Y}$, that gives $g_y = \sigma g_n$. From (27), one gets also $g_n = g_{L^R} = \nu$, and thus $g_y = \sigma\nu$. Finally, since $g_n = \delta L_t^R/n$, one obtains $L_t^R/n_t = \nu/\delta$. The problem is to determine the ratios L_t^Y/n_t and L_t/n_t which, in the Jones paper, depend on s.

As in 2.2 above, we first consider an equilibrium in which all markets are competitive, and where innovations are financed by the government at Lindahl prices levels. Note that, in this model, innovations are only used by the final good sector (see (26)). As previously, we denote by v_t^i the Lindahl price corresponding to the firm i. Thus, at each time t, the value of an innovation is $V_t = \int_t^\infty v_s e^{-\int_t^s r u du} ds$, where $v_s = \sum_{i=1}^I v_s^i$ (see (5)). In the final sector, each firm i has a profit $\pi_t^i = n_t^\sigma L_t^i - w_t L_t^i$, that gives the wage level

$$w_t = n_t^\sigma, \tag{30}$$

and the Lindahl price

$$v_t^i = \frac{\partial \pi_t^i}{\partial n_t} = \sigma n_t^{\sigma-1} L_t^i. \tag{31}$$

From (31), one gets $v_t = \sigma n_t^{\sigma-1} L_t^Y$, which is here also independent of the number of firms in this sector.

In the research sector, the profit on innovations at t of the firm h is $\pi_t^h = \delta L_t^h V_t - w_t L_t^h$, that gives

$$w_t = \delta V_t. \tag{32}$$

Finally, the household behavior leads to the standard condition : $\dot{c}_t/c_t = (r_t - \rho)/\varepsilon$.

From (30) and (32), one gets $g_w = \sigma g_n = g_V = \sigma\nu$. Moreover, since $v_t = \sigma n_t^{\sigma-1} L_t^Y$ and $V_t = w_t/\delta = n_t^\sigma/\delta$, one gets $v_t/V_t = \delta\sigma L_t^Y/n_t$. Finally, the standard condition $\rho + \varepsilon g_c = \dot{V}_t/V_t + v_t/V_t$ (see (20)) becomes $\rho + \varepsilon\sigma\nu = \sigma\nu + \delta\sigma L_t^Y/n_t$, that gives

$$\frac{L_t^Y}{n_t} = \frac{\rho + \sigma\nu(\varepsilon - 1)}{\delta\sigma} \tag{33}$$

$$\text{and} \qquad \frac{L_t}{n_t} = \frac{\rho + \sigma\nu\varepsilon}{\delta\sigma} \tag{34}$$

It is easy to verify that the social planner program leads also to (33) and (34). As previously, this result is a direct consequence of the first welfare theorem (see Appendix B).

3.2 Imperfect competition

Now we study an equilibrium without government. Thus the production of knowledge is directly and privately financed by the firms of the final sector, which are in this model the only ones using it. As in 2.4, we assume that the imperfect competition in this sector leads to two conditions. The first one says that all profits (including now the payment of innovations) are nil :

$$\pi_t^i = n_t^\sigma L_t^i - w_t L_t^i - v_t^i n_t = 0, \quad i = 1, \ldots, I. \tag{35}$$

Second, the prices ratio is equal to the corresponding rate of substitution :

$$\frac{v_t^i}{w_t} = \frac{\sigma L_t^i}{n_t}, \quad i = 1, \ldots, I. \tag{36}$$

As in 3.1, we have $w_t = \delta V_t$ (see (32)) in the research sector and $g_c = (r_t - \rho)/\varepsilon$ from the household behavior.
Combining (35) and (36), and eliminating L_t^i, gives $w_t = n_t^\sigma/(1+\sigma)$. Using this result and (32), one gets $g_w = g_V = \sigma g_n = \sigma\nu$. Moreover, we have $V_t = n_t^\sigma/\delta(1+\sigma)$. From (36), one gets $v_t = \sum_i v_t^i = w_t \sigma L_t^Y = \sigma n_t^{\sigma-1} L_t^Y/(1+\sigma)$, and thus $v_t/V_t = \delta\sigma L_t^Y/n_t$. Finally, the standard condition $\rho + \varepsilon g_c = \dot{V}_t/V_t + v_t/V_t$ becomes $\rho + \varepsilon g_c = \sigma\nu + \delta\sigma L_t^Y/n_t$, that gives the results obtained before (see (33) and (34)) :

$$\frac{L_t^Y}{n_t} = \frac{\rho + \sigma\nu(\varepsilon - 1)}{\delta\sigma} \quad \text{and} \quad \frac{L_t}{n_t} = \frac{\rho + \sigma\nu\varepsilon}{\delta\sigma}.$$

In this model, introducing imperfect competition in the final good sector to directly finance research does not fundamentally modify the equilibrium. More precisely, the quantities of goods and the rates of growth are unchanged, but the repartition of the output is modified.

First we can observe that the rate of return in research, $\dot{V}_t/V_t + v_t/V_t$, is unchanged, that explains why the growth rate is unchanged. Indeed, in the two equilibria we have $g_V = \sigma\nu$. In the first best equilibrium, we have $v_t = \sigma n_t^{\sigma-1} L_t^Y$ and $V_t = n_t^\sigma/\delta$. In the imperfect competition equilibrium case, we have obtained $v_t = \sigma n_t^{\sigma-1} L_t^Y/(1+\sigma)$ and $V_t = n_t^\sigma/\delta(1+\sigma)$: imperfect competition depresses simultaneously the instantaneous payment for any innovation (v_t) and its value (V_t). However, the ratio of these two prices is unchanged, that explains why the rate of return in research is not modified.

The main differences between the two equilibria concern the repartition of the output. This can be seen by looking at the budget constraint of the households.

Let us consider the first best equilibrium. At each time t, the value of the firms in the research sector, that is to say the value of the bonds (B) on the financial market, is $B_t = n_t V_t = n_t^{\sigma+1}/\delta$: it is the value of knowledge, which is the only one asset in this economy. The interests paid by the research sector to the households are $r_t B_t = (\rho + \varepsilon\sigma\nu) n_t^{\sigma+1}/\delta$. The new bonds issued at t are $\dot{B}_t = (\sigma+1) n_t^{\sigma} \dot{n}_t/\delta = (\sigma+1)\nu n_t^{\sigma+1}/\delta$.

Finally, at time t, the households receive the wages $w_t L_t = (\rho+\varepsilon\sigma\nu) n_t^{\sigma+1}/\delta\sigma$, and the interests on bonds $r_t B_t = (\rho + \varepsilon\sigma\nu) n_t^{\sigma+1}/\delta$. They pay the taxes $T_t = v_t n_t = (\rho+\sigma\nu(\varepsilon-1)) n_t^{\sigma+1}/\delta$, they consume $C_t = (\rho+\sigma\nu(\varepsilon-1)) n_t^{\sigma+1}/\delta\sigma$, and they buy the new bonds $\dot{B}_t = (\sigma+1)\nu n_t^{\sigma+1}/\delta$.

Let us now consider the imperfect competition equilibrium. In this case, there is no tax, since knowledge is privately financed. Moreover, the level of consumption is unchanged. Finally, the preceding levels of wages $(w_t L_t)$ interest on bonds $(r_t B_t)$, and borrowings $(\dot{B}_t)$, are divided by $(1+\sigma)$.It can be easily verified that the algebric variation of these terms, $\Delta(w_t L_t + r_t B_t - \dot{B}_t)$ is exactly equal to the taxe T_t of the first best equilibrium.

4 Complete markets and imperfect competition in the Romer's model

Our objective now is to use our methodology in the basic Romer's model. The main difference with the two preceding models is the presence of intermediate goods. As it is well known, in the standard literature these goods are sold by monopolies, that allows to indirectly finance research. In this type of equilibrium, knowledge is not directly priced, and this incompleteness of markets explains the presence of externalities (see for instance Barro-Sala-I-Martin (1995) and Aghion-Howitt (1998)). As in sections 2 and 3, we consider now an equilibrium with complete markets, that is to say in which knowledge and intermediate goods are separately priced.

We keep the same notations as in the two previous sections and, in particular, $[0, n_t]$ is the set of innovations. But we assume now that at each innovation j in $[0, n_t]$ is associated with one intermediate good, denoted also by j. However, these two goods are very different : an innovation is an indivisible, public, and infinitely durable good (see 2.1), when each associate private good is divisible and private [5].

The final good is produced along with $Y_t = A(L_t^Y)^\alpha \int_0^{n_t} x_t(j)^{1-\alpha} dj$, which can be desagregated in I firms : $Y_t^i = A(L_t^i)^\alpha \int_0^{n_t} x_t^i(j)^{1-\alpha} dj, i = 1, \ldots, I$.
The aggregated technology of research is $\dot{n}_t = \delta n_t L_t^R$, and it can be also

[5] Remember for instance the example of the introduction where the innovation is the new theory described in the scientific report, and the intermediate good the engin in which this theory is used.

desagregated as follows : $\dot{n}_t^h = \delta n_t L_t^h, h = 1, \ldots, H$. The technology of production of intermediate goods is linear : $x_t(j) = y_t(j)/a, j \in [0, n]$. A possible simple desagregation is $x^k(j) = y^k(j)/a, j \in [0, n_t], k = 1, \ldots, K$. Finally, the representative household utility is $\int_0^\infty ((c_t)^{1-\varepsilon}/(1-\varepsilon))e^{-\rho t}dt, \varepsilon > 0$.

In this section, we directly study the case of imperfect competition (see Appendix C for the first best case).

The research sector behavior has already been studied in sub-section 2.4. Starting from $\pi_t^h = \delta L_t^h n_t V_t - w_t L_t^h - v_t^h n_t = 0$ (see (17)) and $v_t^h/w_t = L_t^h/n_t$ (see (19)), one gets

$$V_t = \frac{2w_t}{\delta n_t} \tag{37}$$

$$\text{and} \qquad v_t^R = \sum_h v_t^h = \frac{\delta V_t L_t^R}{2} \tag{38}$$

In the intermediate goods sector, the marginal profitability of an innovation is nil. Indeed, if a firm k does not produce, its profit is nil. If it uses an innovation to produce a good, it it also nil. Then we have

$$p_t(j) = p_t = a, \quad \text{for all } j. \tag{39}$$

Consider now the final good sector. Observe that the symmetry of intermediate goods allows to have $x_t^i(j) = x_t^i$ for all j, and thus $\sum_i x_t^i(j) = \sum_i x_t^i = x_t$ for all j.
The first condition, saying that all profits are nil, is here

$$\pi_t^i = A(L_t^i)^\alpha \int_0^{n_t} x_t^i(j)^{1-\alpha} dj - w_t L_t^i - p_t n_t x_t^i - v_t^i n_t = 0, i = 1, \ldots, I.$$

The marginal profitability of an innovation for the firm i is $\partial Y_t^i/\partial n_t - p_t x_t^i = A(L_t^i)^\alpha (x_t^i)^{1-\alpha} - p_t x_t^i$: the second term of the difference comes from the fact that in order to benefit by an innovation, the firm i is obliged to buy the intermediate good in which it is embodied. Similarly, we have also $\partial Y_t^i/\partial L_t^i = \alpha A(L_t^i)^{\alpha-1} n_t (x_t^i)^{1-\alpha}$, and $\partial Y_t^i/\partial x_t^i(j) = (1-\alpha)A(L_t^i)^\alpha (x_t^i)^{-\alpha}$. Now we can write the conditions saying that the prices ratios are equal to the corresponding marginal rates of substitution.
The first condition is $\frac{p_t}{w_t} = \frac{\partial Y_t^i/\partial x_t^i(j)}{\partial Y_t^i/\partial L_t^i} = \frac{(1-\alpha)L_t^i}{\alpha n_t x_t^i}$. Since $p_t = a, \sum_i L_t^i = L_t^Y$, and $\sum_i x_t^i = x_t$, this condition becomes

$$(1-\alpha) w_t L_t^Y = \alpha a n_t x_t. \tag{40}$$

The second condition is $\frac{v_t^i}{w_t} = \frac{\partial Y_t^i/\partial n_t - p_t x_t^i}{\partial Y_t^i/\partial L_t^i} = \frac{x_t^i (L_t^Y/x_t)^{1-\alpha}}{\alpha A n_t}\left(A\left(\frac{L_t^Y}{x_t}\right)^\alpha - p_t\right)$. Using (39) and (40), one gets $v_t^i = \frac{a x_t^i}{(1-\alpha)A}\left(A - a\left(\frac{x_t}{L_t^Y}\right)^\alpha\right)$, that gives

$$v_t^Y = \sum_i v_t^i = \frac{a x_t}{(1-\alpha)A}\left(A - a\left(\frac{x_t}{L_t^Y}\right)^\alpha\right). \tag{41}$$

To eliminate the term $(x_t/L_t^Y)^\alpha$ in this expression, we write that $\sum_i \pi_t^i = 0$. We have $A(L_t^Y)^\alpha n_t x_t^{1-\alpha} - w_t L_t^Y - a n_t x_t - n_t v_t^Y = 0$. Then, using (40), one gets

$$v_t^Y = x_t\left(A\left(\frac{L_t^Y}{x_t}\right)^\alpha - \frac{a}{1-\alpha}\right). \tag{42}$$

Now we use the two expressions of v_t^Y, given by (41) and (42), that gives

$$\frac{a}{(1-\alpha)A}\left(A - a\left(\frac{x_t}{L_t^Y}\right)^\alpha\right) = A\left(\frac{L_t^Y}{x_t}\right)^\alpha - \frac{a}{1-\alpha},$$

and, finally, the following second degree equation

$$\frac{1}{A(1-\alpha)}\left(a\left(\frac{x_t}{L_t^Y}\right)^\alpha\right)^2 - \frac{2}{1-\alpha}\left(a\left(\frac{x_t}{L_t^Y}\right)^\alpha\right) + A = 0,$$

where the two roots are $a(\frac{x_t}{L_t^Y}) = A(1 \underset{-}{+} \sqrt{\alpha})$. In order to have v_t^Y positive, we keep the lower root, that gives (see (41) :

$$v_t^Y = \frac{a x_t \sqrt{\alpha}}{1-\alpha}. \tag{43}$$

As usual, in order to calculate the rate of growth at steady state, we start from $r_t = \rho + g_Y^{IC} = \dot{V}_t/V_t + v_t/V_t$ (g_Y^{IC} is the growth rate in this equilibrium with imperfect competition).
From (37) and (40), we have $\dot{V}_t/V_t = \dot{w}_t/w_t - \dot{n}_t/n_t = 0$. (38) gives $v_t^R/V_t = \delta L^R/2$. From (43), (37) and (40), one gets $v_t^Y/V_t = \delta L^Y/2\sqrt{\alpha}$. Thus we have $\rho + \varepsilon g_Y^{IC} = \delta L^Y/2\sqrt{\alpha} + \delta L^R/2$. Since $L^Y = L - L^R$ and $\delta L^R = g_Y^{IC}$, one gets finally

$$g_Y^{IC} = \frac{\delta L/2\sqrt{\alpha} - \rho}{\varepsilon + \frac{1/\sqrt{\alpha}-1}{2}}. \tag{44}$$

This result is different from the usual formula obtained in the standard literature where research is indirectly financed by the profits on intermediate goods, that is [6] :

$$g_Y = \frac{\delta(1-\alpha)L - \rho}{\varepsilon + 1 - \alpha}.$$

It is also different from the first best one : $g_Y^{FB} = (\delta L - \rho)/\varepsilon$. It can be shown that g_Y^{IC} can be higher or lower than g_Y^{FB}. One gets (see appendix C) :

$$\begin{array}{lll}
\text{if} \quad \alpha > 1/4, & g_Y^{IC} < g_Y^{FB}. & \\
\text{if} \quad \alpha < 1/4, & \text{two cases can occur :} & \\
 & g_Y^{IC} < g_Y^{FB}, & \text{if } \varepsilon < \tilde{\varepsilon}, \\
 & g_Y^{IC} > g_Y^{FB}, & \text{if } \varepsilon > \tilde{\varepsilon}, \\
\text{where} & \tilde{\varepsilon} = \frac{(1-\sqrt{\alpha})(\delta L - \rho)}{\delta L(1-2\sqrt{\alpha})}. &
\end{array}$$

As in the model of section 2 (but unlike to the results obtained in the Jones's Isaac Newton semi-endogenous growth model), imperfect competition can increase or decrease the rate of return in research, and thus the output growth rate.

This section shows that even if we stay inside one of the more standard models of the literature, the Romer's one, it is possible to study an equilibrium with complete markets which is very different from the usual one. Then, in order to choose between different equilibrium concepts, we have to answer in particular the following question : how is new knowledge financed ?

A possible intuitive answer is the following. We know that patents are given to ideas, which are public goods. Then we can consider two polar cases. In the first one, the innovator is able to extract only a little part of the willingnesses to pay of the potential users of his innovation : see for instance sub-section 2.3 above. In this case, he does not sells the patent and he monopolistically sells the good in which the innovation is embodied : the standard equilibrium prevails and, in a sense, incompleteness is endogenous. We can conjecture that more important are the information problems, more likely is the standard equilibrium. In the second case, the innovator is able to extract a large part of the willingnesses to pay and this revenue is larger than the potential profits on the intermediate goods : thus knowledge is directly financed and the equilibrium with complete markets prevails.

[6] In this example, this growth rate is lower than the optimal one. However, as shown by Benassy (1998), it suffices to modify the production function of the final sector to have too much research at equilibrium.

5 Conclusion

This paper shows how to construct equilibria with imperfect competition, which are different from the standard ones generally used in growth models with an expanding variety of products. In these models, markets are incomplete and patents are given to the intermediate goods in which innovations are embodied. In the equilibrium studied in this paper, patents directly protect knowledge, which is a public good.

A first concern of this analysis is to shed a new light on the question of research financing, the difficulty of which comes from the public good nature of knowledge. A second concern comes from the fact that growth theory is often used in different fields of economic theory : see for instance Aghion-Howitt (1998). One important problem of these studies concerns their technical difficulties. They are due to the fact that new questions (unemployment, natural resources, environment, agency problems, . . .) are introduced within models that are already in their initial form not very simple. The methodology presented in this paper seeks to simplify these basic growth models. The main reason is that, since knowledge is directly priced, it is no longer necessary to introduce intermediate goods.

Appendix A : a simple model without intermediate goods

1 The research sector

In sub-section 2., we have said that each firm h maximizes $q^h(L_t^h, n_t)V_t - w_t L_t^h$. Clearly, it is a shortened. A more standard presentation is to say that the firm maximizes the sum of the present values of its current profits, $\int_0^\infty (v_t n_t^h - w_t L_t^h) e^{-\int_0^t r u du} dt$, subject to the constraint $\dot{n}_t^h = q^h(L_t^h, n_t)$. The Hamiltonian of this problem is :

$$H = (v_t n_t^h - w_t L_t^h) e^{-\int_0^t r u du} dt + \nu_t q^h(L_t^h, n_t).$$

The conditions $\partial H / \partial L_t^h = 0$ and $\partial H / \partial n_t^h = -\dot{\nu}_t$ yield

$$-w_t e^{-\int_0^t r u du} + \nu_t q_L^h = 0 \tag{A.1}$$

and

$$v_t e^{-\int_0^t r u du} dt = -\dot{\nu}_t. \tag{A.2}$$

Integrating (A.2) between t and $+\infty$ gives $\int_t^\infty v_s e^{-\int_0^s r u du} ds = \nu_t$. Indeed, we have $\lim_{t \to \infty} \nu_t = 0$ owing to the transversality condition, $\lim_{t \to \infty} \nu_t n_t^h = 0$, in which n_t^h is bounded below (we can assume $n_0^h > 0$, and n_t^h is not decreasing).

Using this result, (A.1) can be written $w_t e^{-\int_0^t r u du} = q_L^h \int_t^\infty v_s e^{-\int_0^s r u du} ds$, and thus $w_t = q_L^h V_t$, where $V_t = \int_t^\infty v_s e^{-\int_t^s r u du} ds$ is the value of an innovation at t (see (5) above) : we obtain the condition (9) of the main text.

Differentiating the hamiltonian with respect to n_t gives

$$\frac{\partial H}{\partial n_t} = \nu_t q_n^h = q_n^h \int_t^\infty v_s e^{-\int_0^s r u du} ds = q_n^h V_t e^{-\int_0^t r u du},$$

that can be written $\frac{\partial H}{\partial n_t} e^{\int_0^t r u du} = q_n^h V_t$: we obtain the expression (11) of the main text. Now, we have $\frac{\partial H}{\partial n_t} = v_t^h e^{-\int_0^t r u du}$: it is the present value at $t = 0$ of the Lindahl price v_t^h paid at t for an innovation.

2 Welfare

The social planner maximizes $\int_0^\infty u(c_t) e^{-\rho t} dt$ subject to the constraints

$$\sum_i F^i(L_t^i, n_t) - Lc_t = 0, \dot{n}_t^h - \sum_h q^h(L_t^h, n_t) = 0,$$

and
$$\sum_i L_t^i + \sum_h L_t^h - L = 0.$$

The current value hamiltonian is

$$H = u(c) + \lambda(\sum_i F^i(L^i, n) - Lc) + \mu(\sum_h q^h(L^h, n)) + \nu(\sum_i L^i + \sum_h L^h - L).$$

The first-order conditions, $\partial H/\partial c = 0, \partial H/\partial L^i = 0$, and $\partial H/\partial L^h = 0$ yield $u'(c) - \lambda L = 0$ (a), $\lambda F_L^i + \nu = 0$ (b) and $\mu q_L^h + \nu = 0$ (c).
Differentiating (a) with respect to t gives $\dot{\lambda}/\lambda = u''\dot{c}/u'$ (a'). From (b) and (c), we have $F_L^i = F_L$ $\forall i$ and $q_L^h = q_L$ $\forall h$. Differentiating the equality $\lambda F_L = \mu q_L$ with respect to t gives $-\dot{\lambda}/\lambda = \dot{F}_L/F_L - \dot{\mu} q_L/\lambda F_L - \dot{q}_L/q_L$ (b').
The condition $\partial H/\partial n = \rho\mu - \dot{\mu}$ gives $\lambda \sum_i F_n^i + \mu \sum_h q_n^h = \rho\mu - \dot{\mu}$ and thus $-\dot{\mu}/\lambda = \sum_i F_n^i + (q_L/F_L)(\sum_h q_n^h - \rho)$. Plugging this expression in (b'), and using (a'), gives finally

$$\rho - \frac{u''\dot{c}}{u'} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{q_L}{F_L}\left(\sum_i F_n^i + \frac{F_L}{q_L}\sum_h q_n^h\right).$$

This condition is exactly the basic condition (13) obtained at equilibrium. If we consider the particular specification of the main text, we get the results already obtained at equilibrium, in particular the rate of growth given by (14).

Appendix B : the Jones' Isaac Newton model

Normalizing L_0 to one, we have $L_t = e^{\nu t}$. Since $Y_t = n_t^\sigma L_t^Y$, the per capita consumption is $c_t = Y_t/L_t = n_t^\sigma e^{-\nu t} L_t^Y$. Finally, we have $\dot{n}_t = \delta L_t^R = \delta(L_t - L_t^Y) = \delta(e^{\nu t} - L_t^Y)$.
The social planner maximizes $\int_0^\infty (u(c_t)/M) e^{(\nu-\rho)t} dt$, subject to the constraints $c_t = n_t^\sigma e^{-\nu t} L_t^Y$, and $\dot{n}_t = \delta(e^{\nu t} - L_t^Y)$. The current value hamiltonian is

$$H = (u(c)/M)e^{(\nu-\rho)t} + \lambda(n^\sigma e^{-\nu t} L^Y - c) + \mu\delta(e^{\nu t} - L^Y).$$

The first-order conditions $\partial H/\partial c = 0$ and $\partial H/\partial L^Y = 0$ yield :

$$(u'(c)/M)e^{(\nu-\rho)t} - \lambda = 0 \quad \text{(a)}$$
$$\text{and} \qquad \lambda n^\sigma e^{-\nu t} - \mu\delta = 0 \quad \text{(b)}.$$

Differentiating (a) with respect to time gives $\dot{\lambda}/\lambda = u''\dot{c}/u' - \rho + \nu$ (a'). Similarly, differentiating (b) gives $\dot{\lambda} n^\sigma + \lambda\sigma n^{\sigma-1}\dot{n} = \delta\dot{\mu}e^{\nu t} + \delta\mu\nu e^{\nu t}$ (b'). The condition $\partial H/\partial n = -\dot{\mu}$ yields $\lambda\sigma n^{\sigma-1} e^{-\nu t} L^Y = -\dot{\mu}$ (c).

Using (b) and (c), (b') becomes $\dot{\lambda} n^\sigma + \lambda\sigma n^{\sigma-1}\dot{n} = -\delta\lambda\sigma n^{\sigma-1}L^Y + \lambda\nu n^\sigma$. Finally, plugging (a') in this last equation, one gets

$$\frac{L^Y}{n} = \frac{\rho + \sigma\nu(\varepsilon - 1)}{\delta\sigma},$$

that is exactly the condition (33), obtained in an equilibrium where all markets are competitive and innovations are financed by the government at Lindahl prices levels.

Appendix C : the Romer's model

Using the notations of section 4, we know that the first best rate of growth is $g_Y^{FB} = (\delta L - \rho)/\varepsilon$, and that the rate of growth in an equilibrium with patents on intermediate goods and knowledge non directly priced is $g_Y = (\delta(1-\alpha)L - \rho)/(\varepsilon + 1 - \alpha)$: these results are standard. It suffices here to verify that an equilibrium with complete markets, perfect competition everywhere, and Lindahl prices to finance knowledge, leads to the first best. As in section 4, we have by symmetry $x_t^i(j) = x_t^i$ and $\sum_i x_t^i(j) = x_t$. In the final good sector, the profit of firm i is

$$\pi_t^i = A(L_t^i)^\alpha \int_0^{n_t} x_t^i(j)^{1-\alpha} dj - w_t L_t^i - \int_0^{n_t} p_t x_t^i(j) dj.$$

The first-order conditions, $\partial\pi_t^i/\partial L_t^i = 0$ and $\partial\pi_t^i/\partial x_t^i(j) = 0$, implies $\alpha A(L_t^i)^{\alpha-1} n_t (x_t^i)^{1-\alpha} - w_t = 0$ and $(1-\alpha)A(L_t^i)^\alpha (x_t^i)^{-\alpha} - p_t = 0$, that gives

$$\alpha A n_t \left(\frac{x_t}{L_t^Y}\right)^{1-\alpha} - w_t = 0 \tag{C.1}$$

$$\text{and} \qquad (1-\alpha)A\left(\frac{L_t^Y}{x_t}\right)^{1-\alpha} - p_t = 0 \tag{C.2}$$

The willingness to pay for an innovation, that is to say the Lindahl price, is $v_t^i = \partial\pi_t^i/\partial n_t = A(L_t^i)^\alpha (x_t^i)^{1-\alpha} - p_t x_t^i$, that implies

$$v_t^Y = \sum_i v_t^i = x_t \left(A\left(\frac{L_t^Y}{x_t}\right)^\alpha - p_t\right). \tag{C.3}$$

In the research sector, the profit of firm h is $\pi_t^h = \delta L_t^h n_t V_t - w_t L_t^h$, where V_t is given by (5). The maximization of π_t^h gives

$$\delta n_t V_t - w_t = 0. \tag{C.4}$$

The Lindahl price corresponding to an innovation is $v_t^h = \partial \pi_t^h / \partial n_t = \delta L_t^h V_t$, that implies

$$v_t^R = \sum_h v_t^h = \delta L_t^R V_t. \tag{C.5}$$

In the intermediate goods sector (which here is perfectly competitive), the profit of firm k is $\pi_t^k = \int_0^{n_t} (p_t y_t^k(j)/a - y_t^k(j))dj$. The constant returns to scale assumption implies

$$p_t = a. \tag{C.6}$$

Moreover, the Lindahl price for an innovation is nil. Indeed, if a firm in this sector does not use an innovation, it does not produce the corresponding intermediate goods : its profit on this innovation is nil. If it uses the innovation to produce the good, the profit is also nil. That is why the Lindahl price is nil : $v_t^k = 0$, for all k.

In order to study the steady state, we use as usual the condition $r_t = \rho + g_Y = \dot{V}_t/V_t + v_t/V_t$, where $v_t = v_t^Y + v_t^R$.

From (C.1), we have $g_w = g_n$. Thus, (C.4) gives $\dot{V}_t/V_t = 0$.

From (C.5), one gets $v^R/V = \delta L^R$.

From (C.2 and (C.6), one has $(L^Y/x) = (a/(1-\alpha)A)^{1/\alpha}$. Plugging this result in (C.3) gives $v^Y = a\alpha x/(1-\alpha)$. Then, using (C.1) and (C.3), one gets $v^Y/V = \delta L^Y$.

Finally, the initial condition becomes $\rho + \varepsilon g_Y = \delta L^Y + \delta L^R$. Since $L = L^Y + L^R$ and $\delta L^R = g_Y$, we obtain

$$g_Y = \frac{\delta L - \rho}{\varepsilon}, \tag{C.7}$$

that is exactly the standard first best solution in this model.

Now we can compare the rate of growth obtained in an economy with complete markets and imperfect competition (see g_Y^{IC} given by (44)) and the first best one (g_Y^{FB} given by (C.7)). One gets

$$\begin{aligned} g_Y^{IC} - g_Y^{FB} &= \frac{\delta L/2\sqrt{\alpha} - \rho}{\varepsilon + \frac{1/\sqrt{\alpha}-1}{2}} - \frac{\delta L - \rho}{\varepsilon} \\ &= \frac{\frac{\delta L \varepsilon}{2\sqrt{\alpha}} - \rho\varepsilon - \delta L\varepsilon - \frac{\delta L}{2}\left(\frac{1}{\sqrt{\alpha}} - 1\right) + \rho\varepsilon + \frac{\rho}{2}\left(\frac{1}{\sqrt{\alpha}} - 1\right)}{\varepsilon\left(\varepsilon + \frac{1/\sqrt{\alpha}-1}{2}\right)} \\ &= \frac{\delta L(\varepsilon - 2\varepsilon\sqrt{\alpha} - 1 + \sqrt{\alpha}) + \rho(1-\sqrt{\alpha})}{\varepsilon(2\varepsilon\sqrt{\alpha} + 1 - \sqrt{\alpha})} \\ &= \frac{\delta L\varepsilon(1 - 2\sqrt{\alpha}) + (\rho - \delta L)(1 - \sqrt{\alpha})}{\varepsilon(2\varepsilon\sqrt{\alpha} + 1 - \sqrt{\alpha})} \end{aligned}$$

The denominator of this expression is positive. Let us assume $(\delta L - \rho)$ positive, that is to say g_Y^{FB} positive. Then, if $\alpha > 1/4$, we have $1 - 2\sqrt{\alpha} < 0$, and thus $g_Y^{IC} < g_Y^{FB}$ if $\varepsilon < \tilde{\varepsilon}$, and $g_Y^{IC} > g_Y^{FB}$ if $\varepsilon > \tilde{\varepsilon}$, where $\tilde{\varepsilon} = \frac{(1-\sqrt{\alpha})(\delta L - \rho)}{\delta L(1-2\sqrt{\alpha})}$.

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