Discounting and Growth

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In a growing economy, investing in safe projects raises intergenerational inequalities. This deteriorates social welfare because of inequality aversion, as expressed by decreasing marginal utility of consumption. The social discount rate can be interpreted as the minimum rate of return that is necessary to compensate for the increased inequality generated by the investment. For an intuitive precautionary argument, this growth effect is reduced if growth is uncertain. To complete the picture, if the investment raises the collective risk, this discount rate should also contain a risk premium. Recent developments (Weitzman (1998, 2001, 2013), Gollier (2008), Arrow et al. (2013)) converge towards recommending using a smaller discount rate for safe assets maturing later. In this paper, we show that this recommendation applied to the risk free rate relies on the assumption that shocks on the growth rate of consumption exhibit some degree of persistence. We also show that this implies in parallel an increasing term structure for the risk premium. Globally, the risk-adjusted discount rate will have a decreasing term structure only if the asset's beta is small enough.

I. The benchmark asset pricing model

Consider a potentially risky benefit F_t occurring t years from now. Its social value P_0 today is the sure increase in current consumption that has the same effect on intertemporal welfare. Assuming the project to be marginal and using the standard Discounted Expected Utility welfare functional, we obtain

$$P_0 = e^{-\delta t} EF_t u'(c_t)/u'(c_0), \tag{1}$$

where δ is the rate of pure preference for the present, u is the increasing and concave utility function of the representative agent, and c_t is the consumption per capita at date t. Obviously, this can be rewritten as a NPV rule $P_0 = \exp(-r_t t)EF_t$ using discount rate

$$r_t = \delta - t^{-1} \ln \frac{EF_t u'(c_t)}{u'(c_0)EF_t}. \tag{2}$$

We determine the condition under which the discount rate r_t is decreasing with maturity *t*.

The benchmark model is the consumption-based CAPM of Lucas (1978). Let us assume that relative inequality aversion is a positive constant γ, i.e., $u'(c) = c^{-\gamma}$. Let us also assume that the asset's beta is constant β, implying $E\left[F_t \left| c_t \right.\right] = c_t^{\beta}$. Under these restrictions, equation (2) can be rewritten as follows:

$$r_t = \delta - t^{-1} \ln \frac{E \exp\left((\beta - \gamma)\ln(c_t / c_0)\right)}{E \exp\left(\beta \ln(c_t / c_0)\right)}. \quad (3)$$

Because $E \exp ax = \exp\left(aEx + 0.5a^2 Var(x)\right)$ when x is normally distributed, the expectations appearing in this equation have an analytical expression when c_t is lognormal. This implies that, under this restriction, equation (3) is equivalent to $r_t = r_{ft} + \beta \pi_t$, where

$$r_{ft} = \delta + \gamma \frac{E \ln c_t / c_0}{t} - \frac{1}{2}\gamma^2 \frac{Var\left(\ln c_t / c_0\right)}{t} \quad (4)$$

is the risk free rate and

$$\pi_t = \gamma \frac{Var\left(\ln c_t / c_0\right)}{t} \quad (5)$$

is the CCAPM systematic risk premium. We see from (4) and (5) that the risk free rate is the sum of the rate of impatience δ, a growth effect which is proportional to the annualized expectation of the cumulative change in log consumption, and a negative precautionary effect which is proportional to its annualized variance, as is the risk premium. If we suppose that consumption follows a geometric Brownian motion with drift μ and volatility σ, then the annualized expectation and the annualized variance of the cumulative change in log consumption are constant across maturities. In particular, we have that

$$\frac{Var\left(\ln c_t / c_0\right)}{t} = \frac{\sigma^2 t}{t} = \sigma^2. \quad (6)$$

This implies that the term structures of the risk free rate and of the risk premium are flat in that classical CCAPM case:

$$r_{ft} = \delta + \gamma\mu - 0.5\gamma^2\sigma^2 \quad (7)$$

$$\pi_t = \gamma\sigma^2. \quad (8)$$

In the absence of serial correlation in the growth rate of consumption, the accumulation of uncertainty affecting c_t when *t* goes from zero to infinity should induce us to exponentially discount sure benefits at a rate reduced by the constant precautionary term $0.5\gamma^2\sigma^2$. Similarly, the accumulation of uncertainty affecting the benefits of a project with a constant β should induce us to exponentially discount their expected value at a rate that is increased by a constant premium $\beta\gamma\sigma^2$ above the risk free rate.

II. Persistent shocks

From the above results, it can easily be anticipated that if uncertainty accumulates faster than in the geometric Brownian case, then prudence justifies biasing the evaluation of safe projects towards those which generate more sure benefits for more uncertain maturities, i.e., for distant maturities. This is implemented through a decreasing term structure of the risk free discount rate. Uncertainty accumulates faster than in the classical case if shocks to changes in log consumption exhibit some degree of persistency, i.e., if these changes exhibit positive auto-correlation. This implies that

$$\frac{Var\left(\ln c_t / c_0\right)}{t} \geq Var\left(\ln c_1 / c_0\right), \qquad (9)$$

so that, by (4), the long risk-free rate is smaller than the short one. In Gollier (2012), we illustrate this by introducing mean-reversion or parameter uncertainty in the consumption process. Outside the Gaussian world examined here, Markov switches processes yield the same result.

We believe that any modeling of long-term economic growth should recognize the persistence of shocks at different frequencies. Remember in particular that the trend of economic growth was basically zero over the last 7 millenniums, except for the last two centuries. Bansal and Yaron (2004) for example document persistency at higher frequencies.

The persistence of shocks to growth magnifies the long term macroeconomic uncertainty. It also magnifies the additional risk generated by any long-term investment with a positive beta. By risk aversion, this implies an increasing term structure of the risk premium, as can be inferred from combining equations (5) and (9).

Although most papers mentioned above focused on the risk free discount rate, the recommendation to use a decreasing term structure for the discount rate was often meant to be true for all investment projects, in particular in climate change. But most long term projects yield highly uncertain benefits. Whether one should actually use a decreasing or increasing term structure for their associated discount rate depends upon their beta. It is immediate from combining definition $r_t = r_{ft} + \beta\pi_t$ with equations (4), (5) and (9) that it should be decreasing only if the project's β is smaller than $\gamma / 2$. For larger betas, the risk aversion effect (increasing term structure of the risk premium) dominates the precautionary effect (decreasing term structure of the risk free rate), so that the associated discount rate should in fact be increasing with maturity.

III. Interdependent growth rates

In the previous sections, we assumed that future consumption is lognormally distributed. This specification does not allow for fat tails and catastrophes for example. To generalize our findings, we need to do a small detour to stochastic dominance theory. Consider a pair of random variables (x_1, x_2). We compare two distribution functions *F* and *G* representing our beliefs on (x_1, x_2). Following Epstein and Tanny (1980), we say that there is "more interdependence" between x_1 and x_2 under *G* than under *F* if and only if they have the same marginal cdfs, and $G(a_1, a_2) \geq F(a_1, a_2)$ for all $(a_1, a_2) \in \mathbb{R}^2$. An increase in interdependence can always be obtained through a sequence of symmetric transfers of probability masses from any two points A and B in $\mathbb{R}^2$ to the corresponding points on the main diagonal characterized by the rectangle defined by (A,B). It is easy to check that an increase in interdependence raises the covariance and the correlation indices, but is stronger than these conditions. Epstein and Tanny showed that an increase in interdependence is necessary and sufficient to raise the expectation of any supermodular function *h* of (x_1, x_2). Applying this result to functions *h* that are a function of $x_1 + x_2$, we obtain that an increase in interdependence in (x_1, x_2) raises $Eh(x_1 + x_2)$ if and only if h is convex. In other words, an increase in interdependence in (x_1, x_2) implies an increase in risk of $x_1 + x_2$.

We now go back to our discounting problem. Let us decompose period $[0, t]$ into two subperiods $[0, \tau]$ and $]\tau, t]$ for some date $\tau \in]0, t[$. Let $x_1 = \ln c_\tau / c_0$ and $x_2 = \ln c_t / c_\tau$ denote the change in log consumption in the two subperiods. As explained above, an increase in interdependence among them generates an increase in risk for $\ln c_t$ without affecting the risk on $\ln c_\tau$.

Let us first examine the consequence of this increase in long-term risk on the long-term discount rate. Taking $F_t = 1$ in equation (2) implies that the long discount rate is reduced by this if $u'(\exp x)$ is convex, or equivalently if relative prudence $-cu'''(c)/u''(c)$ is larger than unity. Because the short discount rate is not affected, this implies that the slope of the term structure of the risk free rate measured by $r_{ft} - r_{f\tau}$ is reduced. Thus, we have proved the following result, which is also in Gollier (2012).

Proposition 1: *An increase in serial interdependence in growth rates reduces the slope of the term structure of the risk free* *discount rate if and only if relative prudence* $-cu'''(c)/u''(c)$ *is uniformly larger than unity.*

If the benchmark has a flat term structure, as is the case under constant relative risk aversion and no serial correlation, this proposition provides a justification for a decreasing term structure when the growth process is positively serially correlated. In the special case $u'(c) = c^{-\gamma}$, relative prudence equals $\gamma + 1$, which is larger than unity.

The analysis of the slope of the risk-adjusted discount rate when benefits have a non-zero beta is more complex because the two terms $EF_t u'(c_t) = E\exp(\beta - \gamma)\ln c_t$ and $Ec_t^{\beta} = E\exp\beta\ln c_t$ appearing in equation (2) are affected by the serial interdependence of growth rates. The implied increased in risk in $\ln c_t$ raises both expectations, so that its effect on the risk-adjusted discount rate is ambiguous, as explained for example by Gollier (1995) and Abel (2002). Using second-order Taylor expansions à la Arrow-Pratt to approximate these expectations yields the following result.

Proposition 2: *Suppose that* $u'(c) = c^{-\gamma}$, $F_t = c_t^{\beta}$ *and that the risk on* c_t *is small. An increase in serial interdependence in growth rates raises the slope of the term structure of the risk-adjusted discount rate if and only* β *is larger than* $\gamma/2$.

IV. Concluding remarks

If the uncertainty on future consumption accumulates faster than in the geometric Brownian case, for example because shocks to the growth rate are persistent, then the rate at which one should discount future safe cash flows has a decreasing term structure. This result is driven by the assumption that the representative agent is prudent. But by risk aversion, the risk premium has an increasing term structure in this world. We showed that this implies that risk-adjusted discount rates have an increasing term structure if the beta of the future benefit is larger than half the relative risk aversion. This also implies that economists should devote more energy to estimate the beta of long-dated investments, for example in the case of environmental policies. In particular, we believe that it is now crucial to focus the debate related to the social cost of carbon on the “climate beta”. Do we believe that most of the benefits of fighting climate change will materialize when future consumption will be large (for example because of the large associated level of emissions), or when future consumption will small?

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