Forced saving, redistribution and nonlinear social security schemes[1]

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Abstract

This paper studies the design of a nonlinear social security scheme in a society where individuals differ in two respects: productivity and degree of myopia. Myopic individuals may not save "enough" for their retirement because their "myopic self" emerges when labor supply and savings decisions are made. The social welfare function is paternalistic: the rate of time preference of the far-sighted (which corresponds to the "true" preferences of the myopics) is used for both types. We show that the paternalistic solution does not necessarily imply forced savings for the myopics. This is because paternalistic considerations are mitigated or even outweighed by incentive effects. Our numerical results suggest that as the number of myopic individuals increases, there is less redistribution and more forced saving. Furthermore, as the number of myopic increases, the desirability of social security (measured by the difference between social welfare with and without social security) increases.

1 Introduction

Social security systems typically fulfill several functions. They force myopic individuals (who are inclined to save less than what is reasonable given their life expectancy) to save an appropriate amount. They also contribute to redistributing resources. Finally, they provide insurance, in particular for the longevity risks by providing an annuity.

In this paper we focus on the first two functions. The "forced saving" argument is rarely disputed. What is disputed is whether one needs social security to ensure that everyone saves enough; after all the government needs only to require that individuals save the desired amount. This would be a valid objection if first-best redistribution were available. However, in a world of asymmetric information, where productivity and degree of myopia are not publicly observable there may well be a case for a social security scheme that pursues both functions.

We adopt a two-period model: individuals work in the first period and retire in the second. They save part of their earnings for their consumption in retirement. Individuals differ in two respects, productivity on the one hand and degree of myopia on the other hand. Myopic individuals may not save "enough" for their retirement because their "myopic self" emerges when labor supply and savings decisions are made. In other words, they use a discount factor which does not reflect their "true" preferences.[1] When they retire, they regret their earlier decisions. Consequently, if they could be forced to save a certain amount, they would be in favor of such an imposed commitment. We assume that the government has a paternalistic view and wants to help these individuals to overcome their myopia problem; in measuring social welfare it uses the rate of time preference of the individuals whose myopic self never emerges. *Ex post*, myopic individuals will be grateful to the government for such forced saving.[2]

In our model, both productivity and time preference are not observable. The government will design a tax transfer policy based on what is observable: gross earnings, disposable income and saving.

[1] For earlier work on this, see Feldstein (1985), Imrohoroglu *et al.* (2003) and recently Diamond and Koszegi (2003).

[2] Without this late realization, there would be little ground for paternalism.

Anticipating the results, we show that the paternalistic solution does not necessarily imply forced savings for the myopics. This is because paternalistic considerations are mitigated or even outweighed by incentive effects. In other words, the interaction between paternalism and redistribution is rather complex and may bring about results which are in contradiction to conventional wisdom. Our numerical results suggest that as the number of myopic individuals increases, there is less redistribution and more forced saving. Furthermore, as the number of myopic increases, the desirability of social security (measured by the difference between social welfare with and without social security) increases.

This paper is part of an ongoing research on social security and myopia. It focuses on non-linear schemes. In companion papers, Cremer *et al.* (2007, 2008b), we study the same problem using a linear schedule and taking both a normative and a positive viewpoint. The closest predecessor in the literature is probably Diamond (2003, Ch. 4). He studies income taxation with time-inconsistent preferences in a two period model which provides arguments in favor of a progressive social security system. In his setting myopia only affects labor supply. We assume that myopia also affects savings decisions and provide an explicit model of optimal social security scheme with individuals differing in both productivity and far-sightedness. In another closely related paper Tenhunen and Tuomala (2007) also analyze the design of nonlinear pension schemes with myopic individuals. There are, however, some important differences between our paper and theirs. First and foremost, our analytical results are both more precise and more general. Second, the questions dealt with in the simulation are quite different. For instance, they concentrate on comparison between paternalistic and non-paternalistic case while we study the impact of the degree of myopia and the proportion of myopics. Furthermore, they concentrate on inequality in consumption measured with Gini and Lorenz criteria (which is not consistent with the utilitarian paternalistic welfare function they use) while we look at inequality in *utility* levels.

The rest of the paper is organized as follows. The basic model is introduced in the next section. Then the second-best optimum is discussed first in general and then in a three-type setting. Section 4 provides numerical simulations.

2 The model

2.1 Myopic and farsighted individuals

Individuals' utility is given by

$$U(c_i, d_i, l_i) = u(c_i) + \beta u(d_i) - v(\ell_i), \tag{1}$$

where c_i and d_i are first- and second-period consumption while ℓ_i is labor supplied in the first period. Observe that we can think of ℓ_i as the retirement age. Gross earnings are given by $y_i = w_i \ell_i$ and are obtained in the first period. Individuals differ in their wage rate, $w_i \in \{w_L, w_H\}$ with $w_L < w_H$. Individuals can save part of first period income at a zero interest rate.

For all individuals the "true" time-discount factor is given by β. However not all individuals will make their labor supply and consumption decisions according to this parameter. For some individuals, their "myopic self" emerges when labor supply and saving are chosen. They take all decisions according to a time discount parameter $\beta_0 < \beta$. Formally, savings and labor supply are chosen according to

$$U_i(c_i, d_i, l_i) = u(c_i) + \beta_i u(d_i) - v(\ell_i). \tag{2}$$

For myopic individuals we have $\beta_i = \beta_0$, while $\beta_i = \beta$ holds for the far-sighted.[3]

To sum up, there are four types of individuals as represented on Figure 1. Type-1 and type-3 individuals are the far-sighted with low and high abilities respectively. Type-2 (low ability) and type-4 (high ability) individuals on the other hand are myopic. Total population size is normalized at one and the proportion of type $i = 1, \ldots, 4$ individuals is denoted by π_i. In the analytical second-best part we provide general expressions but for their interpretation concentrate on a three type setting. The fully-fledged four type case is then solved in numerical examples (Section 4).

[3] These preferences are intertemporally additive. Cremer *et al.* (2008) use preferences in which the utility of the old depends on the level of consumption they had when young. In other words there is "habit formation". This specification, coupled with myopia, can lead to unexpected late retiring or even "unretiring".

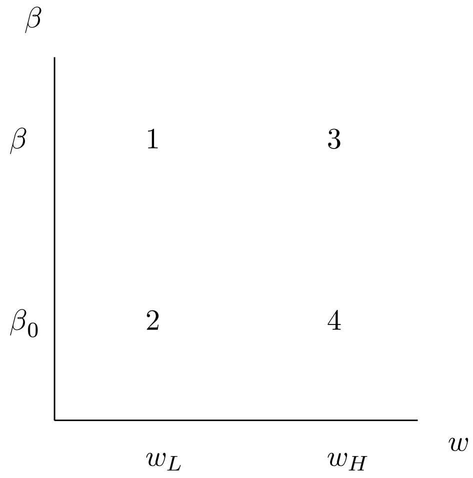

Figure 1: Types of individuals

2.2 First-best solution

We take a paternalistic approach and consider the utilitarian optimum *based on individuals' true preferences*. The corresponding Lagrangian expression is given by

$$\mathcal{L}_{FB} = \sum_i \pi_i \left[u(c_i) + \beta u(d_i) - v\left(\frac{y_i}{w_i}\right) \right] - \mu \sum_i \pi_i (c_i + d_i - y_i),$$

where μ is the Lagrangian multiplier associated with the budget constraint. This yields

$$\begin{aligned} c_1 &= c_2 = c_3 = c_4, \\ d_1 &= d_2 = d_3 = d_4, \\ \ell_1 &= \ell_2 < \ell_3 = \ell_4. \end{aligned}$$

With separable preferences the utilitarian solution implies that consumption levels are equalized across types and periods and that the able individuals work more than the unable. This first-best allocation can be decentralized by using two instruments. First, we need lump-sum transfers to redistribute from high to low productivity individuals. In addition a "Pigouvian" (corrective) subsidy at rate $1 - \beta_0/\beta$ on the savings of the myopics is required to induce them to save the appropriate amount. As an alternative to the savings subsidy, one can also use a pension scheme to force myopics individuals to save. Either way, in a full information setting, there is no conflict between paternalism and redistribution. The two objectives are addressed by separate instruments. Any redistributive impact of corrective policies can be neutralized through lump-sum transfers.

3 Second-best solution with nonlinear schemes

In reality this solution may not be feasible because some key variables are not publicly observable. We adopt the standard assumption in the Mirrlees' model of optimal income taxation according to which an individual's wage and labor supply are not observable, while gross earnings $y_i = w_i \ell_i$ are observable. In addition we assume that an individual's degree of myopia is not observable either. We assume for simplicity that saving is observable so that the (possibly nonlinear) pension benefits scheme is based on both y_i and s_i. The case where saving is not observable is more complicated but yields the same main results.[4]

To interpret the properties of the optimal allocations derived below, let us now look at the problem of implementing a given allocation.

3.1 Implementation

Recall that the government observes s_i and y_i and can tax the individuals non-linearly on the basis of these two variables. The policy instruments are $T(y_i, s_i)$ and $p(y_i, s_i)$ corresponding to the payroll tax and the pension benefit, respectively. Taking these two policy instruments into account the individual problem is given by

$$\max_{y_i, s_i} u(y_i - s_i - T(y_i, s_i)) + \beta_i u(s_i + p(y_i, s_i)) - v\left(\frac{y_i}{w_i}\right).$$

[4] A technical appendix analyzing this case is available from the authors (or can be found on Helmuth Cremer's webpage at www.idei.fr). Yet another specification is to assume that the tax on savings is restricted to be linear (because only anonymous transactions are observable). One can show that any allocation that can be achieved with observable savings can also be implemented with a linear tax. To do this it is sufficient to set a very high tax rate so that *private* savings is completely crowded out and to control second period consumption through the pensions scheme.

the first-order conditions

$$\begin{aligned}\frac{u'(c_i)}{u'(d_i)} &= \beta_i \frac{1+p_s(y_i,s_i)}{1+T_s(y_i,s_i)} = \beta_i(1-\Theta_i), && (3)\\ \frac{v'(l_i)}{u'(c_i)} &= w_i\left(1-T_y(y_i,s_i)+\frac{1+T_s(y_i,s_i)}{1+p_s(y_i,s_i)}p_y(y_i,s_i)\right) = w_i(1-\Gamma_i). && (4)\end{aligned}$$

Define

$$\begin{aligned}\Theta_i &= 1-\frac{1+p_s(y_i,s_i)}{1+T_s(y_i,s_i)} = \frac{T_s(y_i,s_i)-p_s(y_i,s_i)}{1+T_s(y_i,s_i)}, && (5)\\ \Gamma_i &= T_y(y_i,s_i)-\frac{1+T_s(y_i,s_i)}{1+p_s(y_i,s_i)}p_y(y_i,s_i), && (6)\end{aligned}$$

which represent the implicit marginal tax (or subsidy) on savings and on labor implied by the tax and pension schemes. When $\Theta_i < (>) 0$ type-i individual faces a marginal subsidy (tax) on savings. When $\Gamma_i > 0$ type-i individual faces a marginal tax on income.

These two wedges have been widely discussed in the theoretical and empirical literature on social security. Early retirement that is observed in many OECD countries is often explained by a positive Γ_i called the implicit tax on prolonged activity. Recall that ℓ_i can be considered here as determining the activity rate or even the retirement age of type i individuals.[5] Insufficient saving for retirement is also often explained by the presence of an implicit tax on saving and the aim of tax breaks for retirement saving is to generate a negative Θ_i.

In this paper we are interested in the design of a social security system summarized by the functions T and p. Such a system can be approached in two ways. First, we can look at net lifetime benefit which are given by $-T(y_i,s_i)+p(y_i,s_i)$.[6] Alternatively, we can concentrate on (dis)incentives to work and save and study the sign of marginal taxes Θ_i and Γ_i. Analytically, we can only deal with the latter. To study the former, we will have to resort to numerical examples.

[5] In other words, people would work for ℓ years and would retire thereafter.

[6] Recall that the interest rate is zero.

3.2 Second-best solution

With the considered information structure feasible allocations must satisfy a set of incentive constraints that take the following form

$$u(c_i) + \beta_i u(d_i) - v\left(\frac{y_i}{w_i}\right) \geq u(c_j) + \beta_i u(d_j) - v\left(\frac{y_j}{w_i}\right), \tag{7}$$

The Lagrangian (Kuhn-Tucker) expression associated with the second-best problem is given by

$$\begin{aligned} \mathcal{L}_{SB} = & \sum_i \pi_i \left[u\left(c_i\right) + \beta u\left(d_i\right) - v\left(\frac{y_i}{w_i}\right) - \mu\left(c_i + d_i - y_i\right)\right] \\ & + \sum_{i \neq j} \lambda_{ij} \left[u\left(c_i\right) + \beta_i u\left(d_i\right) - v\left(\frac{y_i}{w_i}\right) - u\left(c_j\right) - \beta_i u\left(d_j\right) + v\left(\frac{y_j}{w_i}\right)\right], \end{aligned} \tag{8}$$

where $\lambda_{ij} \geq 0$ are the multipliers associated with the self-selection constraints where the first subscript denotes the mimicker and the second the mimicked.

The FOCs for this problem are

$$\frac{\partial \mathcal{L}_{SB}}{\partial c_i} = \left[\pi_i + \sum_{j:i \neq j} \lambda_{ij} - \sum_{j:i \neq j} \lambda_{ji}\right] u'(c_i) - \pi_i \mu = 0, \tag{9}$$

$$\frac{\partial \mathcal{L}_{SB}}{\partial d_i} = \left[\beta \pi_i + \sum_{j:i \neq j} \beta_i \lambda_{ij} - \sum_{j:i \neq j} \beta_j \lambda_{ji}\right] u'(d_i) - \pi_i \mu = 0, \tag{10}$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{SB}}{\partial y_i} = & - \left[\pi_i + \sum_{j:i \neq j} \lambda_{ij}\right] v'\left(\frac{y_i}{w_i}\right) \frac{1}{w_i} \\ & + \sum_{j:i \neq j} \lambda_{ji} v'\left(\frac{y_i}{w_j}\right) \frac{1}{w_j} + \pi_i \mu = 0. \end{aligned} \tag{11}$$

Note that to have an interior solutions for c_i and d_i we need

$$\pi_i + \sum_{j:i \neq j} \lambda_{ij} - \sum_{j:i \neq j} \lambda_{ji} > 0, \tag{12}$$

$$\beta \pi_i + \sum_{j:i \neq j} \beta_i \lambda_{ij} - \sum_{j:i \neq j} \beta_j \lambda_{ji} > 0. \tag{13}$$

to be satisfied. We will need these conditions for our further analysis.

Combining and rearranging the FOCs one obtains

$$\frac{v'\left(\frac{y_i}{w_i}\right)}{u'(c_i)} = w_i \frac{\pi_i + \sum_{j:i\neq j}\lambda_{ij} - \sum_{j:i\neq j}\lambda_{ji}}{\pi_i + \sum_{j:i\neq j}\lambda_{ij} - \sum_{j:i\neq j}\lambda_{ji}\frac{v'\left(\frac{y_i}{w_j}\right)\frac{1}{w_j}}{v'\left(\frac{y_i}{w_i}\right)\frac{1}{w_i}}}, \tag{14}$$

$$\frac{u'(c_i)}{u'(d_i)} = \beta_i \frac{\pi_i + \sum_{j:i\neq j}\lambda_{ij} - \sum_{j:i\neq j}\frac{\beta_j}{\beta_i}\lambda_{ji}}{\pi_i + \sum_{j:i\neq j}\lambda_{ij} - \sum_{j:i\neq j}\lambda_{ji}} + \frac{\pi_i\left(\beta - \beta_i\right)}{\pi_i + \sum_{j:i\neq j}\lambda_{ij} - \sum_{j:i\neq j}\lambda_{ji}}. \tag{15}$$

When individuals differ in more than one characteristic, nonlinear taxation problems are often rather complex. This is due to the difficulty of knowing *a priori* which are the incentive constraints that bind. Observe that the main hurdle is not to solve the problem. This we have already done because expressions (14) and (15) are valid for any pattern of binding incentive constraints. The difficult part is to interprete (and sign) these expressions. We provide some general results without making any specific assumptions about the pattern of binding incentive constraints. Then, we illustrate these properties by discussing a three type setting and by providing numerical examples for the four type case.

Combining (15) and (5) one obtains the following expression for the marginal implicit tax on savings[7]

$$\Theta_i = \frac{\sum_{j:i\neq j}(\beta_j - \beta_i)\lambda_{ji}}{\beta_i(\pi_i + \sum_{j:i\neq j}\lambda_{ij} - \sum_{j:i\neq j}\lambda_{ji})} - \frac{\pi_i\left(\beta - \beta_i\right)}{\beta_i(\pi_i + \sum_{j:i\neq j}\lambda_{ij} - \sum_{j:i\neq j}\lambda_{ji})}. \tag{16}$$

This distortion can be interpreted in two ways depending on the way the solution is implemented. The implementation considered in Subsection 3.1 relies on a nonlinear taxation of *private* saving which is in line with standard optimal tax models. However, one can also think about a direct control of second period consumption d through the pension benefits (with no private savings at all). And of course any intermediate scheme between these two extremes is conceivable. Now, when we adopt the pension scheme interpretation, a marginal subsidy on "savings" effectively means that the pension system forces individuals to save more than they would otherwise do.

Intuitively, one would expect $\Theta_i < 0$ for all myopic individuals but this conjecture does not necessarily appear to be confirmed by equation (16). The expression consists

[7] Similarly, (14) and (6) can be combined to yield an expression for marginal labor income tax rates Γ_i.

of two terms. The second term is clearly the paternalistic term. It is negative for myopic individuals ($\beta_i < \beta$) while it vanishes for the far-sighted ($\beta_i = \beta$).[8] When all the λ's are zero (i.e., we return to a first best solution) it reduces to $1 - \beta/\beta_i$, which is the Pigouvian subsidy discussed above. When $\lambda_{ji} = 0$ for all j, we can think of individual i as a "top" individual. When individual i is far-sighted we have $\Theta_i = 0$ (no distortion at the top). Interestingly, however, when i is myopic, the second term does not reduce to the first best Pigouvian level; it is not equal to $1 - \beta/\beta_i$ as long as at least one $\lambda_{ij} > 0$.[9]

The first term is a "traditional" optimal tax (incentive) or redistributive term. More precisely it provides the expression for Θ_i that arises if the government is not paternalistic and welfare depends on individuals' short run preferences (with the β_i's) rather than on their "true" preferences. In other words, the second term vanishes when we return to a Paretian social welfare function. To show this one has to replace β's by β_i's in the Lagrangian (8) and rederive the first-order conditions. It then turns out that all the terms that currently form the second term of (16) drop out. Note that in this reformulated problem β_i's no longer represent the degree of myopia, but simply the weight attached in utility to the second period consumption. In other words, the problem is one of nonlinear commodity taxes where individuals differ in productivity and preferences; see Cremer et al. (1998).[10]

As discussed by Cremer et al. (1998) the sign of this term depends on the pattern of binding incentive constraints. If incentive constraints are binding between individuals with the same β and from higher β's to lower β's then the term is positive. If they bind from lower to higher (or identical) β's it is negative. One would expect the first case to be "more likely", but this will ultimately depend on the joint distribution of w's and β's. Specifically, if myopic individuals are on average more productive, the second

[8] It follows directly from the first-order conditions that the denominator of both terms is positive

[9] This is in line with the result obtained by Cremer et al. (1998) within the context of environmental taxation, namely that the second-best *levels* of environmental taxes faced by the "top" individuals are different from their first-best counterparts.

[10] This problem is studied by Cremer et al. (1998) within a different context (and with in addition an atmosphere externality). The first term in (16) effectively corresponds to expression (10a) in Cremer et al. (1998). To see this one has to make the appropriate changes in notation, set the atmosphere externality term ϕ'/μ to zero and use the first order condition to eliminate the μ from (10a) in Cremer et al (1998).

pattern could well arise.

To illustrate the type of results that can follow from the interplay between paternalistic and redistributive considerations, let us consider a special case. Assume that there are only three types ($\pi_2 = 0$) and that only downward incentive constraints are binding. In other words we have either of the following two cases:

1. $\lambda_{34} > 0$, $\lambda_{41} > 0$ and $\lambda_{31} > 0$, while $\lambda_{ij} = 0$ for all other constraints,

2. $\lambda_{34} > 0$ and $\lambda_{41} > 0$, while $\lambda_{ij} = 0$ for all other constraints.[11]

When the binding incentive constraints are those associated with the Lagrange multipliers λ_{34}, λ_{41} and λ_{31} one can easily check (by combining the three constraints) that $d_4 = d_1$. In the other case, when the binding incentive constraints are associated with λ_{34} and λ_{41}, we have $d_1 < d_4$. In both cases substituting into (16) and simplifying yields the following expression

$$\Theta_3 = 0 \tag{17}$$

$$\Theta_4 = \frac{\beta - \beta_0}{\beta_0} \frac{\lambda_{34}}{\pi_4 + \lambda_{41} - \lambda_{34}} - \frac{\beta - \beta_0}{\beta_0} \frac{\pi_4}{\pi_4 + \lambda_{41} - \lambda_{34}} \tag{18}$$

$$\Theta_1 = -\frac{\beta - \beta_0}{\beta_0} \frac{\lambda_{41}}{\pi_1 - \lambda_{31} - \lambda_{41}} \tag{19}$$

Equation (17) means that high-ability far-sighted individuals face no distortion on their savings (they face a zero marginal tax rate). Equation (19) implies $\Theta_1 < 0$ so that savings of low-ability (far-sighted) individuals' are subsidized. This is not due to paternalism but to incentive considerations (to relax an otherwise binding incentive constraint). Subsidizing saving by type 1 individuals makes their consumption bundle less attractive to type 4 individuals (who have a lower β_i).

Turning to the myopic (type 4), the analysis of Θ becomes much more interesting. Intuitively, one might expect $\Theta_4 < 0$ so that the system forces these individuals to save. Interestingly, however, it turn out that Θ_4 can be positive as well as negative because the two terms in (18) are of opposite sign. The optimal tax term is positive since the relevant binding incentive constraint goes from type 3 to type 4 and we have

[11] Recall that in a Kuhn-Tucker problem $\lambda_{ij} > 0$ means that the associated constraint is binding.

$\beta_3 = \beta > \beta_4 = \beta_0$. The paternalistic term, one the other hand is negative (as discussed above). Which case occurs depends on the sign of $\pi_4 - \lambda_{34}$; when $\pi_4 - \lambda_{34} > (<) 0$, Θ_4 is negative (positive). Here we thus have a conflict between paternalistic and redistributive considerations. Intuitively, correcting for myopia (though forced savings) benefits the rich myopic at the expense of the poor far-sighted.

At this point we have shown that (18) has two conflicting terms that may imply taxes or subsidies on savings of the high-ability myopic individuals. The numerical examples in the next section show that both cases are possible. Observe that in any case the under-savings problem of the myopics is never fully corrected; *i.e.* we always have $u'(c_4)/u'(d_4) < \beta$.[12]

4 Numerical results

We now turn to numerical simulations. They provide illustration of the analytical results. In addition, they are useful to study some issues that cannot be dealt with analytically. In particular, they show how the presence of myopic individuals (and a variation in their share) affects welfare and the design of the tax and pension system. The comparison between an all myopic and an all far-sighted society should not be too difficult. One expects that the role of the government is more important in the all-myopic case because it then pursues two objectives: achieving more equality and fostering savings. In a far-sighted society, on the other hand, the role of the government is purely redistributive. At the same time, the task of the government is more difficult in the all myopic case. Can we expect monotonicity between those two polar cases?

The simulations are based on the following utility function

$$u(c_i, d_i, \ell_i) = \sqrt{c_i} + \beta_i \sqrt{d_i} - (\ell_i)^2,$$

[12] As an alternative to the three-type case we have considered here one could assume $\pi_4 = 0$. Consequently, there would then be low productivity far-sighted and myopics and high productivity far-sighted individuals. This case (though not necessarily less realistic) appears to be less suitable to illustrate our results regarding Θ_i. As a matter of fact, the impact of myopia can be easily neutralized here by pooling types 1 and 2 (i.e., one forces type 2 to save and work as much as its far-sighted counterpart). We then return to a two-type model with separable and identical preferences and (16) implies $\Theta_1 = 0$ (which is simply the traditional Atkinson and Stiglitz result). Summing up, in this special case we have no conflict between redistribution and paternalism..

Table 1: Basic parameters

	$w_L = 4$	$w_H = 8$	Relative share
$\beta = 1$	type-1	type-3	$1 - \delta$
$\beta_0 = 0.2$ or 0.8	type-2	type-4	δ
Relative share	0.6	0.4	1

with a distribution of types as indicated in Table 1.

This utility exhibits some complementarity between the two levels of consumption, c_i and d_i. Complementarity is crucial here; it makes myopia more costly and liquidity constraints more penalizing than if there were a lot of substituability. In the extreme case of perfect substituability: $u(c, d, \ell) = c + \beta b - \ell^2$, the problem would be just one of standard redistribution across wage classes. The scenarios we consider differ in the share of myopic individuals (in total population). Observe that the share of high-ability individuals is constant and the same for the myopic and the far-sighted groups. Productivities are given by $w_H = 8$ and $w_L = 4$. The far-sighted have a $\beta = 1$ and the myopic a $\beta_0 = 0.2$ or 0.8. When $\beta_0 = 0.2$, we expect that the difference in time preference dominates that in productivity and when $\beta_0 = 0.8$, the productivity gap should dominate.

Tables 2 and 3 show the *laissez-faire* solution and the paternalistic first-best. In the *laissez-faire* we distinguish the case of $\beta_0 = 0.2$ and 0.8. In the paternalistic first-best the time discount factor of the myopic does not count. In these tables, we distinguish two levels of utility for the myopic: the utility perceived in the first-period with β_0 (denoted by U_i) and the *ex post* utility with β (denoted by $\widetilde{U}_i$).

Figures 2 and 3 depict the level of social welfare in the *laissez-faire* as a function of the proportion of myopic individuals. Not surprisingly, it decreases particularly when $\beta_0 = 0.2$.

We now turn to the second-best solution for different values of δ. Keeping in mind that the first-best welfare is independent of δ, we see from Table 5 and Figures 2–3 that

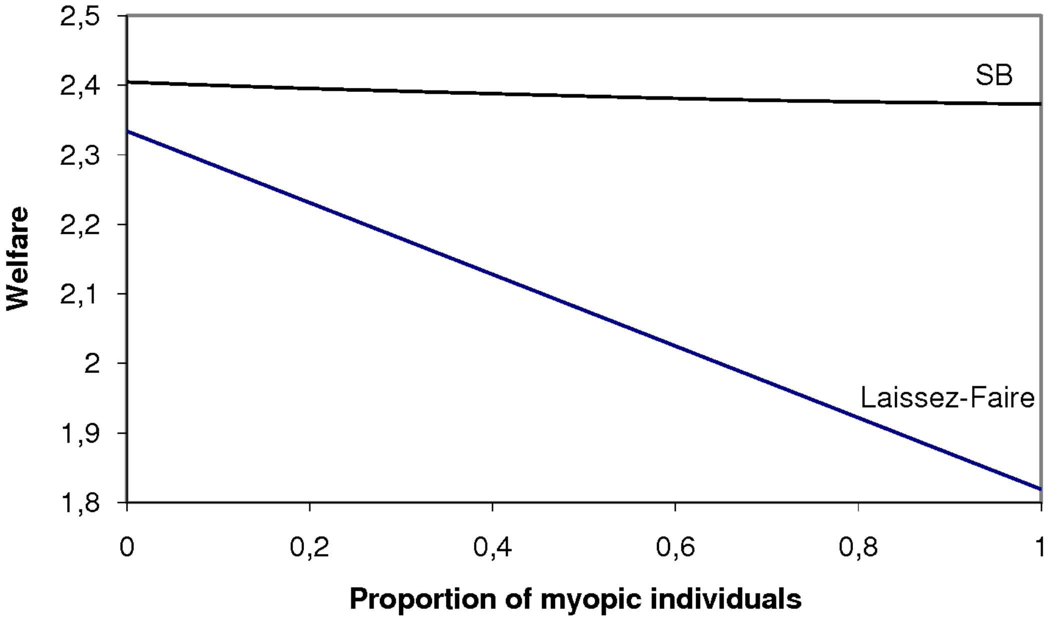

Figure 2: Welfare as a function of δ when $\beta_0 = 0.2$

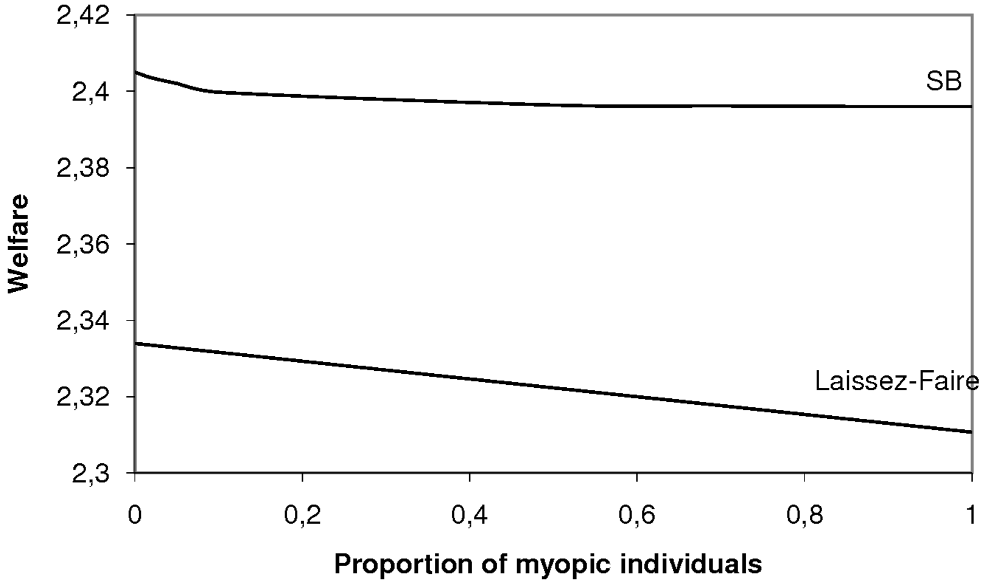

Figure 3: Welfare as a function of δ when $\beta_0 = 0.8$

Table 2: Laissez-faire

$\beta_0 = 0.2$					
Type	c_i	d_i	ℓ_i	U_i	$\widetilde{U}_i$
1	1.587	1.587	0.794	1.890	1.890
2	2.455	0.098	0.638	1.222	1.473
3	4.000	4.000	1.000	3.000	3.000
4	6.186	0.247	0.804	1.940	2.338

$\beta_0 = 0.8$					
Type	c_i	d_i	ℓ_i	U_i	$\widetilde{U}_i$
1	1.587	1.587	0.794	1.890	1.890
2	1.812	1.160	0.743	1.656	1.871
3	4.000	4.000	1.000	3.000	3.000
4	4.566	2.922	0.936	2.628	2.970

Table 3: First-best

Type	c_i	d_i	ℓ_i	U_i $\beta_0 = 0,2$	U_i $\beta_0 = 0,8$	$\widetilde{U}_i$
1	2.685	2.685	0.610	2.905	2.905	2.905
2	2.685	2.685	0.610	1.594	2.577	2.905
3	2.685	2.685	1.221	1.788	1.788	1.788
4	2.685	2.685	1.221	0.477	1.460	1.788
Welfare	2.458					

social welfare decreases with δ, particularly with $\beta_0 = 0.2$. The relation between δ and the gap between welfare in the Second-best and in the *Laissez-faire* is also instructive; the same figures show that this gap increases as δ increases showing that the desirability of social security increases with δ. When δ increases, the difference between second and first-period consumption $(d_i - c_i)$ of both types of poor individuals and of the myopic rich individuals steadily increases. In other words, myopia not only brings about forced saving, but the degree of forced saving also increases with the share of myopics.

Concerning redistribution, we observe that the utility gap between the poor and the rich individuals increases with δ as it is shown by Table 5. Similarly the net lifetime benefits that the poor individuals receive are also decreasing in the proportion of myopic individuals as the column $-T_i + p_i$ in Tables 4a and 4b show. Consequently, the poor

Table 4a: Second-best solution with $\beta_0 = 0.2$

	Type	c_i	d_i	ℓ_i	$-T_i + p_i$	U_i	$\widetilde{U}_i$	Γ_i	Θ_i
δ=0	1	1.838	1.838	0.662	1,028	2.273	2.273	0.102	0.000
	3	3.503	3.503	1.069	-1,546	2.602	2.602	0.000	0.000
δ=0.1	1	1.771	1.904	0.667	1,007	2.266	2.266	0.113	-0.037
	2	1.771	1.904	0.667	1,007	1.163	2.266	0.113	-4.184
	3	3.501	3.501	1.069	-1,550	2.600	2.600	0.000	0.000
	4	4.471	1.904	0.946	-1,193	1.496	2.600	0.000	-2.263
δ=0.5	1	1.569	2.122	0.681	0,967	2.245	2.245	0.147	-0.163
	2	1.569	2.122	0.681	0,967	1.080	2.245	0.147	-4.816
	3	3.493	3.493	1.070	-1,574	2.593	2.593	0.000	0.000
	4	4.285	2.122	0.966	-1,321	1.428	2.593	0.000	-2.519
δ=0.9	1	1.448	2.140	0.691	0,824	2.188	2.188	0.168	-0.216
	2	1.448	2.140	0.691	0,824	1.018	2.188	0.168	-5.079
	3	3.564	3.564	1.059	-1,344	2.653	2.653	0.000	0.000
	4	4.116	2.547	0.986	-1,225	1.376	2.653	0.000	-2.933
δ=1	2	1.430	2.132	0.693	0,790	1.008	2.176	0.172	-5.105
	4	4.087	2.641	0.989	-1,184	1.368	2.668	0.000	-3.020

Table 4b: Second-best solution with $\beta_0 = 0.8$.

	Type	c_i	d_i	ℓ_i	$-T_i + p_i$	U_i	$\widetilde{U}_i$	Γ_i	Θ_i
δ=0.1	1	1.772	1.894	0.667	0,998	2.263	2.263	0.113	-0.034
	2	1.772	1.894	0.667	0,998	1.988	2.263	0.113	-0.292
	3	3.507	3.507	1.068	-1,530	2.605	2.605	0.000	0.000
	4	4.393	2.014	0.954	-1,225	2.321	2.605	0.000	0.154
δ=0.5	1	1.728	1.855	0.670	0,903	2.228	2.228	0.120	-0.036
	2	1.728	1.855	0.670	0,903	1.955	2.228	0.120	-0.295
	3	3.560	3.560	1.060	-1,360	2.650	2.650	0.000	0.000
	4	3.733	3.201	1.035	-1,346	2.292	2.650	0.000	-0.158
δ=0.9	1	1.722	1.850	0.670	0,892	2.223	2.223	0.120	-0.036
	2	1.722	1.850	0.670	0,892	1.951	2.223	0.120	-0.296
	3	3.566	3.566	1.059	-1,340	2.655	2.655	0.000	0.000
	4	3.662	3.363	1.045	-1,335	2.288	2.655	0.000	-0.198
δ=1	2	1.722	1.850	0.670	0,892	1.951	2.223	0.120	-0.296
	4	3.653	3.383	1.046	-0,877	2.288	2.656	0.000	-0.203

Table 5: Welfare and utility gap in the second-best

	$\beta_0 = 0.2$		$\beta_0 = 0.8$	
δ	Welfare	$\widetilde{U}_3 - \widetilde{U}_1$	Welfare	$\widetilde{U}_3 - \widetilde{U}_1$
0,02	2,4035	0,3296	2,4035	0,3296
0,05	2,4021	0,3310	2,4021	0,3310
0,10	2,3997	0,3332	2,3997	0,3418
0,20	2,3953	0,3374	2,3977	0,3906
0,50	2,3843	0,3482	2,3964	0,4220
0,70	2,3784	0,3922	2,3961	0,4281
0,90	2,3744	0,4648	2,3960	0,4316
0,95	2,3736	0,4790	2,3960	0,4323
0,98	2,3731	0,4870	2,3960	0,4326

far-sighted workers are penalized by the presence of myopic (rich) individuals. In other words, myopia implies a less redistributive tax and pension system. Not surprisingly those effects are stronger with $\beta_0 = 0.2$ (when myopia is more severe) than with $\beta_0 = 0.8$.

The tables also report the distortion in labor supply (measured by Γ_i) which were not discussed in the analytical section. There is no such distortion for types 3 and 4, namely the productive individuals.[13] For types 1 and 2, the unskilled workers, there is a positive and identical marginal tax which increases as δ decreases. Turning to the saving choice, things are different. First, only type 3, the far-sighted skilled workers, are not subject to distortion. The others are subject to a subsidy that is particularly high for type 2 (myopic and unskilled) when $\beta_0 = 0.2$. When $\beta = 0.8$, that is when the degree of myopia is small, the implicit subsidies are also small. Types 1 and (to a more significant extent) 2 are subject to a subsidy but for $\delta = 0.10$, type 4 is subject to a tax. Observe that the tax-subsidy rate is different for all types.

5 Conclusion

This paper has studied the design of an optimal non linear social security scheme in a setting where individuals differ in both productivity and myopia and where the government acts paternalistically in attributing to all individuals the same far-sighted time

[13] We have $\lambda_{34} > 0$, but since these two types of individuals have the same wage, this constraint cannot be relaxed by distorting labor supply.

preferences. The main analytical result we obtain is that the paternalistic utilitarian solution does not necessarily imply forced savings for the myopics. While the Pigouvian (corrective) term calls for such forced saving, it is mitigated (or outweighed) by an incentive term which calls for a tax on savings (inducing a reduction in savings). Our numerical results suggest that as the number of myopic individuals increases, there is less redistribution and more forced saving. Furthermore, as the number of myopic agents increases, the desirability of social security (measured by the difference between social welfare with and without social security) increases.

In two companion papers, we have examined the same issue restricting government intervention to linear schemes studied both from a normative point of view (Cremer *et al.* 2008b) and in a political economy setting (Cremer *et al.* 2007). Each of these studies sheds light on the same underlying issue but from a different perspective. A basic lesson that emerges from the three papers is that the interplay between redistribution and forced saving is both complex and interesting. In the absence of myopia, the problem would be "straightforward" (we have a standard Mirrlees problem); without heterogeneity in wage, it would be trivial (the first-best can easily be achieved). Combining these two features brings about an intricate interaction which yields some rather counterintuitive results.

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