Competition between banks and channels of monetary policy

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Abstract

This paper introduces strategic competition between banks into a standard keynesian model. Banks offer horizontally differentiated goods on the two markets of deposits and loans. At the two symmetric Nash equilibria, the three interest rates (on loans, deposits and bonds) are positively correlated. We study the effects of the fiscal and monetary policies under two diffrent assumptions : the central bank pegs the monetary base or it pegs the interest rate. We show that the model exhibits several non standard results.

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Résumé

La caractéristique essentielle de cet article est de prendre en compte explicitement la concurrence imparfaite entre les banques à l'intérieur d'un modèle keynésien standard. Les banques offrent des biens différenciés horizontalement sur les deux marchés des dépôts et des crédits. Aux deux équilibres de Nash symétriques, les troix taux d'intérêt (sur les crédits, les dépôts et les titres) sont corrélés positivement. L'étude des effets des politiques budgétaire et monétaire est alors menée en considérant deux cas : ou bien la banque centrale fixe la base monétaire, ou bien elle fixe le taux d'intérêt. Dans chacun des cas, le modèle conduit à des résultats non standards.

1 Introduction

Monetary policy influences the economy by the intermediary of the money and credit channels. Banks are major actors in both channels. On the one hand, they borrow and lend on the bonds market, on the other hand, they collect deposits from households and lend to firms. Thus, banks intervene in the determination of three interest rates (on bonds, deposits and loans) which play an important role in the monetary transmission mechanism. We study this influence in the context of a standard Keynesian macro-economic model, under the assumption that banks are strategic.

The model is presented in section 2. In several respects it is close to Bernanke-Blinder (1988), the main difference is that we study imperfect competition between banks in the deposits and loans markets. By explicitly studying imperfect competition between banks in a macroeconomic framework, we endeavour to bring closer standard macro–economic models and the growing literature on channels of monetary policy (see for instance Bernanke-Gertler (1995)). In standard macro-economic models, such as IS-LM, there are three types of agents (households, firms, government) and three goods (output, money, bonds). To study the strategic behavior of banks, and its consequences on the money and credit channels, we consider five types of agents (households, firms, government, banks, and the central bank) and five goods (output, deposits, loans, monetary base, bonds). Both on the loans and on the deposits markets, banks are assumed to offer horizontally differentiated goods, and to strategically set interest rates. In the symmetric Nash equilibrium of this game, the three interest rates (on loans, deposits and bonds) are positively correlated.

In the following sections of the paper we plunge this Nash equilibrium in a macro-economic setting, under two types of monetary policy : pegging the monetary base or pegging the interest rate.

In section 3 we assume that the central banks pegs the monetary base. This is similar to IS-LM. Bernanke and Blinder (1988) refer to this model as the CC-LM model, because the CC curve (along which the two markets of goods an loans are simultaneously equilibrated) is substituted for the IS curve. In contrast with Bernanke and Blinder (1988), in the present model, the willingness of households to deposit their savings at the bank (i.e., the demand for deposits) is assumed to be increasing in the interest rate served on deposits. This plausible assumption can lead to surprising results. If the demand for deposits is not very sensitive to the rate on deposits, the standard results are preserved. As this sensitivity increases, monetary policy becomes more efficient while the fiscal policy becomes less efficient. If the reactiveness of the demand for deposits to the interest rate on deposits is large, the LM curve slopes downward. In this case, the effects of the monetary and fiscal policies depend on the relative slopes of the IS and LM curves, and they are non standard. For instance, if LM is less sloped than IS (in absolute value), an increase in government purchases leads to a recession. And if LM is more sloped than IS, an increase in the money supply raises the interest rates and lowers output : there is a liquidity trap effect.

In section 4, we assume that the central bank pegs the interest rate on the interbank market. Then the monetary base becomes endogenous and the LM curve disappears. Here also the model exhibits some non standard results. In particular, increases in government spendings have no crowding out effects on private spendings since the interest rates are unchanged. In fact the output expansion leads to the creation of new money by banks. Since this money is also a new resource for banks, they can lower their borrowings (or increase their loans) on the financial market, which can be reequilibrated with an unchanged interest rate.

2 The model

We consider a closed economy with five types of agents : firms, households, banks, the central bank and the government. Output Q can be used for consumption C, investment I and government spending G. Firms can finance their investment either by issuing bonds B^e on financial market or by using banks loans L. Households receive the income Q, pay taxes T, and can hold three assets : currency Cu, deposits D and bonds B^m. Banks receive households deposits D and must hold reserves $R = \tau D$ (τ is the reserves ratio). They lend L to firms. Their (net) borrowings on financial market are B^b. They directly borrow H from the central bank on the interbank market. The central bank produces currency Cu, holds banks reserves R, lends H to banks and can lend L^g to government (seignoriage). Finally, government can finance its spendings by taxes T, bonds B^g and direct borrowings from the central bank L^g. We could assume that households directly borrow from banks to buy durable items and houses. This assumption would reinforce the credit channel mechanism.

As usual in keynesian short-term models, we assume that the price of the good is fixed (normalized to one) and that there is an excess supply on the good market. We could distinguish a first interest rate, r_B, on financial market and another, ρ, on interbank market. Since banks can arbitrage between the two markets, we assume $\rho = r_B$.

2.1 Banks

There are n identical banks ($j = 1, \ldots, n$). For each bank j, deposits are denoted by D_j, reserves by $R_j = \tau D_j$, net borrowings on the financial market by B_j^b, direct borrowings from the central bank by H_j, and loans by L_j.

We assume that deposits D_j and loans L_j are substitute differentiated goods between banks. For deposits, this assumption seems rather natural because banks are spatially dispersed, which creates a differentiation due to transportation costs. For loans, there are many characteristics that lead to differentiation : flexibility of usage and reimbursement, application procedure, default risk insurance, ... In order to formalize this differentiation, we assume that the demand of loans to bank j is

$$L_j^d = L_j^d(\underset{+}{r_L^1}, \underset{+}{r_L^2}, \ldots, \underset{-}{r_L^j}, \ldots, \underset{+}{r_L^n}, \underset{+}{r_B}) \tag{1}$$

where $+$ and $-$ are the signs of partial derivatives, and where r_L^j is the interest rate of bank's j loans, for $j = 1, \ldots, n$. In the same way, we assume that the demand of deposits to bank j is

$$D_j^d = D_j^d(\underset{-}{r_D^1}, \underset{-}{r_D^2}, \ldots, \underset{+}{r_D^j}, \ldots, \underset{-}{r_D^n}, \underset{-}{r_B}) \tag{2}$$

where r_D^j is the interest rate of bank's j deposits, for $j = 1, \ldots, n$.

Then each bank j chooses interest rates r_D^j and r_B^j that maximize its profit

$$\pi_j = r_L^j L_j - r_D^j D_j - r_B(B_j^b + H_j) - C_L(L_j) - C_D(D_j)$$

where $C_L(L_j)$ and $C_D(D_j)$ are costs functions. Since $B_j^b + H_j = L_j + R_j - D_j = L_j - D_j(1 - \tau)$, we have

$$\pi_j = L_j(r_L^j - r_B) + D_j((1 - \tau)r_B - r_D^j) - C_L(L_j) - C_D(D_j) \tag{3}$$

Using (1) and (2) we obtain the following first order conditions

$$\begin{aligned} L_j + (r_L^j - r_B - C_L'(L_j))\frac{\partial L_j^d}{\partial r_L^j} &= 0 \\ -D_j + ((1-\tau)r_B - r_D^j - C_D'(D_j))\frac{\partial D_j^d}{\partial r_D^j} &= 0 \end{aligned}$$

where $C_L'(L_j)$ and $C_D'(D_j)$ are marginal costs.

Denoting by $\epsilon_{jj} = \frac{-\partial L_j^d}{\partial r_L^j}\frac{r_L^j}{L_j^d}$ and $\eta_{jj} = \frac{\partial D_j^d}{\partial r_D^j}\frac{r_D^j}{D_D^j}$ the direct price elasticities of loans and deposits demand, these conditions can be rewritten

$$\frac{r_L^j - r_B - C_L'(L_j)}{r_L^j} = \frac{1}{\epsilon_{jj}} \tag{4}$$

$$\text{and} \quad \frac{(1-\tau)r_B - r_D^j - C_D'(D_j)}{r_D^j} = \frac{1}{\eta_{jj}} \tag{5}$$

To obtain simple results, we assume that the functional forms of demands $L_j^d()$ and $D_j^d()$ are the same for all banks[1]. In the same way, we assume constant marginal costs : $C_L'(L_j) = h_L$ and $C_D'(L_j) = h_D$. Then we can consider the two symmetric Nash equilibria on the two markets for loans and deposits. Denoting by ϵ and η the elasticities of loans and deposits demands, we obtain from (4) and (5) the two equilibrium interest rates[2] :

$$r_L = \frac{r_B + h_L}{1 - 1/\epsilon} \tag{6}$$

$$\text{and} \quad r_D = \frac{(1-\tau)r_B - h_D}{1 + 1/\eta} \tag{7}$$

These results show how the competition betweeen banks leads to a positive correlation between the three interest rates on loans, deposits and bonds.

[1]This assumption is usually made in the standard differentiation models, as for instance Hotelling or Salop.

[2]If we consider that loans and deposits are two homogeneous products and that banks are price takers on these two competitive markets, (6) and (7) become $r_L = r_B + h_L$ and $r_D = (1-\tau)r_B - h_D$.

2.2 Firms and households

Firms produce the output Q which is used for consumption C, investment I and government spendings G. Since they can finance their investments either from financial markets or from banks loans, we assume that I is a decreasing function of interest rates, namely

$$I = I(\underset{-}{r_L^1}, \ldots, \underset{-}{r_L^j}, \ldots, \underset{-}{r_L^n}, \underset{-}{r_B}) \tag{8}$$

Moreover, the demands of loans to any bank j is given by (1) (see above).

If all the interest rates of loans are equal ($r_L^j = r_L, \forall j$), which is the case in the symmetric Nash equilibrium, we assume that investment I is given by[3] :

$$I = I(\underset{-}{r_L}, \underset{-}{r_B}) \tag{9}$$

In the same case, the total loans demand $L^d = \sum_{j=1}^{n} L_j^d$ is given by :

$$L^d = L^d(\underset{-}{r_L}, \underset{+}{r_B}) \tag{10}$$

Households receive income Q from firms, and their consumption is :

$$C = C(\underset{+}{Q - T}) \tag{11}$$

where T are taxes.

They hold three assets : currency, deposits and bonds. We assume that these assets are substitute in their portfolio. More precisely, if all the interest rates on deposits are equal ($r_D^j = r_D, \forall j$), which is the case in the symmetric Nash equilibrium on deposits market, we have

[3]For instance, it is an implicit assumption in Bernanke-Blinder (1988). More generally, I could represent housing and durable expenditures. See for instance Mishkin (1995).

$$Cu = Cu(\underset{+}{Q}, \underset{-}{r_D}, \underset{-}{r_B}) \tag{12}$$

$$\text{and} \quad D = D(\underset{+}{Q}, \underset{+}{r_D}, \underset{-}{r_B}) \tag{13}$$

Remark : in a more standard way, we could assume that the total demand of money $M^d = Cu + D$ is a function of Q, r_D and r_B. For instance we could assume $C_u + D = M^d(\underset{+}{Q}, \underset{+}{r_D}, \underset{-}{r_B})$ with $Cu = cM^d$, and thus $D = (1-c)M^d, 0 < c < 1$. This formulation would lead to the usual monetary base mutiplier. On the contrary, under assumptions behind (12) and (13), there is no constant multiplier between money $Cu + D$ and base $M_0 = Cu + R = H + L^g$.

2.3 Equilibria

There are five goods to the model : output (Q), deposits (D), loans (L), monetary base ($M_0 = Cu + R$), and bonds (B). Using the Walras law, we can eliminate bonds and we obtain the four following equilibrium equations :

$$\begin{aligned} Q &= C(Q-T) + I(r_L, r_D) + G \\ r_D &= \frac{(1-\tau)r_B - h_D}{1 + 1/\eta} \\ r_L &= \frac{r_B + h_L}{1 - 1/\epsilon} \\ M_0 &= H + L^g = Cu(Q, r_D, r_B) + \tau D(Q, r_D, r_B) \end{aligned}$$

After substitution for r_D and r_B, this leads to

$$Q = C(Q-T) + I\left(\frac{r_B + h_L}{1 - 1/\epsilon}, r_B\right) + G \tag{14}$$

$$H + L^g = Cu\left(Q, \frac{(1-\tau)r_B - h_D}{1 + 1/\eta}, r_B\right) + \tau D\left(Q, \frac{(1-\tau)r_B - h_D}{1 + 1/\eta}, r_B\right) \tag{15}$$

This model can be studied under two assumptions. On the one hand, we can assume that authorities peg the monetary base M_0. Then the system (14)-(15)

determines Q and r_B, and we obtain r_L and r_D from (6) and (7). On the other hand, we can assume that monetary policy consists in pegging the interest rate $\rho = r_B$ in the interbank market. In this case, the system (14)-(15) determines Q and H, and also r_B and r_D from (6)-(7) : clearly, the monetary base is now endogenous.

3 Monetary policy by pegging monetary base

We present here the main results (see the appendix for computations).

3.1 The LM curve

Equation (14) implicitly defines a decreasing relation between Q and $r_B = \rho$ that can be called (CC) curve. In some sense, in our model, CC plays a role similar to that of the standard IS curves, On each point of this curve, the two markets for the output and for loans are simultaneously equilibrated (see for instance Bernanke-Blinder (1988)).

Equation (15) defines the LM curve that can be upward or downward sloping. In fact, the slope of LM is positive if and only if

$$A = \left(\frac{\partial Cu}{\partial r_B} + \tau\frac{\partial D}{\partial r_B} + \frac{\partial Cu}{\partial r_D}\frac{1-\tau}{1+1/\eta}\right) + \frac{\partial D}{\partial r_D}\frac{\tau(1-\tau)}{1+1/\eta}$$

is negative, i.e. if $\frac{\partial D}{\partial r_D}$ is not too large (note that the first term in brackets is negative). The economic intuition is the following : if output Q increases, the money demand $(Cu + D)$ also increases, as well as the demand for the money base $(Cu + \tau D)$. In the standard IS-LM model (where the interest rate r_D is exogenously fixed), equilibrium of the money market requires an increase of $\rho = r_B$. In the present model, if $\frac{\partial D}{\partial r_D}$ is sufficiently large, it is possible that a decrease in $\rho = r_B$, and a corresponding decrease $r_D = \frac{(1-\tau)r_B - h_D}{1+1/\eta}$, leads to a decrease in deposits D, which reequilibrates the money and the base markets : in this case, LM is downward sloping.

3.2 Economic policies : liquidity trap and crowding out

Recall that if we consider the determinant $\Delta = (1 - C_Q)A + JF$ where $J = \frac{\partial I}{\partial r_L}/(1-1/\epsilon) + \frac{\partial I}{\partial r_B} < 0$ and $F = \frac{\partial Cu}{\partial Q} + \tau \frac{\partial D}{\partial Q} > 0$, the effects of economic policies are given by

$$\frac{dQ}{dG} = \frac{A}{\Delta}, \tag{19}$$

$$\frac{dr_B}{dG} = -\frac{F}{\Delta}, \tag{20}$$

$$\frac{dQ}{dM_0} = \frac{J}{\Delta}, \tag{21}$$

$$\frac{dr_B}{dM_0} = \frac{1-C_Q}{\Delta}. \tag{22}$$

Clearly, all these expressions can be positive or negative. Results are summarized in Table 1.

<table>
<tr><td>A</td><td colspan="3">< 0 0 > 0 $\tilde{A}$ > 0</td></tr>
<tr><td>LM slope</td><td>> 0</td><td>< 0</td><td>< 0</td></tr>
<tr><td>Δ</td><td>< 0</td><td>< 0</td><td>> 0</td></tr>
<tr><td>Monetary policy</td><td colspan="2">$\frac{dQ}{dM_0} > 0$ $\frac{dr_B}{dM_0} < 0$</td><td>$\frac{dQ}{dM_0} < 0$ $\frac{dr_B}{dM_0} > 0$</td></tr>
<tr><td>Fiscal Policy</td><td>$\frac{dQ}{dG} > 0$ $\frac{dr_B}{dG} > 0$</td><td>$\frac{dQ}{dG} < 0$ $\frac{dr_B}{dG} > 0$</td><td>$\frac{dQ}{dG} > 0$ $\frac{dr_B}{dG} < 0$</td></tr>
</table>

Table 1 : monetary and fiscal policies

It would be tedious to go into the details of all possible cases. The main results can be presented distinguishing two cases.

1. $A < 0$

   In this case the interest rate r_D does not have a significant impact on deposits. Hence LM is upward sloping. Then the standards results of $IS - LM$ model are qualitatively preserved. Yet we can make some remarks on monetary policy and fiscal policy.

   - In this model with financial intermediation, *monetary policy* impacts the real sector simultaneously through two channels. First, the standard money (or financial) channel : an increase in the monetary base leads to a decrease is the interest rates ρ and r_B. Second, the credit channel : due to the competition between banks, the decrease in the interest rate r_B on financial markets induces a decrease in r_L, the loans interest rate (see (6) above). This competition also leads to a decrease of r_D, the deposits interest rate (see (7) above). In fact, as we have seen above, simultaneous competition on the loans market and the deposits market leads to a positive correlation between the three interest rates on financial market ($\rho = r_B$), loans market (r_L) and deposits markets (r_D).

     Finally, it can be seen that when $\frac{\partial D}{\partial r_D}$ increases (the slope of LM decreases), monetary policy is more efficient : see the proof in appendix. For instance, if the base M_0 increases, as we have seen before, all the interest rates decrease. If r_D has an important impact on D, households want to hold less money and more bonds, which reinforces the decrease of r_B, so that monetary policy is more efficient.

Remark : it is possible to obtain here a more standard credit channel by assuming that banks cannot lend or borrow on financial markets. In this case, their assets are $L + \tau D$ and their liabilities are $D + H$. If we assume that $Cu = cM, 0 < c < 1$, we obtain $M = kM_0$, where $k = 1/(c + \tau(1 - c))$ is the usual monetary base multiplier. Then we have $L = D(1 - \tau) + H = (1 - \tau)(1 - c)kM_0 + H$, where M_0 and H are pegged by the central bank, which thus controls the loan supply L.

- If A is negative, *fiscal policy* has standard effects on output and the interest rate r_B (see table 1). Here also, all interest rates are positively correlated. But, when $\frac{\partial D}{\partial r_D}$ increases, fiscal policy is less efficient (see the proof in appendix).

  For instance, if G increases, there can be a very small increase in Q, because r_B and r_D simultaneously go up. Therefore the money market is reequilibrated and private investment is crowded out.

2. $A > 0$

If the interest rate r_D has a significant impact on deposits, LM is downward sloping and the model exhibits non standard results. As can be seen in Table 1 (see also appendix), there are two cases. The first case is when $A < \tilde{A} = \frac{-JF}{1-C_Q}$: Δ is negative, i.e. CC is (in absolute value) steeper than LM. In the other case, where $A > \tilde{A}$, Δ is positive and CC is less steep than LM. Consider two characteristics cases :

If $0 < A < \tilde{A}$, fiscal policy leads to (see figure 1) :

$$\frac{dQ}{dG} < 0 \qquad \text{and} \qquad \frac{dr_B}{dG} > 0$$

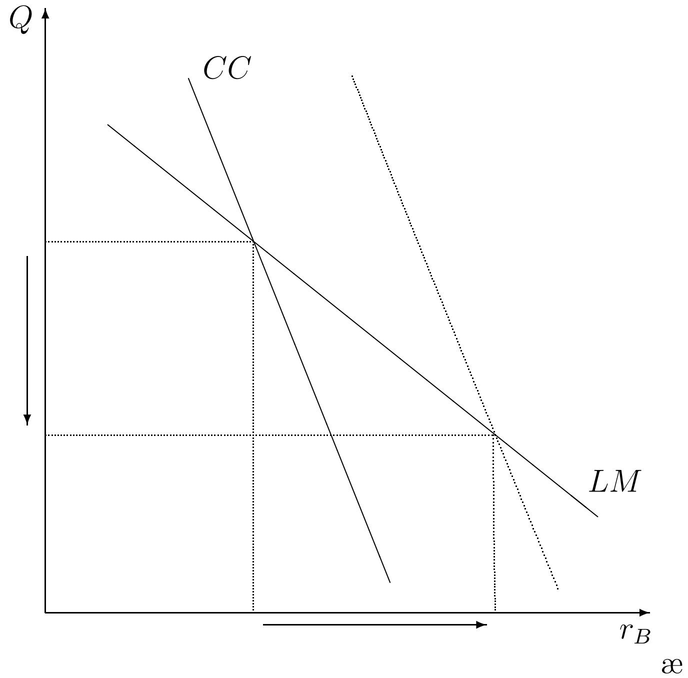

Figure 1 : fiscal policy and strong crowding out

In this case, an increase in G leads to a recession because there is strong crowding out of private demand. In fact, since Q decreases, money demand decreases, thus the monetary base demand also decreases. But, as r_B increases, r_D also increases, and here, the demand of deposits strongly increases. At last, the market of monetary base is reequilibrated.

If $A > \tilde{A}$, monetary policy leads to (see figure 2) :

$$\frac{dQ}{dM_0} < 0 \qquad \text{and} \qquad \frac{dr_B}{dM_0} > 0$$

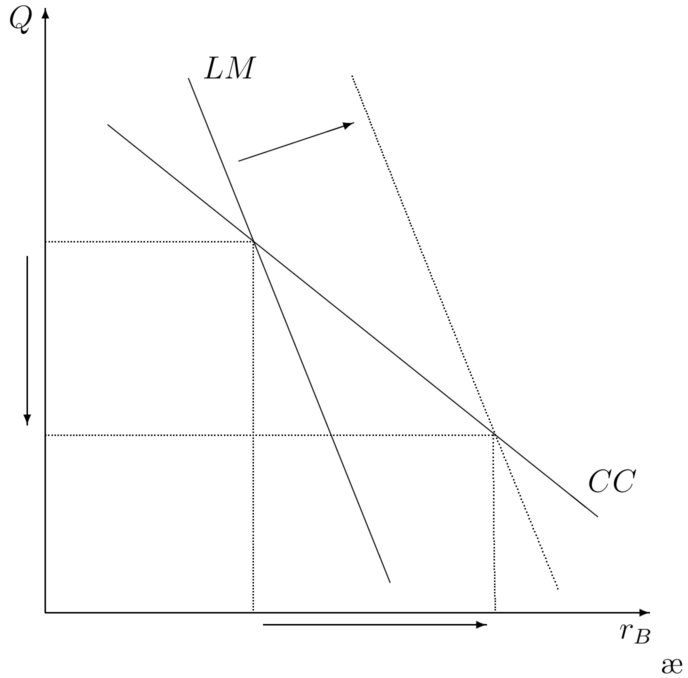

Figure 2 : monetary policy and liquidity trap

In the standard $IS - LM$ model, an increase of monetary base M_0 leads to a decrease in r_B and an increase in Q. Thus money demand increases and the base market is reequilibrated. Here, the same monetary policy leads to a recession and an increase in interest rates : the decrease in Q and the increase in r_B lead to a decrease in money demand, but the increase in r_D has such a strong positive effect on money demand that the monetary base market is reequilibrated. Here we observe a mechanism that can be called the liquidity trap because, on account of the increase of r_D, all the new money stays inside deposits.

4 Monetary policy by pegging interest rate

If the central bank pegs the interest rate ρ on the interbank market, the two endogenous variables given by (14)-(15) are Q and H. Of course r_L and r_D are still given by (6) and (7). The main point here is that *the LM curve disappears* since the monetary base is endogenous. Then it is easy to study monetary and fiscal policies in this context (see computations in appendix).

4.1 Monetary policy

If, for instance, the central bank lowers ρ, there is an increase of I. Therefore there is an increase of Q. As before, due to competition between banks, all the interest rates (r_B, r_L, r_D) increase. Thus monetary policy affects economic activity by the two previous channels : the money channel (r_B decreases) and the credit channel (r_L decreases). Let us notice that the monetary base $C_u + H$ and the quantity of money $C_u + D$ can increase or decrease because the fall of r_D reduces the demand of deposits. In the same way, the loans demand $L^d(r_L, r_B)$ can also increase or decrease because r_L and r_B both decrease.

4.2 Fiscal policy

Fiscal policy leads to very simple but surprising results. The main point here is that *the government-purchases multiplier* $\Delta Q/\Delta G = 1/(1 - C_Q)$ *is maximum* because the interest rates, specially r_B and r_L, are constant. Certainly, this result is trivial in the standard $IS - LM$ model. But in our model where banks create money, it is useful to understand how the bonds market equilibrates after the government borrowings.

Let us assume that there is no new tax and no seignoriage. Then the new expenditures ΔG are financed by a new flow of bonds. Since the interest rates are unchanged, the investment I is constant and we have $\Delta Q = \Delta C + \Delta G$, namely $\Delta Q - \Delta C = \Delta G$: the flow of new bonds is buyed by households.

Let us now consider the consolidated balance sheet of the central bank and the commercial banks. Assets are L^g and L. Liabilities are $M = Cu + D$ and B^b. Since L^g and L are constant, we have $\Delta M + \Delta B^b = 0$, with $\Delta M = -\Delta B^m$ in the households portfolio. Thus we obtain

$$\Delta M = -\Delta B^b = -\Delta B^m \tag{23}$$

where the new endogenous money is nearly given by $\Delta M \simeq \left(\frac{\partial Cu}{\partial Q} + \frac{\partial D}{\partial Q}\right) \Delta Q = \left(\frac{\partial Cu}{\partial Q} + \frac{\partial D}{\partial Q}\right) \frac{\Delta G}{1-C_Q}$.

After the increase in G and Q, households hold fewer bonds ($\Delta B^b < 0$) and more money ($\Delta M > 0$). But this new endogenous money is a new resource for banks which can decrease their borrowings on the financial market ($\Delta B^b < 0$). The financial market can thus be reequilibrated without any variation of the interest rate.

5 Conclusion

In the standard macroeconomic models, it is generally assumed that the government creates money and that lenders and borrowers can have all access to a perfect financial market. In fact, we know that deposits are held by banks, which moreover have an important intermediation role. The model we have presented here introduced these two characteristics of banks activity in a standard general equilibrium framework with five types of agents (firms, households, banks, central bank, government) and five goods (output, deposits, loans, monetary base, bonds). The main result of the competition between banks in the two markets of deposits and loans is that the different interest rates (on financial market, deposits and loans) are positively correlated equilibrium.

This analysis allows to study the effects of fiscal and monetary policies when the central bank can use two types of monetary policy : pegging monetary base of pegging interest rate. Results are often non standard and sometimes surprising.

However, several formulas in the text are "black boxes" (see for instance the costs in bank's profit or the investment demand (8)). At these places of the model, it is probably possible to introduce more formally the modern ideas on the credit channel, in particular by studying in detail the complex relations between firms and banks.

6 Appendix

The basic model is

$$Q = C(Q - T) + I\left(\frac{r_B + h_L}{1 - 1/\epsilon}, r_B\right) + G \tag{14}$$

$$H + L^g = Cu\left(Q, \frac{(1-\tau)r_B - h_D}{1 + 1/\eta}, r_B\right) + \tau D\left(Q, \frac{(1-\tau)r_B - h_D}{1 + 1/\eta}, r_B\right) \tag{15}$$

with

$$r_L = \frac{r_B + h_L}{1 - 1/\epsilon}, \tag{6}$$

$$r_D = \frac{(1-\tau)r_B - h_D}{1 + 1/\eta} \tag{7}$$

and $r_B = \rho$.

6.1 Monetary policy by pegging monetary base

Differentiating the system (14)-(15), we obtain

$$dQ(1 - C_Q) - dr_B J = dG - C_Q dT$$

$$dQF + dr_B A = dM_0$$

where

$$J = \frac{\partial I}{\partial r_L}\bigg/(1 - 1/\epsilon) + \frac{\partial I}{\partial r_B} < 0,$$

$$F = \frac{\partial C_u}{\partial Q} + \tau\frac{\partial D}{\partial Q} > 0,$$

and

$$A = \left(\frac{\partial C_u}{\partial r_B} + \tau\frac{\partial D}{\partial r_B} + \frac{\partial C_u}{\partial r_D}\frac{1-\tau}{1+1/\eta}\right) + \frac{\partial D}{\partial r_D}\frac{\tau(1-\tau)}{1+1/\eta}$$

which can be positive or negative.

The slope of CC is

$$\left.\frac{dQ}{dr_B}\right|_{CC} = \frac{J}{1 - C_Q} < 0 \tag{16}$$

and the LM one is

$$\left.\frac{dQ}{dr_B}\right|_{LM} = \frac{-A}{F} \tag{17}$$

Thus we have

$$\left.\frac{dQ}{dr_B}\right|_{LM} > 0 \Leftrightarrow A < 0 \tag{18}$$

If we denote by $\Delta = (1 - C_Q)A + JF$ the determinant of the differentiated system, we can study the effects of fiscal and monetary policies.

Let us consider a fiscal policy such that $dG = dB^g$. Then we obtain

$$\frac{dQ}{dG} = \frac{A}{\Delta} = \frac{1}{1 - C_Q + JF/A} \tag{19}$$

and

$$\frac{dr_B}{dG} = \frac{-F}{\Delta} \tag{20}$$

If we consider a monetary policy characterized by a variation of the monetary base, we have

$$\frac{dQ}{dM_0} = \frac{J}{\Delta} \tag{21}$$

and

$$\frac{dr_B}{dM_0} = \frac{1 - C_Q}{\Delta} \tag{22}$$

In order to characterize the effects of these policies, it is useful to distinguish two cases : $A < 0$ and $A > 0$.

First case : $A < 0$ (LM has a positive slope)

In this case, $\Delta = (1 - C_Q)A + JF$ is negative since we know that $J < 0$ and $F > 0$. Therefore we have

$$\frac{dQ}{dG} > 0 \; ; \frac{dr_B}{dG} > 0 \; ; \frac{dQ}{dM_0} > 0 \; ; \frac{dr_B}{dM_0} < 0 \; ;$$

Let us examine how the effects of fiscal and monetary policies are affected by the sensitiveness of deposits demand to interest rate r_D.

First, it is clear that, if $\frac{\partial D}{\partial r_D}$ increases, then A increases (the slope of LM decreases). Thus we have

- $\frac{dQ}{dG}$ decreases and $\frac{dr_B}{dG}$ increases, with

$$\lim_{A \to 0} \frac{dQ}{dG} = 0$$

  and

$$\lim_{A \to 0} \frac{dr_B}{dG} = \frac{-1}{J}.$$

- $\frac{dQ}{dM_0}$ increases and $\frac{dr_B}{dM_0}$ decreases, with

$$\lim_{A \to 0} \frac{dQ}{dM_0} = \frac{1}{F}.$$

  and

$$\lim_{A \to 0} \left| \frac{dr_B}{dM_0} \right| = \frac{1 - C_Q}{JF}.$$

  In other words, when $\frac{\partial D}{\partial r_D}$ increases, fiscal policy is less efficient and monetary policy is more efficient.

Second case : $A > 0$ (LM has a negative slope).

In this case, Δ can be negative or positive and we must distinguish two sub-cases according to A is smaller or greater than $\tilde{A} = \frac{-JF}{1-C_Q}$.

1. $A < \tilde{A}$

   In this sub-case, Δ is negative. This can be interpreted by the fact that CC is (in absolute value) more sloped than LM since $A < \frac{-JF}{1-C_Q} \Leftrightarrow \frac{-J}{1-C_Q} > \frac{A}{F}$ (see (16) and (17)). Then we have

- $\frac{dQ}{dG} < 0$ and $\frac{dr_B}{dG} > 0$ for fiscal policy,
- $\frac{dQ}{dM_0} > 0$ and $\frac{dr_B}{dM_0} < 0$ for monetary policy.

2. $A > \tilde{A}$

   In this sub-case Δ is positive, that can be interpreted by the fact that CC is (in absolute value) less sloped than LM. Then we have

   - $\frac{dQ}{dG} > 0$ and $\frac{dr_B}{dG} < 0$ for fiscal policy,
   - $\frac{dQ}{dM_0} < 0$ and $\frac{dr_B}{dM_0} > 0$ for monetary policy.

6.2 Monetary policy by pegging interest rate

From (14)-(15) we obtain

- Monetary policy

$$\frac{dQ}{d\rho} = \frac{\frac{\partial I}{\partial r_L}/(1-1/\epsilon) + \frac{\partial I}{\partial r_B}}{1 - C_Q} < 0$$

$$\frac{dH}{d\rho} = \left(\frac{\partial Cu}{\partial Q} + \tau\frac{\partial D}{\partial Q}\right)\frac{dQ}{d\rho} + \left(\frac{\partial Cu}{\partial r_D}\frac{(1-\tau)}{1+1/\eta} + \frac{\partial Cu}{\partial r_B} + \tau\frac{\partial D}{\partial r_B}\right) + \tau\frac{\partial D}{\partial r_D}\cdot\frac{(1-\tau)}{1+1/\eta}$$

  The first and the second terms are negative. But the third one is positive because $\frac{\partial D}{\partial r_D}$ is positive. Thus $\frac{dH}{d\rho}$ can be positive or negative.

  In the same way, it can be easily verified that $\frac{dM}{d\rho} = \frac{d(C_u+D)}{d\rho}$ can be positive or negative.

- Fiscal policy

$$\begin{aligned} \frac{dQ}{dG} &= \frac{1}{1-C_Q} > 0 \\ \frac{dM}{dG} &= \left(\frac{\partial Cu}{\partial Q} + \frac{\partial D}{\partial Q}\right)\frac{dQ}{dG} > 0 \\ \frac{dH}{dG} &= \left(\frac{\partial Cu}{\partial Q} + \tau\frac{\partial D}{\partial Q}\right)\frac{dQ}{dG} > 0 \end{aligned}$$

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