Competing Mechanism Games of Moral Hazard: Communication and Robustness *

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Abstract

In multiple-principal multiple-agent models of moral hazard, we provide sufficient conditions for the outcomes of pure-strategy equilibria in simple mechanisms to be preserved when principals can offer indirect communication schemes. The conditions include strong robustness in the simple mechanism game, as developed in the literature on competing mechanisms by Peters (2001) and Han (2007), and a no-correlation property we define. We show via an example that the no-correlation condition is tight in moral hazard models. Finally, we provide a rationale for restricting attention to take-it or leave-it offers, as is typically done in applications.

Key words: Moral hazard, multiple principal, multiple agent, simple mechanisms.
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1 Introduction

We consider multiple-principal, multiple-agent models of pure moral hazard. That is, principals compete through mechanisms in a scenario where there is complete information about the types of agents, but agents' effort is not contractible. Our goal is to establish conditions under which equilibria sustained by simple communication mechanisms are robust to the possibility that any principal may deviate to a richer communication scheme (i.e., an indirect mechanism) to interact with agents.

With multiple principals, it is well-established that there is a loss of generality in focusing on incentive compatible direct mechanisms (see, for example, Peck (1997), Martimort and Stole (2002), and Peters (2001)). That is, there exist equilibrium outcomes sustained by indirect communication schemes that are not replicable through direct mechanisms. In such contexts, it is thus important to understand whether there is a rationale for the restriction to simple mechanisms which have typically been postulated in much of the literature on competing principals.[1] It turns out (see Theorems 1 and 2 in Peters (2003)) that, whenever multiple principals interact in the presence of a single agent, pure strategy equilibria in simple mechanisms are robust in the sense that they remain equilibria when richer communication schemes are feasible.

We provide a similar result for multiple-principal, multiple-agent games of pure moral hazard. Such settings are characterized by the simultaneous presence of two sources of correlation. By privately communicating with each agent, a principal can induce correlated equilibria in the effort game played by agents, as is well known since the work of Aumann (1974) and Myerson (1982). At the same time, each agent may act as a correlating device among principals, as shown for instance by Peters (2001) and Martimort and Stole (2002). That is, principals' decisions depend on messages sent by agents, and agents' choices in turn depend on recommendations they receive from principals.

At first glance, the idea of a communication scheme in a complete information setting may seem strange. The notion of private communication between a principal and an agent is also developed by Rahman and Obara (2009), in the context of a partnership, and Martimort and Moreira (2008), in a common agency game with informed principals. Rahman and Obara (2009) consider a single principal who can offer "mediated contracts," which are contingent on the private recommendations sent to agents in pre-

[1] See Peters (2003) and Han (2007) for a discussion. If there is complete information and the effort is contractible, in a direct mechanism a single principal is restricted to offering a single pay-for-effort contract to each agent.

vious stages of the game. Such mediated communication helps to restore efficiency in partnership problems. Martimort and Moreira (2008) consider a public good problem in which principals privately know their own willingness to pay, and communicate this to an agent. Thus, the agent endogenously has private information about principal j that is payoff-relevant to principal i.

In our setting each competing principal is allowed to send private recommendations to the agents. An agent uses the recommendations she receives to update her beliefs over allocations. Hence, by communicating privately with an agent, a principal can create asymmetric information among agents, before they choose effort. In a multi-principal framework, if such private communication is relevant to describe equilibrium behavior, a principal may want to ask agents to reveal the information conveyed by their private communication with other principals. That is, he may have an incentive to set up a further stage of communication. We argue that absence of correlation between allocations and recommendations in the decision rule of every principal makes this additional communication useless. Hence, in these cases we can naturally extend the original Myerson (1982) framework to multiple-principal, multiple-agent models of moral hazard.

We define a simple mechanism to be the one in which, for each agent, a principal chooses a message space equal to the agent's type space, and a recommendation space equal to the space of feasible actions. We first show that, if the principals' allocations do not vary with the messages they receive, and if there is no correlation between the allocations they offer and the recommendations they make, the best response of each principal can be characterized by a simple mechanism. That is, there is no benefit to a principal from asking an agent to report on the recommendations the agent receives from other principals. Thus, the infinite regress problem mentioned by McAfee (1993) and Epstein and Peters (1999) is broken.

We then consider the notion of strong robustness introduced by Peters (2001).[2] An equilibrium of a given multiple-principal multiple-agent game is said to be strongly robust if there does not exist a continuation equilibrium in the agents' game that would induce a principal to deviate. We show that strong robustness and no-correlation are together sufficient to imply that equilibria in simple mechanisms remain equilibria when principals can offer more complex communications schemes. We highlight the role of the no-correlation condition via an example.

[2]Han (2007) considers strong robustness in a model with complete information and contractible effort, and Han (2008) examines the common agency case.

Much work has been done on moral hazard with common agency (i.e., with multiple principals and a single agent).[3] In addition, Ishiguro (2005) develops a framework of competing principals with a finite number of agents who take a non-contractible action. These papers typically restrict attention to a situation in which no form of communication between principals and agents is considered, with principals proposing take-it or leave-it offers. For the most part, the theoretical literature on competing mechanisms does so.[4] Take-it or leave-it offers satisfy the no-correlation condition we introduce. It follows that an equilibrium in take-it or leave-it offers that can be sustained as a strongly robust equilibrium in a simple mechanism game can also be sustained as a strongly robust equilibrium in the indirect mechanism game. We provide an example of such an equilibrium in the context of a standard linear production economy subject to moral hazard.

As yet, little is known about the features of equilibrium contracts in multiple-principal, multiple-agent models. Yamashita (2007) and Peters and Valverde (2009) provide folk theorems for such games, with a large number of allocations supported in equilibrium. However, the menu theorems of Martimort and Stole (2002) and Peters (2001) do not extend straightforwardly to a general multiple-principal setting.[5] The methodology proposed by Pavan and Calzolari (2008) has also not yet been extended to multiple-principal multiple-agent games. Attar, Campioni, Piaser, and Rajan (2009) provide two examples showing a failure of the revelation principle in competing mechanism models of moral hazard.

Our results represent a step forward toward a characterization of equilibrium contracts in this framework. Since we consider an environment with non-contractible effort, our findings complement those of Han (2007) who restricts attention to the situation in which principals can write contracts contingent on agents' actions

[3]See, for example, Kahn and Mookherjee (1998), Parlour and Rajan (2001), Bisin and Guaitoli (2004) and Attar, Campioni, and Piaser (2006).

[4]Exceptions include Epstein and Peters (1999) who introduce private recommendations in a model without any moral hazard.

[5]Han (2006) extends the menu theorems to a restricted class of multiple-principal multiple-agent games, in which the contract between a principal and agent is essentially bilateral, and separate from the contract with any other principal or agent.

2 Model

The model is similar to that outlined in Attar, Campioni, Piaser, and Rajan (2009). There are n principals dealing with ℓ agents, where $n \geq 2$ and $\ell \geq 2$. Each agent i chooses an unobservable effort $e^i \in E^i$, where E^i is a finite set. We denote the vector of efforts as $e = \left(e^1, e^2, ..., e^\ell\right) \in E = \times_{i=1}^{\ell} E^i$. Each array of efforts supports a joint distribution on the space of final outcomes $Z = Z_1 \times \ldots \times Z_n$. We take $(z_1, z_2, \ldots, z_n)$ to be a generic element of Z, and we denote by $g(z_1, \ldots, z_n | e_1, \ldots, e_\ell)$ the probability of the outcome $(z_1, \ldots, z_n)$ induced by the efforts array $(e_1, \ldots, e_\ell)$. Let X_j be the set of actions available to principal j and x_j a generic element of that set, with $X = \times_j X_j$. Preferences of the players are defined over the set $X \times Z \times E$. We denote by v_j the utility function of the principal j and by u^i the utility function of the agent i.

Principal j only observes the realization z_j. An allocation rule chosen by principal j can therefore be represented as the measurable mapping $y_j : Z_j \to X_j$. We denote Y_j the set of such mappings for principal j and we let y_j be a generic element of this set.

Given the array of choice rules $y \in Y_1 \times \ldots \times Y_n$ and the vector of efforts $e = (e_1, e_2, \ldots, e_\ell)$, the expected utility of principal j and agent i is given by:

$$V_j(y,e) = \int_{z \in Z} g(z|e) v_j(y(z), z, e). \tag{1}$$

$$U^i(y,e) = \int_{z \in Z} g(z|e) u^i(y(z), z, e). \tag{2}$$

We extend the general communication structure for principal-agent models introduced by Myerson (1982). Each principal j chooses a message space M_j^i and a recommendation space R_j^i for each agent i. Let $R_j = \times_{i=1}^{\ell} R_j^i$ denote the set of recommendations principal j can make, and $M_j = \times_{i=1}^{\ell} M_j^i$ the set of messages he can receive. The allocations and recommendations chosen by principal j depend on the messages received from the agents. All relevant sets are taken to be compact and measurable.

A mechanism offered by principal j is thus given by $\gamma_j = \left(M_j, R_j, \pi_j\right)$ and $\pi_j : M_j \to \Delta(Y_j \times R_j)$ is the relevant choice rule. Mechanisms are publicly observed, but the message from agent i to principal j, and the recommendation from principal j to agent i, are observed only by i and j. That is, principal j chooses a realization from the lottery π_j, and communicates the realized recommendations r_j to the agents. Conditional on observing r_j^i, agent i updates her belief about the stochastic allocation rule y_j, but need not know the actual realization of the rule. Since recommendations are private, two agents i and i' may have different posterior beliefs about principal j's chosen allocation rule, y_j. Potentially, this allows a principal to induce a correlated equilibrium in the continuation game in which agents choose efforts. As is usual in the literature, principals commit to their mechanisms before agents send messages.

There are two stages at which agent i moves in the game. First, she sends a message array $m^i = \left(m_1^i, \ldots, m_n^i\right)$ to the principals. Then, after observing only her private recommendations $r^i = \left(r_1^i, \ldots, r_n^i\right)$, she chooses an effort $e^i \in E^i$. Given the offered mechanisms, let $\mu^i \in \Delta\left(M^i\right)$ denote the message strategy of agent i, and let $\delta^i : M^i \times R^i \to \Delta\left(E^i\right)$ be her strategy in the effort game, where $M^i = \times_{j=1}^n M_j^i$ and $R^i = \times_{j=1}^n R_j^i$.

Since each principal j commits to his mechanism (including his strategy π_j) at the start of the game, agents' best responses will depend on the array of offered mechanisms $\gamma = (\gamma_1, \ldots, \gamma_n)$. Let $\beta^i = (\mu^i, \delta^i)$ represent agent i's strategy, with $\beta = (\beta^1, \ldots, \beta^\ell)$ denoting the joint strategy of the agents.

The time structure of the interaction follows the one considered in Myerson (1982), and is provided in Figure 1.

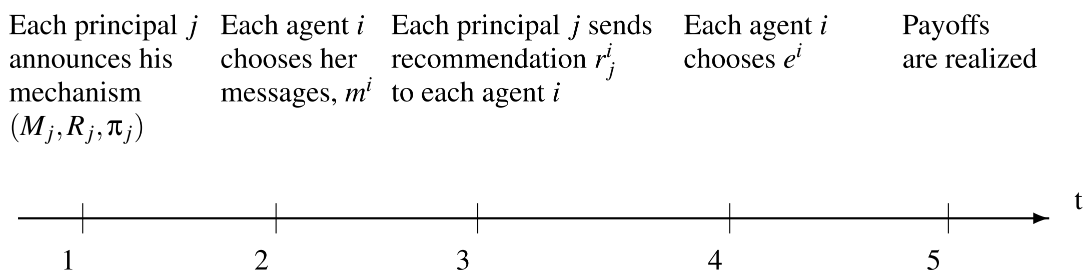

Figure 1: Timing of the generalized communication game

As noted by Epstein and Peters (1999), it is important to appreciate that, although there is no explicit communication between the agents and any principal after the agents have received their recommendations, the structure nevertheless permits principals to choose allocation rules that depend on other principals' recommendations. For example, suppose there are two principals, and principal 2 chooses a recommendation space R_2. Principal 1 can choose a recommendation space $R_1 = E \times R_2$. Then, a recommendation r_1^1 offered by principal 1 to agent 1 is interpreted as a contingent recommendation: it recommends an action strategy based on principal 2's recommendation. In Example 1 below, we demonstrate how a principal can in our framework use such contingent recommendations.

Agent i's payoff from a final outcome (y,e) is given by the von Neumann–Morgenstern utility function $U^i(y,e)$ and principal j's payoff is given by $V_j(y,e)$.[6] The mechanisms offered by principals, γ, and strategies played by agents, β, induce a distribution over the outcome space $Y \times E$. With a standard abuse of notation, let $U^i(\gamma,\beta)$ denote agent i's expected utility given γ and β, and let $V_j(\gamma,\beta)$ be principal j's expected utility.

In this complete information framework, we define a simple mechanism as follows. Principals set the message space to be equal to the type space for each agent, and directly suggest the action each agent should take. That is, $M_j^i = \Theta^i$ and $R_j^i = E^i$ for every $j = 1,\ldots,n$ and for every $i = 1,\ldots,\ell$. Since the type space is a singleton for each agent, without loss of generality, strategies of principals and agents in a simple mechanism may be defined independently of messages. That is, $\pi_j \in \Delta\left(Y_j \times E\right)$, and a mixed strategy for an agent is given by $\delta^i : \left(E^i\right)^n \to \Delta\left(E^i\right)$. We refer to any mechanism in which, for any principal j and any agent i, either $M_j^i \neq \Theta^i$ or $R_j^i \neq E^i$, or both, as an indirect mechanism.[7]

3 Robustness

This section provides our result on the robustness of pure strategy equilibria of simple mechanism games to the introduction of communication. In our setting, principals are playing a game with each other, and their choices of mechanisms must correspond to a Nash equilibrium of this game. Further, agents' choices of messages and efforts must represent continuation equilibria, given the mechanisms chosen by the principals and recommendations received by the agents.

Ex ante, this is a complete information game: no participant has a non-trivial type. However, since agents receive private recommendations from principals, agents may have private information when they play the effort game. Hence, in the spirit of perfect Bayesian equilibrium, we require that each agent i plays a best response following any recommendation array $r^i = \left(r_1^i,\ldots,r_n^i\right)$ she may receive.

[6] Since the allocation rule is an incentive scheme, $U^i(y,e)$ and $V_j(y,e)$ must be thought of as expected utilities.

[7] Thus, a simple mechanism in our framework corresponds to a "direct mechanism" as defined by Myerson (1982). A different route to define direct mechanisms is suggested in Epstein and Peters (1999), who include the communication about other principals' mechanisms in the set of messages available to each single agent. This general formulation leads to an infinite regress, and hence makes it difficult to characterize equilibrium mechanisms.

Recall that a mechanism offered by principal j is defined by (M_j, R_j, π_j). A simple mechanism is defined by (Θ, E, π_j), where $\pi_j \in \Delta(Y_j \times E)$. Let $\Gamma_{\mathcal{D}}$ be the simple mechanism game among the principals. In this game, each principal j chooses a simple mechanism $\pi_j \in \Delta(Y_j \times E)$ at stage 1 (see Figure 1), and each agent i plays a strategy δ^i. Let $\Gamma_{\mathcal{G}}$ be the indirect mechanism game, in which each principal j chooses (M_j, R_j, π_j), where (with a slight abuse of notation) $\pi_j : M_j \to \Delta(Y_j \times R_j)$, and each agent i plays a strategy $\beta^i = (\mu^i, \delta^i)$.

In an equilibrium of either $\Gamma_{\mathcal{D}}$ or $\Gamma_{\mathcal{G}}$, we require that (i) each principal plays a best response, given other principals' strategies and agents' strategies, and (ii) each agent i plays a best response for every recommendation array r^i she may receive, given principals' strategies and other agents' strategies. Observe that a mechanism (M_j, R_j, π_j) is a pure strategy for principal j (even though the choice rule π_j may provide a lottery over allocation rules and recommendations). A mixed strategy for principal j is then defined as a probability distribution over mechanisms. For convenience, we refer to an equilibrium of $\Gamma^{\mathcal{D}}$ or $\Gamma^{\mathcal{G}}$ in which principals play pure strategies as a pure strategy equilibrium.

We start by identifying conditions under which the best response of a principal is a simple mechanism. When the agents' continuation game is played, an agent may (via his private recommendations) have more information about the allocation rules offered by other principals, compared to the information available to some principal j. This possibility is ruled out if the allocation strategies of the $-j$ principals do not depend on the messages they receive, and if their recommendations do not provide any information about their allocation rules. The latter condition implies that for every array of messages sent to any of the principals $k \neq j$, the conditional densities over k's recommendations and allocation rules are independent.

Definition 1 *A mechanism offered by principal j, $\gamma_j = (M_j, R_j, \pi_j)$, exhibits no correlation between recommendations and allocations if the allocation rule π_j is such that $\pi_j : M_j \to \Delta(Y_j) \times \Delta(R_j)$.*

In some games, strongly robust equilibria may not exist. For example, suppose there are just two principals, and, for a given choice of mechanisms and realized recommendations, there are two continuation equilibria in the agents' effort game. Continuation equilibrium 1 yields principal 1 his highest payoff in the overall game, and principal 2 his lowest payoff. Continuation equilibrium 2 yields principal 1 his lowest payoff in the overall game, and principal 2 his highest payoff. Then, equilibrium selection in the agents' game may be critical in determining principals' best responses, so that the overall equilibrium of the game fails to be strongly robust.

A special case of recommendations uncorrelated with allocations is when recommendations are deterministic rather than stochastic. For example, assume that each agent can put in a binary effort, say high or low. In addition, suppose that in equilibrium, each principal recommends that each agent should choose high effort. Then, recommendations are deterministic, and regardless of allocation strategies, satisfy our definition of being uncorrelated with allocations. With stochastic recommendations, even if uncorrelated with allocations, principals can still induce a correlated equilibrium in the agents' effort game. Although agents have symmetric information about allocation rules, the recommendations serve the role of a private randomization device, as in Aumann (1974).

Theorem 1 *Suppose each principal $k \neq j$ offers a mechanism $\tilde{\gamma}_k = (\tilde{M}_k, \tilde{R}_k, \tilde{\pi}_k)$ in which $\tilde{\pi}_k$ is invariant across messages m_k and exhibits no correlation. Then, any expected payoff principal j can obtain in an equilibrium of the continuation game after offering an indirect mechanism $\gamma_j = (M_j, R_j, \pi_j)$ can be obtained in an equilibrium of the continuation game after principal j offers a simple mechanism $\hat{\gamma}_j = (\Theta, E, \hat{\pi}_j)$.*

Proof. Suppose principal j offers an indirect mechanism $\gamma_j = (M_j, R_j, \pi_j)$. Consider any continuation equilibrium in which agent i plays the strategy $\tilde{\beta}^i = (\tilde{\mu}^i, \tilde{\delta}^i)$. The mechanisms and agents' strategies induce a (possibly correlated) distribution over allocations y and efforts e. Let $\tilde{\nu}(y,e)$ denote this distribution.

Now, every principal $k \neq j$ has an allocation strategy that is invariant to the messages he receives, and is using recommendations uncorrelated with allocations, i.e. $\tilde{\pi}_k \in \Delta(Y_k) \times \Delta(R_k)$. Since each agent i observes only the mechanisms, his own message m^i_k to principal k, and his own recommendation array $r^i = (r^i_1, \ldots, r^i_k, \ldots, r^i_n)$, the efforts chosen must remain uncorrelated with the allocation rules of principals $k \neq j$. Hence, we can write $\tilde{\nu}(y,e) = \nu_j(y_j, e) \cdot \prod_{k \neq j} \tilde{\pi}_k(y_k)$ where $\tilde{\pi}_k(.)$ is the marginal distribution over allocations of any principal $k \neq j$.

The remainder of the proof replicates the arguments in Myerson (1982) for the single-principal case. It is now straightforward for principal j to induce the same joint distribution over allocation rules and efforts by offering the simple mechanism $\hat{\gamma}_j = (\Theta, E, \nu_j)$. Since this strategy induces the same joint distribution over efforts and allocation rules as in the continuation equilibrium when principal j offered the indirect mechanism γ_j, it must be a best response for each agent i to (i) play the same message strategy $\tilde{\mu}^i$ as earlier, and (ii) obey the recommendation of principal j, and to ignore the recommendations of the others. That is, it is a best response for each agent i to take the action e^i_j recommended by principal j. Let $\hat{\delta}^i$ denote agent i's obedient strategy at the effort stage, and let $\hat{\beta}^i = (\tilde{\mu}^i, \hat{\delta}^i)$ denote her overall strategy in the game.

Now, the mechanisms $(\tilde{\gamma}_1, \cdots, \hat{\gamma}_j, \cdots, \tilde{\gamma}_n)$ and continuation equilibrium $(\hat{\beta}^1, \ldots, \hat{\beta}^\ell)$ induce the same joint distribution over allocation rules and efforts as the mechanisms $(\tilde{\gamma}_1, \cdots, , \gamma_j, \cdots, \tilde{\gamma}_n)$ and continuation equilibrium $(\tilde{\beta}^1, \ldots, \tilde{\beta}^\ell)$. Hence, the expected payoffs of all principals and agents must be similar in the two cases. ■

The theorem shows that if agents have no private information about the allocation rules offered by other principals, a principal cannot benefit from a complex communication scheme which seeks to uncover private communication between agents and the other principals. To interpret this result, consider an alternative extensive form of the game. Suppose that, after the agents have received their recommendations, principals are allowed to communicate again with the agents and to (possibly) change their original offers. A principal could ask to each agent what are the recommendations that he has received and modify the allocation accordingly. One can interpret this additional stage as a possibility of renegotiation. Imagine that $n-1$ principals play an uncorrelated strategy, is there any incentive for the n-th one to use this new communication opportunity? Theorem 1 provides a negative answer. Since recommendations and allocations are not correlated for $n-1$ principals, the n-th one would learn nothing from the new round of communication. Hence, whatever payoff he reaches at any continuation equilibrium, can also be obtained by means of a simple communication mechanism.

In particular, if the recommendations that agents receive from other principals communicate no private information about their allocations, there is no benefit for principal j from choosing a recommendation space such as $E \times R_{-j}$ (where $R_{-j} = \times_{k \neq j} R_k$) in order to send contingent recommendations to agents. Thus, the infinite regress problem is broken. However, if there is correlation between allocation rules and recommendations in principals' strategies, simple communication mechanisms are no longer sufficient to characterize a principal's best response, as we show later in Example 1.

Observe that the theorem cannot be extended to mixed strategies for principals. The reason is that whenever a principal plays a mixed strategy the agent can observe the realization of that mixed strategy. This, in turn, constitutes relevant information for the other principals. This intuition applies to the single-agent case as well (see Peters

(2003)), and an example in this direction is provided in Han (2007) for the case with contractible effort.

We now turn to our main question: under what conditions does an equilibrium outcome of a simple game $\Gamma_{\mathcal{D}}$ survive the introduction of more complex communication mechanisms? In a simple mechanism, (trivially) allocation rules do not depend on messages. If the no correlation property is satisfied, we know that every principal can recreate the payoff from any deviation to an indirect mechanism via a suitable simple mechanism. The difficulty, however, is that different principals may wish to replicate different correlated equilibria in the agents' effort game, potentially leading to conflicting recommendations being sent to the agents.

To rule out this possibility, we require equilibria in the mechanism design game to be strongly robust in the sense of Peters (2001) and Han (2007). An equilibrium of the simple (indirect) mechanism game is defined to be strongly robust if, regardless of the continuation equilibrium that is chosen in the agents' effort game, no principal j can improve his own payoff by deviating to some other simple (indirect) mechanism.

Definition 2 *(i) Let* (π^*, δ^*) *be an equilibrium of the simple mechanism game* $\Gamma^{\mathcal{D}}$*. The equilibrium is strongly robust if, for every principal* j*, every simple mechanism* $\tilde{\pi}_j$ *and every continuation equilibrium* $\tilde{\delta}$*,* $V_j(\pi^*, \delta^*) \geq V_j((\tilde{\pi}_j, \pi^*_{-j}), \tilde{\delta})$*.*
(ii) Let (γ^*, β^*) *be an equilibrium of the indirect mechanism game* $\Gamma^{\mathcal{G}}$*. The equilibrium is strongly robust if, for every principal* j*, every indirect mechanism* $\tilde{\gamma}_j$ *and every continuation equilibrium* $\tilde{\beta}$*,* $V_j(\gamma^*, \beta^*) \geq V_j((\tilde{\gamma}_j, \gamma^*_{-j}), \tilde{\beta})$*.*

When will an equilibrium of the simple mechanism game be strongly robust? If there is a single principal, the optimal incentive compatible simple mechanism, and the associated continuation equilibrium in the agents' effort game, constitute a strongly robust equilibrium. In any mechanism he offers, the principal can induce (via his recommendations) the continuation equilibrium that maximizes his payoff. Thus, given an optimal mechanism, there cannot exist another incentive compatible simple mechanism and an associated continuation equilibrium that leave the principal strictly better off.

In a pure moral hazard setting, incentive compatibility is equivalent to agents "obeying" the recommendations offered by a principal. With multiple principals, the notion of obedience is troublesome, since an agent may receive conflicting recommendations from different principals. Nevertheless, the following are some situations in which strongly robust equilibria will exist:
(i) The agents' continuation game has a unique equilibrium for every set of mechanisms

offered by principals and for every realization of recommendation strategies.
(ii) The agents' continuation game has a Pareto-dominant equilibrium for every set of mechanisms offered by principals and for every realization of recommendation strategies, and this equilibrium is selected in each case. More precisely, suppose, for every $j \in n$, every mechanism offered by j (while keeping fixed the other $n-1$ principals' mechanisms), and every realization of the recommendation strategies, there is an equilibrium in the agents' continuation game which is preferred by all principals to any other equilibrium in the continuation game. Suppose further that this equilibrium is selected as the continuation equilibrium in all cases. Then, the original equilibrium is strongly robust.
(iii) In the equilibrium of the overall game, all principals play weakly dominant strategies. Then, a unilateral deviation cannot make any principal better off.

Using the no-correlation property, we now state our theorem on robustness of pure strategy equilibria. The theorem provides sufficient conditions for an equilibrium outcome of a simple mechanism game to remain an equilibrium outcome of an indirect mechanism game. Formally,

Theorem 2 *Suppose the simple mechanism game $\Gamma_{\mathcal{D}}$ has a strongly robust equilibrium (π^*, δ^*) in which, for each principal j, π_j^* is a pure strategy that satisfies the no-correlation property. Then, in the indirect mechanism game $\Gamma_{\mathcal{G}}$, it remains a strongly robust equilibrium for each principal j to offer the simple mechanism (Θ, E, π_j^*) and for each agent i to play δ^{i*}. Thus, the joint distribution over allocation rules and efforts that obtains in the equilibrium of the simple mechanism game remains an equilibrium outcome of the indirect mechanism game.*

Proof. Consider the indirect mechanism game $\Gamma_{\mathcal{G}}$. Suppose that, in this game, every principal j offers a mechanism $\left(M_j, R_j, \pi_j\right) = (\Theta, E, \pi_j^*)$, where π_j^* is his equilibrium pure strategy in the simple mechanism game $\Gamma_{\mathcal{D}}$. It is immediate that $\delta^* = (\delta^{1*}, \cdots, \delta^{\ell*})$ must remain a continuation equilibrium in the agents' efforts game.

Now, suppose the equilibrium associated with π^* and δ^* is not strongly robust in the game $\Gamma_{\mathcal{G}}$. Then, there exists a unilateral deviation by some principal j' to an indirect mechanism $\tilde{\gamma}_{j'} = \left(\tilde{M}_{j'}, \tilde{R}_{j'}, \tilde{\pi}_{j'}\right) \neq \left(\Theta, E, \pi_{j'}^*\right)$, and a continuation equilibrium $\tilde{\beta} = (\tilde{\mu}, \tilde{\delta})$, such that principal j' earns a strictly greater utility than in the equilibrium of the simple mechanism game (π^*, δ^*). Since principal j' deviates to an indirect mechanism, in general in the continuation equilibrium agents may be both sending messages according to $\tilde{\mu}$ and then choosing efforts according to $\tilde{\delta}$.

Now, every principal $k \neq j'$ has offered a mechanism in which his allocation rules do not depend on the messages he receives (since his message space is a singleton) and in which there is no correlation between his allocation rules and recommendations (by assumption). Hence, from Theorem 1, principal j' can achieve the same expected payoff as he achieves by offering $\tilde{\gamma}_{j'}$ if he instead offered a suitable simple mechanism. However, this implies that (π^*, δ^*) is not a strongly robust equilibrium of the game $\Gamma_{\mathcal{D}}$, which is a contradiction. ■

We show in Example 1 that the theorem fails to obtain when a principal correlates his allocations with his recommendations. Attar, Campioni, Piaser, and Rajan (2009) provide an example in the same spirit to demonstrate a failure of the revelation principle in this setting. Here, we focus on the role of the no-correlation condition.

Example 1 (No-correlation property)

There are two principals and two agents, i.e $n = 2$ and $\ell = 2$. The allocation spaces are $Y_1 = \{y_1\}$ and $Y_2 = \{y_{21}, y_{22}\}$ for principal 1 and 2, respectively. Here, we assume that the output space is a singleton, so that an allocation rule is directly identified with an allocation. The effort spaces are $E^1 = \{a_1, a_2\}$ and $E^2 = \{b_1, b_2\}$, for agents 1 and 2. The payoffs of the game are given in the following matrix. The first payoff is that of principal 1 (P1), who has only one allocation and chooses the row in the table; the second payoff is that of principal 2 (P2), who chooses the column, and the last two payoffs are those of agent 1 and 2, respectively.

		y_{21}			y_{22}	
		b_1	b_2		b_1	b_2
y_1	a_1	$(-100, 2, 0, 0)$	$(0, 2, 8, 3)$	a_1	$(-100, 2, 0, 0)$	$(0, 2, 8, 3)$
	a_2	$(0, 2, 3, 8)$	$(70, 2, 6, 6)$	a_2	$(0, 2, 3, 8)$	$(70, 2, 7, 7)$

Table 1: Payoffs in Example 1

Consider the following choice rules for the principals.

$$\pi_1 = \begin{cases} (y_1, a_1, b_2) & \text{with probability } \frac{2}{7} \\ (y_1, a_2, b_1) & \text{with probability } \frac{2}{7} \\ (y_1, a_2, b_2) & \text{with probability } \frac{3}{7} \end{cases}$$

$$\pi_2 = \begin{cases} (y_{21}, a_1, b_1) & \text{with probability } \frac{1}{2} \\ (y_{22}, a_2, b_2) & \text{with probability } \frac{1}{2} \end{cases}$$

Observe that, given the offered mechanisms, there is a continuation equilibrium where both agents follow the recommendations of P1. The corresponding payoff profile is $(30, 2, \frac{83}{14}, \frac{83}{14})$.

We show that these choice rules constitute a strongly robust equilibrium in the direct mechanism game. Since P2 is indifferent across all outcomes, it is sufficient to consider only deviations by P1. We argue that, following any deviation by P1 to another simple mechanism, there is no continuation equilibria in the agents' game that yields P1 a payoff strictly greater than 30.

Suppose P1 deviates to another simple mechanism. First, consider continuation equilibria in which agents obey the recommendation of P1. In a simple mechanism, P1 cannot offer recommendations that are contingent on the recommendations of P2. Thus, P1 is limited to inducing the same correlated equilibrium in the agents' game, regardless of whether P2 offers y_{21} or y_{22}. If both agents obey the recommendations of P1, the maximal payoff he can attain is then 30, via the correlated equilibrium which places probability $\frac{3}{7}$ on (y_1, a_2, b_2), and $\frac{2}{7}$ on each of (y_1, a_2, b_1) and (y_1, a_1, b_2).

Next, consider continuation equilibria in which agents ignore the recommendation of P1 in one column of the large matrix in Table 1. If agents ignore the recommendation of P1, they will instead coordinate on a Nash Equilibrium of their continuation game. Each of the cells of the large matrix has exactly three Nash equilibria (two pure strategy equilibria and a mixed strategy one), each of which offer a payoff of zero to P1.

Now, it is straightforward to show that when P2 offers y_{21}, the continuation equilibrium that maximizes the payoff of P1 is again the correlated equilibrium that places probability $\frac{3}{7}$ on (y_1, a_2, b_2), and $\frac{2}{7}$ on each of (y_1, a_2, b_1) and (y_1, a_1, b_2). P1 earns a payoff of 30 in this case. Similarly, when P2 offers y_{22}, the payoff of P1 is maximized by the correlated equilibrium that places probability $\frac{3}{5}$ on (y_1, a_2, b_2), and $\frac{1}{5}$ on each of (y_1, a_2, b_1) and (y_1, a_1, b_2). P1 earns a payoff of 42 in this case. Given the randomization π_2, therefore, the maximal payoff P1 can earn if agents ignore his recommendation in at least one column of the large matrix is 21.

Thus, there is a pure strategy equilibrium in simple mechanisms which supports the profile of payoffs $(30, 2, \frac{83}{14}, \frac{83}{14})$.

Next, suppose that, when P2 offers the direct mechanism π_2, P1 can instead deviate to an indirect mechanism. We show that there exists an indirect mechanism that yields P1 an expected payoff of 36, by inducing the optimal correlations separately for each allocation P2 may choose.

Consider the following indirect mechanism offered by P1. The recommendation sets for agents 1 and 2 are, respectively $R = \{r_1, r_2, r_3, r_4\}$ and $S = \{s_1, s_2, s_3, s_4\}$. Agent 1 plays the following strategies after he receives the recommendation of P1:

r_1: play a_1 regardless of the recommendation offered by P2.

r_2: play a_1 if P2 sends recommendation a_1 and a_2 if P2 sends recommendation a_2

r_3: play a_2 if P2 sends recommendation a_1 and a_1 if P2 sends recommendation a_2

r_4: play a_2 regardless of the recommendation offered by P2.

Agent 2's strategy is similar (substitute s_i for r_i and b_j for a_j above).

Since P1 has only one allocation to offer (y_1), we specify her allocation rule only in terms of probabilities over recommendations sent to the agents. Consider the following probabilities over R and S.

	s_1	s_2	s_3	s_4
r_1	0	0	0	$2/35$
r_2	0	0	$2/35$	$6/35$
r_3	0	$2/35$	0	$3/35$
r_4	$2/35$	$6/35$	$3/35$	$9/35$

Table 2: Probabilities over recommendations in indirect mechanism

Suppose each agent obeys the recommendation of P1. It is straightforward to check that for each of the two allocations P2 can offer, the indirect mechanism above induces a correlated equilibrium in the agents' effort game. In particular, it induces the correlated equilibrium that maximizes P1's expected payoff following each allocation offered by P2. P1 earns an expected payoff of 36 from the indirect mechanism, higher than the 30 she can earn from a direct mechanism. ■

This example shows that, in the absence of the no-correlation property, a principal may wish to deviate to an indirect mechanism even if other principals offer direct mechanisms. In the example, P2 offers a mechanism with a recommendation space $R_2 = E$, and the best response of P1 to the mechanism of P2 involves a recommendation space $E \times R_2$. From the viewpoint of P1, the recommendation offered by P2 is equivalent to an unknown type for each agent, and the best response of P1 is contingent on this unknown parameter.

In our example, P2 is indifferent across all outcomes. It is easy to see that, if P2 had non-trivial preferences over outcomes, she may, in turn, wish to make recommendations contingent on the (contingent) recommendations of P1, and so on, leading to the infinite regress problem mentioned by McAfee (1993) and the universal message space of Epstein and Peters (1999).

4 Games without communication

Standard models of multiple principals with complete information on the agents' side typically do not consider communication. Instead, principals simply propose allocation rules, and agents take a non-contractible effort. We call such a game a "game without communication." If the equilibrium outcomes generated in a game without communication were replicable in simple mechanism games with communication, with strategies that satisfied the conditions of our theorem, we would be confident that no principal could gain by a unilateral deviation to an indirect mechanism. The result is hence a direct implication of Theorem 2.

In a game without communication, let $\sigma_j \in \Delta(Y_j)$ denote the strategy of principal j, and let $\rho^i : \prod_{j=1}^{k} \Delta(Y_j^i) \to \Delta(E^i)$ denote the strategy of agent i. Let $\sigma = (\sigma_1, \ldots, \sigma_n)$, and $\rho = \left(\rho^1, \ldots, \rho^k\right)$.

Theorem 3 *Let (σ^*, ρ^*) be a pure strategy equilibrium in the game without communication. If the corresponding equilibrium outcome can be supported as a strongly robust equilibrium in $\Gamma_{\mathcal{D}}$, then it remains an equilibrium outcome in the indirect mechanism game $\Gamma_{\mathcal{G}}$.*

Proof. Consider a pure strategy equilibrium (σ^*, ρ^*) of the game without communication. We can construct strategies (π, δ) in the game $\Gamma_{\mathcal{D}}$ that replicate the outcome of the equilibrium in the game with no communication. For example, let $\pi_j = \sigma_j^* \times e_1^1 \times \cdots \times e_1^k$ for each principal j. That is, each principal offers the allocation lottery σ_j and the recommendation array $(e_1^i, \cdots, e_1^k)$ to the agents. Set $\delta^i(\pi_1^i, \ldots, \pi_n^i) = \rho^{i*}(\sigma_j^i, \ldots, \sigma_j^n)$ for each i.

By construction, π satisfies the property that recommendations are uncorrelated with allocation rules for each principal j. Notice that (π, δ) induces the same distribution over terminal payoffs as (σ^*, ρ^*).

Therefore, from Theorem 2, if it is a strongly robust equilibrium in $\Gamma_{\mathcal{D}}$ for each principal j to offer π_j and for each agent i to play δ^i, it remains a strongly robust equilibrium in $\Gamma_{\mathcal{G}}$ for each principal j to offer $\gamma_j = (\Theta, E, \pi_j)$, and for each agent to play δ^i. ■

Theorem 3 shows that pure strategy equilibria of games without communication can survive the introduction of complex forms of communication if they can be supported as strongly robust equilibria in games with communication.

One implication of our analysis is that, in the game $\Gamma_{\mathcal{D}}$, at the deviation stage it is sufficient to analyze incentive compatible (i.e., obedient) behavior of the agents, since only one principal deviates to a mechanism with communication. In other words, we only need to consider deviations where the principal's recommendations are followed by agents in the continuation game. If those deviations are not profitable, then none is and the equilibrium will be strongly robust. Even in a multiple-principal context where the notion of obedience is not helpful to characterize equilibria, we provide a rationale for considering incentive compatibility at the deviation stage in games without communication.

We conclude with an example that considers an equilibrium in take-it or leave-it offers in a standard production economy. In the example, we identify conditions under which the equilibrium in take-it or leave-it offers is sustainable as a strongly robust equilibrium in the corresponding simple mechanism game.

Example 2 (Robustness of equilibrium in take-it-or-leave-it offers).

There are two identical principals ($j = 1, 2$), each of whom obtains a profit (or payoff) equal to his total production minus the amount he transfers to the agents. Production is risky: output has the value $f > 0$ or 0. The probability of success (i.e., obtaining f) depends on the efforts made by two agents ($i = 1, 2$). Let $p\left(e^1, e^2\right)$ be the probability of success of a principal, where e^i is the effort of agent i, for $i = 1, 2$. For each agent i, the effort choice e^i lies in the set $E = \{e_1, e_2\}$, where $e_1, e_2 \in \mathbb{R}_+$ with $e_1 < e_2$. Given e_1, e_2, successes are independent across principals. Further, the probability of success is symmetric (so $p(e_1, e_2) = p(e_2, e_1)$) and increasing in both arguments. Finally, we assume that

$$p(e_2, e_2) - p(e_1, e_2) > p(e_2, e_1) - p(e_1, e_1) > 0, \tag{3}$$

which implies there are complementarities in production: inducing high effort from agent 1 is more beneficial when the other agent is also providing high effort.

All players have limited liability. Thus, for each principal, an allocation rule y_j is defined by a pair of functions (y_j^1, y_j^2), where $y_j^i(f) = T_j^i \in [0, f]$ is a monetary transfer to agent i if production is successful, and $y_j^i(0) = 0$ is the transfer to agent i if production fails. Each player has a reservation utility of zero.

All players are risk-neutral. Principal j's expected utility is represented as $p(e^1, e^2)\,(f - T_j^1 - T_j^2)$, and, since (given efforts) successes are independent across principals, agent i's expected utility is $p\left(e^1, e^2\right) T_1^i + p\left(e^1, e^2\right) T_2^i - e^i = p(e^1, e^2)[T_1^i + T_2^i] - e^i$.

Define T^* to be the compensation level that leaves each agent indifferent between choosing high and low effort, if both principals offer T^* and if the other agent chooses e_2. That is, T^* is defined by the equation

$$2\,p\,(e_2, e_2)\,T^* - e_2 = 2\,p\,(e_1, e_2)\,T^* - e_1. \tag{4}$$

That is: $T^* = \frac{e_2 - e_1}{2[p(e_2, e_2) - p(e_1, e_2)]}$.

Consider a game with no communication, in which principals can only make take-it-or-leave-it offers to agents. Suppose, as an extreme case, $p(e_2, e_2)\,(f - 2T^*) > p(e_1, e_2) f$. Then, it follows that (i) if principal 2 offers the take-it-or-leave-it offer T^* to each agent, it is a best response for principal 1 to do the same, provided agents coordinate on the continuation equilibrium (e_2, e_2), and (ii) the equilibrium generates a higher utility for each principal than any other equilibrium of the game (since even without compensating the agents, a principal is worse off if any agent chooses e_1).

Thus, each principal offering T^* to each agent, with the continuation equilibrium (e_2, e_2), constitutes a strongly robust equilibrium in the take-it-or-leave-it offer game.

Now, suppose principal 2 makes a take-it-or-leave-it offer, and principal 1 deviates to a mechanism with communication. From Theorem 1, in determining the best response of principal 1, we only need to consider simple mechanisms with a continuation equilibrium in which both agents obey the recommendations they receive. Let π denote the recommendation and allocation strategy of principal 1 in a simple mechanism. Recall that π induces a correlated equilibrium in the agents' effort game. Let π_{ij} denote the probability that agent 1 is recommended action e_i and receives allocation T_{ij}^1, and agent 2 is recommended action e_j and receives allocation T_{ij}^2.

Note that we consider only deterministic transfers. In principle, we could allow principal 1 to make a recommendation associated with a stochastic transfer (i.e., a distribution over $[0, f]$). However, since all parties are risk-neutral, this restriction is without loss of generality.

Suppose agent 1 receives the private recommendation e_2. She believes that agent 2 receives e_1 with probability π_{21} and e_2 with probability π_{22}. Thus, she obeys the recommendation only if:

$$\begin{aligned} &\pi_{21}\left[p\left(e_2,e_1\right)\left(T_{21}^1+T^*\right)\right]+\pi_{22}\left[p\left(e_2,e_2\right)\left(T_{22}^1+T^*\right)\right]-e_2 \\ &\qquad \geq \pi_{21}\left[p\left(e_1,e_1\right)\left(T_{21}^1+T^*\right)\right]+\pi_{22}\left[p\left(e_1,e_2\right)\left(T_{22}^1+T^*\right)\right]-e_1. \end{aligned} \tag{5}$$

Now, note that if $\pi_{21}=0$, (5) implies $T_{21} \geq T^*$. Similarly, if $\pi_{22}=0$, the assumption that $p(e_2,e_1)-p(e_1,e_1)<p(e_2,e_2)-p(e_1,e_2)$ implies that $T_{22}>T^*$. Therefore, for any array of probabilities (π_{21},π_{22}), it must be that $\pi_{21}T_{21}^1+\pi_{22}T_{22}^1 \geq T^*$.

Finally, recall that principal 1 prefers to induce e_2 than e_1, provided he can transfer only T^* to each agent. It follows that the following simple mechanism is optimal for P1: $\pi_{11}=\pi_{12}=\pi_{21}=0$ and $\pi_{22}=1$, with $T_{11}^i=T_{12}^i=T_{21}^i=0$ and $T_{22}^i=T^*$ for each i. Hence, the maximal payoff that principal 1 can achieve with communication is equivalent to his maximal payoff with take-it-or-leave-it offers. By symmetry, the same applies to principal 2. Thus, the outcome with take-it-or-leave-it offers can be sustained as a strongly robust equilibrium of the simple mechanism game. It follows from Theorem 3 that the same outcome can also be supported at a strongly robust of the indirect mechanism game.

Note that if $p(e_2,e_2)-p(e_1,e_2)<p(e_2,e_1)-p(e_1,e_1)$, principal 1 can induce agent 1 to play e_2 even with a transfer less than T^*. Thus, the equilibrium in take-it-or-leave-it offers need not be robust in the simple mechanism game. ■

5 Conclusion

Applied moral hazard models typically ignore communication between principals and agents. We consider the robustness of pure strategy equilibria in a game with pure moral hazard. We show that if principals do not correlate their allocation rules with the recommendations they use with the agents, then a strongly robust equilibrium in a game in which principals use simple mechanisms, remains a strongly robust equilibrium when principals can instead use complex communication schemes.

It is worth emphasizing that the no-correlation condition provides for sufficiency only. If an equilibrium in direct mechanisms fails the no-correlation property, it may yet be feasible to sustain the induced outcome as an equilibrium in indirect mechanisms. Strong robustness instead is a property that, it seems, must be required: if a principal can profitably deviate in direct mechanisms, it is trivial that she can profitably deviate to an indirect mechanism as well.

As we show in Example 2, equilibria in models without communication may indeed be sustainable as strongly robust equilibria in the direct mechanisms game, in which case the restriction to mechanisms without communication is inconsequential. However, other applications may fail to have strongly robust equilibria, with the payoff rankings of principals differing across the continuation equilibria in the agents' game. In the latter case, a principal can profitably deviate to a mechanism with communication.

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