

Multiple Lenders, Strategic Default and the role of Debt Covenants ^{*}

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Abstract

This paper investigates the relationship between competition and contract design in capital markets subject to moral hazard. We consider the stylized representation of the capital market introduced by Holmstrom and Tirole (1997, 1998), and we explicitly model competition among investors as an extensive form game. Financial contracts are taken to be non-exclusive, which guarantees that entrepreneurs can trade with several investors at a time. In such a context, we provide a full characterization of the set of equilibrium allocations and we show that the features of market equilibria crucially depend on the financial contracts made available to financiers. If lenders make use of debt contracts only, the equilibrium outcome is always efficiently, and unique when the moral hazard problem is severe. Then, the aggregate of lenders earn monopoly profits. If covenants contingent on the project's cash-flow can be included in financial contracts, then every feasible allocations can be supported at equilibrium: market equilibria are indeterminate and Pareto-ranked. The introduction of institutional mechanisms which prevent borrowers from strategically defaulting on their loans can restore the competitive outcome as the unique equilibrium allocation.

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1 Introduction

This paper contributes to the analysis of capital markets in which investors compete to provide funds to entrepreneurs in the presence of moral hazard. We consider the situation where none of the investors can monitor entrepreneurs' transactions with his competitors, which we regard as a salient feature of several modern financial markets. That is, exclusivity of financial relationships cannot be enforced and every single entrepreneur has the opportunity to raise funds from several financiers at a time. Our objective is to analyze the role of covenants in coping with non exclusivity.

In a moral hazard scenario, non exclusivity of contracts generates a fundamental externality among financiers: An entrepreneur's effort choices depend on the total amount of external capital she raises, but each of the investors is only able to monitor his own trades with entrepreneurs. By raising additional funds, each entrepreneur becomes more willing to misbehave, which reduces the expected repayments to investors, as originally documented by Bizer and DeMarzo (1992). More recently, several works emphasized the perverse effects of such contractual externalities in credit markets. Market equilibria feature lenders earning positive profits and providing a smaller amount of loans, compared with the situation where exclusive clauses can be enforced at no cost (see Parlour and Rajan (2001), Bisin and Guaitoli (2004), Bisin and Rampini (2006) and Bennardo, Pagano, and Piccolo (2009)). The corresponding allocations, however, have typically shown to be constrained efficient: a social planner who does not control either the entrepreneurs' effort choices or their trades *cannot* perform better than markets.¹

The present paper proposes a further examination of these markets, supporting the view that the set of financial instruments which are available to competitors crucially affects the features of market equilibria. We develop a framework where contracts are non-exclusive but they can be made contingent on publicly observable financial results (cash-flows, assets...). This allows financiers to specify covenants with some targeted investment level, and to design penalties when

¹The efficiency notion corresponding to a situation where the planner cannot enforce exclusivity clauses is referred to as third best optimality. See Bisin and Guaitoli (2004), Attar, Campioni, and Piaser (2006) and Attar and Chassagnon (2009) for a formal definition of this concept and for an extensive discussion of the efficiency results.

such covenants are violated. Such instruments are *not* equivalent to controlling the number of traded contracts ex ante, since the ability of investors to punish departures from targeted ratios is reduced by entrepreneurs' limited liability. We fully investigate the strategic role of such covenants on investors' market power and credit rationing for firms. One could in principle argue that an explicit consideration of such additional clauses makes competition more fierce, driving lenders' profits to zero. Quite on the contrary we stress that these additional instruments provide new profit opportunities for lenders as well as new threats against their opponents opportunistic behaviors. As a result, market equilibria will be indeterminate and Pareto-ranked.

The possibility that lenders can issue payments' schedule contingent on aggregate variables has typically been disregarded in recent analysis of credit relationships under non exclusivity. In this respect, we show that the (constrained) efficiency features of market equilibria emphasized in these analyses tightly depend on the restriction to the set of financial instruments available to lenders. The presence of additional clauses indeed generates new coordination problems: each of the lenders can use such clauses to protect himself against unilateral deviations by his competitors. This makes possible to support a large set of allocations at equilibrium. Our results therefore suggest that the presence of credit market institutions as well as their regulatory role should be explicitly considered when focusing on non-exclusive credit markets.

The starting point of our analysis is the credit economy considered by Holmstrom and Tirole (1997, 1998), where entrepreneurs need funds to invest in a project which operates under a linear technology subject to moral hazard. With reference to such a standard setting, we model competition amongst financial intermediaries as an extensive form game: investors simultaneously post their offers; after observing them, entrepreneurs take their portfolio and effort decisions.

We first consider the situation where lenders' offers can be made contingent on the "success" or "failure" of the project, but not on its cash-flow. Since this restriction has been postulated in all recent attempts at modeling non exclusive competition in credit markets, we look at this scenario as our reference benchmark. In such a context, we show that competition delivers an extreme result: When the moral hazard problem is severe enough, the only aggregate allocation supported at equilibrium is the one where lenders earn a monopolistic profit. None of the financiers has

a unilateral incentive to propose loans at a smaller rate, because this would always induce the entrepreneur to trade several contracts at a time, and to select the low effort, which makes the deviation not profitable in the first place. When the moral hazard problem is mild, all efficient allocations can be sustained at equilibrium. One should observe that the possibility to support monopolistic (or positive profit) allocations at equilibrium has been emphasized in a number of recent works (see Parlour and Rajan (2001) and Bennardo, Pagano, and Piccolo (2009) among others). Here we show that, as long as exclusivity of contracts cannot be enforced, this extreme non competitive result can also arise in linear environments. In addition, linearity exacerbates the market power of lenders and allows us to pin down monopoly as the unique equilibrium allocation (when severe moral hazard problems prevail).

This result provides a natural starting point to develop a framework where contracts can also be made contingent on aggregate cash-flows. We interpret these contracts as debt contracts with financial covenants. In practice, covenants are designed to induce efficient decisions from the borrower and thereby reduce potential agency problems between borrowers and lenders. Such financial covenants are typically contingent on verifiable and contractible variables such as balance sheet, income statement or cash flow items². A single lender may gain from the introduction of these covenants because they allow to punish an entrepreneur who deviates from some targeted investment level. Intuitively, covenants should enhance competition. Our main result is that introducing covenants does not help lenders to coordinate on some competitive outcome: We prove that every feasible allocation can be sustained as a pure strategy equilibrium of our competition game. Intuitively, covenants make coordination easier, and deter entry of passive investors, which renders all feasible allocations sustainable.

This calls for the explicit consideration of how institutional mechanisms can remove this indeterminacy. In particular, we discuss to what extent financial institutions can induce lenders to coordinate on constrained efficient allocations. The specific contractual externality arising in our setting creates incentives for borrowers to make "false" promises, i.e. to accept more loans than those that can effectively be repaid. We investigate to what extent this opportunity of strategic de-

²See for example Paglia and Mullineaux (2006), Demiroglu and James (2008).

fault affects equilibrium outcomes. Exogenously constraining the entrepreneur not to make false promises does not affect much the equilibrium outcome when only debt contracts are allowed. But it sharply modifies the nature of competition when contracts can include covenants on investment. In that case, we find that the only equilibrium allocation is the competitive one. In light of this analysis, we argue that offering the entrepreneur below market rate financing can alleviate her incentives to strategically default and restore a competitive equilibrium and characterize such a market subsidy. Our analysis thus provides a rationale for the joint use of covenants in debt contracts and institutions that offer cheap financing to entrepreneurs to relax temptations of strategic default.

2 The model

We build on the stylized representation of the capital market popularized by Holmstrom and Tirole (1997, 1998), and Tirole (2006) .

Agents, technology and preferences. We consider a production economy that lasts two periods. It is populated by a single representative entrepreneur and a finite number N of investors. The entrepreneur owns a project and can ask money to investors to expand the scale of her project. Production takes place through a linear technology which realizations are subject to uncertainty over two aggregate states denoted "success" and "failure". An investment of $I \in \mathbb{R}_+$ yields a final output (cash-flow) of GI , with $G \in \mathbb{R}_+$, in case of success and of zero in case of failure. Output can be verified at no cost. The probability distribution over final outcomes depends on an unobservable effort $e = \{L, H\}$ chosen by the entrepreneur. Denote $(\pi_e, 1 - \pi_e)$ the probability distribution induced by the effort choice e , where π_e is the probability of success. We let $e = H$ represent the high level of effort, and assume that $\pi_H > \pi_L$. If the entrepreneur misbehaves (selecting $e = L$), she receives a private benefit $B \in \mathbb{R}_+$ per unit invested in the project.

In line with Holmstrom and Tirole (1998), we introduce the following two assumptions. First, the investment project has a positive net present value if and only if the entrepreneur selects $e = H$;

that is:

$$\pi_H G > 1 > \pi_L G + B, \quad (1)$$

Second, if the entrepreneur chooses $e = H$, the unitary revenue from the project is smaller than the unitary agency cost, that is:

$$0 < \pi_H G - \frac{\pi_H B}{\Delta\pi} < 1, \quad (2)$$

where $\Delta\pi = \pi_H - \pi_L$. Were (2) not true, the moral hazard problem would have no incidence on the entrepreneur's ability to raise funds.

The entrepreneur is risk-neutral and protected by limited liability. She has an initial endowment of $A \in \mathbb{R}_+$ and can raise additional funds by trading financial contracts issued by competing investors. If she raises I units of investment in exchange for an aggregate repayment of R in case of success, and zero in case of failure, her expected utility is:

$$U(I, R, e) = \begin{cases} \pi_H \max \{ (G(I + A) - R), 0 \} - A & \text{if } e = H \\ \pi_L \max \{ (G(I + A) - R), 0 \} + B(I + A) - A & \text{if } e = L \end{cases}$$

If $G(I + A) < R$, default takes place and the entrepreneur always prefers to choose the low effort $e = L$ and earns the private benefit $B(I + A)$.³ If the entrepreneur decides not to enter into a credit relationship, she is left with the option to carry out the production process using her endowment only. The corresponding reservation payoff is $U(0) = (\pi_H G - 1)A$, which is strictly positive given (1).

For every aggregate allocation (I, R) traded by the entrepreneur, we let $e(I, R) \in \arg \max_e U(I, R, e)$ be any corresponding optimal effort choice. We denote $\mathcal{H} = \{(I, R) \in \mathbb{R}_+^2 : e(I, R) = H\}$ the set of aggregate allocations inducing $e = H$ as an optimal effort choice, and $\mathcal{L} = \{(I, R) \in \mathbb{R}_+^2 : e(I, R) = L\}$ its complement. Finally, $\Psi = \mathcal{H} \cap \mathcal{L} = \{(I, R) \in \mathbb{R}_+^2 : \pi_H (G(I + A) - R) =$

³ A is observable, so that the entrepreneur has to invest A even if he chooses $e = L$. Assuming that the entrepreneur does not invest A if he exerts effort $e = L$ does not modify the results.

$\pi_L(G(I + A) - R) + B(I + A)\}$ is the set of aggregate allocations that make the entrepreneur indifferent between $e = H$ and $e = L$. We call Ψ the incentive frontier.

If there is no default, the payoff to investor i only depends on his contract (I^i, R^i) with the entrepreneur. Since investors are risk-neutral, we define investor i 's expected utility as:

$$V^i(I^i, R^i, e) = \pi_e R^i - I^i \quad \text{with } e \in \{L, H\},$$

and his reservation utility is set to zero.

If default takes place, a single investor may not be repaid according to contractual terms, and his payoff is determined by the relevant repayment rule, as we explain below.

Contracts and strategic default. Investors compete by offering non-exclusive, bilateral contracts to the entrepreneur. A financial contract proposed by investor i is an array $(I^i, R^i(\theta))$, where $I^i \in \mathbb{R}_+$ is an amount of investment and $R^i(\theta) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a schedule of repayments contingent on the cash-flow realization $\theta = \{0, GI\} \forall I \in \mathbb{R}_+$. Given the entrepreneur's limited liability, one necessarily has $R^i(0) = 0$. To the extent that the final cash-flow perfectly reflects total outside financing I in case of success, this amounts to contracting on I : Without loss of generality, we hence say that the schedule $R^i(I)$ associates any possible aggregate outside investment I raised by the entrepreneur to a repayment for investor i .⁴

The entrepreneur can simultaneously accept contracts from several investors, optimally choosing the amount of investment and the repayments to be made given the array of offers. But in our setting, $R^i(I)$ does not necessarily coincide with the payment received by investor i if the entrepreneur raises I and the project succeeds, because the entrepreneur can strategically default. This happens whenever she commits to repay more than the cash-flow generated by the project in case of success. In this case, the entrepreneur makes 'false promises', entering into multiple investors' relationships even though not all repayments will be honored. The possibility of strate-

⁴This is not equivalent to contracting on the number of contracts accepted ex ante, because of the entrepreneur's limited liability. This limits the set of available punishments if the entrepreneur deviates from the targeted investment level.

gic default is a natural implication of non-exclusive competition: When the entrepreneur trades with multiple investors, she might have an incentive to accept contracts involving conflicting prescriptions. Financial contracts could in principle specify seniority rules to determine the order of repayments in case of default. However, such clauses do not always preclude conflicts. For instance, it can happen that two accepted contracts entail the same level of seniority.⁵ For this reason, we do not treat priority rules as given,⁶ but we determine them as part of the entrepreneur's optimal behavior.

The following timing of events describes our non-exclusive competition game:

1. Each investor i offers a financial contract, i.e. an array $(I^i, R^i(.))$.
2. Having observed the array of offered contracts, the entrepreneur chooses a subset of investors to trade with and she takes the corresponding optimal effort. In addition, if her portfolio decisions induce strategic default, she determines a priority rule for lenders.
3. Cash flow are realized and payments are made.

Throughout the paper, and unless stated otherwise, the equilibrium concept is pure strategy subgame perfect equilibrium.

Observe that strategic default never takes place at equilibrium. Indeed, if the entrepreneur expects to default, he necessarily chooses $e = L$, in order to earn a positive private benefit. However, (1) guarantees that if $e = L$ is selected, at least one investor earns a strictly negative payoff. At any pure strategy equilibrium, we must have $e = H$.

Constrained efficiency: competitive and monopoly allocations. We now define some aggregate allocations that provide useful benchmarks for our analysis. The competitive allocation (I^c, R^c) maximizes the entrepreneur's expected utility subject to incentive compatibility and participation

⁵As anecdotal evidence, consider the example of ABN Amro and JPMorgan who disputed over their claim on Dutch investor Louis Reijtenbagh's art collection after he defaulted on both institutions' loans. Reijtenbagh apparently used his art collection as collateral in the two banks.

⁶This contrasts with Bennardo, Pagano, and Piccolo (2009), who focus mainly on pro-rata rules, and with Bizer and DeMarzo (1992) who impose priority rules.

constraints. Given (1) and (2), (I^c, R^c) solves:

$$\begin{aligned} (I^c, R^c) &\in \arg \max_{(I, R)} U(I, R, e) \\ \text{s.t.} \quad &U(I, R, H) \geq U(I, R, L) \end{aligned} \quad (3)$$

$$\pi_H R - I \geq 0, \quad (4)$$

where (3) is the entrepreneur's incentive compatibility constraint, and (4) the (aggregate) participation constraint of investors. At the optimum, (3) and (4) are binding. The optimal investment-repayment pair is therefore

$$I^c = A \left(\frac{G - \frac{B}{\Delta\pi}}{\frac{B}{\Delta\pi} - G + \frac{1}{\pi_H}} \right) \quad \text{and} \quad R^c = \frac{1}{\pi_H} I^c \quad (5)$$

Similarly, define the monopolistic allocation (I^m, R^m) as that prevailing when financial investors maximize their joint utility. It is determined by

$$\begin{aligned} (I^m, R^m) &\in \arg \max_{(I, R)} \pi_H R - I \\ \text{s.t.} \quad &U(I, R, H) \geq U(I, R, L) \\ &U(I, R, H) \geq U(0) \end{aligned}$$

which implies that

$$I^m = A \left(\frac{G\Delta\pi}{B} - 1 \right) \quad \text{and} \quad R^m = G I^m. \quad (6)$$

Both the competitive and the monopoly allocations are feasible choices for a planner who cannot observe the entrepreneur's effort and retains a full control of her trades. The whole set of constrained efficient (I, R) allocations is identified by the linear relationship

$$R = \left(G - \frac{B}{\Delta\pi} \right) (I + A), \quad (7)$$

which represents the incentive frontier Ψ defined earlier.

3 Credit market equilibrium with plain debt contracts

This section analyzes the situation whereby investors offer contracts contingent on the aggregate success or failure state, but not on the total cash-flow available in the success state. We call these contracts plain debt contracts and investors are indifferently denominated lenders. A plain debt contract offered by lender i is any array $C_i = (I_i, R_i) \in \mathbb{R}_+^2$, where I_i is a loan amount, and R_i is the corresponding fixed repayment in case of success. It is useful for the analysis to denote $p_i = \frac{R_i}{I_i}$ the price of this contract.

We provide here a full characterization of the set of pure strategy equilibria of the competition game when lenders are restricted to make use of plain debt contracts only. This assumption is standard in the literature on non-exclusive credit markets subject to moral hazard (see for instance Bizer and DeMarzo (1992), Parlour and Rajan (2001), Bisin and Guaitoli (2004), Bisin and Rampini (2006), Attar, Campioni, and Piaser (2006), and Bennardo, Pagano, and Piccolo (2009)). It is thus useful to introduce the plain debt benchmark to compare our results to those of the literature.

Let us first emphasize a simple property of the entrepreneur's optimal choices. Take any array of lenders' offers and let $C = (I, R)$ be the corresponding aggregate investment-repayment pair optimally chosen by the entrepreneur. Also, denote $\tau(C)$ the entrepreneur's marginal rate of substitution evaluated at C . For a given level of effort, $\tau(C)$ reflects the maximum price that the entrepreneur is willing to pay for an additional unit of investment for her utility to remain constant. In particular, one gets $\tau(C) = G$ for every $C \in \mathcal{H}$ and $\tau(C) = G + \frac{B}{\pi_L}$ for every $C \in \mathcal{L}$. In words, the entrepreneur does not modify her utility by trading additional contracts at a price $p = G$ (resp. $p = G + \frac{B}{\pi_L}$) and selecting $e = H$ (resp. $e = L$).

Given the linearity of preferences, the entrepreneur always has an incentive to trade contracts which price is strictly smaller than her marginal rate of substitution. This is formalized in the following:

Lemma 1 *Let $\{C_1 = (I_1, R_1), \dots, C_n = (I_n, R_n)\}$ be any array of offers such that the corre-*

sponding investment-repayment pair $C = (I, R)$ optimally chosen by the entrepreneur does not lie on the incentive frontier Ψ . Then, if default does not take place, each contract C_i such that $p_i < \tau(C)$ is accepted, and each contract C_i such that $p_i > \tau(C)$ is rejected. If default occurs, any offer $C_i = (I_i, R_i)$ such that $I_i > 0$ is accepted by the entrepreneur.

As long as default does not take place, the entrepreneur accepts (resp. rejects) all contracts issued at a price strictly smaller (resp. greater) than her corresponding marginal rate of substitution. If her behavior induces default, though, the entrepreneur accepts all contracts involving a strictly positive level of investment, since this increases her private benefit.

Lemma 1 has one main implication. Since the marginal rate of substitution evaluated in \mathcal{L} is greater than that evaluated in \mathcal{H} , each contract accepted if $e = H$ is chosen is also accepted if $e = L$ is chosen. Therefore, at any optimal choice, all contracts which price is strictly smaller than G are accepted. This property is crucial to characterize pure strategy equilibria, which is done in the two following propositions.

To characterize the equilibrium allocations, we first need to understand what incentives to deviate the agents of our economy might have. Consider an aggregate allocation $C \in \mathcal{H} - \Psi$. Such an allocation implies that the borrower strictly prefers to choose $e = H$, and that there are incentives for additional trades (i.e. deviations). Indeed, (1) and (2) guarantee that if an additional contract $(x, (1/\pi_H + \epsilon)x)$ is offered, where x and ϵ are sufficiently small, the entrepreneur strictly prefers to accept this contract along with C , rather than accepting C only. Such a contract is also profitable for the lender because as long as $e = H$, the project's net present value is positive. Therefore, for the allocation C to be supported at equilibrium, it must be that, following the described above deviation, the borrower has a strict incentive to select $e = L$. Since both x and ϵ can be arbitrarily small, we can derive the following indifference condition.

Proposition 1 *If $C = (I, R)$ is an equilibrium allocation, then $C \in \Psi$.*

Proposition 1 states that every equilibrium allocation belongs to the incentive frontier. The intuition for the result is that if the threat of selecting $e = L$ is not sufficiently strong (i.e., if given the

equilibrium allocation, the entrepreneur strictly prefers $e = H$) there is always at least one active lender who could profitably increase total investment. This however is not enough to state that any equilibrium allocation is on the incentive frontier. It might be that the preferred allocation of the entrepreneur in the \mathcal{L} state lies on the same indifference curve as the equilibrium allocation, but that no allocation is on Ψ . We show in the appendix that in such a case, an inactive lender would have an incentive to remove his offer, weakening the threat of low effort, and offer instead a contract that increases investment and social surplus.

Proposition 2 *If the moral hazard is mild (in the sense that $\pi_H G - B \leq 1$), then every aggregate allocation $C \in \Psi$ can be supported at equilibrium.*

If the moral hazard problem is sufficiently severe (in the sense that $\pi_H G - B > 1$), the monopoly allocation (I^m, R^m) is the unique equilibrium allocation of the competition game.

An important result of proposition 2 is that if the moral hazard problem is sufficiently severe, only the monopoly allocation can be sustained at the equilibrium. The reason is the following. We know from proposition 1 that any equilibrium allocation must lie on the incentive frontier Ψ , i.e. that the entrepreneur must be indifferent between supplying efforts H or L , given the equilibrium accepted contracts. This means that no investor can deviate and offer a contract that is profitable and increases total investment: any such contract would induce the entrepreneur to select $e = L$. An active investor can however deviate and offer a more expensive contract that reduces the entrepreneur's utility as well as the amount of loans. This deviation is profitable for the investor if the entrepreneur trades the contract and chooses $e = H$. We show that when the moral hazard problem is severe, such deviations are always possible, except if all contracts traded at equilibrium have a price equal to G .

When the moral hazard problem is mild, one can characterize equilibria such that the above deviations are not possible. This is the case if the contracts offered by lenders are such that the entrepreneur is indifferent between choosing effort $e = H$, choosing effort $e = L$ and not defaulting on the promised repayments in case of success, and choosing $e = L$ and defaulting in case of success. If B is small, deviations that reduce the entrepreneur's utility immediately induce

the latter to accept all loans and strategically default. When the moral hazard problem is mild, we therefore cannot rule out these equilibria and all allocations on the incentive frontier can be sustained.

That positive profit equilibria (and in particular monopoly equilibria) emerge in credit markets subject to moral hazard has already been documented in the literature (see Parlour and Rajan (2001) and Bennardo, Pagano, and Piccolo (2009)). In those papers, monopoly equilibria are sustained with concave production functions and conditions on the minimum number of investors. The first contribution of our paper is to show that these assumptions are not necessary: A monopoly equilibrium can arise with a linear production technology and irrespectively of the number of investors. The second contribution is to characterize the impact of moral hazard on aggregate investment and surplus. If moral hazard worsens (i.e. if B increases), total investment decreases, and total surplus is reduced (if one takes a utilitarian social welfare criterion).

4 Capital market equilibrium with covenants contingent on cash-flows

The essential feature of markets where investors compete through non-exclusive contracts is that none of the competitors can make his offers contingent on the proposals of his rivals. In the context of capital markets, this has often translated into assuming that financial contracts cannot be contingent on total investment, or total assets (see for instance Bizer and DeMarzo (1992), Bisin and Guaitoli (2004) or Bisin and Rampini (2006)). This is the approach we followed in section 3.

We now extend our analysis by assuming that financial contracts can be contingent on the entrepreneur's total cash-flow which is itself related to the total amount invested initially. This corresponds to the idea that in a production context, firms' cash-flows or assets are verifiable and contractible.

The main implication of expanding the set of contracts available to each of the investors is to provide them with an additional set of punishments, or coercive clauses, in case the entrepreneur

deviates from some targeted investment level. It is thus natural to interpret such contingent contracts as debt contracts with covenants (we therefore continue to indifferently use the denomination "investors" or "lenders"). The question is whether one's ability to write covenants contingent on cash-flows ex post is equivalent to one's ability to control for the number of contracts accepted ex ante, as it would be the case in a setting of exclusive competition. Intuitively, the two might differ because investors' ability to "punish" departures from targeted investment levels is limited. In our model, the entrepreneur's limited liability sets an upper bound on penalties that lenders can impose.

Considering contracts contingent on total cash-flow amounts to conditioning repayment R_i on total investment I rather than on the single investment I_i proposed by investor i . In contrast, exclusive competition amounts to conditioning I_i on total investment I before investment is actual sunk. The next proposition states that introducing cash-flow-contingent covenants dramatically changes the set of equilibrium allocations in our competition game.

Proposition 3 *When repayments R_i can be contingent on the total investment $I = \sum_{i \in \mathcal{I}} I_i$, if the number of investors N is large enough, the equilibrium allocation is indeterminate in the sense that:*

- *if $\pi_H G - 1 < B$, any feasible aggregate allocation $C = (I, R) \in \mathcal{H}$ (i.e. such that $U(I, C, H) \in [U(I^m, R^m, H), U(I^c, R^c, H)]$) can be supported at equilibrium,*
- *if $\pi_H G - 1 \geq B$, any aggregate allocation $C = (I, R) \in \mathcal{H}$ such that $U(I, C, H) \in [U_B^*, U(I^c, R^c, H)]$, with $U_B^* > U(I^m, R^m, H)$ defined in the appendix, can be supported at equilibrium.*

Proposition 3 establishes that when the moral hazard problem is severe enough (so that $B > \pi_H G - 1$) each feasible allocation can be sustained at equilibrium with covenants contingent on the final cash-flow. When the moral hazard problem is less severe, only those allocations granting the entrepreneur a utility strictly greater than a threshold U_B^* can be sustained. This result holds under some assumptions regarding the number of agents in the economy. The corresponding equilibrium

strategies can be described as follows. A first set of lenders offer contracts that collectively grant the entrepreneur his equilibrium utility: These contracts are active, i.e. accepted at equilibrium. To achieve this, each offer is formulated so that the entrepreneur obtains his equilibrium utility when accepting all (or all but one) contracts: If the entrepreneur accepts any other set of contracts, she is punished by having to repay all the cash-flow. Such contracts can be interpreted as debt contracts, with covenants specifying a targeted investment level. When a covenant is violated, lenders capture the firm's assets.⁷ An important feature is that any equilibrium utility can be sustained by this set of active contracts. A second set of lenders offer high price contracts that are not accepted at equilibrium. Such passive contracts ensure that the entrepreneur would obtain his equilibrium utility, were she to accept all contracts and exert effort $e = L$.

If agents select such strategies, every feasible allocation ((or every allocation such that $U(I, R) \geq U_B^*$) can be supported at equilibrium. Firstly, the entrepreneur has an incentive to accept only the offers of the first group of lenders and to choose $e = H$. Secondly, no lender has a unilateral incentive to deviate, irrespectively of his equilibrium profits. The intuition goes as follows. A single lender cannot gain by offering a contract if this is the only one traded by the entrepreneur at the deviation stage: in this case, the amount of loans will necessarily be large, which provides incentives to the entrepreneur to default on all offered contracts. Given the covenants issued at equilibrium, a single lender cannot deviate by offering a contract if it is traded together with additional offers.

In our construction, market equilibria are supported by a large number of inactive lenders. This is because the credit individually proposed by each inactive lender needs to be small, while the amount of loans available in the aggregate needs to be large. Finally, the equilibria are constructed so that a deviating investor cannot make profits if the entrepreneur chooses $e = L$. This is done by assuming, as in proposition 2, that the deviating investor is repaid after the others, in case of strategic default. The result also holds when investors are repaid according to a pro rata rule. With no doubt, priority rules matter for equilibria to exist. If one investor could deviate and make his contract senior to any other, such deviations would be hard to deter.

⁷In our model, asset value is simply the final realized cash-flow.

The above discussion sheds light on the strategic role of covenants. When the contract space is reduced, as it is the case when only the state of nature can be contracted upon, only efficient allocations can be sustained at equilibrium. When the moral hazard problem worsens, the set of equilibrium allocations reduces to the monopoly allocation. By contrast, expanding the contract space creates an indeterminacy that can be explained as follows. On the one hand, introducing cash-flow contingent covenants increases the ability of investors to punish deviations, which should enhance competition. On the other hand, cash-flow contingent contracts make coordination easier, and deter entry of passive investors, which renders all feasible allocations sustainable. This is because covenants increase the entrepreneur's incentives to strategically default following any deviation by a passive lender: This in turn undermines effective competition.

The indeterminacy stated in proposition 3 has important consequences for credit market efficiency. Inefficient equilibria can be sustained here (i.e. equilibria such that the allocation $C = (I, R)$ is not on Ψ). This suggests the need to consider some institutional mechanism to coordinate lenders' behaviors.

5 Institutional features to restore efficiency

A main feature of our credit economy is the lack of any institution which prevents the entrepreneur from strategically defaulting over the contracts she accepts. That is, the entrepreneur has the opportunity to make "false" promises, accepting a number of contracts greater than those she will be able to honor. In such cases, even before production is realized, the entrepreneur knows that she will be bankrupt with certainty. The ability to default strategically can influence the nature of competition, because some investors might have an interest to induce a default, and benefit from it if their claim is senior to that of others. To explore further to what extent strategic default affects the equilibrium outcome, the following paragraph studies the game when strategic default is precluded. The next paragraph describes an institutional setting to alleviate the entrepreneur's incentives to default and restore efficiency.

5.1 The impact of strategic default on equilibrium outcomes

In order to clarify the strategic role of this decision, consider a situation where strategic default is ruled out. We thus impose that the array of contracts accepted by a single borrower satisfies the inequality $\sum_{i \in \mathcal{I}} (GI_i - R_i) + GA \geq 0$. That is, the entrepreneur cannot accept an array of contracts such that the sum of repayments is greater than the maximum achievable cash-flow. Such an assumption modifies the nature of competition among financiers, as stated in the following proposition.

Proposition 4 *The following holds.*

- (i) *If repayments R_i are contingent on the total investment $I = \sum_{i \in \mathcal{I}} I_i$ chosen by the entrepreneur, and if strategic default is precluded in the sense that accepted contracts are such that $\sum_{i \in \mathcal{I}} (GI_i - R_i) + GA \geq 0$, then the competitive allocation $C^c = (I^c, R^c)$ is the unique equilibrium allocation.*
- (ii) *If only plain debt contracts are allowed, then the monopoly allocation (I^m, R^m) is the unique equilibrium allocation.*

Ruling out strategic default when contracts are contingent on the final cash-flow removes the equilibrium indeterminacy and leads to the competitive allocation as the unique equilibrium allocation. It is worth pointing out that when only plain debt contracts are allowed, precluding strategic default does not affect much our previous result: The monopoly allocation is now the unique equilibrium allocation, even when the moral hazard problem is mild. The intuition goes as follows. Introducing contingent covenants and precluding strategic default allows to restore the basic mechanism of price competition. A cash-flow contingent contract allows an investor to deviate towards a contract that is profitable as long as it is the only one accepted by the entrepreneur. If strategic default is precluded, there is always room for a single deviator to serve the whole market by marginally undercutting the equilibrium offers without triggering $e = L$. This indeed supports the competitive outcome at equilibrium. The same situation cannot arise if strategic default is allowed because following each unilateral deviations the entrepreneur has an incentive to accept all contracts and

shirk. This is the undesirable consequence of using covenants. Because covenants punish the entrepreneur for any deviation from a targeted investment level, they exacerbate the latter's incentive to default strategically if one investor deviates and tries to undercut his competitors. This in turn favors coordination among investors on (possibly inefficient) outcomes.

A natural idea to preclude strategic default is to impose unlimited liability to the entrepreneur if she makes false promises (because strategic default can be observed *ex post*). In our setting this can be achieved by introducing a court that lenders can refer to after payments are received. Assume that the court can impose additional (non pecuniary) punishment to the entrepreneur in case she is found guilty of strategic default.⁸ At equilibrium, the court effectively acts as a threat that ruins the entrepreneur's incentives to undertake strategic default. This solution however relies on the ability to remove limited liability on the entrepreneur's side, which is somehow outside our basic model. We propose below a market mechanism that alleviates the issue of strategic default by offering extra financing at below market rate (i.e. "cheap money"). Such a mechanism prevents investors from coordinating on inefficient equilibria by making deviations from these equilibria possible.

5.2 Cheap financing to rule out inefficient equilibria

When covenants are included in financial contracts, inefficient equilibria arise because such clauses impede deviations that undercut equilibrium offers: By punishing the entrepreneur when he departs from a target investment level, covenants exacerbate her incentive to accept all contracts and default strategically following any profitable deviation. To make such deviations possible, one solution would be to offer a very attractive (i.e. below market rate) financial contract to the entrepreneur, so that she has an incentive to accept a deviating contract together with the cheap contract, instead of accepting all available contracts and default. The basic reason why such a contract has to be below market rate is that we know from proposition 3 that inefficient equilibrium allocations are robust to any profitable deviation. Last, if cheap contracts restore efficiency, we need to ensure that they are feasible, i.e. that investors collectively accept to grant such contracts.

⁸The court can for instance send the borrower to prison, ruining her reputation.

We explore these questions below.

Consider the following mechanism. After investors offer contracts to the entrepreneur, an institution offers an additional contract $(\tilde{I}, \tilde{R}(\theta)) = \tilde{C}$. Then the population of investors collectively agrees (or not) to provide \tilde{C} , if chosen by the entrepreneur along with (part of) initial offers. After offers are chosen, as before, investment takes place, effort is selected and the final cash-flow is realized. The equilibria of this game have the following feature.

Proposition 5 *If an institution can offer a contract $\tilde{C} = (\tilde{I}, \tilde{R}(\theta))$ after investors offer their own contracts, there always exists $(\tilde{I}, \tilde{R}(\theta))$ such that the only equilibrium allocation is the competitive one (I^c, R^c) .*

The intuition of the proof is the following. Proposing an additional offer (\tilde{I}, \tilde{R}) below market rate (i.e. such that $\tilde{R} \leq \frac{\tilde{I}}{\pi_H}$) induces a passive investor to deviate from his equilibrium offer and propose a contract that increases the entrepreneur's utility when the latter selects $e = H$. The entrepreneur is not induced to strategically default and select $e = L$ because she also accepts the very attractive contract (\tilde{I}, \tilde{R}) . In some cases, this contract can even take the form of a subvention based on output with $\tilde{I} = 0$ and $\tilde{R} < 0$. Last, it is important to notice that investors collectively agree to finance the contract (\tilde{I}, \tilde{R}) because if they refuse, the presence of the deviation induces the entrepreneur to accept all contracts and default. By definition such a strategy has a negative present value, and investors are willing to pay to avoid this loss.

6 Conclusion

In this paper, we have presented a model of a capital market in which investors compete to provide funds to a single entrepreneur who is taking some unobservable actions. We consider a situation where competition is non-exclusive: Investors cannot monitor each other's trades with the borrower. In such a scenario, we argue that the feature of market equilibria crucially depend on the set of financial instruments made available to investors.

If we restrict investors to use plain debt contracts only, i.e. contracts contingent on the project's

success or failure only, then only efficient aggregate allocations can be supported at equilibrium. Also, as moral hazard worsens, the set of equilibrium allocations reduces to monopolistic profit to financiers. However, if investors can condition their required payments on the project's cash-flows, then a new set of strategic interactions is introduced. As a result, market equilibria are indeterminate and Pareto-ranked.

The fact that market outcomes crucially depend on the instruments available to financiers suggests that the positive and normative implications of non-exclusivity in credit markets should be carefully examined. Recent researches in this area have stressed that market equilibria can involve positive profits for the active financiers. At the same time, they have stated a constrained efficiency result: A social planner who does not control either entrepreneur's effort choices or her trades *does not* indeed perform better than markets (see Bizer and DeMarzo (1992), Parlour and Rajan (2001), Bisin and Guaitoli (2004), Bisin and Rampini (2006)). Our analysis indicates that this last conclusion importantly depends on the restriction to debt contracts which has been postulated in all these contributions.

We also suggest that capital market's institutions play a fundamental role to enforce competition among investors. In particular, designing mechanisms that offer cheap financing to entrepreneurs alleviate the latter's incentive to strategically default, and rule out inefficient equilibria.

Appendix

Proof of Lemma 1

Assume first that default does not occur. The proof is developed by contradiction. Let $C_i = (I_i, R_i)$ be a contract not traded by the entrepreneur at her optimal choice C . If $p_i < \tau(C)$, then one can directly check that

$$U(I + I_i, R + R_i, e(I, R)) > U(I, R, e(I, R)).$$

That is, the entrepreneur always has an incentive to trade the C_i contract, without changing her effort choice. This contradicts that C is an optimal choice for the entrepreneur. A similar argument can be used to show that all offers C_i with $p_i > \tau(C)$ are not traded by the entrepreneur.

Second, if default occurs, the entrepreneur's utility is: $U(I, R, L) = BI - A$, which is strictly increasing in the aggregate investment.

■

Proof of Proposition 1

Assume by contradiction that the equilibrium allocation $C = (I, R) \in \mathcal{H} - \Psi$. The proof proceeds in two steps.

1. No deviations for active lenders.

Given the equilibrium offers, we denote \tilde{C} any aggregate allocation optimally chosen by the entrepreneur if $e = L$ is selected.

In this step, we prove that active lenders do not have an incentive to deviate only if $U(\tilde{C}, e = L) = U(C, e = H)$. The threat of the entrepreneur accepting \tilde{C} and shirking prevents such deviations.

Suppose by contradiction that

$$U(C, e = H) > U(\tilde{C}, e = L). \tag{8}$$

Let C_i be the equilibrium offer of an active lender i , and suppose he deviates offering $C'_i = (I_i + \epsilon, R_i + \frac{\epsilon}{\pi_H} + \epsilon^2)$, for some strictly positive number ϵ . If the offer C'_i is accepted and the effort $e = H$ is selected, the deviation is profitable. Indeed we have that $\pi_H(R_i + \frac{\epsilon}{\pi_H} + \epsilon^2) - I_i - \epsilon > \pi_H R_i - I_i$. Let us prove that for $\epsilon > 0$ small enough, the entrepreneur has an incentive to accept contract C'_i and choose $e = H$. First see that the entrepreneur always has an incentive to accept the deviating contract C'_i if ϵ is sufficiently small since its price is strictly smaller than G . Following this deviation, the entrepreneur strictly prefers $e = H$ if and only if:

$$\pi_H(G(I + A) - R) + \pi_H(G\epsilon - \frac{\epsilon}{\pi_H} - \epsilon^2) > U(\tilde{C} + (\epsilon, \frac{\epsilon}{\pi_H} + \epsilon^2), e = L) + A, \quad (9)$$

since by lemma 1, any contract accepted when $e = H$ is chosen is also accepted when $e = L$ is chosen.

Denote \bar{I} the aggregate investment provided by inactive lenders and $C_L = (I_L, R_L)$ the aggregate allocation that maximizes the entrepreneur's utility when she selects $e = L$ and does not default when the project succeeds. By definition, we have:

$$U(\tilde{C}, e = L) = \max\{U(C_L, e = L), B(I + \bar{I} + A) - A\}. \quad (10)$$

Consider the case where $U(C_L, e = L) \geq B(I + \bar{I} + A) - A$, condition (9) becomes:

$$\pi_H(G(I + A) - R) + \pi_H(G\epsilon - \frac{\epsilon}{\pi_H} - \epsilon^2) > \pi_L(G(I + A) - R) + \pi_L(G\epsilon - \frac{\epsilon}{\pi_L} - \epsilon^2) + B(I + A + \epsilon).$$

Given (8), the above condition holds if ϵ is small enough. The same logic applies when $U(C_L, e = L) < B(I + \bar{I} + A) - A$. C'_i constitutes a profitable deviation for investor i , which contradicts the claim that C is an equilibrium allocation. At equilibrium we have: $U(\tilde{C}, e = L) = U(C, e = H)$. Also, see that since by assumption $C \notin \Psi$, \tilde{C} includes contracts that are not accepted on the equilibrium path, that is, there are inactive lenders.

2. No deviation for inactive lenders.

In this second step, we prove that if $C \notin \Psi$, inactive lenders have an incentive to deviate, which concludes the proof. Consider first the case in which $U(\tilde{C}, e = L) = U(C_L, e = L)$. We partition the set J of inactive lenders as follows: $J = J_1 \cup J_2 \cup J_3$ with:

1. $J_1 = \{j \in J : p_j = G\}$
2. $J_2 = \{j \in J : p_j \in (G, G + \frac{B}{\pi_L})\}$
3. $J_3 = \{j \in J : p_j \geq G + \frac{B}{\pi_L}\},$

We show below that there must be one C_L allocation that we denote \bar{C}_L , such that $U(C, e = H) = U(\bar{C}_L, e = L) = U(\bar{C}_L, e = H)$. We proceed as follows. If \bar{C}_L is not available, we show that the set J of lenders who are inactive at equilibrium necessarily includes at least one lender j offering a contract $C_j = (I_j, R_j)$ of price $p_j \in (G, G + \frac{B}{\pi_L})$. Next, we show that this lender j can profitably deviate inducing the entrepreneur to select $e = H$, which contradicts the fact that \bar{C}_L is not available. Last, we show that an inactive lender j offering a contract of price $p_j = G$ has also an incentive to deviate. The first two steps are established in the following lemma.

Lemma 2 *At any equilibrium, the aggregate allocation \bar{C}_L must be available to the borrower.*

Proof of Lemma 2. We first show that if \bar{C}_L is not available, then there must be at least one (inactive) lender j offering a contract at a price $p_j \in (G, G + \frac{B}{\pi_L})$, i.e $j \in J_2 \neq \emptyset$. Indeed, assume that $J_2 = \emptyset$ then the aggregate allocation C_L satisfies $U(C_L, e = L) = U(C + \sum_{j \in J_1} C_j, e = L)$ implying in turn that $U(C_L, e = L) = U(C_L, e = H)$. This contradicts the fact that \bar{C}_L is not available. Second, we argue that any s lender in J_2 can profitably deviate. Observe that $U(C, e = H) = U(C + \sum_{j \in J_1, j \in J_2} C_j, e = L) > U(C + \sum_{j \in J_1, j \in J_2, j \neq s} C_j, e = L)$. Suppose now that lender $s \in J_2$ withdraws his offer and proposes the contract $C'_s = (\epsilon, \frac{\epsilon}{\pi_H} + \epsilon^2)$ for some strictly positive ϵ . Given Lemma 1, the borrower will always accept the C'_s contract when $e = H$ is selected. In addition, using a continuity argument one gets

$$U(C + C'_s, e = H) > U(C + \sum_{j \in J_1, j \in J_2, j \neq s} C_j + C'_s, e = L),$$

which guarantees that the borrower strictly prefers to select $e = H$ at the deviation stage. This ensures that C'_s is a profitable deviation for lender s . ■

An immediate implication of Lemma 2 is that at any equilibrium $J_2 = \emptyset$. If this is not the case, \bar{C}_L cannot be an optimal choice for the borrower when $e = L$ is chosen. It follows that $U(C, e = H) = U(C + \sum_{j \in J_1} C_j, e = L)$. Consider now any lender $s \in J_1$. As before, $U(C, e = H) > U(C + \sum_{j \in J_1, j \neq s} C_j, e = L)$. Suppose now that lender $s \in J_1$ withdraws his offer and proposes the contract $C'_s = (\epsilon, \frac{\epsilon}{\pi_H} + \epsilon^2)$ for some strictly positive ϵ . The same argument than in Lemma 2 yields that C'_s is a profitable deviation for lender s . Since $J_1 = J_2 = \emptyset$ it follows from Lemma 1 that $U(C, e = H) = U(C + \sum_{j \in J_3} C_j, e = L) = U(C, e = L)$, that is $C \in \Psi$.

It remains to consider the case in which $U(C, e = H) = B(I + \bar{I} + A) > U(C_L, e = L)$. The inequality implies that $J_3 \neq \emptyset$. Consider an inactive lender $s \in J_3$ who withdraws his offer (I_s, R_s) . We have that either $U(C, e = H) > U(C_L, e = L) > B(I + \bar{I} - I_s + A)$, either $U(C, e = H) > B(I + \bar{I} - I_s + A) > U(C_L, e = L)$. In both cases lender s can deviate as before. We conclude that $U(C, e = H) = U(C, e = L) = B(I + \bar{I} + A)$, that is $C \in \Psi$. ■

Proof of Proposition 2

CASE 1: $B \leq \pi_H G - 1$

Let $C = (I, R) \in \Psi$ be the candidate aggregate equilibrium allocation. Consider the following profile of strategies. Each lender $i = 1, 2, \dots, N$ offers: $C_1 = (I, R)$, $C_2 = (\bar{I}, \bar{R})$ and, for $i > 2$, $C_i = (0, 0)$ where \bar{I} and \bar{R} are chosen in such a way that

$$U(C, e = H) = U(C, e = L) = B(I + \bar{I} + A) - A \quad (11)$$

and $\frac{\bar{R}}{I} > G + \frac{B}{\pi_L}$. Given these offers, we prove that it is optimal for the borrower to accept only the contract C_1 and to select $e = H$. We argue that there is no unilateral profitable deviation for lenders if $B \leq \pi_H G - 1$.

As before, we proceed in two steps. In the first step, we show that no inactive investor $i \geq 2$ has an incentive to deviate. In the second step, we show that lender 1 has no incentive to deviate either.

1. No deviation for inactive lenders.

Let (I'_2, R'_2) be a deviation of lender 2. We show below that any such deviation induces the entrepreneur to select $e = L$, and that this deviation is not profitable.

By contradiction, suppose that this deviation induces the entrepreneur to select $e = H$. To yield a strictly positive profit we must have: $R'_2 > \frac{1}{\pi_H} I'_2$. Let $K \in \mathcal{H}$ be the aggregate allocation optimally selected by the entrepreneur at the deviation stage. We first show that if $K \in \mathcal{H}$, then $K = (I'_2, R'_2)$.

If the entrepreneur accepts both (I, R) and (I'_2, R'_2) , we have:

$$G(I + I'_2 + A) - (R + R'_2) = \frac{B(I + A)}{\Delta\pi} + (G I'_2 - R'_2),$$

since (I, R) belongs to the set Ψ . Use condition (2) as well as the fact that (I'_2, R'_2) must be profitable to get: $\frac{R'_2}{I'_2} > \frac{1}{\pi_H} > G - \frac{B}{\Delta\pi}$. It follows that:

$$G(I + I'_2 + A) - (R + R'_2) < \frac{B(I + A)}{\Delta\pi} + G I'_2 - (G - \frac{B}{\Delta\pi}) I'_2 = \frac{B(I + I'_2 + A)}{\Delta\pi},$$

i.e. $(I + I'_2, R + R'_2) \in \mathcal{L}$. One hence gets: $K = (I'_2, R'_2) \in \mathcal{H}$.

Second, we show that following the deviation (I'_2, R'_2) , the entrepreneur strictly prefers to select $e = L$. We know that:

$$\pi_H(G(I + I'_2 + A) - (R + R'_2)) < \pi_L(G(I + I'_2 + A) - (R + R'_2)) + B(I + I'_2 + A). \quad (12)$$

From (12), it is always possible to find $\mu \in [0, 1[$ such that:

$$\pi_H(G(\mu I + I'_2 + A) - (\mu R + R'_2)) = \pi_L(G(\mu I + I'_2 + A) - (\mu R + R'_2)) + B(\mu I + I'_2 + A).$$

That is, if the entrepreneur accepts contracts $(\mu I + I'_2)$ paying back the amount $(\mu R + R'_2)$ to the aggregate of investors, she is indifferent between selecting $e = L$ and $e = H$. Since $\frac{R}{I} < G + \frac{B}{\pi_L}$, one gets:

$$U(I + I'_2, R + R'_2, e = L) > U(\mu I + I'_2, \mu R + R'_2, e = L) = U(I'_2, R'_2, e = H).$$

It follows that $K = (I'_2, R'_2) \in \mathcal{H}$ cannot be an optimal choice: the entrepreneur strictly prefers $e = L$, which contradicts our assumption.

Suppose next that, following the deviation (I'_2, R'_2) , the entrepreneur chooses $e = L$. Given (2), the deviation can only be profitable if the entrepreneur decides to strategically default,⁹ in which case she accepts all offered contracts. Given (I'_2, R'_2) , the entrepreneur defaults whenever her corresponding payoff is greater than her equilibrium utility, i.e.:

$$B(I + A + I'_2) \geq \pi_H(G(I + A) - R) = \frac{\pi_H B}{\Delta\pi}(I + A). \quad (13)$$

Which is equivalent to:

$$I'_2 \geq \frac{\pi_L}{\Delta\pi}(I + A). \quad (14)$$

Suppose that I'_2 satisfies (14), then the entrepreneur's strategy can always be constructed in such a way that the deviation is not profitable. Fix the following priority rule for the entrepreneur: lender 1 is repaid first, and lender 2 is the residual claimant. Lender 2 earns a strictly positive profit if:

$$\pi_L(G(I + I'_2 + A) - R) - I'_2 > 0 \iff I'_2 < \frac{1}{1 - \pi_L G} \frac{\pi_L}{\Delta\pi} B(I + A). \quad (15)$$

⁹It follows from (2) that $\pi_L G - 1 < 0$, which implies that any lender who is repaid according to the contractual premises earns a strictly negative profit if $e = L$ is selected.

It can be directly checked that, given (1), the inequalities (14) and (15) never hold simultaneously.

Finally, the same equilibrium allocation can be supported by several alternative repayment strategies. Take as an example the situation where the entrepreneur decides to repay all defaulted loans according to some *pro-rata* rule. In this case, (15) becomes :

$$\pi_L G(I + I'_2 + A) \frac{I'_2}{I + I'_2} > I'_2 \Leftrightarrow I'_2 < \frac{\pi_L G(I + A) - I}{1 - \pi_L G}. \quad (16)$$

This is consistent with condition (14) if:

$$(I + A) \frac{\pi_L(1 - \pi_L G)}{\Delta\pi} < \pi_L G(I + A) - I. \quad (17)$$

See that (17) implies:

$$(I + A) \frac{\pi_L B}{\Delta\pi} < \pi_L G(I + A) - I \Leftrightarrow I < \pi_L \left(G - \frac{B}{\Delta\pi}\right)(I + A). \quad (18)$$

Using the fact that $R = (G - \frac{B}{\Delta\pi})(I + A)$, (18) becomes:

$$\frac{R}{I} > \frac{1}{\pi_L},$$

which is inconsistent with $\frac{R}{I} \leq G$. Therefore, (17) does not hold, and even under a prorata repayment rule in case of default, no inactive lender wants to deviate from the initially defined equilibrium strategy.

2. No deviation for active lenders.

Let us now consider deviation from the active lender 1. Let (I'_1, R'_1) be a deviation of lender 1. We show below that any such deviation induces the entrepreneur to select $e = L$, and that this deviation is not profitable. By contradiction, suppose that this deviation induces the entrepreneur to select $e = H$. To yield a strictly positive profit we must have: $R'_1 > \frac{1}{\pi_H} I'_1$. In addition, since $C \in \Psi$ and given (2), the deviation is profitable if $I'_1 < I_1$, and $R'_1 < R_1$. Any such profitable

deviation by lender 1 can be written as $C'_1 = (I - \epsilon, R - p\epsilon)$ with $\epsilon > 0$ and $p \in (G - \frac{B}{\Delta\pi}, \frac{1}{\pi_H})$.

At the deviation stage, if $e = H$ is chosen, the borrower only accepts the deviating contract C'_1 by lemma 1. If $e = L$ is selected, the borrower prefers to default if:

$$U(C'_1, e = L) = \pi_L (G(I - \epsilon + A) - (R - p\epsilon)) + B(I - \epsilon + A) - A < B(I + \bar{I} + A - \epsilon) - A,$$

that is, using (11), $\pi_L(G - p) > 0$ which is always satisfied since $G > \frac{1}{\pi_H}$.

To complete the proof, we have to show that the borrower always prefers to strategically default following any deviation to C'_1 . This latter point requires that the moral hazard is mild. One has

$$\begin{aligned} U(C'_1, e = H) &= \pi_H(G(I - \epsilon) - (R - p\epsilon)) - A = B(I + \bar{I} + A) - \pi_H(G - p)\epsilon - A \\ &< B(I + \bar{I} + A - \epsilon) - A \end{aligned} \quad (19)$$

that is, $B < \pi_H(G - p)$ which holds whenever moral hazard is mild.

CASE 2: $B > \pi_H G - 1$

To complete the proof of proposition 2, let us consider $C \in \Psi - \{(I^m, R^m)\}$ and, by contradiction, let us assume that it is an equilibrium allocation. Observe that, since $C \in \Psi - \{(I^m, R^m)\}$, there exists at least one active lender i who proposes a contract of price $p_i < G$.

Let us first consider the case where $U(C, e = H) = U(C, e = L) = B(I + \bar{I} + A)$. See that, because moral hazard is severe, inequality (19) is reversed so that an active lender can profitably deviate, the borrower accepts the deviation and selects $e = H$. Thus $C \in \Psi - \{(I^m, R^m)\}$ cannot be an equilibrium allocation.

Let us now consider the case where $U(C, e = H) = U(C, e = L) > B(I + \bar{I} + A)$. We show below that, irrespective of the severity of the moral hazard problem, an active lender i who proposes a contract C_i with price $p_i < G$ can profitably deviate and reduce the entrepreneur's utility. Suppose that investor i deviates to $C'_i = (I_i - \epsilon, R_i - \epsilon(G - \frac{B}{\Delta\pi}) - \epsilon^2)$ for some strictly

positive ϵ . Under condition (2), if the deviation is accepted and the entrepreneur chooses $e = H$, such a deviation increases investor i 's profit for ϵ sufficiently small. We check below that the deviation is accepted and the entrepreneur chooses $e = H$.

By Lemma 1, the entrepreneur always has an incentive to trade a contract whose price is strictly lower than G . This yields

$$\begin{aligned} R_i - \epsilon \left(G - \frac{B}{\Delta\pi} \right) - \epsilon^2 &< G(I_i - \epsilon) \\ \Leftrightarrow \quad R_i - GI_i + \epsilon \left(\frac{B}{\Delta\pi} - \epsilon \right) &< 0 \end{aligned}$$

Since $R_i - GI_i < 0$, the above condition holds when ϵ is small enough.

Next, we show that in every continuation game following the deviation to C'_i the entrepreneur selects $e = H$. Observe that the entrepreneur's maximal utility when $e = L$ amounts to $U(C - C_i + C'_i, e = L)$, furthermore

$$\begin{aligned} U(\bar{C} - C_i + C'_i, e = H) &> U(\bar{C} - C_i + C'_i, e = L) \\ \Leftrightarrow \quad \pi_H \left\{ G(\bar{I} - \epsilon) - (\bar{R} - \epsilon(G - \frac{B}{\Delta\pi}) - \epsilon^2) \right\} &> \pi_L \left\{ G(\bar{I} - \epsilon) - (\bar{R} - \epsilon(G - \frac{B}{\Delta\pi}) - \epsilon^2) \right\} + B(\bar{I} - \epsilon) \\ \Leftrightarrow \quad \epsilon^2 &> 0. \end{aligned}$$

Therefore, following the deviation C'_i , the entrepreneur strictly prefers to choose $e = H$. We conclude that $C \in \Psi - \{(I^m, R^m)\}$ cannot be an equilibrium allocation.

Last, we verify that (I^m, R^m) is indeed an equilibrium allocation. Consider the following strategies: $(I_1, R_1) = (I^m, R^m)$ and $(I_i, R_i) = (0, 0)$ for $i = 2, \dots, n$. Then, it is a best reply for the entrepreneur to trade the contract (I_1, R_1) and to select $e = H$. This in turn provides her with the reservation utility $U(I_1, R_1, e = H) = \pi_H GA - A$. We now show that none of the lenders has a unilateral incentive to deviate. Since lender 1 is earning the monopoly profit, only deviations of the inactive lenders must be considered. Follow step 1 above to show that this deviation is not profitable.

■

Proof of Proposition 3.

Let $C = (I, R) \in \mathcal{H}$ be a feasible aggregate allocation. Define $\overset{\circ}{I} \in \mathbb{R}_+$ as the investment level such that

$$\pi_H[G(I + A) - R] = B(I + \overset{\circ}{I} + A). \quad (20)$$

According to equation (20), the entrepreneur obtains the equilibrium utility if she borrows $\overset{\circ}{I}$ in addition to the equilibrium investment I , chooses $e = L$ and is left with his private benefit only.

Recall that, to guarantee borrower's participation, it must be $R \leq GI$. It follows that

$$B \overset{\circ}{I} = \pi_H[G(I + A) - R] - B(I + A) \geq \pi_L[G(I + A) - R] \geq \pi_L GA > 0 \quad \forall (I, R) \in \mathcal{H},$$

which guarantees $\overset{\circ}{I} > 0$. Throughout the proof of Proposition 3 we shall use repeatedly the above relation (20) together with the following Lemma.

Lemma 3 *Let us consider the function f defined on $[\pi_H GA, U^c]$ by the relation $f(U) = U^c - BI^c - U$, where $U^c = U(I^c, R^c, H)$ denotes the entrepreneur's expected utility under high effort obtained with the competitive allocation (I^c, R^c) . The following holds*

(i) *If $B \leq \pi_H G - 1$, then there exists a unique $U_B^* \in [\pi_H GA, U^c]$ such that $f(U(I, R, H)) < 0$ for any feasible allocation $(I, R) \in \mathcal{H}$ satisfying $U(I, R, H) > U_B^*$. Furthermore, any feasible allocation $(I, R) \in \mathcal{H}$ such that $U(I, R, H) > U_B^*$ satisfies the relation $R < \frac{1}{\pi_H}(1 + B)I$.*

(ii) *If $B > \pi_H G - 1$, then $f(U(I, R, H)) < 0$ for any feasible allocation $(I, R) \in \mathcal{H}$.*

Observe that for any feasible allocation (I, R) in \mathcal{H} we have that $f(U(I, R, H)) = U^c - B(I^c + I + \overset{\circ}{I} + A)$. The proof of the Lemma relies then on a straightforward computation. We show below that, (i) if $B \leq \pi_H G - 1$ then any aggregate feasible allocation $C = (I, R) \in \mathcal{H}$ such that $U(I, R, H) > U_B^*$ can be supported at the equilibrium, (ii) if $B > \pi_H G - 1$ then any aggregate feasible allocation $C = (I, R) \in \mathcal{H}$ can be supported at the equilibrium.

The equilibrium we construct involves a large number of both active and inactive lenders. Consider the following profile of strategies. Each lender $i = 1, 2, \dots, M$ offers:

$$\left\{ \begin{array}{ll} (\frac{I}{M}, \frac{R}{M}) & \text{if the total investment is } I \\ (\frac{I}{M}, \frac{\mathring{R}}{M-1}) & \text{if the total investment is } \frac{(M-1)I}{M} \\ (\frac{I}{M}, G(\hat{I} + A)) & \text{if the total investment is } \hat{I} \notin \{\frac{(M-1)I}{M}, I\} \end{array} \right. \quad (21)$$

where \mathring{R} is defined by:

$$\pi_H[G(I + A) - R] = \pi_H[G(\frac{M-1}{M}I + A) - \mathring{R}], \quad (22)$$

which implies $\mathring{R} = R - \frac{GI}{M}$.

Each lender $j = M + 1, \dots, N$ proposes the pair $(\frac{\mathring{I}}{N-M}, G(\hat{I} + A)) \forall \hat{I} \in \mathbb{R}_+$.

Let us show that these strategies constitute an equilibrium in which the entrepreneur accepts the offers of investors $1, \dots, M$, rejects those of investor $M + 1, \dots, N$ and selects $e = H$. Therefore, only M investors are active. If $B \leq \pi_H G - 1$ we restrict the feasible aggregate allocations $(I, R) \in \mathcal{H}$ to those satisfying $U(I, R, H) > U_B^*$. If $B > \pi_H G - 1$, (I, R) represents any feasible allocation in \mathcal{H} . We shall assume that the number of investors N is sufficiently large. As we shall see the borrower's equilibrium strategy is constructed in such a way that, if default takes place, she first repays all non-deviating investors. We shall also see that a situation in which lenders are repaid to some pro-rata rules is consistent with the above borrower's equilibrium strategy.

1. We first show that given the investors' offers, it is a best reply for the entrepreneur to accept the proposals of investors $1, \dots, M$ and to reject those of investors $M + 1, \dots, N$. Following the equilibrium strategy, the entrepreneur obtains the payoff $\pi_H[G(I + A) - R] \geq \pi_H GA$. See that no other portfolio choice associated to $e = H$ provides the entrepreneur with a strictly higher payoff: indeed, accepting $M - 1$ contracts yields the same utility for the entrepreneur. Accepting any of the $N - M$ contracts triggers $e = L$. Accepting less than $M - 1$ contracts also triggers $e = L$. Next, see that the entrepreneur cannot increase her utility by choosing $e = L$ and ac-

cepting all N contracts. She would then obtain an aggregate loan of $I + \overset{\circ}{I}$ and a utility level of $B(I + \overset{\circ}{I} + A) = \pi_H[G(I + A) - R]$, by (20).

2. We next show that given the equilibrium strategies, none of the inactive investors $M + 1, \dots, N$ can profitably deviate.

- Consider an inactive investor j . Let us establish that he cannot propose a deviation (I'_j, R'_j) such that only his offer is accepted out of equilibrium. Define (i, r) as the investment-repayment pair that lies at the intersection between the zero-profit line (of investors) of slope $\frac{1}{\pi_H}$ and the entrepreneur's equilibrium indifference curve given (I, R) . Check that $i = \frac{\pi_H(GI - R)}{\pi_H G - 1}$ and $r = \frac{i}{\pi_H}$. Denote also (i_I, r_I) the intersection between the entrepreneur's indifference curve and Ψ , and the competitive allocation (I^c, R^c) defined by the intersection between the zero-profit line and Ψ . Any deviation (I'_j, R'_j) that provides a strictly positive profit to lender j and (weakly) increases the utility of the entrepreneur if only (I'_j, R'_j) is accepted must lie in the triangle defined by (i, r) , (i_I, r_I) and (I^c, R^c) . We show below that any such deviation induces the entrepreneur to accept all contracts and exert $e = L$ if N is sufficiently large. To do so, we prove that, if N is large enough then the function

$$F(I'_j, R'_j; I, R) = \pi_H (G(I'_j + A) - R'_j) - B \left(I'_j + I + \overset{\circ}{I} \frac{N - M - 1}{N - M} + A \right)$$

is negative at (i, r) . This will in turn imply that F is negative at any point in the triangle $(i, r), (i_I, r_I), (I^c, R^c)$ which corresponds to the set of admissible deviations (I'_j, R'_j) . Consider therefore that investor j deviates and proposes the pair (i, r) , the entrepreneur prefers to choose $e = L$ if:

$$\begin{aligned} & \pi_H (G(i + A) - r) - B \left(i + I + \overset{\circ}{I} \frac{N - M - 1}{N - M} + A \right) \leq 0 \\ \iff & B \left(i - \frac{\overset{\circ}{I}}{N - M} \right) \geq 0. \end{aligned} \tag{23}$$

We use then the definitions of i and \bar{I} to rewrite (23) as:

$$(N - M) \frac{\pi_H(GI - R)}{\pi_H G - 1} - \pi_H \frac{G(I + A) - R}{B} + (I + A) \geq 0. \quad (24)$$

If N is large enough, then (24) is satisfied for all feasible allocations (I, R) : F is negative at the point (i, r) . Next, see that for a given I'_j , the minimum value of R'_j such that the deviation is in the admissible triangle $(i, r), (i_I, r_I), (I^c, R^c)$ is defined by $R'_j = \frac{I'_j}{\pi_H}$ and observe that

$$\left. \frac{dF}{dI'_j} \right|_{R'_j = \frac{I'_j}{\pi_H}} = \pi_H G - 1 - B. \quad (25)$$

First consider the case where the private benefit B satisfies $B \leq \pi_H G - 1$ then the function $F(I'_j, \frac{I'_j}{\pi_H}; I, R)$ increases with I'_j and reaches its maximum at $I'_j = I^c$. Because we assume in this case that the allocation (I, R) satisfies $U(I, R, H) \geq U_B^*$ this maximum is negative from Lemma 3. This yields $F(I'_j, R'_j; I, R) < 0$ for any pair $(I'_j, \frac{I'_j}{\pi_H})$ with $I'_j \in [i, I^c]$. Finally, since F is decreasing in R'_j one gets that F is negative at any point in the triangle $(i, r), (i_I, r_I), (I^c, R^c)$ satisfying $U(I, R, H) \geq U_B^*$. Thus, for every feasible allocation (I, R) with $U(I, R, H) \geq U_B^*$ there always exists a sufficiently high number N of investors such that none of the inactive investors has a unilateral incentive to deviate as long as the entrepreneur accepts only his offer out of equilibrium. If the private benefit B is such that $B > \pi_H G - 1$ then the function $F(I'_j, \frac{I'_j}{\pi_H}; I, R)$ decreases with I'_j and the conclusion clearly holds for any feasible allocation (I, R) in the triangle $(i, r), (i_I, r_I), (I^c, R^c)$.

- Let us now establish that no inactive investor j has an incentive to deviate so that the entrepreneur trades several contracts out of equilibrium. For a deviation to be profitable, the entrepreneur must choose $e = H$ following the deviation. This is because the deviating investor is repaid after the others in case of default. Observe that the entrepreneur has an incentive to select $e = H$ only if the investment she trades in the aggregate is either I , or $\frac{(M-1)I}{M}$. In all other cases, the entrepreneur optimally chooses $e = L$ since all the cash-flow is paid to investors. It follows that only a subset of the active investors' offers must be traded out of equilibrium (and no offer from passive investors).

Let (I'_j, R'_j) be a unilateral deviation by investor j , and let $m < M$ be the number of additional contracts which are traded following the deviation.

We first consider the case where the aggregate level of investment is equal to I ; one we must have:

$$I = I'_j + \frac{m}{M}I \iff I'_j = \frac{M-m}{M}I \quad (26)$$

The borrower will accept the deviating contract (I'_j, R'_j) and choose $e = H$ if

$$\pi_H \left[G(I + A) - R'_j - \frac{m}{M}R \right] > B \left(I + \overset{\circ}{I} + A + I'_j - \frac{\overset{\circ}{I}}{N-M} \right), \quad (27)$$

which, given (20) and (26), corresponds to

$$\begin{aligned} \pi_H \left[\left(1 - \frac{m}{M}\right)R - R'_j \right] &> B \left(\frac{M-m}{M}I - \frac{\overset{\circ}{I}}{N-M} \right) \\ \Leftrightarrow B \frac{\overset{\circ}{I}}{N-M} &> \frac{M-m}{M} \left(BI - \pi_H R + \pi_H R'_j \frac{M}{M-m} \right). \end{aligned} \quad (28)$$

For the deviation (I'_j, R'_j) to be profitable for the lender one must have $R'_j \pi_H > I'_j$. Then, given (28), we have

$$B \frac{\overset{\circ}{I}}{N-M} > \frac{M-m}{M} \left(BI - \pi_H R + I'_j \frac{M}{M-m} \right)$$

which, using (26), yields

$$B \frac{\overset{\circ}{I}}{N-M} > \frac{M-m}{M} (I(1+B) - \pi_H R). \quad (29)$$

Assume that $B \leq \pi_H G - 1$. In this case the feasible allocation $(I, R) \in \mathcal{H}$ that we consider satisfies $U(I, R, H) > U_B^*$ which implies from Lemma 3 that $R < \frac{1}{\pi_H}(1+B)I$. It follows that condition (29) does not hold if N is high enough. This in turn contradicts (27), showing that there is no profitable deviation. If $B > \pi_H G - 1$, none of the feasible allocation $(I, R) \in \mathcal{H}$ satisfies (27).

We now consider the situation where the aggregate investment chosen at the deviation stage is

equal to $\left(\frac{M-1}{M}\right)I$. The corresponding I'_j is such that:

$$\left(\frac{M-1}{M}\right)I = I'_j + \frac{m}{M}I \iff I'_j = \left(\frac{M-m-1}{M}\right)I \quad (30)$$

with $m < M-1$. A necessary condition for the deviation to (I'_j, R'_j) to be profitable is

$$\pi_H \left[G \left(\frac{M-1}{M}I + A \right) - R'_j - \frac{m}{M-1} \overset{\circ}{R} \right] > B \left(I + \overset{\circ}{I} + A + I'_j - \frac{\overset{\circ}{I}}{N-M} \right), \quad (31)$$

which, given (20), corresponds to

$$B \frac{\overset{\circ}{I}}{N-M} > B I'_j - \pi_H \frac{M-m-1}{M-1} \left(R - \frac{GI}{M} \right) + \pi_H R'_j \quad (32)$$

Using $\pi_H R'_j > I'_j$ and (30) together with the fact that $\frac{M-m-1}{M-1} \left(R - \frac{GI}{M} \right) \leq \frac{M-m-1}{M} GI$ because $R \leq GI$, we deduce from (32) that

$$B \frac{\overset{\circ}{I}}{N-M} > \frac{M-m-1}{M} I (1 + B - \pi_H G). \quad (33)$$

As previously, if $B \leq \pi_H G - 1$, the inequality $R < \frac{1}{\pi_H} (1+B)I$ holds because we focus on allocation (I, R) such that $U(I, R, H) > U_H^*$. It follows that the (RHS) of (33) is positive. This implies that if N is large enough (31) is violated and there is no profitable deviation. If $B > \pi_H G - 1$, (31) is violated for any feasible allocation $(I, R) \in \mathcal{H}$. This concludes the analysis of deviations by passive investors.

3. We now turn to the proof that no active investor $(1, \dots, M)$ can profitably deviate. Consider any of the investors who is active at equilibrium, say the k -th one.

- Consider first the situation where only the offer (I'_k, R'_k) of investor k is accepted out of equilibrium. Let us reformulate (i, r) as the investment-repayment pair that lies at the intersection between investor k 's profit line and the entrepreneur's equilibrium indifference curve. Similarly,

define (i_I, r_I) the intersection between the entrepreneur's indifference curve and Ψ , and (I^c, R^c) the competitive allocation defined by the intersection between investor k 's profit line and Ψ . See that:

$$\begin{cases} \pi_H [G(I + A) - R] = \pi_H [G(i + A) - r] \\ \pi_H r - i = \pi_H \frac{R}{M} - \frac{I}{M} \end{cases} \quad (34)$$

It follows that $i = \frac{GI - R + (1/M)(R - I/\pi_H)}{G - 1/\pi_H}$. As before, we need to show that the function $F(I'_k, R'_k; I, R) = \pi_H (G(I'_k + A) - R'_k) - B \left(I'_k + I + \frac{I}{M} + A \right)$ is negative at each point in the triangle $(i, r), (i_I, r_I), (I^c, R^c)$. To prove this, we first show that F is negative at (i, r) . Assume that investor k deviates and offers (i, r) , the entrepreneur prefers to choose $e = L$ if:

$$\pi_H (G(i + A) - r) - B \left(I + \frac{I}{M} + A + i - \frac{I}{M} \right) = -B \left(i - \frac{I}{M} \right) \leq 0. \quad (35)$$

Given the definition of i , (35) can be rewritten as:

$$\frac{(GI - R) \left(1 - \frac{1}{M}\right)}{G - \frac{1}{\pi_H}} \geq 0, \quad (36)$$

which is always satisfied since $GI \geq R$.

Next, see that for a given I'_k , the minimum value of R'_k such that the deviation is in the admissible triangle $(i, r), (i_I, r_I), (i_P, r_P)$ is defined by $R'_k = \frac{I'_k + V}{\pi_H}$, where V represents the equilibrium profit of investor k . See that:

$$\left. \frac{dF}{dI'_k} \right|_{R'_k = \frac{I'_k + V}{\pi_H}} = \pi_H G - 1 - B. \quad (37)$$

If $B < \pi_H G - 1$ then the function $F(I'_k, \frac{I'_k + V}{\pi_H}; I, R)$ increases with I'_k and reaches its maximum at $I'_k = I^c$. Because the allocation (I, R) is such that $U(I, R, H) > U_B^*$ we obtain from Lemma 3 that this maximum is negative.¹⁰ This yields that $F(I'_k, R'_k; I, R) < 0$ for any pair $(I'_k, \frac{I'_k + V}{\pi_H})$ with $I'_k > i$. Note also that F is decreasing in R'_k . This yields that F is negative at any point (I, R)

¹⁰Very precisely, this easy computation also uses the relation $BI < V$ that follows from the inequality $\pi_H R > I$ together with the inequality $B < 1$ that is implied by relation (1).

in the triangle $(i, r), (i_I, r_I), (I^c, R^c)$ such that $U(I, R, H) > U_B^*$. It follows that none of the active investors has a unilateral incentive to deviate if the entrepreneur accepts only his offer out of equilibrium. Here again if $B > \pi_H G - 1$ then it is no more necessary to impose the condition $U(I, R, H) > U_B^*$ to get the result.

• Consider next the situation where an active investor k deviates and the entrepreneur trades several contracts out of equilibrium. As before, for a deviation to be profitable, default should necessarily be avoided. This can only happen if the aggregate investment selected by the entrepreneur at the deviation stage is either I , or $\frac{(M-1)I}{M}$.

We first consider the situation where such aggregate investment is set equal to I . Let m be the number of additional contracts which are traded following the deviation, we have: $I'_k = \frac{M-m}{M}I$. A necessary condition for the deviation to be profitable is

$$\pi_H \left[G(I + A) - R'_k - \frac{m}{M}R \right] > B(I + I + A + I'_k - \frac{I}{M})$$

or equivalently using (20),

$$\pi_H \left[-R'_k + \frac{M-m}{M}R \right] > B \frac{M-m-1}{M}I. \quad (38)$$

For the deviation (I'_k, R'_k) to be profitable for the lender one must have $\pi_H R'_k - I'_k > \pi_H \frac{R}{M} - \frac{I}{M}$. It follows that, if (38) holds then, necessarily

$$\frac{M-m-1}{M} \pi_H R > \frac{M-m-1}{M} (B+1)I. \quad (39)$$

Again, if $B \leq \pi_H G - 1$, then the allocations (I, R) that we consider satisfy $U(I, R, H) > U_H^*$. From Lemma 3 this implies that $\pi_H R < (B+1)I$. If $B \leq \pi_H G - 1$, the inequality $\pi_H R < (B+1)I$ is always satisfied for any feasible allocation $(I, R) \in \mathcal{H}$. This contradicts condition (39).

Finally, if the aggregate investment chosen at the deviation stage is equal to $\left(\frac{M-1}{M} \right) I$, we have

$I'_k = \frac{M-m-1}{M}I$. A necessary condition for the deviation to be profitable is

$$\pi_H \left[G \left(I + A - \frac{I}{M} \right) - R'_k - \frac{m}{M-1} \mathring{R} \right] > B(I + \mathring{I} + A + I'_k - \frac{I}{M}),$$

equivalently, using (20) and the definition of \mathring{R} ,

$$\pi_H \left[- (M-m-1) \frac{GI}{M(M-1)} + \frac{M-m-1}{M-1} R - R'_k \right] > BI \left(\frac{M-m-2}{M} \right) \quad (40)$$

The deviation (I'_k, R'_k) which must be profitable for the lender satisfies $\pi_H R'_k - I'_k > \pi_H \frac{\mathring{R}}{M-1} - \frac{I}{M}$.

It follows that if (40) holds then necessarily,

$$\pi_H \left[R - \frac{1}{M} GI \right] \frac{M-m-2}{M-1} > (B+1)I \left(\frac{M-m-2}{M} \right). \quad (41)$$

Observe that the (LHS) of (41) is lower than $\pi_H \frac{M-m-2}{M} GI$, so that given $\pi_H R < I(1+B)$, the inequality (40) cannot be satisfied.

This completes the proof of Proposition 3. When $B < \pi_H G - 1$, any allocation $C = (I, R) \in \mathcal{H}$ such that $R \in [\frac{1}{\pi_H}I, GI]$ with $U(I, R, H) > U_B^*$ can be supported at equilibrium. When $B < \pi_H G - 1$ any allocation $C = (I, R) \in \mathcal{H}$ can be supported at the equilibrium.

Remark: Proposition 3 also holds if defaulted bonds are repaid *pro rata*. To prove this let us first consider an inactive lender j who deviates and thus proposes (I'_j, R'_j) such that the borrower earns a utility strictly greater than the equilibrium one by selecting $e = L$ accepting all contracts and defaulting even in the success state. That is,

$$B(I'_j + I + \frac{N-M-1}{N-M} \mathring{I} + A) > B(I + \mathring{I} + A)$$

or equivalently, using (20)

$$I'_j > \frac{\mathring{I}}{N-M}. \quad (42)$$

In a *pro rata* environment, the deviation is profitable for the lender iff

$$\begin{aligned} \pi_L G \left(I'_j + I + \frac{N-M-1}{N-M} \overset{\circ}{I} + A \right) \frac{I'_j}{I'_j + I + \frac{N-M-1}{N-M} \overset{\circ}{I}} &> I'_j \\ \iff (\pi_L G - 1) \left(I + I'_j + \frac{N-M-1}{N-M} \overset{\circ}{I} \right) + \pi_L G A &> 0. \end{aligned} \quad (43)$$

Relation (42) together with $B \overset{\circ}{I} \geq \pi_L G A$ implies

$$\begin{aligned} &(\pi_L G - 1) \left(I + I'_j + \frac{N-M-1}{N-M} \overset{\circ}{I} \right) + \pi_L G A \\ < (\pi_L G - 1) \left(I + \frac{1}{N-M} \frac{1}{B} \pi_L G A + \frac{N-M-1}{N-M} \frac{1}{B} \pi_L G A \right) + \pi_L G A \\ = &(\pi_L G - 1) I + \pi_L G A \left(\frac{1}{B} (\pi_L G - 1) + 1 \right) < 0 \end{aligned}$$

where the last inequality comes from (1) and (2). This shows that relations (42) and (43) are not compatible.

Second let consider an active lender j who deviates and proposes (I'_j, R'_j) . Observe that, necessarily, $I'_j > I_j = \frac{I}{M}$. Indeed, if it was not the case, because of Equation (20), the borrower's utility at the deviation stage would be lower than her equilibrium utility. Formally,

$$B \left(I'_j + \frac{M-1}{M} I + \overset{\circ}{I} + A \right) < B(I + \overset{\circ}{I} + A) = \pi_H(G(I + A) - R).$$

We thus have that $I'_j > \frac{I}{M}$. A direct computation that uses Equation (1) shows that relation $I'_j > \frac{I}{M}$ cannot hold together with inequality

$$\pi_L G \left(I'_j + \frac{M-1}{M} I + \overset{\circ}{I} + A \right) \frac{I'_j}{I'_j + \frac{M-1}{M} I + \overset{\circ}{I}} > I'_j. \quad (44)$$

This means that the deviation is not profitable for the lender j under a *pro rata* rule.

■

Proof of Proposition 4.

The proof follows the standard logic of price competition. Let $C \neq C^c$ be a equilibrium allocation where the entrepreneur selects $e = H$. Consider first the situation where investors are earning a strictly positive profit at equilibrium, which implies that $R > \frac{1}{\pi_H}I$. Take any of the investors earning the smallest profit at equilibrium (the same logic goes through if one considers a deviation by a passive investor), say the i -th one, and suppose he deviates proposing:

$$\begin{cases} (I'_i, R'_i) = (I, R - \varepsilon) & \text{if the aggregate level of investment is } I \\ (I'_i, R'_i) = (I, G(2I - I_i + \overset{\circ}{I} + A)) & \text{if the aggregate level of investment is } \neq I, \end{cases}$$

where ε has been taken to be small enough. The deviation guarantees that investor i earns (almost) the aggregate equilibrium payoff. The entrepreneur has a clear incentive to accept the offer, since she can achieve a payoff strictly greater than the equilibrium one, and to accept only investor i 's offer: any alternative choice would indeed induce her to reject the offer of investor i (because strategic default is ruled out). Also, the entrepreneur clearly selects $e = H$ at the deviation stage, which guarantees that the deviation is profitable in the first place.

If aggregate profits are zero at equilibrium, i.e. if $R = \frac{1}{\pi_H}I$, it is always possible for any of the investors, say the i -th one, to profitably deviate proposing:

$$\begin{cases} (I'_i, R'_i) = (I + \epsilon, R + G\epsilon^2) & \text{if the aggregate level of investment is } I \\ (I'_i, R'_i) = (I + \epsilon, G(2I + \epsilon - I_i + \overset{\circ}{I} + A)) & \text{if the aggregate level of investment is } \neq I, \end{cases}$$

where ϵ has been taken to be small enough. The deviation guarantees that investor i earns a strictly positive profit if $e = H$ is chosen. The entrepreneur has a clear incentive to accept the offer, since she can achieve a payoff strictly greater than the equilibrium one. In addition, the optimal level of investment she will be trading at the deviation stage is exactly I'_i : any alternative choice would indeed induce her to reject the offer of investor i . It follows that $e = H$ is selected at the deviation stage, which guarantees that the deviation is profitable in the first place. Assertion (i) is proven. Assertion (ii) directly follows from the proof of Proposition 1.

■

Proof of Proposition 5.

Assume that there exists an equilibrium allocation $C = (I, R)$ different from the competitive one. Let us show that there always exists a profitable deviation for an investor (I', R') together with a contract $\tilde{C} = (\tilde{I}, \tilde{R})$ such that C is not an equilibrium. Throughout the proof we assume there is a large number of both active and inactive lenders. This will allow us to consider deviations from active (resp. inactive) lenders¹¹ who provide a negligible initial investment $\frac{I}{M}$ (resp. $\frac{\bar{I}}{N-M}$).

Consider the deviation $C' = (I', R')$ defined by

$$\begin{aligned} & (I', \frac{I'}{\pi_H}) \quad \text{if the total investment } I = \tilde{I} + I' \text{ where } I' = \frac{\pi_H(GI - R)}{\pi_H G - 1} \\ & (I', G(I + A)) \quad \text{if the total investment } I \neq \tilde{I} + I'. \end{aligned}$$

The deviation C' is constructed in order to guarantee to the entrepreneur its equilibrium utility. The covenant clause guarantees that the entrepreneur has no incentives to take all contracts and to default. We show below that there always exists a contract $\tilde{C} = (\tilde{I}, \tilde{R})$ with $\tilde{I} > 0$ and $\tilde{R} \leq \frac{\tilde{I}}{\pi_H}$ such that (i) the entrepreneur's optimal choice is to accept C' and \tilde{C} (ii) investors collectively agree to offer (\tilde{I}, \tilde{R}) rather than to do nothing. Note that by construction $U(C' + \tilde{C}, H) > U(C, H)$. This is because C' guarantees to the entrepreneur its equilibrium utility together with the fact that the price of the contract \tilde{C} is lower than G since $\tilde{R} \leq \frac{\tilde{I}}{\pi_H}$.

The entrepreneur prefers to choose the deviation (I', R') together with (\tilde{I}, \tilde{R}) if:

$$U(C' + \tilde{C}, H) > B(I + \bar{I} + \tilde{I} + I' + A) - A. \quad (45)$$

The RHS of condition (45) represents the entrepreneur's utility if she accepts all available contracts and shirks. Again we implicitly assume that the initial investment¹² offered by the deviator is negligible. Explicitly considering the initial deviator's offer $\frac{I}{M}$ (if active) or $\frac{\bar{I}}{N-M}$ (if inactive) in the condition above does not change the demonstration but burdens the expressions.

Investors collectively accept to grant the allocation \tilde{C} if:

¹¹ Observe that if $C \notin \Psi$, then there are some inactive investors. This is not always the case if $C \in \Psi$.

¹² $\frac{I}{M}$ for an active lender and $\frac{\bar{I}}{N-M}$ for an inactive lender.

$$\pi_H(R' + \tilde{R}) - (I' + \tilde{I}) \geq \pi_L G(I' + I + \bar{I} + A) - (I' + I + \bar{I}). \quad (46)$$

The RHS of condition (46) represents the aggregate profit of investors if the deviation (I', R') is offered and investors refuse to finance (\tilde{I}, \tilde{R}) .

Conditions (45) and (46) yields

$$\pi_H(R' + \tilde{R}) \leq \pi_H G(I' + \tilde{I} + A) - B(I + \bar{I} + \tilde{I} + I' + A)$$

$$\pi_H(R' + \tilde{R}) > \pi_L G(I' + I + \bar{I} + A) - (I + \bar{I}) + \tilde{I}$$

using the relation $R' = \frac{I'}{\pi_H}$ we obtain

$$\pi_H \tilde{R} \leq \pi_H G(I' + \tilde{I} + A) - I' - B(I + \bar{I} + \tilde{I} + I' + A) \quad (47)$$

$$\pi_H \tilde{R} > \pi_L G(I' + I + \bar{I} + A) - (I + \bar{I} + I') + \tilde{I} \quad (48)$$

which leads to the condition

$$\begin{aligned} & \pi_L G(I' + I + \bar{I} + A) - (I + \bar{I} + I') + \tilde{I} \\ & < \pi_H G(I' + \tilde{I} + A) - I' - B(I + \bar{I} + \tilde{I} + I' + A) \end{aligned}$$

or equivalently

$$\tilde{I}(\pi_H G - B - 1) > (\pi_L G - 1 + B)(I + \bar{I}) + (-\Delta\pi G + B)(I' + A) \quad (49)$$

See that the RHS of (49) is always negative. Therefore two cases can happen. If $\pi_H G - B - 1 > 0$ then any $I' > 0$ satisfies (49). If $\pi_H G - B - 1 < 0$ condition (49) defines an upper bound on \tilde{I} according to I' . In all cases, it is always possible to find feasible values for (I', R') and (\tilde{I}, \tilde{R}) satisfying (47) and (48). Finally, by continuity the above reasoning holds for $R' = \frac{I'}{\pi_H} + \epsilon$ with

$\epsilon > 0$ sufficiently small. This in turn ensures a strictly positive profit to the deviating lender. Providing cheap financing through a contract (\tilde{I}, \tilde{R}) thus rules out any equilibrium allocation except, by construction, the competitive equilibrium.

■

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